NUMERICAL STUDY OF EFFECT ON DYNAMIC RESPONSES OF STRUCTURES DUE TO FAULT DISPLACEMENT AND SEISMIC MOTION

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Abstract

To estimate the effect of fault displacement underneath reinforce concrete structures, static analysis has mostly been used in previous studies. However, when fault displacement occurs, not only the fault displacement itself but also the seismic motion related to it simultaneously act on the structures. Although there are previous studies that handle both fault displacement and seismic motion in a dynamic analysis, effective and concise methods cannot be found. For example, a method requires a complicated procedure of analysis, such as combining static and dynamic analyses in a single step, and another method realizes the simultaneous loads by replacing fault displacement with an external force, but it is thought to be an expedient method.

This paper aims at establishing a method to evaluate the effect on the dynamic responses of reinforced-concrete structures due to the superimposed load from fault displacement and seismic motion. In our numerical method, it is assumed that ground motion, which is the source of the superimposed load, can be clearly separated into fault displacement and seismic motion. This assumption yields some modifications of the equation of motion in which fault displacement and seismic motion are treated as imposed displacement and inertial force, respectively.

The proposed method is tested by a multiple degree-of-freedom model with contact nonlinearity and a two-dimensional finite element model with material and contact nonlinearity. The former model is used to solve an ordinary earthquake response problem, and the numerical results are compared with those obtained by the conventional method. The latter model solves a superposed loading problem where the structure is assumed to be affected by fault displacement partially beneath the foundation and seismic motion, and the numerical results are compared with those obtained via static analysis. In addition, some numerical studies are conducted, where the dominant period of displacement and the location of the fault displacement are varied, so that the effects of the superimposed load can be investigated.

Keywords: fault displacement; finite element; nonlinear; reinforced concrete; superimposition; transient

1. Introduction

Fault displacement has been observed on surface ground during some earthquakes even though it does not appear during most earthquakes. This paper describes the numerical method that is needed for evaluating the effect of fault displacement on structures when such fault displacement appears beneath the structures. Note that the degree of fault displacement treated in this study is assumed to be at most the degree of fault displacement that branches from the main fault displacement.

Most studies evaluating the effects of fault displacement on structures have focused on the static behaviors of the displacement. However, when fault displacement occurs for real, the shaking generated by the earthquake and the displacement occurring beneath the structure are superimposed to affect the structures. Thus, in order to evaluate the effect of fault displacement on structures, a numerical method that can superimpose fault displacement and earthquake motion is required.

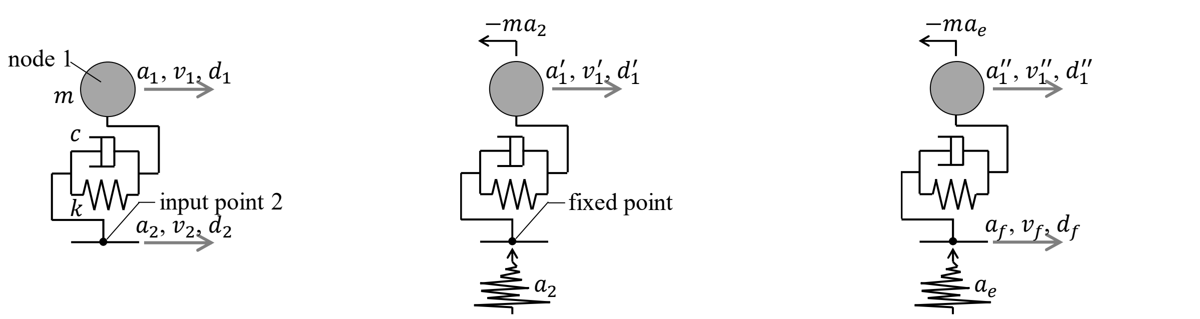
Some previous studies treat both fault displacement and earthquake motion in terms of dynamics, but they are neither effective nor concise. For example, there is a method [1] in which fault displacement is treated as static behavior, and static analysis and dynamic analysis are combined within a single step. However, the required procedure is complicated. Another method [2] interprets fault displacement as an external force, but proper mass and stiffness are required for each model; in other words, a parametric study is required to determine the mass and stiffness for each model.

Hence, in this study, a concise numerical method in which fault displacement and earthquake motion are superimposed is proposed to establish a method to evaluate the effect of the superposition of fault displacement and earthquake motion on the dynamic responses of reinforced-concrete structures.

2. Formulation

It is assumed throughout this paper that the ground motion acting on a building can be separated into earthquake motion and fault displacement components. With this assumption, earthquake motion and fault displacement are treated as an inertial force and imposed displacement, respectively. In the rest of this section, the equation of motion is formulated with respect to the system of a single degree of freedom in which the inertial force and the imposed displacement are simultaneously considered.

Let us consider the response of a single degree-of-freedom system as shown in Fig. 1 (a) to ground motion. The mass of the node is denoted as m, the coefficient of the dashpot as c, and the stiffness of the spring as k. The acceleration of the node is denoted as a_1 , the velocity of the node as v_1, and the displacement of the node as d_1. In the same manner, the acceleration, velocity, and displacement of the input point, which are exactly the input motion of the ground, are a_2, v_2, and d_2, respectively. The coordinate of the system used to express these response quantities is called the absolute system.



(a) absolute coordinate system (b) relative coordinate system I (c) relative coordinate system II

Fig. 1 – Single degree-of-freedom models and coordinate systems to describe response quantities

2.1 Inertial force only case

First, the ordinary formulation of the problem is described here. As shown in Fig. 1 (b), the input point where the ground motion is prescribed is fixed, and the responses of the node are expressed by the reference coordinate system based on the input point. In the rest of this paper, the response quantities described in this coordinate system are indicated with a single dash. In this coordinate, the inertial force, which is calculated as the product of the mass of the node and the negative of the ground acceleration ($-ma_2$), is thought to act on the node. Then, the equation of motion in this coordinate system is written as

$$ma_1' + cv_1' + kd_1' = -ma_2. \tag{1}$$

The method to obtain the response based on Eq. (1) is called the Inertial Force Method in this paper.

2.2 Imposed displacement case

Second, the same system can be seen as the system of two degrees of freedom. In this perspective of the view, the equation of motion can be written with the response quantities in the absolute coordinate system in the matrix form as

$$\begin{pmatrix} m & 0 \\ 0 & 0 \end{pmatrix}\begin{pmatrix} a_1 \\ a_2 \end{pmatrix} + \begin{pmatrix} c & -c \\ -c & c \end{pmatrix}\begin{pmatrix} v_1 \\ v_2 \end{pmatrix} + \begin{pmatrix} k & -k \\ -k & k \end{pmatrix}\begin{pmatrix} d_1 \\ d_2 \end{pmatrix} = \begin{pmatrix} 0 \\ r \end{pmatrix}, \tag{2}$$

where r denotes the reaction force of the input point.

In Eq. (2), the response quantities of node 1 are unknown, whereas those of input point 2 are given. This means the first row of Eq. (2) is essential and equivalent to Eq. (1), while the second row of Eq. (2) is used to obtain the reaction force r, which is unknown.

2.3 Superposition of inertial force and imposed displacement case

Let us separate the ground motion into a pair of earthquake motion and fault displacement as shown below.

$$\begin{cases} a_2 = a_g = a_e + a_f \\ v_2 = v_g = v_e + v_f \, , \\ d_2 = d_g = d_e + d_f \end{cases} \tag{3}$$

where the subscript g means ground motion, e represents the portion of earthquake motion, and f represents the portion of fault displacement.

Let us consider the coordinate system that is relative to earthquake motion (a_e, v_e, d_e). The response quantities described in this coordinate system are written with a double dash. With this notation, the response quantities of node 1 are written as

$$\begin{cases} a_1 = a_e + a_1'' \\ v_1 = v_e + v_1'' . \\ d_1 = d_e + d_1'' \end{cases} \tag{4}$$

Substituting Eqs. (3) and (4) into Eq. (2) yields the modified equation of motion as below.

$$\begin{pmatrix} m & 0 \\ 0 & 0 \end{pmatrix}\begin{pmatrix} a_1'' \\ a_f \end{pmatrix} + \begin{pmatrix} c & -c \\ -c & c \end{pmatrix}\begin{pmatrix} v_1'' \\ v_f \end{pmatrix} + \begin{pmatrix} k & -k \\ -k & k \end{pmatrix}\begin{pmatrix} d_1'' \\ d_f \end{pmatrix} = \begin{pmatrix} -ma_e \\ r \end{pmatrix}. \tag{5}$$

Similar to the case of Eq. (2), the first row of Eq. (5) is used to obtain the response quantities of node 1, whereas the second row is used for the reaction force r. Thus, only the first row of Eq. (5) is substantially solved.

Note that, in Eq. (5), the inertial force due to ground acceleration a_e invoked by earthquake motion is treated as an external force on the right hand side, whereas ground velocity v_f and ground displacement d_f with regard to fault displacement are treated as the imposed displacement at the input point. The situation corresponding to this description is shown in Fig. 1 (c).

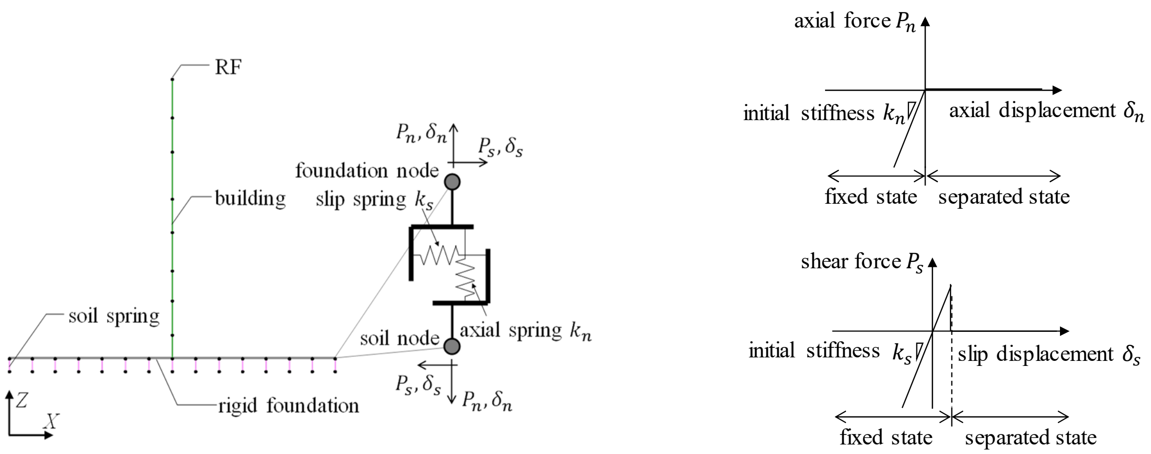
The method to obtain the response based on Eq. (5) is called the Proposed Method in this paper. Note that the equation set with respect to a multiple degree-of-freedom system can also be derived and extended to non-linear problems as in the reference [3].

3. Fundamental Study

3.1 MDOF case

To verify the Proposed Method, let us solve a problem involving a structure-soil system affected by a single ground motion using the Inertial Force Method and Proposed Method.

The numerical model is shown in Fig. 2. The assumption of the problem is described as follows. The upper part of the structure consists of eight nodes as shown in (a), whereas the foundation of the structure is assumed to be rigid. The structure stands on the soil via gap elements. The gross spring stiffness of the soil in both the sway and rocking components is calculated based on the admittance theory and is transformed into a set of scalar springs in the horizontal and vertical directions, respectively. The soil properties to be used are listed in Table 1. To represent the uplift behavior of the bottom of the foundation from the surface of the ground, a gap element is placed between a node of the rigid foundation and a node of each soil spring. Each gap element can represent contact and separation in the axial direction in accordance with the constitutive model described in Fig. 2 (b). The initial state of the gap elements is set by the static analysis to evaluate the self-weight of the structure prior to the transient analysis. Note that no friction slip occurs, and the shear force is released at the transition from the fixed state to the separated state. The damping model is given by the stiffness proportional damping model, in which the damping coefficient is set to 3% with respect to the first natural period of the system except for the gap elements. The damping coefficient of the gap elements is set to zero so that no damping force is carried during the separated state.



(a) eight node-rigid foundation model and a gap element

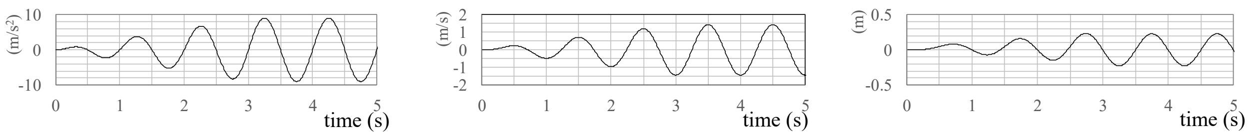
(b) gap element model: (upper) axial component and (lower) slip component

Fig. 2 – Multiple degree-of-freedom model for verification of the proposed formulation

Table 1 – Soil properties for calculating soil springs

	layer thickness (m)	unit volume weight (kN/m³)	shear wave velocity (m/s)	Poisson ratio
rigid bedrock	infinity	23	1,500	0.36

Input acceleration wave is defined by a 5-second long sine curve with a one-second period, which is tapered over the first three seconds. The amplitude of the input wave is set as the maximum becomes an amplitude of 9 m/s^2 so that the minimum ground contact ratio of the rigid foundation slightly undergoes 50%. The velocity and displacement waves are obtained by integrating the acceleration wave. These three curves are shown in Fig. 3. Note that the portion of earthquake motion (inertial force) and fault displacement (imposed displacement) are 50:50 in the Proposed Method so that the same problem is solved by the Inertial Force Method and the Proposed Method, where the results can be compared.

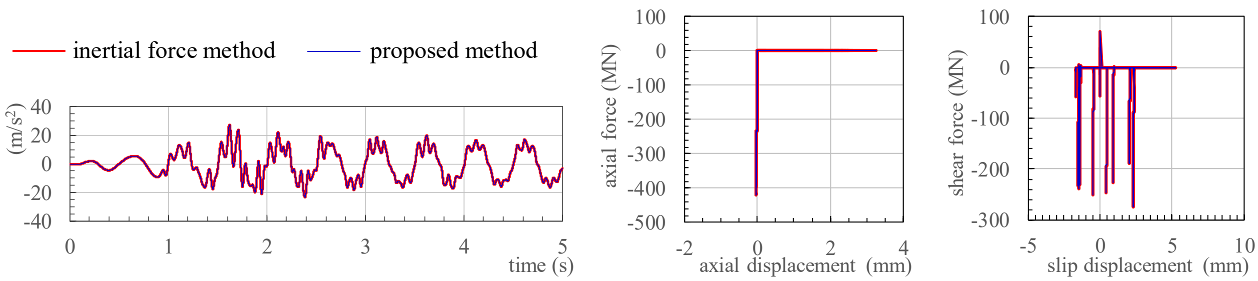


(a) acceleration (b) velocity (c) displacement

Fig. 3 – Tapered sine wave with a one-second period as input motion

The Newmark-β method (β = 1/4, γ = 1/2) [4] is used as the direct time integration method. The time increment is chosen to be 1/4,000 of a second so that the response waves become stable after the re-contact of the gap elements.

The results obtained by the computation are shown in Fig. 4. The response acceleration in the horizontal direction at the roof floor (RF) is shown in (a). The result obtained by the Proposed Method overlaps that obtained by the Inertial Force Method. Slightly high frequent waves can be seen in the response, which is thought to be caused by separation and re-contact between the foundation and the soil. The response hysteresis of the gap element at the left-most position is shown in (b). It can be observed that separation has occurred; meanwhile, large slip displacement and re-contacts occur several times. In these figures, again, the result obtained by the Proposed Method overlaps that obtained by the Inertial Force Method.



(a) response acceleration in horizontal direction at roof floor (RF)

(b) response hysteresis of the gap element at the left-most position in (left) axial component and (right) slip component

Fig. 4 – Numerical results of the MDOF case

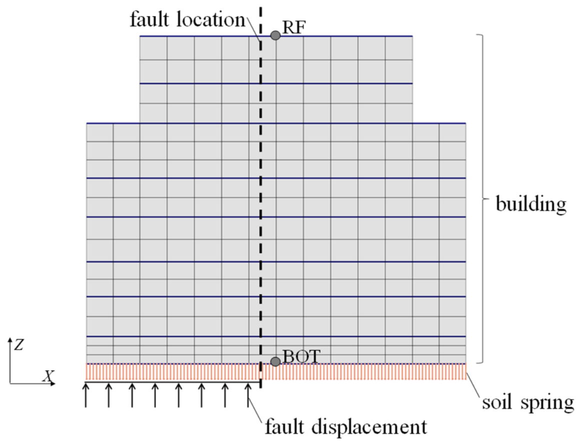
It is concluded that the Proposed Method has been verified through a contact non-linear problem with respect to inertial force.

3.2 Finite Element case

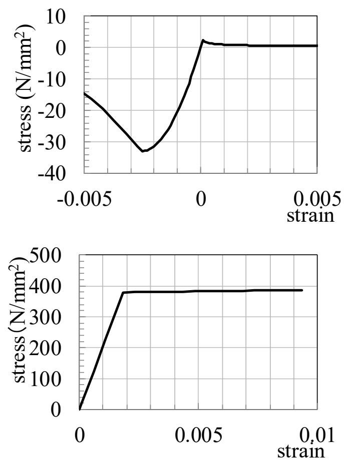
To verify the Proposed Method with a non-linear finite element model, let us solve the problem in two dimensions involving a structure-soil system affected by vertical displacement together with horizontal ground motion using the Proposed Method.

The two-dimensional finite element model used in this study is shown in Fig. 5. The structure is wall-shaped and of reinforced concrete as shown in (a). The three consecutive elements from the bottom are considered to be the foundation of the structure. Plane elements, which represent reinforced concrete, and rod elements, which represent distributed mass of the structure at each floor, are used in the model. Note that the rod elements do not represent stiffness.

Each plane element can represent the nonlinearity of both concrete and reinforcing steel. The material constants are listed in Table 2. The constitutive model of each material [5] is shown in Fig 5 (b). The concrete model is based on the multi-directional smeared crack model [6].



(a) finite element model in two dimensions



(b) constitutive models of (upper) concrete and (lower) reinforcing steel

Fig. 5 – Non-linear finite element model for verification of the proposed formulation

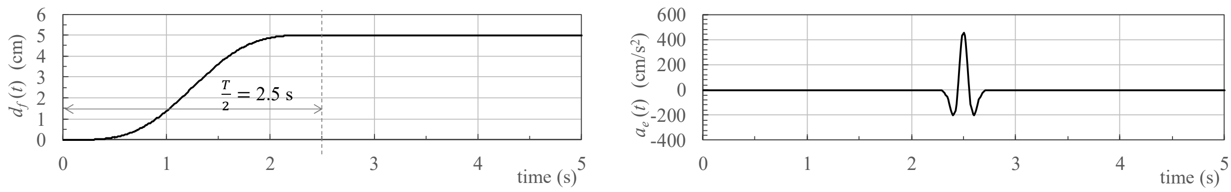
Table 2 – Material constants

Material	unit volume weight (kN/m^3)	Young modulus (N/mm^2)	compressive strength (N/mm^2)	tensile strength (N/mm^2)	Poisson ratio
Concrete	24	25,100	33	2.37	0.2
Steel	77	205,000	379.5	379.5	0.3

The property of the soil used in this section is the same as listed in Table 1 in the previous section. To represent the uplift behavior of the structure, gap elements are again used. However, there are two things modified from the modeling in the previous section to avoid potential numerical instabilities. One is that the gap elements are solely used to model both the soil-structure interaction and the uplift behaviors of the foundation. In this modeling, the initial stiffness of each gap element is set to be that of the corresponding soil spring. The other one is that the number of gap elements is increased from the previous section, as seen in Fig. 5 (a), because the discretization of soil spring may affect the response of the structure. Note that the same assumption as the previous section is applied to the gap elements: in other words, no friction slip is allowed, and the shear force is released at the transition from the fixed state to the separated state. The damping model is given by the instantaneous stiffness proportional damping model in which the damping coefficient is set to 3% with respect to the first natural period of the system. The reason why the instantaneous stiffness is used to define the damping model is to assure that no damping force is carried by a gap element during its separated state.

Upward displacement is applied in the vertical direction to the left half of the model as shown in Fig. 5 (a). The displacement is given at the fixed point of the gap elements as the imposed displacement as described in sections 2.2 or 2.3. The input wave of the displacement $d_f(t)$ is shown in Fig. 6 (a). The corresponding velocity wave $v_f(t)$ is obtained by differentiating the displacement wave. The dominant period T of the input displacement is five seconds, while the maximum displacement occurs at 2.5 seconds.

Horizontal acceleration $a_e(t)$ is superimposed with the above displacement, which is treated as the inertial force as described in sections 2.1 or 2.3. The wave is defined by a Ricker wave, where the central frequency is 4 Hz and the maximum occurs at 2.5 seconds as shown in Fig. 6 (b).



(a) vertical displacement wave: upward is positive (b) horizontal acceleration wave

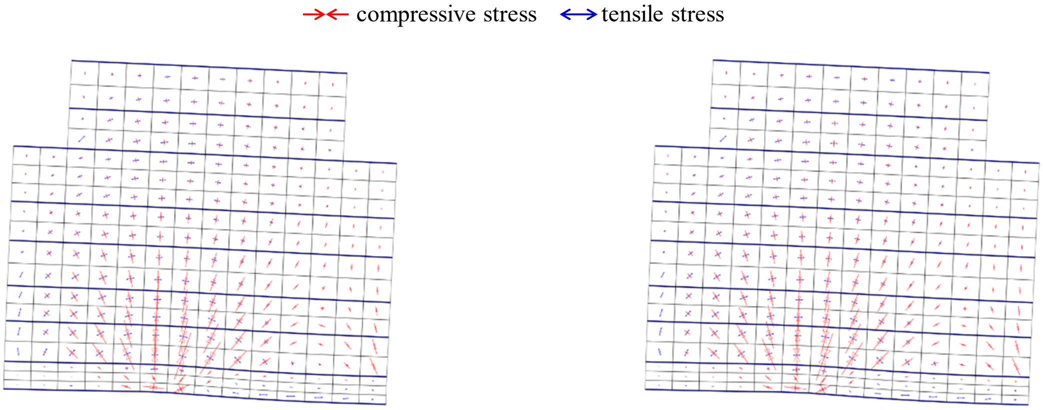
Fig. 6 – Input waves for verification with two-dimensional finite element model

The motion of the input displacement shown in Fig. 6 (a) is relatively slow, which may be treated as quasi-static. It is expected that the response of the structure can be comparable to a response that is subject to static displacement. Therefore, four cases are computed and compared as listed in Table 3 for verifying the Proposed Method. Cases S and D are used for verifying only the loading of the displacement by comparing dynamic displacement with static displacement. Cases S+D and D+D are used to verify the superimposed loading, which is of interest, of displacement and acceleration.

Table 4 – List of cases for two dimensional finite element model

Case	**Displacement**	**Acceleration**
S	Static	N/A
D	Dynamic	N/A
S+D	Static	Dynamic
D+D	Dynamic	Dynamic

The results of Cases S and D are shown in Fig. 7. Both results are comparable. The compressive stress is observed around the fault location. There can also be found the compressive stress on the right-most part of the structure, which is probably because of the uplift and rotational motion of the structure due to the upward fault displacement. It is confirmed that the Proposed Method can provide results for the quasi-static displacement applied in a dynamic problem consistent to static loading.



(a) Case S, static displacement case (b) Case D, dynamic displacement case (T=5s)

Fig. 7 – Comparison of static and dynamic cases for displacement only loading: Deformation, which is magnified by 50 times; and principal stress distribution, in which red and blue arrows denotes compressive and tensile stresses, respectively.

The acceleration response spectrum of the results calculated with 5% damping factor obtained from Cases S+D and D+D is shown in Fig. 8. The result of the dynamic-dynamic case (D+D) overlaps with that of the static-dynamic case (S+D). The peak around 0.2 seconds is due to the natural period of the system, whereas the other peak around 0.05 seconds is due to contact-separation of the foundation. It is also confirmed that the Proposed Method can provide results for the superposition of quasi-static displacement and acceleration consistent with the superposition of static displacement and acceleration.

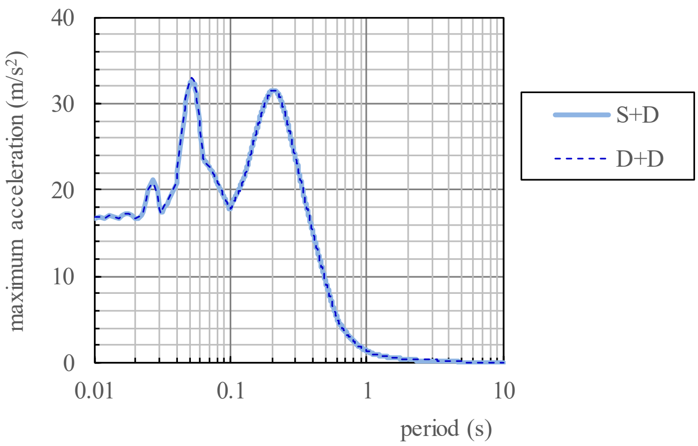


Fig. 8 – Comparison of static and dynamic case for superimposed loading: Acceleration response spectrum with 5% damping factor in horizontal direction at roof floor (RF)

Through these cases, the Proposed Method has been verified to solve the superimposed loading of displacement and acceleration.

4. Numerical Examples

4.1 Effect of dominant period of input displacement

Here is a demonstration to study the effect of the dominant period of the input displacement on the response of a structure. In this study, another upward displacement where the dominant period is one second is incorporated, and the numerical result is compared with that obtained in section 3.2. The input displacement wave is shown in Fig. 9. The same numerical model and the acceleration wave as in section 3.2, except for the input displacement, are used. Note that the superposition of displacement and acceleration is only considered in this study.

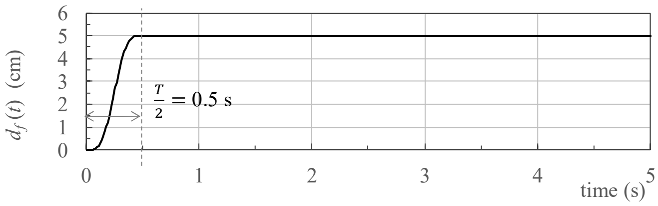
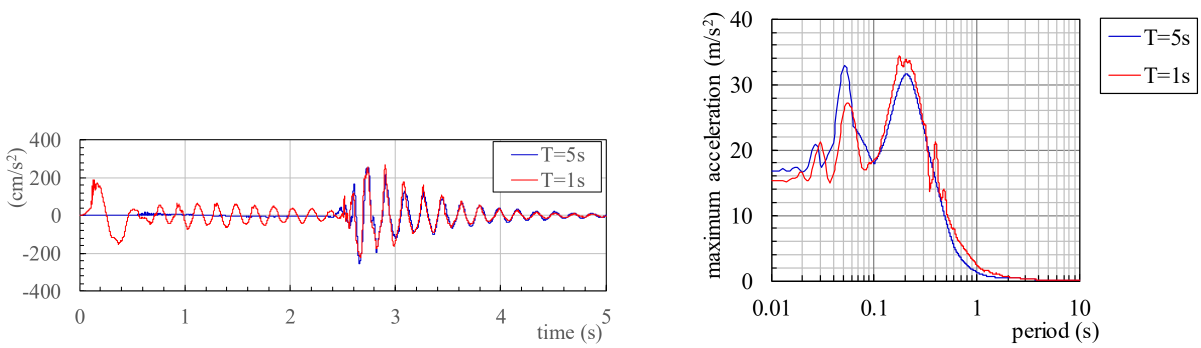


Fig. 9 – Vertical displacement of dominant period one second

The response acceleration in the vertical direction at the bottom of the structure (BOT) is shown in Fig. 10 (a). In the first half of the response (0–2.5 seconds), the one-second case shows significant responses, whereas the five-second case does not. The one-second displacement invokes significant acceleration unlikely for the five-second displacement. This difference indicates that the one-second displacement is no longer quasi-static loading. In the second half of the response (2.5–5.0 seconds), although slight differences between these two cases can be seen, the overall responses are quite close. It may imply that the effect of the input displacement with the amplitude of 5 cm on the structure is not as much as that of the horizontal acceleration with the amplitude of 450 cm/s^2.



(a) response acceleration in vertical direction at bottom of structure (BOT)

(b) acceleration response spectrum with 5% damping factor in horizontal direction at roof floor (RF)

Fig. 10 – Comparison of numerical results of superimposed vertical displacement where the dominant periods are 5 seconds and 1 second and horizontal acceleration

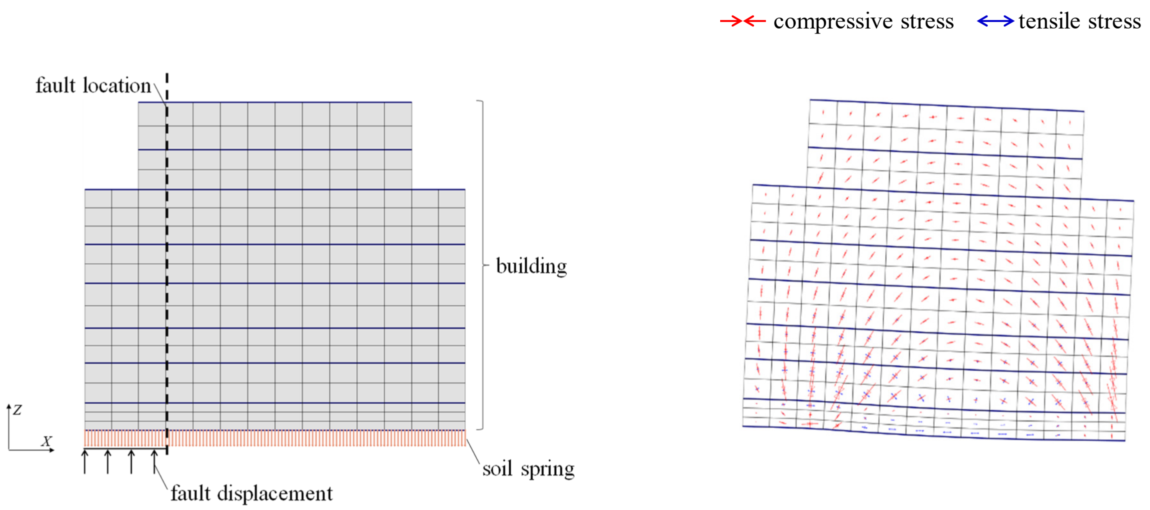
The acceleration response spectrum with 5% damping factor in the horizontal direction at the roof floor (RF) is shown in Fig. 10 (b). It is observed that the peak around 0.2 seconds differs slightly from each other, and the second peak around 0.05 seconds exhibits a bigger difference. The responses of the five-second case and the one-second case do not differ much, which is consistent with the discussion above. The overall responses are still very close.

It can be concluded from this study that the dominant period of the input displacement at this amplitude is not as significant as the horizontal acceleration at this magnitude.

4.2 Effect of location of fault displacement

Here is another demonstration to study the effect of the location of fault displacement on the response of a structure. In this study, the location of fault displacement is modified as it becomes 1/4 from the left as shown in Fig. 11 (a). All other conditions are maintained from section 3.2. Note that the solo displacement case is only computed with the five-second displacement motion.

The deformation and the principal stress distribution are drawn in Fig. 11 (b). Compared to the original cases (Cases S and D in section 3.2) shown in Fig. 7, the compressive stress field, which is an arch-like shape, is organized on the right-hand side in this case. It is observed that the stress distribution could change as the location of fault displacement moves.



(a) location of fault displacement

(b) deformation and principal stress distribution (T=5s)

Fig. 11 – Numerical model and principal stress distribution under fault displacement at 1/4 location

5. Conclusion

In this paper, a concise numerical method in which fault displacement and earthquake motion are superimposed has been proposed to establish a method to evaluate the effect of the superposition of fault displacement and earthquake motion on the dynamic responses of reinforced-concrete structures.

Conclusions obtained in this paper are summarized below.

1) The equation of motion applicable to the simultaneous input of fault displacement and earthquake motion has been formulated.
2) It has been confirmed that the proposed numerical method (Proposed Method) can properly solve a non-linear MDOF problem with contact nonlinearity and a two-dimensional finite element problem with material and contact nonlinearity.
3) Two numerical examples are shown to demonstrate the potential studies of the superimposed displacement and acceleration loading using non-linear finite element models.

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