EFFECT OF RANDOMNESS IN MEMBER STRENGTHS ON SEISMIC PERFORMANCE OF MOMENT RESISTING STEEL FRAMES

Praveen Khandelwal[(1)], Kenichi Ohi[(2,Deceased)]

[(1)] *(Former Graduate Student, University of Tokyo) NTPC Ltd, India, E mail: ksanpraveen@gmail.com*
[(2)] *Former Professor, Institute of Industrial Science, University of Tokyo, Japan*

Abstract

The dynamic analysis for seismic design purpose is generally based on the deterministic (mean) member strengths. In reality, some randomness in member strengths exists due to manufacturing and construction processes with its magnitude depending upon the quality control exercised in such processes.

In this paper, a generalised approach for studying the effect of randomness in member strength on the inelastic dynamic response of moment resisting steel frames is used. The variation in the yield strength of steel is considered up to (+/-) 20% from the mean strength and three cases for variation have been considered viz. 5%, 10% and 20%.

The restoring force characteristics are represented by yield polyhedron model, simulating global elastic-plastic behaviour of the frame, instead of a set of member hysteresis based models usually adopted in the inelastic structural analysis [1]. Yield polyhedron is a convex region bounded on all sides by the failure planes. This approach provides fairly good simulation of response quantities compared to detailed member hysteresis based analysis [1].

The performance function for the failure mode is expressed in terms of the work done by the resistance capacity and the load effects. The random equivalent-static seismic force is given by a linear combination of the deterministic modal force vectors and a random multiplier to each modal load profile.

The reliability index of the various failure modes is evaluated considering the variation in the member strengths and to identify stochastically dominant failure modes for yield polyhedron.

Application of the proposed concepts is studied for a six storey frame to an earthquake ground history.

It is seen that demand envelope is sensitive to higher order randomness in member strengths, which alters the dynamic response significantly from the mean strength response.

It is pertinent to note that for considering various possible combinations of member strengths for columns and beams, more than 3000 inelastic time history analyses were performed in a very short time. Application of global yield polyhedron model offers considerable flexibility to the designer as he can quickly perform several alternatives within the constraints of time and limited resources.

References: [1] Khandelwal P, Ohi K, Fang P, A Simple Proposal for Ultimate Seismic Demand Evaluation of Moment Resisting Steel Frames, Journal of Structural and Construction Engineering, Architectural Institute of Japan, No. 545, July 2001

Keywords: seismic performance, member strength randomness, yield polyhedron, time history analysis, steel frames

1. Introduction

Dynamic analysis for seismic design purpose is generally based on deterministic (mean) member strengths. In reality, some randomness in member strengths exists due to manufacturing and construction processes with its magnitude depending upon the quality control exercised in such processes.

In this paper, a generalised approach for studying the effect of randomness in member strength on the inelastic dynamic response of moment resisting steel frames is used. The restoring force characteristics are represented by yield polyhedron model, simulating global elastic-plastic behaviour of the frame, instead of a set of member hysteresis based models usually adopted in the inelastic structural analysis [1]. Yield polyhedron is a convex region bounded on all sides by the failure planes. This approach provides fairly good simulation of response quantities compared to detailed member hysteresis based analysis.

The variation in the yield strength of steel is considered up to (+/-) 20% from the mean strength and three cases for variation have been considered viz. 5%, 10% and 20%.

Application of the proposed concepts is studied for a six-storey two-bay frame to an earthquake ground history and the results are compared with the member-hysteresis based analytical procedures.

2. Generation of Yield Polyhedron Model

In order to explain construction of yield polyhedron, a two storey single bay frame as shown in Fig. 1 is taken. The failure/collapse modes are shown in Fig. 2. These include local storey collapse mechanisms and combined mechanism. Locations of the plastic hinges are assumed at the top and bottom ends of the columns, which have undergone side sway, and at the floor beams end connected with the side columns. Sequence of yielding before mechanism formation i.e. partial yielding is ignored in the yield polyhedron model. The likely failure modes can be drawn based on the experience or stochastic limit analysis using compact procedure [2, 3,4] may be used for complex situations.

Equation of Motion for MDOF structures for seismic excitation can be written as:

$$[M]\{\ddot{x}\}+[C]\{\dot{x}\}+\{f\}=-[M]\{1\}\{\ddot{y}\} \tag{1}$$

where, {x}, {y} and {f} are vectors of relative displacement, ground displacement and the restoring force, respectively in original coordinates. Matrices [M] and [C] denotes the mass and damping matrix, respectively. Eq. (1) can be uncoupled by the following classical elastic modal transformation equations:

$$\{x\}=[\phi]\{q\} \tag{2}$$

$$\{f\}=[\phi^T]^{-1}\{r\}=[\psi]\{r\}=\{\psi^1\}r^1+\{\psi^2\}r^2+\ldots+\{\psi^n\}r^n \tag{3}$$

where, {q}, {r} and [ϕ] are modal displacement vector, modal restoring force vector and modal participation matrix, respectively. Matrix [ϕ] is normalised such that the participation factor for any mode is unity. Eq. (2) and (3) represent the displacement and force transformation, respectively. Equation of motion in modal coordinates can be written as:

$$\ddot{q}^j+2h^j\omega^j\dot{q}^j+\frac{r^j}{m^{j*}}=-\ddot{y}, \quad (j=1,2,\ldots,n) \tag{4}$$

where, vectors {h}, {ω} and {m^*} represent the modal damping ratio, circular frequency and effective modal mass, respectively. The number of total degrees-of-freedom (d.o.f.) is denoted by *n*.

The f1 and f2 represent the restoring forces (in original space) at the second floor and roof level, respectively. Fig. 3 shows the global yield polyhedron in the original restoring force space. Each column vector of $[\phi]^{T-1}$ represents the lateral loading pattern of each vibration mode and remains constant during the analysis. Equation of motion for seismic excitation is integrated by numerical methods. The locus of restoring forces can move within or tangentially on the boundary of the yield polyhedron, as shown in Fig. 4

for the two-storey frame. The r1 and r2 are components of {r} and represent the restoring forces for the first and second mode, respectively.

Yield polyhedron approach enables a quick examination of the seismic response in terms of the failure modes of the structure, which is quite informative and helpful in understanding critical/ important design situations.

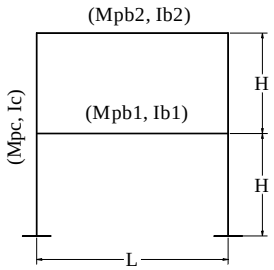
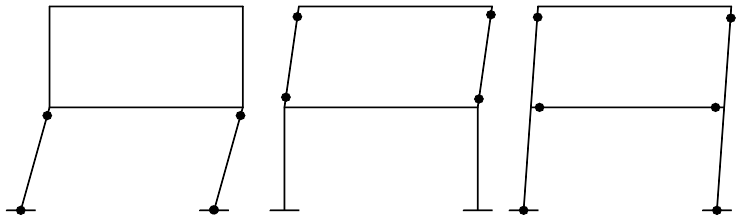


Fig. 1 Two Storey Single Bay Frame



1st Storey Local Collapse 2nd Storey Local Collapse Combined Mechanism

Fig. 2 - Failure Modes of the two-storey frame

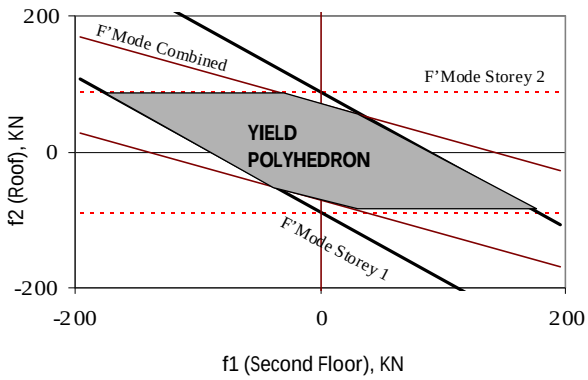


Fig.3 Global Yield Polyhedron in Original Restoring Force Space

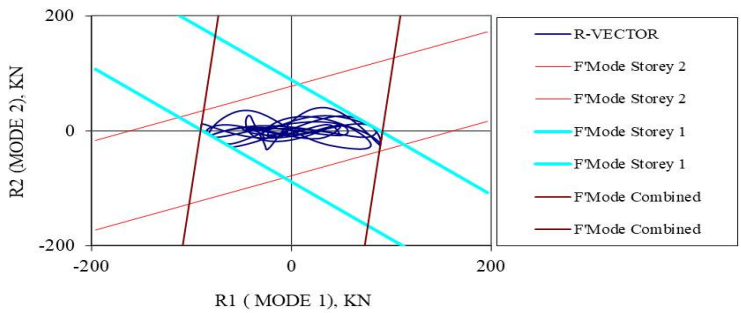


Fig. 4 Locus of Restoring Force in Modal Space

3. Random Pushover Approach

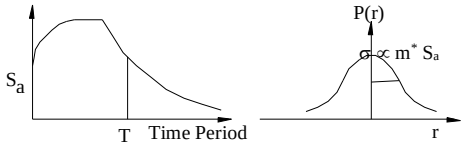
Random pushover approach [1] is proposed to choose important failure modes. It is based on FORM and a simple equivalent-static model of seismic load effects.

3.1 Equivalent random seismic loading model

An equivalent static model of seismic load effects as given by Eq. (3) is used. The {ψ} vector represents the lateral loading pattern and remains constant in the analysis. As the ground motion varies randomly in time, expected value of modal restoring forces is taken as zero and its standard deviation (σ_r^j) is proportional to effective modal mass ($m^{*(j)}$) times the spectral values (S_a^j) read from the acceleration response spectra. The vibration modes are considered independent.

$$E\left(r^j\right)=0\,,\quad E\left(r^i r^j\right)=0\quad (i\neq j)\,,\quad \sigma_r{}^j = c\, m^{*(j)} S_a^j \tag{5}$$

where, c is a constant and S_a^j is dependent on period and damping of the jth vibration mode (refer Fig. 5). The value of the constant in Eq. (5) is taken as one-third [5,6] for all the modes.



Acceleration response spectra

Definition of standard deviation

Fig. 5.Random Loading Model

3.2 Reduction of failure modes

Stochastic limit analysis is applied for identification of important failure modes. It is assumed that the loads and resistances are independent of each other. The performance function for a failure mode can be written as:

$$g = g(X) = \sum_{i=1}^{m} M_{pi}\left|\theta_{pi}\right| - \sum_{j=1}^{n} a_j r^j = g_0 - \sum_{j=1}^{n} a_j r^j \tag{6}$$

where, m and n are number of element resistance (moment) capacity and number of vibration modes respectively. M_{pi} is the basic variable of element moment capacity, $\left|\theta_{pi}\right|$ represents the rotation of plastic hinge and a_j is the coefficient of the jth modal restoring force r^j for the failure mode considered. Mean value (μ_g) and standard deviation (σ_g) are given as:

$$\mu_g = \sum_{i=1}^{m} \bar{M}_{pi}\left|\theta_{pi}\right| - \sum_{j=1}^{n} \bar{r}^j a_j \,, \quad \sigma_g = \sqrt{\sum_{i=1}^{m} \sigma_{mpi}^2 \theta_{pi}^2 + \sum_{j=1}^{n} \left(\sigma_r^j\right)^2 \left(a_j\right)^2} \tag{7}$$

where, $\bar{M}_{pi}, \bar{r}^j$ are mean values and σ_{mpi}, σ_r^j are standard deviations of load and resistance capacity, respectively. Eq. (7) represents the uncertainties in load effects and resistance capacity. Reliability index (β) is given as μ_g/σ_g.

A lower β indicates higher probability of failure. In our view, failure modes with probability less than about 10% of the most likely (β_{min}) failure mode can be excluded from further analysis. Alternatively, failure modes with $\beta \geq \beta_{min} + \Delta$ can be neglected.

Mutual correlation between the failure modes is considered by PNET [7]. In this approach, the failure modes with correlation coefficients more than a demarcating correlation ρ_0 are unified. The maximum number of failure modes for the dynamic analysis should be limited to the number of vibration degrees of freedom. Using these failure modes, global yield polyhedron is constructed and dynamic analysis is performed in this region.

4. Description of Six Storey Frame Model

A six-storey two-bay frame [8] is taken to study the application of the proposed concepts. The storey height is 3.75m and the span of each bay is 6m. The frame is shown in Fig. 6 and the member properties and floor weights are shown in Table 1 and 2, respectively. The modulus of elasticity and yield stress for steel is taken as 205GPa and 235Mpa, respectively. A constant modal-damping ratio of 2% is considered for all the vibration modes. The vibration periods are shown in Table 3. The collapse modes under horizontal loads have been enumerated and are shown in Fig. 7. These include local storey collapse mechanisms and combined mechanisms. Locations of the plastic hinges are assumed at the top and bottom ends of the columns, which have undergone side sway, and at the floor beams end connected with the side columns. At the junction of the floor beams and the central column, plastic hinges are assumed in the beams or in the columns, whichever is having the least summation of the plastic moment capacities at the junction.

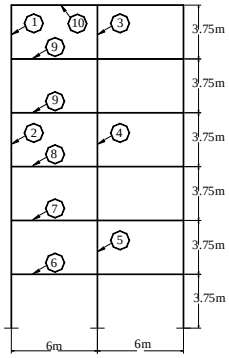


Fig. 6-Six-storey two-bay frame

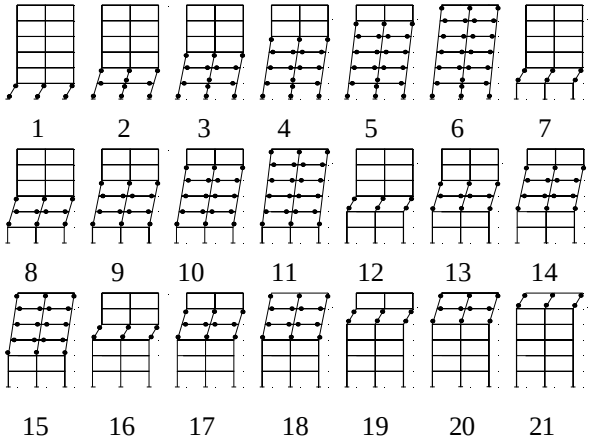


Fig. 7 -. Collapse modes for six-storey frame under lateral loading

Table 1 - Member properties

Mem-ber ID	Moment of Inertia (10^{-8} m^4)	Plastic Moment Capacity (KN. m)
1	2492	83.19
2	8091	194.35
3	5696	151.11
4	11260	247.46
5	14920	301.51
6	23130	307.15
7	16270	239.47
8	11770	188.94
9	8356	147.58
10	3892	86.25

Table 2 - Floor weights

Floor	Weight (KN)
Roof	380.4
6	589.2
5	589.2
4	589.2
3	589.2
2	589.2

Table 3-Vibration periods (sec)

Mode	Time Period
1	2.57
2	0.98
3	0.57
4	0.43
5	0.31
6	0.23

5. Randomness in Member Strength

In general, the response analysis is based on the deterministic (mean) member strengths. In reality, some randomness in member strengths exists due to manufacturing and construction processes with its magnitude depending upon the quality control exercised in such processes. In ref. [9], it is shown that the randomness in the member strengths has a predominant influence on the failure mechanisms and consequently on the system ductility. Such randomness can impair the realisation of the designed (targeted) overall collapse mechanism against earthquakes particularly if the member strengths are non-correlated with its effect becoming more pronounced as the number of the storeys in the frame increases. The assessment of dynamic performance [9] under earthquakes is based on the energy principles.

The effect of randomness in member strength on the dynamic response behaviour is studied by yield polyhedron model for the six-storey frame. The variation in the yield strength of ordinary mild steel is assumed to vary from 5% to 20% from the mean strength. For simplicity and conservatism, it is assumed that the member strength can take either the minimum value or the maximum value, with each having equal probability of occurrence. In order to study the effect of magnitude of randomness, three cases for the variation in plastic moment capacity are considered (refer Table 4).

Table 4 - Variations in Member Strengths for Studying Randomness

Sl. No.	Variation Coefficient	Min. Strength	Max. Strength	Member types	Analysis type
1.	±0.05	0.95 Mean	1.05 Mean	10	Dynamic
2.	±0.10	0.9 Mean	1.1 Mean	10	Dynamic
3.	±0.20	0.8 Mean	1.2 Mean	10	Dynamic

6. Analysis Code

The response of the frame for randomness in member strength is studied with dynamic analyses being performed for each category of variation. As there are 10 types of member cross-sections for the six-storey frame, a 10-digit code is used for the identification of the analysis parameters (refer Fig. 8).

XXXXXXXXXX

Property code For Member 1 Property code For Member 10

Fig. 8 Ten-digit analysis identification no.

The property code can be 1 or 2 denoting the minimum or maximum member strength, respectively.

7. Discussion of Results

The frame is analysed for first 10 sec duration of El Centro (NS), 1940 ground motion with peak ground acceleration as 0.325g.

The envelopes of the storey drifts and floor displacements are plotted in the Figs. 9 and 10, respectively. It can be seen that for COV=0.05, the demand envelope doesn't deviate significantly from the mean response. The deviation in the demand envelope is moderate for COV=0.10 and becomes severe for COV=0.2. In particular, the maximum value of fifth and first storey drifts for COV=0.2 is more than two times the respective values for the mean response.

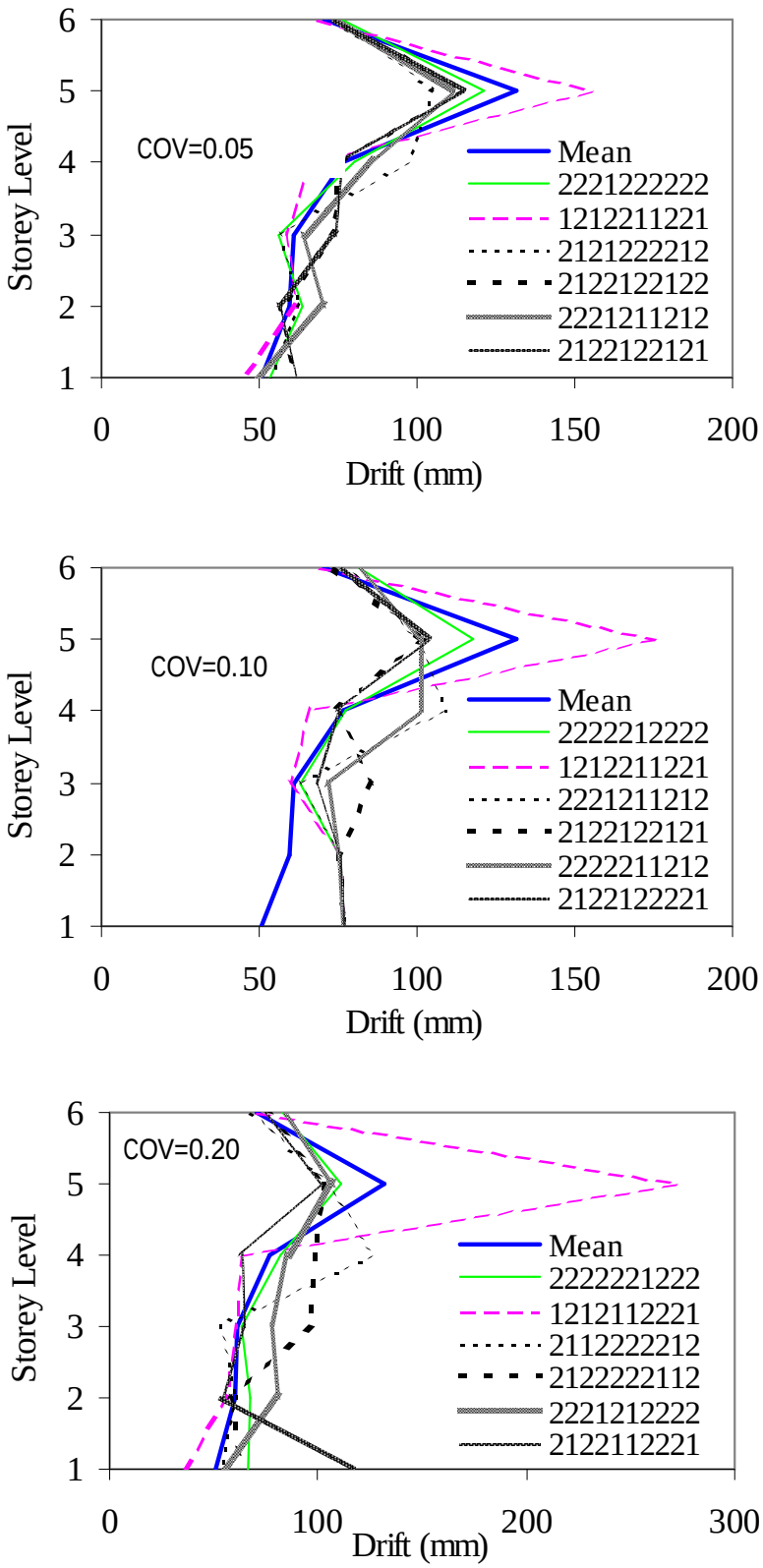


Fig. 9 Envelope of Storey Drifts Considering Randomness in Member Strengths (Dynamic analyses performed in each category of COV)

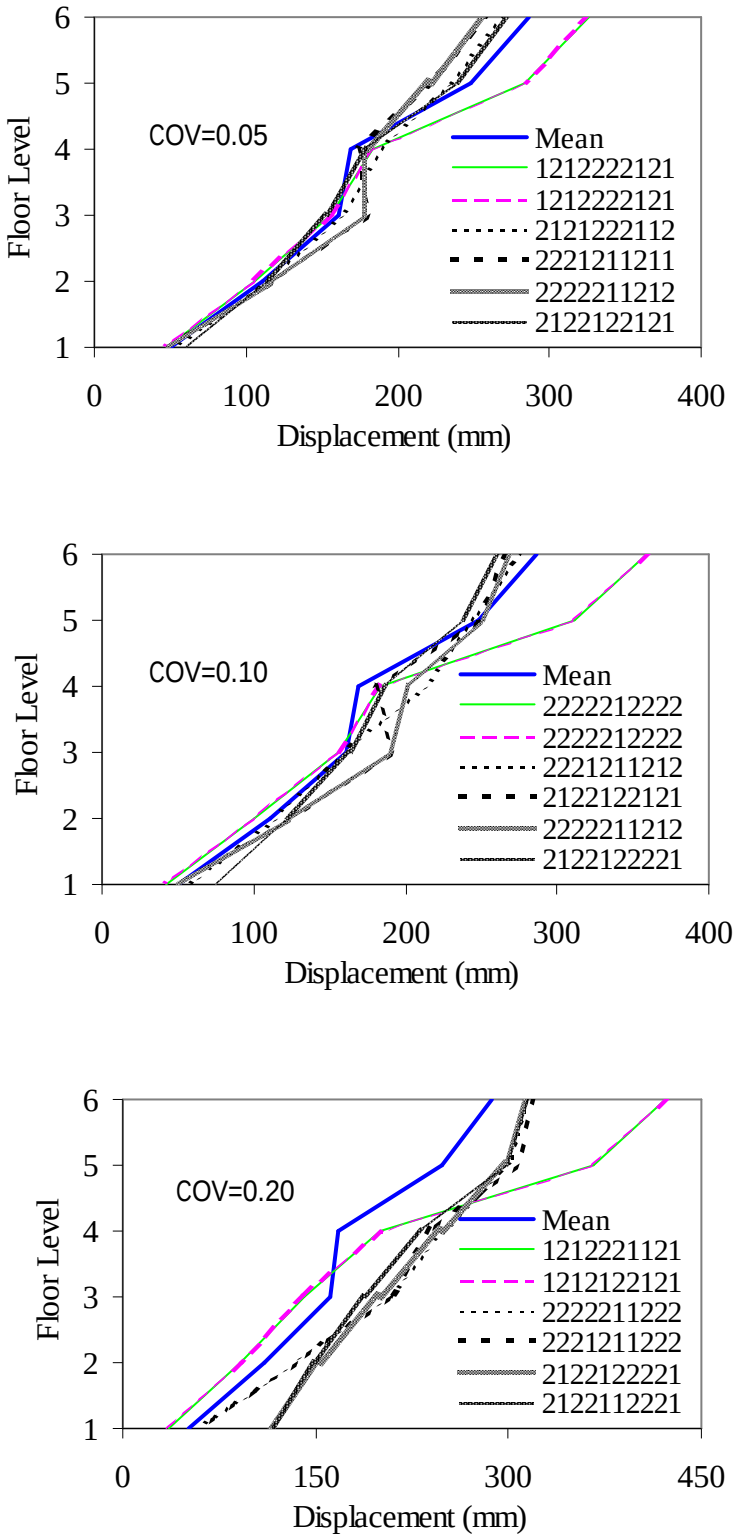


Fig. 10 Envelope of Floor Displacements Considering Randomness in Member Strengths (Dynamic analyses performed in each category of COV)

8. Conclusions

It is seen that demand envelope is sensitive to higher order randomness in member strengths, which alters the dynamic response significantly from the mean strength response.

It is pertinent to note that for considering various possible combinations of member strengths for columns and beams, more than 3000 inelastic time history analyses were performed in a very short time. Performing similar order studies using conventional analytical models may require considerable engineering resources and such studies may not be performed (or viewed unfavourably) from techno-economic considerations. Application of global yield polyhedron model offers considerable flexibility to the designer as he can quickly perform several alternatives within the constraints of time and limited resources.

9. References

[1] Khandelwal P, Ohi K, Fang P (2001): A Simple Proposal for Ultimate Seismic Demand Evaluation of Moment Resisting Steel Frames, *Journal of Structural and Construction Engineering*, Architectural Institute of Japan, No. 545 (July).

[2] Ohi K (1991): Stochastic Limit Analysis of Framed Structures, *Proc. of JCOSSAR' 91*, pp. 675-678, JSCE (In Japanese).

[3] Sun H, Ohi K, Takanashi K (1997): A Proposal and Verification about Plastic Design Process with Target Collapse Mechanism for Structures, *Proc. of 7th ICOSSAR*.

[4] Sun H (1997): *A study on Stochastic Limit Analysis for Ultimate Limit State Design of Steel Frames*, Ph.D. Thesis, The University of Tokyo, Japan.

[5] Davenport AG (1964): Note on the Distribution of the Largest Value of a Random Function with Application to Gust Loading, *Proceedings of the Institution of Civil Engineers*, Vol. 28, pp. 187-196.

[6] Shibata A (1993): *New Earthquake Resistant Structural Analysis*, Morikita Press Co. Ltd.,pp. 188-191 (In Japanese).

[7] Ang AHS, Tang WH (1984): *Probability Concepts in Engineering Planning and Design Volume II Decision , Risk and Reliability*, John Wiley & Sons, pp. 504-506.

[8] Chen WF, Toma S (1994). *Advanced Analysis of Steel Frames*, CRC Press, pp. 341-343

[9] Kuwamura H, Sakai S, Kato B (1989), Mechanism Based Seismic Design Considering Random Yield Strength, *Journal of Structural and Construction Engineering*, Architectural Institute of Japan, No. 401, pp.151-162.

