ROCKING RESPONSE OF FREE-STANDING RIGID BLOCKS ON SLOPES SUBJECTED TO ONE-SINE PULSE

Y. Lv [(1)], X.H. Sun [(2)], J. Ling [(3)], Z.X. Li [(4)], N. Chouw [(5)]

[(1)] *Associate Professor, Tianjin Key Laboratory of Civil Structure Protection and Reinforcement, Tianjin Chengjian University, lvyangtju@163.com*
[(2)] *Master student, Tianjin Key Laboratory of Civil Structure Protection and Reinforcement, Tianjin Chengjian University, steel71@163.com*
[(3)] *Master student, Tianjin Key Laboratory of Civil Structure Protection and Reinforcement, Tianjin Chengjian University, 429697693@qq.com*
[(4)] *Professor, Key Laboratory of Coast Civil Structure Safety of Ministry of Education, Tianjin University, zxli@tju.edu.cn*
[(5)] *Associate Professor, Department of Civil and Environmental Engineering, University of Auckland, n.chouw@auckland.ac.nz*

Abstract

Structure, equipment or storage cask may experience free-standing rocking on slopes because of the non-uniform settlement of the base, construction error or foundation failure. Since the pioneering work by Housner in 1963, the rocking response of a free-standing block on the horizontal rigid surface has been researching all over the world. However, little research considered the rocking response of a block on a slope. In this paper, based on Housner's model, the free-standing rigid block on a slope subjected to one-sine pulse excitation is examined.

First, ignoring the vertical and sliding motions, and assuming the block and base are rigid, the equation of motion of a block on a slope was built and linearized to an ordinary differential equation. By comparing the impacting time with the duration of the excitation, the overturning of the block on a slope under one-sine pulse can be divided into 3 cases. In case 1, the block experiences one impact and the impact happens before the excitation expired. In case 2, the block also experiences one impact and the impact happens after the excitation expired. While in mode 2 (case 3), the block experiences overturning without any impacts. These three cases were judged and solved separately, and the overturning acceleration spectrum was obtained. Then the overturning spectrum under one-sine pulse excitation was verified by the numerical solution based on both linear and nonlinear formulations. The minimum overturning acceleration obtained from the spectrum is the same as the numerical solution based on the linear formulation. However, it will overestimate the minimum overturning acceleration in case 1 and underestimate the minimum acceleration in cases 2 and 3.

Results show that the angle of the slope has significant influences on the overturning mode and the rocking process. All the spectra on different slopes have the same shape but different areas. The angle of the slope will change the safe region of the spectrum between the impact cases (cases 1 and 2) and no impact case (mode 2). The existing of a slope will change the critical frequency between case 1 and case 2.

Keywords: slope; free-standing block; one-sine pulse; overturning acceleration spectrum

1. Introduction

Since the pioneer work proposed by Housner in 1963 [1], the rocking response of free-standing blocks on horizontal rigid surface has been researching world widely. Among which the minimum acceleration rocking spectrum is particularly important. In 1996, Shi et al. [2] rederived and corrected Housner's model to estimate the minimum overturning acceleration of a precarious rocks at different frequencies. Aslam et al. [2] examined the rocking problem of free-standing block through experiment and analysis. In 1999, Anooshehpoor et al. [3] developed an analytical solution for the rocking and overturning response of a two-dimensional, symmetric rigid block subject to a full sine wave of horizontal ground acceleration. Their work was further improved by Zhang and Makris [4] in 2001, they found that under cycloidal pulses, a free-standing block can overturn with two distinct modes. Lenci and Rega [5] proposed a closed form and manageable criteria for overturning of a rocking block resting on a horizontal rigid foundation. Dimitrakopoulos and Fung [6] examined the stability of a block subjected to a family of multi-lobe pulse ground motions. Voyagake et al. [7] also obtained a closed form expression for the dynamic response of a two-dimensional rectangular block on a rigid base.
Later, researchers began to consider more complex rocking system. The flexibility, soil-structure interaction, isolated rocking structures are studied. Makris and Konstantinidis [8] examined the fundamental differences between the oscillatory response of a single-degree-of-freedom oscillator and the rocking response of a slender rigid block. Aiming to characterize and predict the maximum rocking response of large and flexible structures to earthquakes using an idealized structural model, Acikgoz and DeJoug [9] used different earthquake records to evaluate the ground motion intensity measures. Qin and Chouw [10] carried out shake table tests to investigate the effect of soil-structure interaction and structural flexibility on the rocking response of structures. Chen et al. [11] presented the results of free vibrations and shake table tests on a single degree-of-freedom model of a bridge pier with footing uplift on a rigid base. It is revealed that the flexibility of the structure has significant effects on footing uplift duration and amplitude. Masaeli et al. [12] investigated the rocking isolation effect on seismic demands of shear-building structures rested on shallow foundation. DeJong and Dimitrakopoulos [13] presented a methodology to derive equivalence between the single rocking block and various rocking mechanisms, yielding a set of fundamental rocking parameters. Voyagaki et al. [14] revisited the free-standing block to earthquake ground shaking without considering the effect of slipping.
Due to nonuniform settlement of the base, failure of the foundation, the structures, equipment or storage casks may experience free-standing rocking on slopes under vibration. However, none of the above research considered the free-standing rocking on slopes. Plaut et al. [15] systematically examined the free-standing blocks with a flat or concave base resting on a rigid and flat foundation, they carried out a series of numerical calculations, symmetric or asymmetric block, horizontal or tilted foundation, to find out the acceleration that overturns the block. Virgin et al. [16] assumed the block does not slide or bounce off on the foundation, they examined the transient response of the block subjected to a seesawing excitation. However, they didn't get the analytical solution of overturning acceleration spectrum.
This paper examined the free-standing rigid block on slopes subjected to one-sine pulse excitation. First, considering a slope with an angle, the overturning acceleration spectrum under one-sine pulse are built. Then the influences of the slope on the overturning acceleration spectrum are discussed. The analytical solution is compared with the numerical solution based on both linear and nonlinear formulations.

2. Free-standing rigid block on slopes

2.1 Rigid block on slopes

Because of the nonuniform settlement of the base, imprecise construction of the structures, failure of the foundation and etc., the structures may experience free-standing on slopes. The parameters of a free-standing rigid block on a slope are the same as that on a horizontal surface, except for an angle of slope β (see Fig. 1). It is assumed that the angle is positive if the foundation rotates in clockwise.
For a block rotating with respect to its corner points B and A, the equations of motion are

$$\begin{cases} I_0\ddot{\theta}(t) + mgr\sin[\alpha - \theta(t)] + m\ddot{u}_g(t)r\cos[\alpha - \theta(t)] = 0 & \theta > \beta \\ I_0\ddot{\theta}(t) + mgr\sin[-\alpha - \theta(t)] + m\ddot{u}_g(t)r\cos[-\alpha - \theta(t)] = 0 & \theta < \beta \end{cases} \tag{1}$$

where m is the mass and I_0 is inertia of the block with respect to points B and A, respectively. r is the distance of the mass center to the corner point. α is slenderness of the block. $\ddot{u}_g$ is the horizontal ground acceleration. β is the angle of the slope. Positive or negative of $\theta(t)$ depends on the rotation about A or B. In the following sections, $\theta(t)$ is written in a simplified version θ. Adopting the same method as the rigid blocks on horizontal surface [1] and considering the block under dynamic base excitation, Equation (1) can be expressed in a compact form

$$\ddot{\theta} = -p^2(\sin[\alpha\,\mathrm{sgn}(\theta - \beta) - \theta] + \frac{\ddot{u}_g}{g}\cos[\alpha\,\mathrm{sgn}(\theta - \beta) - \theta]) \tag{2}$$

where $p = \sqrt{mgr / I_0}$ is a characteristic frequency parameter of the block.

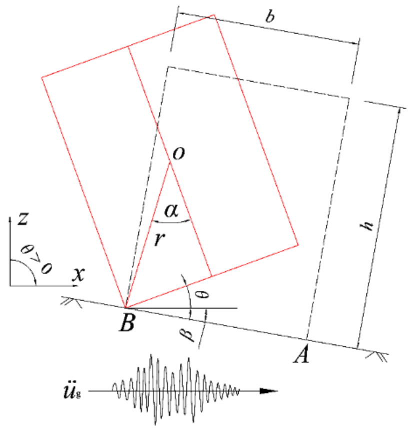


Fig.1 - Schematic of free-standing rigid block on slope

2.2 Rocking response under one-sine pulse

Under dynamic base excitation, the rigid block may initiate rocking or overturn when the acceleration exceeds the corresponding critical acceleration. The critical acceleration is largely dependent on the size and slenderness of the block and the ground motion. Previous research revealed that the required peak ground acceleration for initiating rocking and overturning is a sensitive function of both the block size and the excitation frequency. For the rigid block rocking respect to corner B and A, the free-body diagram is shown in Fig. 2. Based on the equilibrium equations in x, z and rotation θ directions, the angular acceleration at the instant when rocking initiates at corner B and A are expressed as

$$\ddot{\theta}_0 = \begin{cases} -p^2\sin(\alpha + \beta)[\frac{\lambda_B a_p}{g\tan(\alpha + \beta)} - 1], & rocking \quad at \quad B \\ -p^2\sin(\alpha - \beta)[\frac{\lambda_A a_p}{g\tan(\alpha - \beta)} - 1], & rocking \quad at \quad A \end{cases} \tag{3}$$

$$a_{\mathrm{rock}} = \begin{cases} g\tan(\alpha + \beta), & rocking \quad at \quad B \\ g\tan(\alpha - \beta), & rocking \quad at \quad A \end{cases} \tag{4}$$

where λ_B and λ_A are the ratio of the acceleration rocking initiating a_{rock} to the peak acceleration of the excitation a_p respect to B and A, respectively. The ground acceleration of one-sine pulse can be expressed as Equation (5) and shown in Fig.3.

$$\ddot{u}_g(t) = \begin{cases} a_p \sin(\omega_p t + \psi), & 0 \le t \le \dfrac{2\pi}{\omega_p} \\ 0, & otherwise \end{cases} \tag{5}$$

where ψ is the phase angle when rocking initiates. For rocking respect at B and A, the phase angles are $\psi_B=\sin^{-1}[(\alpha+\beta)g/a_p]$ and $\psi_A=\sin^{-1}[(\alpha-\beta)g/a_p]$, respectively. a_p is the acceleration amplitude of the one-sine pulse excitation.

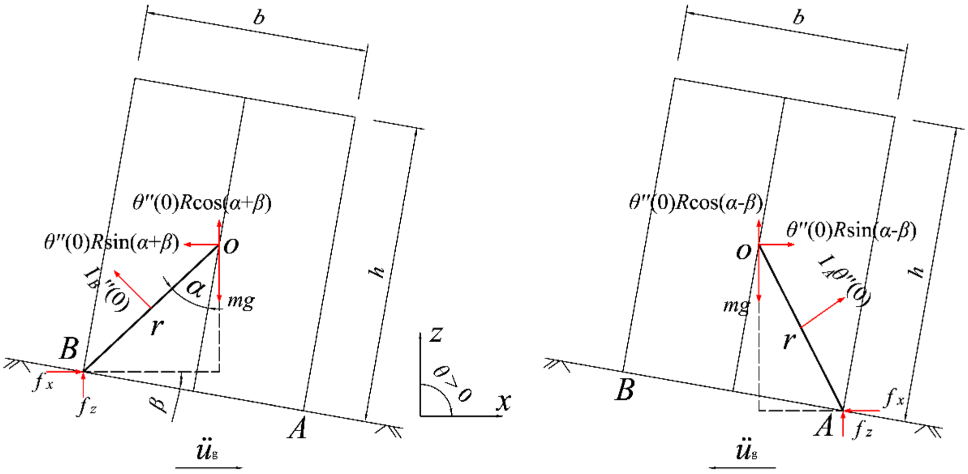


Fig.2 - Free-body diagram of rigid block at entering instant of rocking motion respect to B and A

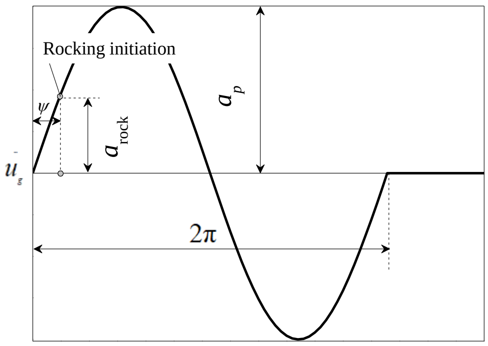


Fig. 3 - Acceleration time history of one-sine pulse

Substituting Equation (5) into Equation (1) and written in piecewise formation gives

$$\ddot{\theta} = \begin{cases} -p^2[\sin(\alpha-\theta) + \dfrac{a_p}{g}\sin(\omega_p t + \psi_B)\cos(\alpha-\theta)], & \theta > \beta \\ -p^2[\sin(-\alpha-\theta) + \dfrac{a_p}{g}\sin(\omega_p t + \psi_A)\cos(-\alpha-\theta)], & \theta < \beta \end{cases} \tag{6}$$

For a large slenderness structure and within the limit of the linear approximation, Equation (6) becomes

$$\ddot{\theta}=\begin{cases}-p^2(\alpha-\theta)-\dfrac{a_p}{g}p^2\sin(\omega_p t+\psi_B), & \theta>\beta\\ -p^2(-\alpha-\theta)-\dfrac{a_p}{g}p^2\sin(\omega_p t+\psi_A), & \theta<\beta\end{cases}\tag{7}$$

The solution of Equation (7) is

$$\theta=\begin{cases}A_1\sinh(pt)+A_2\cosh(pt)-\alpha+\dfrac{1}{1+\dfrac{\omega_p^2}{p^2}}\dfrac{a_p}{g}\sin(\omega_p t+\psi_B), & \theta<\beta\\ A_3\sinh(pt)+A_4\cosh(pt)+\alpha+\dfrac{1}{1+\dfrac{\omega_p^2}{p^2}}\dfrac{a_p}{g}\sin(\omega_p t+\psi_A), & \theta>\beta\end{cases}\tag{8}$$

The time histories of angular velocities obtained from Equation (8) are

$$\dot{\theta}=\begin{cases}pA_1\cosh(pt)+pA_2\sinh(pt)+\dfrac{\omega_p}{1+\dfrac{\omega_p^2}{p^2}}\dfrac{a_p}{g}\cos(\omega_p t+\psi_B), & -\alpha<\theta<\beta\\ pA_3\cosh(pt)+pA_4\sinh(pt)+\dfrac{\omega_p}{1+\dfrac{\omega_p^2}{p^2}}\dfrac{a_p}{g}\cos(\omega_p t+\psi_A), & \beta<\theta<\alpha\end{cases}\tag{9}$$

where the constant parameters A_1, A_2, A_3 and A_4 are calculated by the initial conditions at the instant that the rigid block enters the rocking motion. For the rigid block under one-sine pulse, we have t = 0, $\ddot{u}_g(0) = (\alpha+\beta)g$ when it rocks at corner B and $\ddot{u}_g(0) = (\alpha-\beta)g$ at corner A. The initial angular velocities both at corners B and A are zero. Substituting these values into Equations (8) and (9), the constant parameters can be obtained by

$$\begin{cases}A_1=\dfrac{\dot{\theta}_B}{p}-\dfrac{\omega_p}{p}\dfrac{1}{1+\dfrac{\omega_p^2}{p^2}}\dfrac{a_p}{g}\cos(\psi_B);\quad A_2=\theta_B+\alpha-\dfrac{1}{1+\dfrac{\omega_p^2}{p^2}}\dfrac{a_p}{g}\sin(\psi_B)\\ A_3=\dfrac{\dot{\theta}_A}{p}-\dfrac{\omega_p}{p}\dfrac{1}{1+\dfrac{\omega_p^2}{p^2}}\dfrac{a_p}{g}\cos(\psi_A);\quad A_4=\theta_A-\alpha-\dfrac{1}{1+\dfrac{\omega_p^2}{p^2}}\dfrac{a_p}{g}\sin(\psi_A)\end{cases}\tag{10}$$

When the acceleration amplitude approaches the critical overturning acceleration, the block will take a long time to re-center. For the exact value of the critical acceleration, the block will theoretically spend an infinitely long time to re-center. This condition is the boundary point between rocking and overturning, which can be expressed as

$$\ddot{\theta}(t_{\infty}) = 0 \tag{11}$$

According to Zhang and Makris [4], under one-sine pulse, a free-standing block has two modes of overturning, i.e. mode 1 overturning with one impact either before (case 1) or after (case 2) the expiration of the excitation and mode 2 overturning with no impact. In this study, only the rocking respect to point B is considered.

Denoting t_{fv} as the time when the block enters its free vibration. T_{ex} is the time the excitation expired. t_i is the time impacting happens. The condition for overturning after the block has experienced one impact is

$$\dot{\theta}(t_{fv}) + p[\theta(t_{fv}) - \alpha] = 0 \tag{12}$$

For different blocks under various one-sine pulses, as mentioned before, the impact may happen either before (case 1: $t_i<T_{ex}$) $t_{fv}= T_{ex}$ or after the excitation expires (case 2: $t_i>T_{ex}$) $t_{fv}= t_i$. By solving the above equations, the minimum acceleration spectrum can be obtained.

3. Results and discuss

Choosing a rigid block with characteristic frequency parameter p = 2.14 rad, γ = 0.9 and α = 0.25 as an example, the minimum overturning acceleration spectrum of the free-standing rigid block on slopes under one-sine wave is shown in Fig. 4. It shows that when the frequency is extremely small (ω_p approaching to 0), the block will overturn in mode 2. The minimum overturning acceleration is equal to $g\tan(\alpha+\beta)$ and $g\tan(\alpha-\beta)$ when it is rocking respect to corner B and A, respectively. For the blocks on different slopes, the overturning spectrums are significantly different. The influence of the slope on the overturning spectrum increases with an increase of the frequency of the one-sine wave. In the case ω_p/p is smaller than 4.5, the influence is slight; in the case ω_p/p is larger than 4.5, the minimum overturning acceleration becomes sensitive to the frequency of the one-sine wave. For the angles of the slopes equal to 0.02 rad, 0.01 rad, 0 rad, -0.01 rad and -0.02 rad, when the frequency ratio ω_p/p is larger than 7.37, 6.95, 6.61, 6.28 and 6, the block will only overturn in mode 2.

According to Zhang and Makris [4], a safe area exists between the overturning spectrum in mode 1 and mode 2. It indicates that overturning with one impact and overturning without impact is not immediate. The overturning area become larger with an increase of the angle of the slope. To be exact with ω_p/p=4.0, a block on a slope with an angle respectively equals to 0.02 rad, 0.01 rad, 0 rad, -0.01 rad and -0.02 rad, the overturning amplitude accelerations of one-sine wave in case 1 are 1.7543αg, 1.7743αg, 1.7893αg, 1.8177αg and 1.8412αg; in case 2, these are 4.9764αg, 4.9141αg, 4.8358αg, 4.7844αg and 4.7166αg; and in mode 2, these are 5.2923αg, 5.2656αg, 5.239αg, 5.2124αg and 5.1859αg. For example, fixing the frequency ω_p/p=4.0 and β=0.02 rad, with the acceleration amplitude increasing from 1.0αg to 1.7543αg, the block rocks without overturning; from 1.7543αg to 2.765αg, the block overturns in case 1; from 2.765αg to 4.9764αg, the block overturns in case 2; from 4.9764αg to 5.2923αg, the block does not overturn; when the amplitude of acceleration is larger than 5.2923αg, the block overturns in mode 2. For a determined one-sine wave, by setting the impact time t_i equal to the excitation expiring time T_{ex}, the acceleration amplitude distinguishing the overturning of the block in case 1 and case 2 is located inside the circle (see point O_1 in Fig. 5).

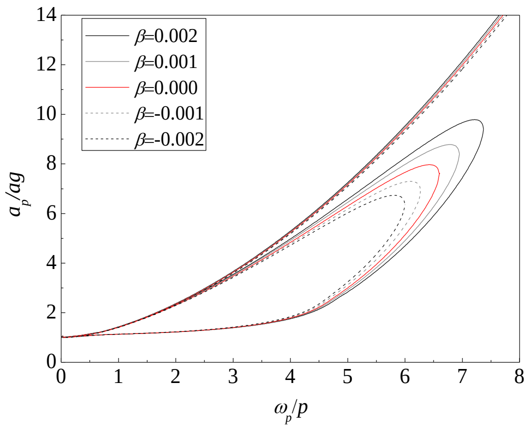


Fig.4 - Overturning acceleration spectrum

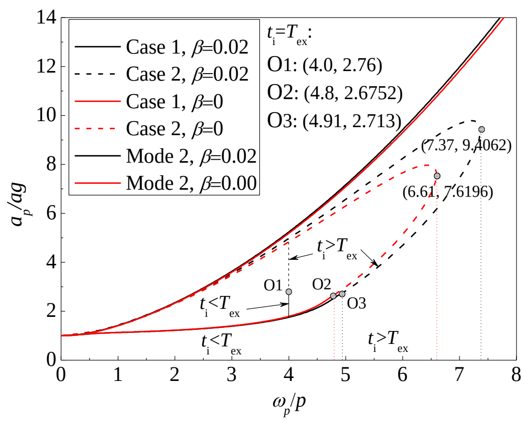


Fig.5 - Components of the overturning acceleration spectrum

The overturning acceleration spectrums of the rigid blocks on horizontal surface and slope with an angle 0.02 rad are further examined. The overturning acceleration spectrum contains three-part, overturning mode in case 1, case 2 and mode 2 (see Fig.5). The slope significantly affects the critical frequency between case 1 and case 2. The critical frequencies of the block on horizontal and slope with an angle 0.02 rad are respectively 4.80p and 4.91p. In the case the frequency of the one-sine wave is smaller than the critical value, the block will overturn in case 1, case 2 and mode 2. However, when the frequency is larger than the critical value, the block will overturn in two types of case 2 (two intersections) and mode 2.

Based on linear and nonlinear formulations, the minimum overturning acceleration to overturn the block is determined by fixing ω_p/p=6 and gradually increasing the acceleration amplitude of the one-sine wave. The minimum acceleration (see Fig. 6) indicates that the numerical result with linear formulation is in full agreement with the analytical result. However, in the case of considering the nonlinear formulation, the numerical result is smaller than the analytical results for case 1, and larger in case 2 and mode 2. It is easy to indicate that if nonlinear formulation is considered, the area surrounded by the minimum acceleration spectrum is larger than that obtained from linear formulation. Therefore, the spectrum simplified by linear formulation is not conservative, which should be careful in practical use.

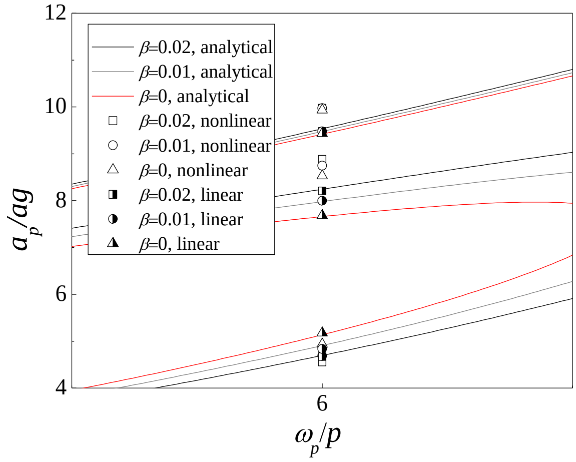
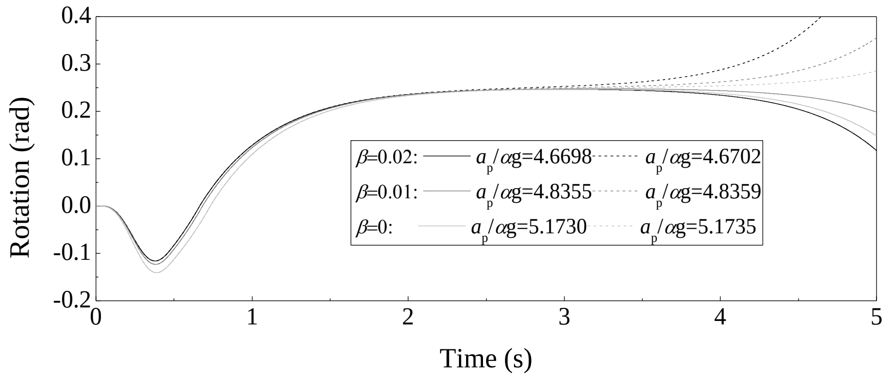
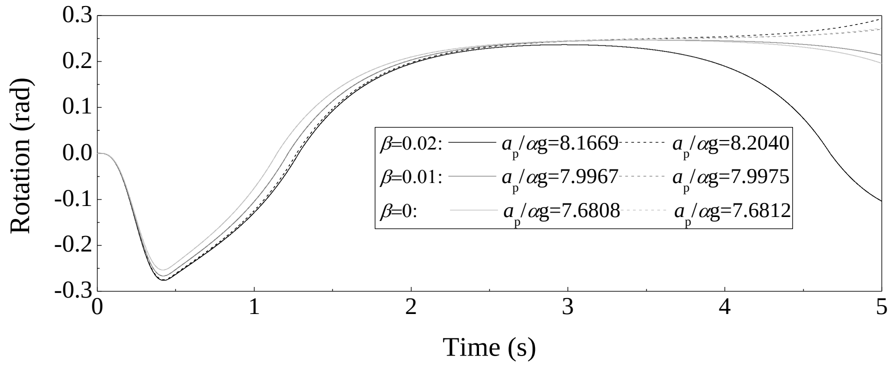


Fig.6 - Comparison between analytical and numerical results

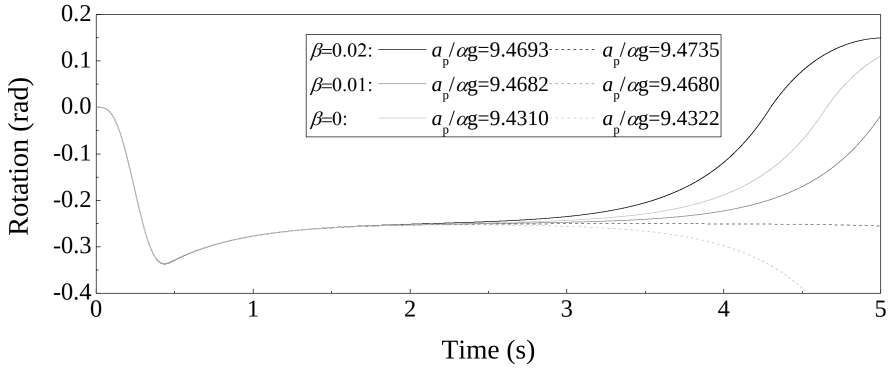
Figures 7 and 8 show the rotation time histories of the free-standing rigid block on slopes subjected to one-sine wave with ω_p/p=6. Fig.7 is computed based on linear formulation simplification and Fig. 8 according to nonlinear formulation. These two formulations have similar rotation time history except, as mentioned in the above, first, the overturning acceleration for the first type of case 2 is smaller and for the second type of case 2 and mode 2 is larger when the nonlinear formulation is considered; second, the first peak rotation computed by the nonlinear formulation is larger in the case of the same slope and overturning mode. Because ω_p/p=6 is larger than the critical frequency between case 1 and case 2, the first two graphs in Fig.7 and Fig.8 are overturned in case 2, i.e. the impact happens after the excitation expires. The duration of the one-sine pulse is 0.4909 s, the earliest impact time is 0.6341 s for the block on the slope with an angle of 0.02 rad. Figs 7 and 8 also indicate that the rotation of the block is very sensitive to the amplitude of the one-sine wave. Another interesting phenomenon is that the first maximum negative rotation in the last two graphs in both Figs 7 and 8 exceeds the slenderness α of the block, however, the block successfully recenters in the deaccelerating motion of the one-sine wave. The maximum negative rotation reaches -0.2962 rad for case 2 and -0.3475 rad for mode 2 in the case the slope is 0.02 rad.



(a)

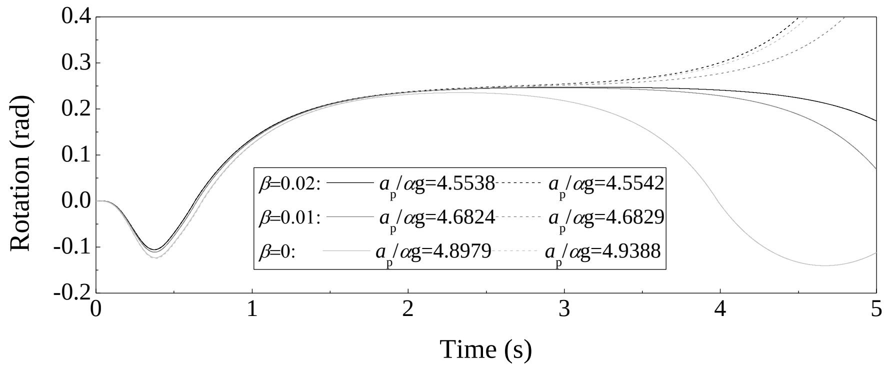


(b)

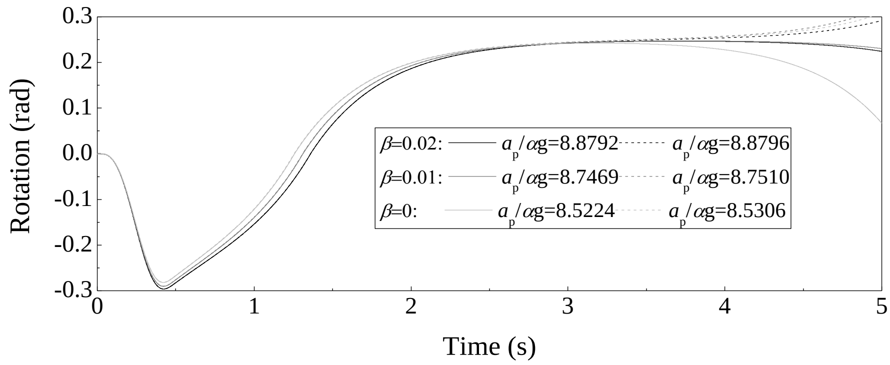


(c)

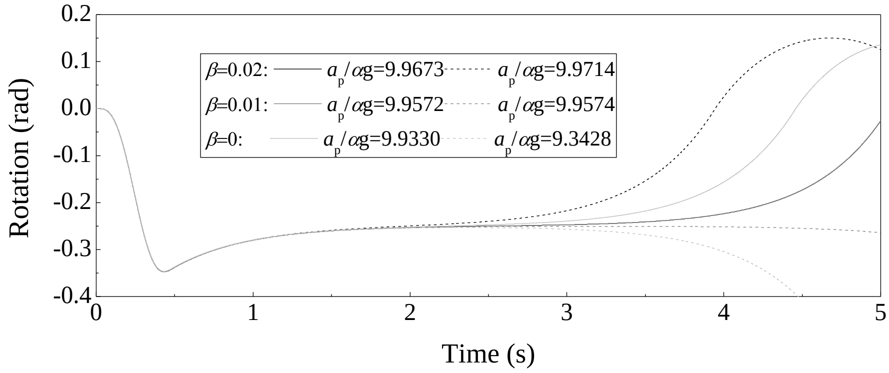
Fig.7 - Rotation time history of rigid block on slopes subjected to one-sine pulse with ω_p/p = 6 based on linear formulation: overturning in (a) case 1, (b) case 2 and (c) mode 2



(a)



(b)



(c)

Fig.8 - Rotation time history of rigid block on slopes subjected to one-sine pulse with ω_p/p = 6 based on nonlinear formulation: overturning in (a) case 1, (b) case 2 and (c) mode 2

4. Conclusions

The structures, equipment or storage casks may experience free-standing rocking on a slope because of the nonuniformed settlement of the base, imprecise construction of the structures, failure of the foundation and *etc.* After examining the free-standing rigid block on slopes subjected to one-sine pulse excitation, the following conclusions can be obtained:

(1) The slope has significant influences on the minimum acceleration spectrum, the overturning mode and the rocking process.

(2) The safe region between the acceleration-frequency plane above the minimum overturning acceleration spectrum and the overturning boundary of mode 2 depends on the angle of the slope.

(3) The minimum acceleration spectrum is built based on the linear formulation simplification; however, it is not conservative. Considering the nonlinear formulation, smaller acceleration amplitude of one-sine wave can overturn the block in case 1, and larger acceleration is required to overturn the block in case 2 and mode 2.

5. Acknowledgements

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6. Copyrights

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