



## ROTATIONAL INPUT MOTION IDENTIFICATION IN NONLINEAR STRUCTURE MODEL

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### **Abstract**

For the safety assessment of buildings following an earthquake, a simulation analysis is an effective evaluation method for areas that cannot be easily evaluated through visual inspection. In such cases, the input motion is set based on the observed earthquake records.

For such analyses, the most convenient method is to input the horizontal acceleration record observed in the foundation of the building into the model without considering the rotation of the foundation. However, this method is not appropriate for the response evaluation of a building highly affected by the rotational input motion at the foundation. To evaluate the response with a large effect of the rotational input motion at the foundation, it is naturally necessary to consider this rotation input motion.

In conducting these analyses, an input motion should be set to reproduce an observed wave at an observed point of the analysis model. If the analysis model is in the linear range, the input motion can easily be set using the frequency-domain method, which is the most popular method. However, this setting is difficult if the analysis model exhibits nonlinear behavior. In these cases, the modal iterative error correction (MIEC) method can be used to set the horizontal input motion from horizontal acceleration records at the top of the building, even if the analysis model exhibits nonlinear behavior.

In this study, a sample problem is used in MIEC to conduct rotational input motion inversion in a nonlinear building model. In this inversion, rotational acceleration input motion is identified from horizontal acceleration records at the base and top of the building model. The results confirm that a rotational input motion that reproduces the horizontal acceleration records with high accuracy can be determined by MIEC, even if the building model exhibits nonlinear behavior. Using this method, safety assessment analyses considering rotational input motions can be carried out in buildings in which rotational input motion has not been observed.

*Keywords: input motion inversion; singular value decomposition; nonlinear model*



## 1. Introduction

For the safety assessment of buildings following an earthquake, a simulation analysis is an effective evaluation method for areas that cannot be easily evaluated through visual inspection. In such cases, the input motion is set based on the observed earthquake records.

For such analyses, the most convenient method is to input the horizontal acceleration record observed in the foundation of the building into the model without considering the rotation of the foundation. However, this method is not appropriate for the response evaluation of a building highly affected by the rotational input motion at the foundation. To evaluate the response with a large effect of the rotational input motion at the foundation, it is naturally necessary to consider this rotation input motion.

Recently, the number of buildings recording the horizontal acceleration of the foundation has increased. In contrast, the rotational acceleration of the foundation is not measured in many buildings, and the rotational acceleration of the foundation is often unknown. Therefore, in many cases, it is necessary to estimate the unknown rotational input motion to conduct a simulation analysis considering the rotation of the foundation in the building.

This rotational input motion can be easily estimated if the response of the building is within a linear range from the horizontal acceleration records at the top and foundation of the building. However, it is difficult to estimate the rotational input motion when the building exhibits nonlinear behavior as a result of a large earthquake. This is because this problem is an inverse problem of nonlinear input/output systems.

In this study, the modal iterative error correction (MIEC) method, which is the inverse analysis method of the nonlinear input/output system [1], was used to estimate the rotational input motion from the horizontal acceleration of the top and foundation of the nonlinear building model. In addition, the accuracy is confirmed, and the applicability is verified. For this object, it is essential to use analytical conditions that enable the analysis model to represent the true behavior of the system for an accurate simulation analysis. This study confirms the accuracy of the MIEC method under the assumption that the conditions used in the analysis model are appropriate.

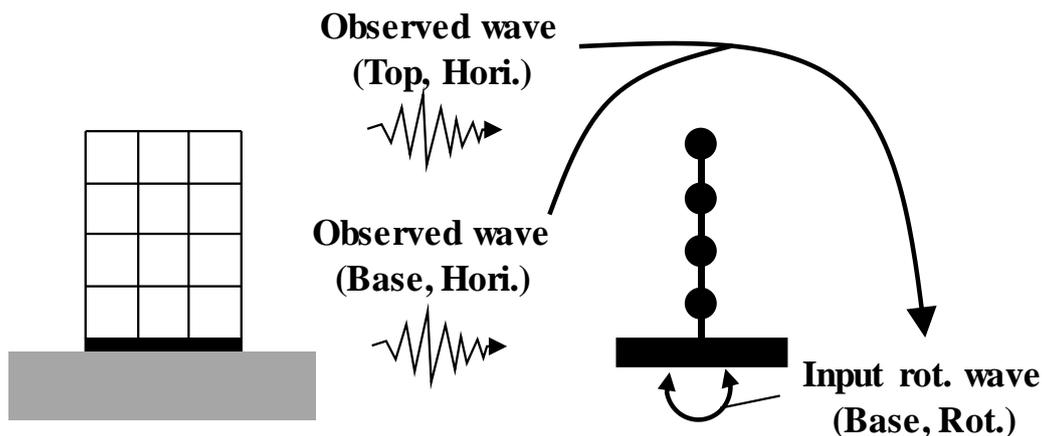


Fig. 1 – Identification of rotational input motion



## 2. Inversion Method

This chapter describes the MIEC method, which is the inverse analysis method used in this study. Table 1 shows the list of symbols used in this study.

Table 1 – Symbol list

$N$ : Number of time steps	$F(\ )$ : Calc. system by analysis
$\{a_i\}$ : Input wave vector	$\{\Delta a_i^j\}$ : $j^{\text{th}}$ time step perturbation input wave vector
$\{a_o\}$ : Output wave vector	$\{a_i^j\}$ : Input wave vector added $j^{\text{th}}$ time step perturbation
$\{a_{\text{dum}}\}$ : Dummy input wave vector	$\{a_o^j\}$ : Output wave vector by input added $j^{\text{th}}$ time step perturbation
$\{a_{\text{tar}}\}$ : Target of output wave vector	$\{\Delta a_o^j\}$ : Variation wave vector by $j^{\text{th}}$ time step perturbation input
$[K]$ : Perturbation impulse matrix	$\{k^j\}$ : Variation coef. vector by $j^{\text{th}}$ time step perturbation input
$[U]$ : Left singular matrix	$\Delta$ : Perturbation magnitude
$[V]$ : Right singular matrix	$\varepsilon$ : Allowable tolerance
$[\Sigma]$ : Singular value matrix	$\{r\}$ : Residual error of output wave vector

The impulse response matrix  $K$  of the MIEC method is shown in Fig. 2. It combines the incremental input vector  $\{\Delta a_i\}$  and the incremental output vector  $\{\Delta a_o\}$  and is calculated by perturbation analyses. Then, the generalized inverse matrix of this impulse response matrix is used to calculate the incremental input vector to correct the error in that iteration. The input vector is corrected by repeating this process.

$$\left\{ \Delta a_o \right\} = \left[ K \right] \left\{ \Delta a_i \right\}$$

Fig. 2 – Identification of rotational input motion

The generalized inverse matrix is used instead of the normal inverse matrix to correspond to the case where  $K$  does not reach full rank. In addition, the convergence performance is improved by reducing the number of modes used.

The calculation procedure is similar to that of the Newton-Raphson method used in the implicit method of elasto-plastic structural analysis. First, the error vector in the case of an input candidate vector is confirmed. Then, the correction input vector to compensate for the error is calculated from "current amount of error" and "effect of perturbation on the output vector". In addition, the error vector is confirmed again using the corrected input vector, and the correction is finished if it is within the allowable range. If it is out of the allowable range, the correction of the input vector is repeated in the same manner until it is within the allowable range. The flow diagram is shown in Fig. 3. Please see Ref. [1] for details of each process.

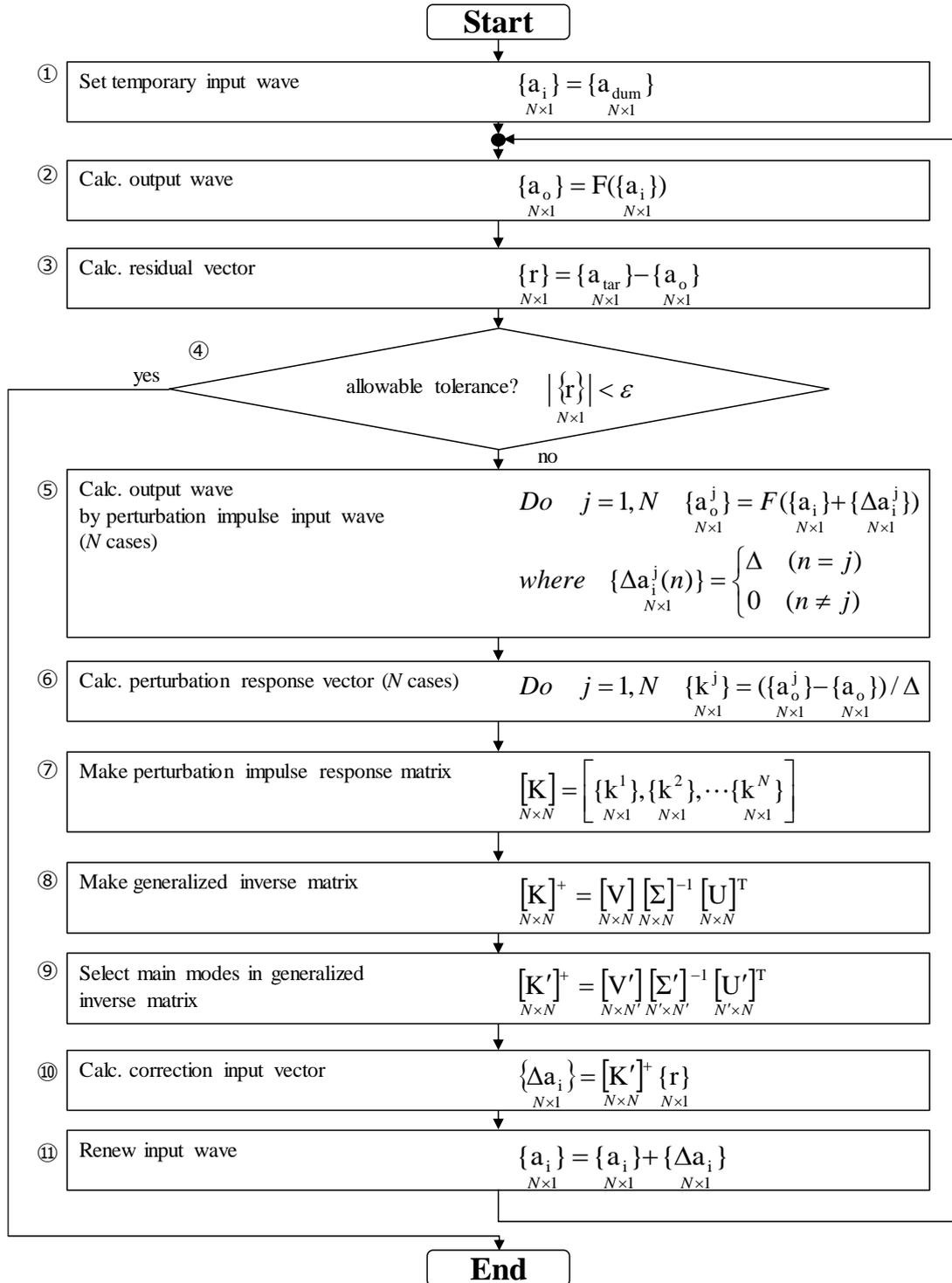


Fig. 3 – Flow diagram



### 3. Sample problem

This section explains the inverse problem used as an example. Fig. 4 provides the analytical model used for the seismic response analysis. Tables 2 and 3 display the properties of the building model. The analytical model is a lumped mass model assuming a nuclear power plant facility with an RC structure. The building height is 30 m, the foundation width is 50 m × 50 m, and the total weight is 90,000 tons. Assuming  $F_c$  24.5, the concrete has a Young's modulus of 22.8 GPa and Poisson's ratio of 0.2. The nonlinear model of shear is defined as the maximum point oriented trilinear according to Ref. [2]. Modal damping is used, and the damping ratio is 3%.

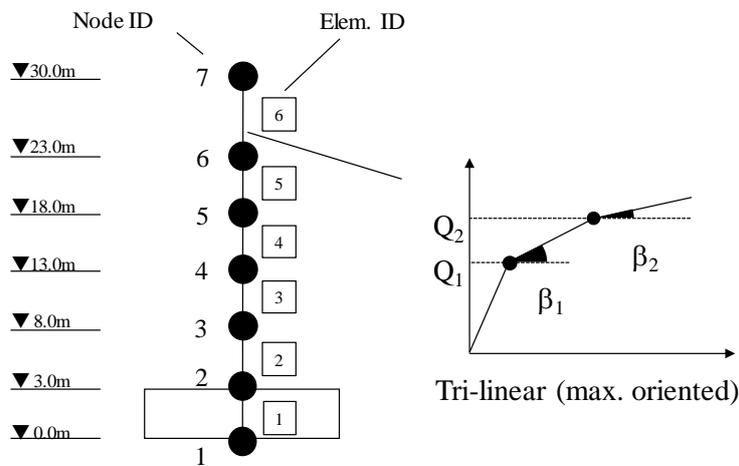


Fig. 4 – Analytical model

Table 2 – Linear property of building model

Node ID	Trans. mass (t)	Rot. mass ( $\times 10^3 \text{ tm}^2$ )	Elem. ID	Sectional area ( $\text{m}^2$ )	Shear sectional area ( $\text{m}^2$ )	Moment of inertia of area ( $\text{m}^2$ )
7	10000	2000	6	300	150	35000
6	12000	3000				
5	13000	3500	5	400	200	50000
4	15000	4000	4	400	200	55000
3	10000	3000	3	500	250	55000
2	15000	4000	2	500	250	55000
1	15000	4000	1	1.0E4	1.0E4	1.0E6



Table 3 – Nonlinear property of building model

Elem.ID	Q <sub>1</sub> (kN)	Q <sub>2</sub> (kN)	$\beta_1$	$\beta_2$
6	300	405	0.175	0.04
5	400	540	0.175	0.05
4	400	540	0.175	0.06
3	500	675	0.175	0.07
2	500	675	0.175	0.08
1	10000	13500	0.175	0.10

First, horizontal and rotational acceleration were input as external forces at the foundation of this analytical model, and the horizontal acceleration of the top of building was calculated. These input acceleration time histories were calculated by the response analysis using Taft 1952 EW earthquake motion (maximum acceleration of 1.76 m/s<sup>2</sup>, duration up to 10 s) as the input earthquake motion for sway-rocking model of this building. In this response analysis, the shear strain of the members reached a maximum of 1000  $\mu$ . Thus, it can be judged that this model reached the nonlinear region.

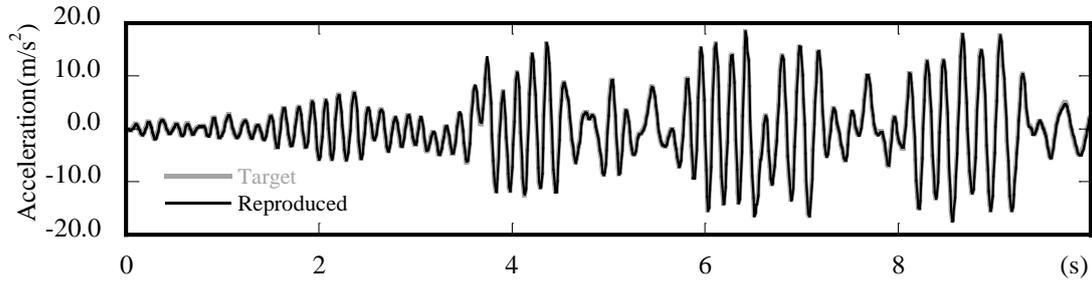
This study confirms the possibility of setting the rotational input acceleration that reproduces the target horizontal acceleration at the top of building using the MIEC method, assuming that the horizontal input accelerations are known.

In this inverse analysis, the allowable error was not set, and the iteration was stopped at 50 times. The perturbation amount used in the perturbation analysis was 0.01 rad/s<sup>2</sup>. The number of selected modes was determined by "norm ratio of the error vector" in Ref. [3]. The threshold value of the norm ratio used in this paper was 0.5.

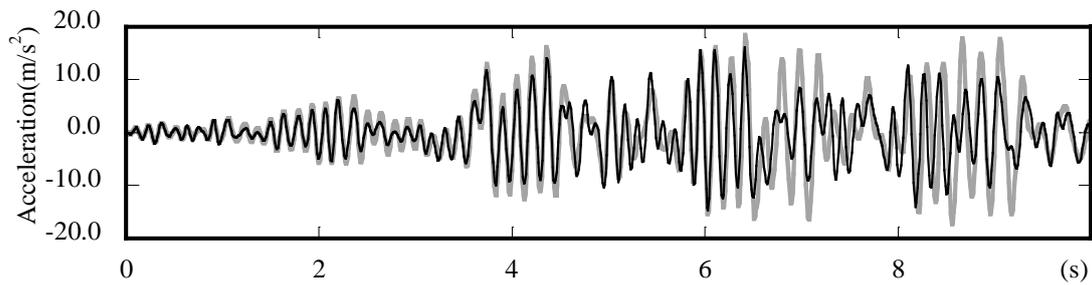
#### 4. Results

Fig. 5(a) shows the top acceleration time history reproduced using the rotational input acceleration identified by the MIEC method. For comparison, the target acceleration time history is also shown. Fig. 5(b) shows the reproduced acceleration time history without the rotational input acceleration. Fig. 5(a) confirms that the target acceleration at the top of the building can be reproduced with high accuracy by considering the rotational input acceleration identified by the MIEC method. In contrast, Fig. 5(b) shows that, without considering the rotational input acceleration, the reproduction accuracy of the top acceleration time history is low. This indicates that it is important to consider rotational input acceleration to perform more accurate simulation analysis.

Fig. 6 shows the rotational input acceleration time history identified by the MIEC method. For comparison, the rotational input acceleration time history observed when calculating the target top acceleration time history is also shown. From Fig. 6, the identification accuracy of the rotational input acceleration is slightly lower than the reproduction accuracy of the target wave in Fig. 5(a). In particular, there is a significant difference in the second half starting at 5 s. The identification accuracy of the rotation input motion is so low because this inverse problem has numerous solutions that can reproduce target waves. Therefore, although the identified rotational input motion is one of the solutions, it is thought that it did not completely match the original rotational input motion.



(a) With rotational input



(b) Without rotational input

Fig. 5 – Reproduced acceleration time history at the top of building

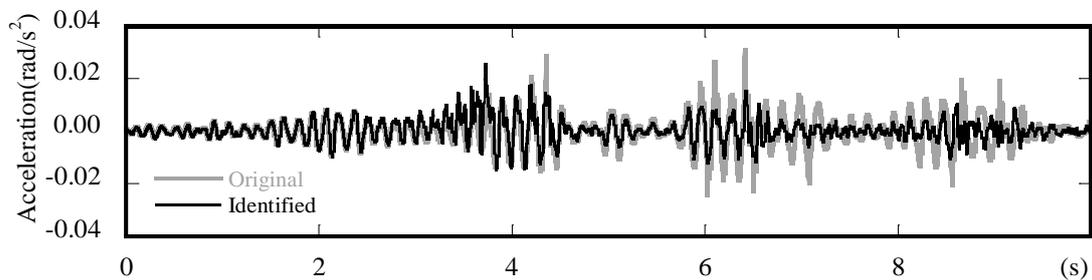


Fig. 6 – Identified rotational acceleration time history at basement

As described, it was confirmed that the MIEC method can set a rotational input motion that can reproduce the target acceleration with sufficient accuracy, even in a nonlinear response analysis of the building.

## 5. Conclusion

In this study, the MIEC method was used to estimate the rotational input acceleration from the acceleration at the top and bottom of a nonlinear building model. It was confirmed that the MIEC method can set the rotational input acceleration, which can reproduce the target acceleration wave with high precision. However, the identification accuracy of the estimated rotational input was slightly low. Simulation analysis considering rotational input is possible with this method, even in a nonlinear building models, without measuring rotational input.



## References

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