



## EVALUATION OF ACCIDENTAL TORSION OF SEISMICALLY-RECORDED BUILDING STRUCTURES USING THE TOTAL RESISTANCE ECCENTRICITY

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### Abstract

The accidental torsion is intended to account for building torsion arising from the discrepancies between the idealized distribution of the mass, stiffness, and strength in analysis and the true distributions at the time of an earthquake; torsional vibrations induced by a rotational motion of the building base; and other sources of torsion not considered explicitly in analysis. Seismic codes require to take into consideration this accidental torsion by following one of two design approaches: (1) the use of accidental eccentricity,  $\eta_{static} = \beta = e_a/b$ , in an equivalent lateral force (static) procedure, (2) the use of dynamic analysis, where the center of mass (CM) at each story is shifted from its idealized location in each direction by a distance equal to  $e_a = \beta b$  with the code-specified value of  $\beta$  being 5%.

Since the primary interest in torsion design is to control the maximum edge-frame drift and the accidental torsional moment in the static procedure is defined with the total resistance eccentricity,  $\eta_y = T_{total}/V_x$  (rather than the resistance eccentricity  $e_y = T_x/V_x$ ), the values of  $\eta_y$  at the instant of the maximum edge-frame drift are used to determine the accidental eccentricity such that; the true accidental eccentricity,  $\eta_{true}$ , is the difference between the value of  $\eta_{rec}$  from the recorded motions and  $\eta_{CM0}$  calculated from the dynamic analysis with the CM at the idealized location (CM0), while the values of the dynamic total accidental eccentricity,  $\eta_{dyn}$ , is determined as the difference between the value of  $\eta_{CM0}$  and the maximum value among  $\eta_{CM1}$ ,  $\eta_{CM2}$ ,  $\eta_{CM3}$ , and  $\eta_{CM4}$  which are obtained by dynamic analyses with shifted locations of the CM, while the value of  $\eta_{ellip}$  is obtained from the proposed ellipsoidal bounding method.

The purpose of this study is to identify the true accidental torsion by examining the torsional behaviors of five building structures recorded in California Strong Motion Instrumentation Program (CSMIP) and to evaluate the amount of the dynamic accidental torsion caused by the above dynamic procedure, and then, to propose an ellipsoidal bounding method to estimate the maximum edge-frame drift and the corresponding design accidental torsion which bounds at least all the maximum edge-frame drifts in dynamic analyses with shifted CMs.

Conclusions are as follows: (a) the values of the maximum edge-frame drifts obtained from the recorded data, those calculated by dynamic analyses with different locations of CM (CM0, CM1, CM2, CM3, and CM4), and those predicted by the ellipsoidal bounding method proposed in this study, appear to be overall similar and (b), however, the range of dynamic accidental eccentricity,  $\eta_{dyn}$ , is 9.11%~27.0% for torsionally regular structures and 18.0%~67.3% for torsionally irregular structures, whereas that of the true accidental eccentricity,  $\eta_{true}$ , is 0.30%~5.00% for torsionally regular structures and 0.45%~7.43% for torsionally irregular structures, with the code-specified static accidental eccentricity,  $\eta_{static} = \beta$ , being 5%. And, (c) the ellipsoidal bounding method provides the range of  $\eta_{ellip}$  is 1.20%~16.0% for torsionally regular structures and 8.47%~32.0% for torsionally irregular structures with the values of the maximum edge-frame drifts bounding reasonably those from dynamic analyses.

*Keywords: Accidental torsion, resistance eccentricity, accidental eccentricity*



## 1. Introduction

Building structures with irregularities are more vulnerable to seismic damage during strong earthquakes. In such buildings, the torsional behavior is one of the most frequent sources of structural damages and failures because the demanded strength and inter-story drift at certain parts of the structure increase due to torsion beyond those required when translational deformation occurs alone. Despite the fact that severe damages or failures of building structures in inelastic torsional responses are of major interest to engineers and researchers, regulations provided by current codes focus only on the elastic behavior.

The accidental torsion is intended to account for building torsion arising from the discrepancies between the idealized distribution of the mass, stiffness, and strength in analysis and the true distributions at the time of an earthquake; torsional vibrations induced by a rotational motion of the building base; and other sources of torsion not considered explicitly in analysis.

ASCE 7-10 [2] specifies two elastic torsion design approaches: One uses an equivalent force (static) procedure as shown in Figure 1(a),

*“Where diaphragms are not flexible, the design shall include the inherent torsional moment resulting from eccentricity between the locations of the center of mass (CM) and the center of rigidity (CR) plus the accidental torsional moments caused by assumed displacement of the center of mass each way from its actual location by a distance equal to 5% of the dimension of the structure perpendicular to the direction of the applied forces. ... Where earthquake forces are applied concurrently in two orthogonal directions, the required 5% displacement of the center of mass need not be applied in both of the orthogonal directions at the same time but shall be applied in the direction that produces the greater effect.”*



Figure 1. Conventional torsion design approaches: (a) Equivalent lateral force (static) analysis and (b) Dynamic analysis

Another uses dynamic analysis in Figure 1(b) such as modal response spectrum analysis or time history analysis, where the CM at each story is shifted from its idealized location in each direction by a distance equal to the accidental eccentricity ( $e_a$ ). The most unfavorable results in terms of member deformations and forces of the structure from dynamic analyses of four positions of the CM in each floor are used for the design. Although ASCE 7-10 [2010] does not explicitly provide any equation for static design torsion moment, the situation of static torsion is described by Figure 2(a). The static design eccentricity,  $e_d$ , is defined with Equations (1) and (2) [2]:

$$e_d = \alpha e_s + \beta b \quad (1)$$

$$e_d = \delta e_s - \beta b \quad (2)$$

where  $e_s$  is the static eccentricity representing the distance between the center of mass (CM) and the center of stiffness or rigidity (CS or CR);  $\beta b$  is the accidental eccentricity ( $e_a$ ) which is included in order to consider torsional effects caused by uncertainties of the CM and the CS, the rotational component of ground motion, and other uncertainties that are not particularly considered;  $b$  is the plan dimension of the building perpendicular to the direction of ground motion; and  $\alpha$ ,  $\beta$ , and  $\delta$  are code-specified coefficients whose values vary among building codes:  $\alpha = \delta = 1$  and  $\beta = 0.05$  in the ASCE 7-10;  $\alpha = 1.5$ ,  $\delta = 1$ , and  $\beta = 0.1$  in the Mexico Federal District Code; and  $\alpha = 1.5$ ,  $\delta = 0.5$ , and  $\beta = 0.1$  in the National Building Code of Canada [3].



The code-specified value of the static accidental eccentricity,  $\eta_{static}=\beta$  ( $=e_a/b$ ), in Equations (1) and (2) for the static analysis (Figure 1(a)) may be different from the value of accidental eccentricity obtained from the dynamic analyses in Figure 1(b). In this study the code-specified value of  $\beta$  is compared to the values of  $\eta$  computed based on both recorded motions and dynamic analytical results for five different structures.

The resistance eccentricity ( $e_y$ ) is defined as the distance between the center of mass and the center of resistance as shown in Figure 2(b), which is conceptually different from the code static design eccentricity,  $e_d$ , defined as the arm length about the center of rotation as shown in Figure 2(a).

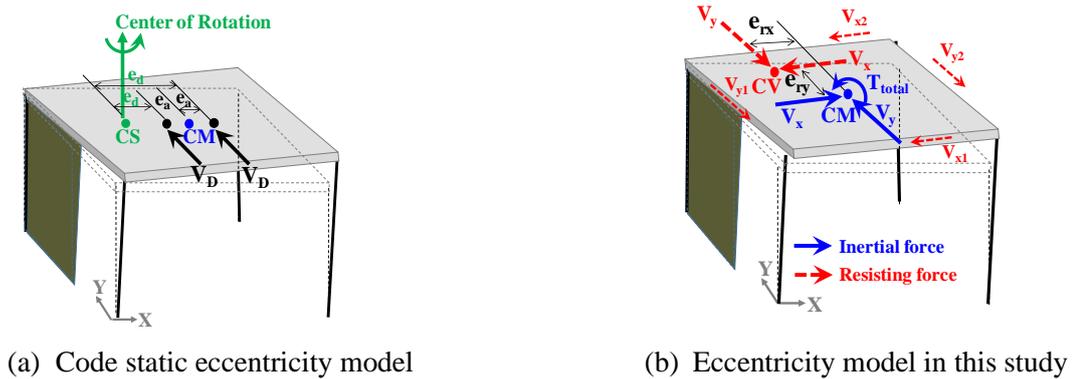


Figure 2. Eccentricity models

The resistance eccentricity model in Figure 2(b) takes advantages of (1) the recognition of the existence of  $T_{total}$  at the CM, (2) the avoidance of the confusion by using  $e_y$  instead of  $e_d$ , and (3) a clear relationship of applied inertial forces at the CM and resisting forces as follows.

$$T_{total} = T_x + T_y = \sum V_{xi}d_{yi} + \sum V_{yi}d_{xi} = e_y V_x + e_x V_y \quad (3)$$

$$e_y = T_x/V_x, e_x = T_y/V_y, \eta_y = T_{total}/V_x \quad (4)$$

where  $i$  represents frame numbers in X and Y directions, respectively;  $T_{total}$  is the sum of torsional moments resisted to by X-directional ( $T_x$ ) and Y-directional ( $T_y$ ) frames; and  $d_{xi}$  and  $d_{yi}$  are distances of the  $i$ -th frame in X and Y directions, respectively, from the CM. Using this resistance eccentricity model, general relationships of forces in a one-story building under earthquake can be described with inertial forces ( $V_x$ ,  $V_y$ ,  $T_{total}$ ) at the CM and resisting resultant shear forces located at the **resistance eccentricities**,  $e_x$  and  $e_y$ , and the **total resistance eccentricity**,  $\eta_y$ , as shown in Equations (3) and (4).

Since the primary interest in torsion design is to control the maximum edge-frame drift and the accidental torsional moment in the static procedure is defined with the total resistance eccentricity,  $\eta_y = T_{total}/V_x$  (rather than the resistance eccentricity  $e_y = T_x/V_x$ ), the values of  $\eta_y$  at the instant of the maximum edge-frame drift are used to determine the accidental eccentricity such that; the true accidental eccentricity,  $\eta_{true}$ , is the difference between the value of  $\eta_{rec}$  from the recorded motions and  $\eta_{CM0}$  calculated from the dynamic analysis with the CM at the idealized location (CM0), while the values of the dynamic total accidental eccentricity,  $\eta_{dyn}$ , is determined as the difference between the value of  $\eta_{CM0}$  and the maximum value among  $\eta_{CM1}$ ,  $\eta_{CM2}$ ,  $\eta_{CM3}$ , and  $\eta_{CM4}$  which are obtained by dynamic analyses with shifted locations of the CM.

The purpose of this study is to identify the true accidental torsion by examining the torsional behaviors of five building structures recorded in California Strong Motion Instrumentation Program (CSMIP) and to evaluate the amount of the dynamic accidental torsion caused by the above dynamic procedure, and then, to propose an ellipsoidal bounding method to estimate the maximum edge-frame drift and the corresponding design accidental torsion which bounds at least all the maximum edge-frame drifts in dynamic analyses with shifted CMs.



## 2. Description of building structures

The first structure identified as CSMIP station No. 58506, is located in Richmond, California, at 111km from the epicenter of the 18 Oct. 1989, Loma Prieta earthquake. The building has a nominally symmetric floor with the area of 1,230m<sup>2</sup>. The total height of the building is 14.1m, and has 3 stories of the story height 4.75m (first through second) and 4.57m(third). A typical plan of the structure is shown in [Figure 3\(a\)](#). The lateral force resisting system in the building consists of perimeter moment-resisting steel frames, on the other hand, the vertical load-carrying system consists of concrete over steel deck supported by steel frames. The foundation system consists of concrete pile caps at each column with grade beams.

The second structure identified as CSMIP station No. 57562, is located in San Jose, California, at 20.0km from the epicenter of the 18 Oct. 1989, Loma Prieta earthquake. The building has a nominally symmetric floor area of 1,610m<sup>2</sup>. The total height of the building is 15.1m, and has 3 stories of the story height 5.49m(first) and 4.80m (second through third). A typical plan of the structure is shown in [Figure 3\(b\)](#). The lateral force resisting system of the building consists of moment-resisting steel frames with exterior aluminum cladding on the sides, on the other hand, the vertical load-carrying system consists of concrete over steel deck supported by steel frames. The foundation system consists of rectangular column footings interconnected by grade beams. The building considered is one of four similar wings, around a central building. Each wing is isolated from the central building by a separation joint and in principle, there is no structural interaction between the wings and the central building.

The third structure identified as CSMIP station No. 24370, is located in Burbank, California, at 22.0km from the epicenter of the 17 Jan. 1994, Northridge earthquake. The steel moment resistant frame structure was designed in 1976 and constructed in 1977. The building has a nominally symmetric floor area of 1,340m<sup>2</sup>. The total height of the building is 25.1m, and has 6 stories of story height 5.33m(first) and 3.96m (second through sixth). A typical plan of the structure is shown in [Figure 3\(c\)](#). The lateral force resisting system of building consists of moment-resisting steel frames on perimeter walls, and on the other hand, the vertical load-carrying system consists of steel beams and columns 82.6mm concrete slab over metal deck. The foundation system consists of isolated footings interconnected by a 533 by 533mm grade beam along the building perimeter and a 102mm slab on grade.

The fourth structure identified as CSMIP station No. 24385, is located in Burbank, California, at 21.0km from the epicenter of the 17 Jan. 1994, Northridge earthquake. The reinforced concrete shear wall residential building was designed and constructed in 1977 in accordance with Uniform Building Code (UBC). The building has a nominally symmetric floor area of 1,520m<sup>2</sup>. The total height of the building is 26.8m, and has 10 stories of story height 3.05m (first) and 2.64m (second through tenth). A typical plan of the structure is shown in [Figure 3\(d\)](#). The lateral force resisting system of building consists of precast concrete shear walls in both x- and y-directions, and on the other hand, the vertical load-carrying system consists of precast and poured in place concrete floor slabs supported by precast concrete bearing walls. The foundation system consists of continuous pile caps 1,070mm deep supported on 202 normal weight concrete piles, 610mm in diameter, and length that varies from 7,620 to 10,700mm.

The fifth and the last structure identified as CSMIP station No. 24601, is located in Los Angeles, California, at 30.0km from the epicenter of the 17 Jan. 1994, Northridge earthquake. The reinforced concrete shear wall residential building was designed and constructed in 1980. The building has an asymmetric floor area of 2,070m<sup>2</sup>. The total height of the building is 45.6m, and has 17 stories of story height 3.35m (first) and 2.64m (second through 17th). A typical plan of the structure is shown in [Figure 3\(e\)](#). The lateral force resisting system of building consists of distributed precast concrete wall panels acting as shear walls. In the transverse direction, 254mm thick partition walls between apartment units act as shear walls. In the longitudinal direction, corridor walls (203mm and 305mm thick) act as shear walls. The vertical load-carrying the system consists of precast, pre-tensioned concrete slabs, 203mm or 102mm (with 102mm topping) thick, supported by precast concrete walls (203mm-254mm thick). The foundation system consists of concrete drilled piles (610mm diameter, 13.4m-16.5m long) supporting grade beams (depth: 1,220mm



transverse, 914mm longitudinal) under the walls. 305mm square concrete tie beams on the perimeter and 102mm slab on grade.

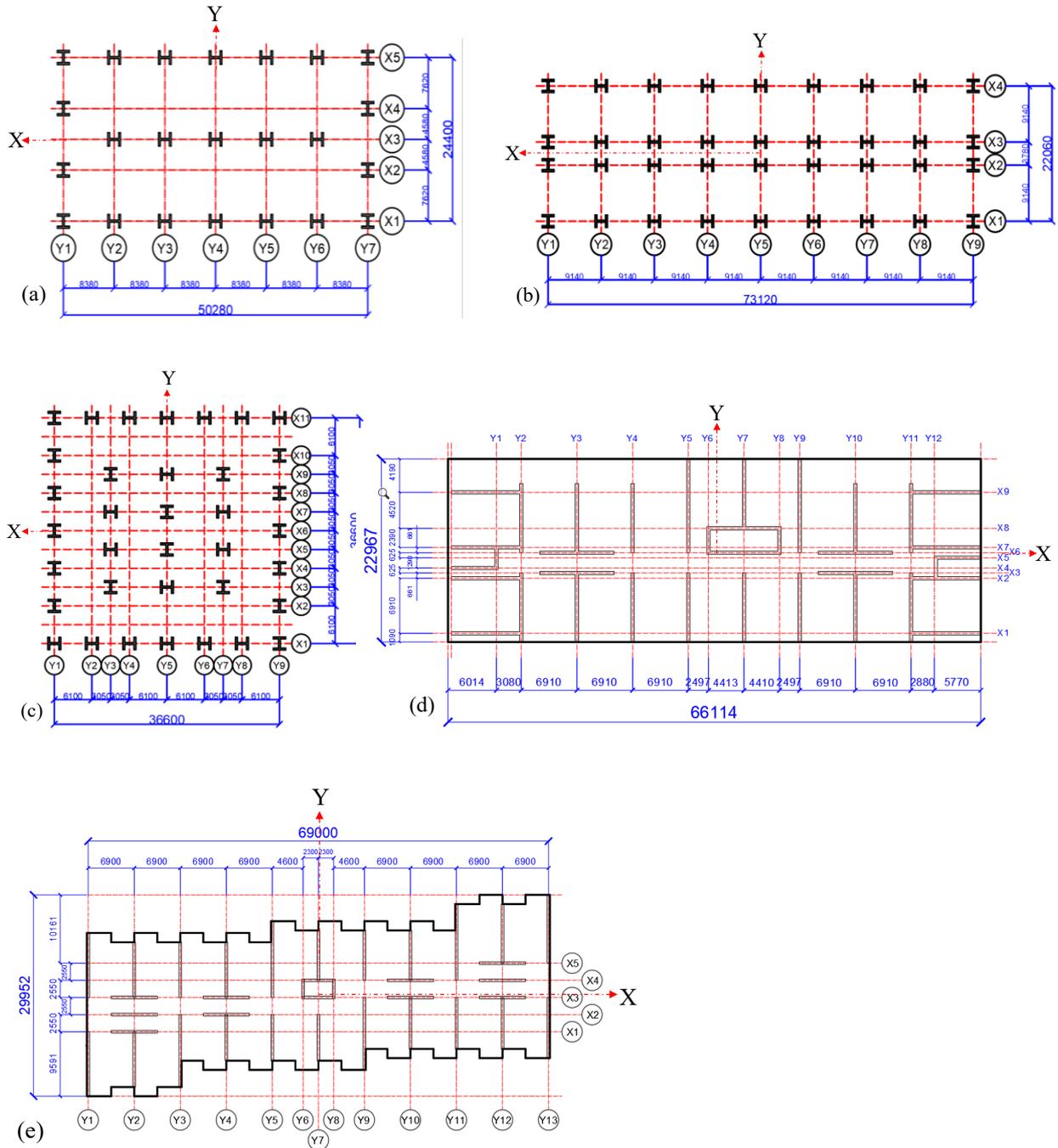


Figure 3. Plan at the ground floor for: (a) Richmond 3-story, (b) San Jose 3-story, (c) Burbank 6-story, (d) Burbank 10-story and (v) LA 17-story. (unit: mm).



### 3. Comparisons of the recorded motions and dynamic analyses results with CM0

Many dynamic analyses, with the CM located at the idealized position CM0, were conducted to best simulate the recorded motions of the five-building models by using ETABS and PERFORM-3D, specifically with respect to story shear force and story drift, wherein the floor diaphragms are assumed to be rigid, and all the buildings are treated as fixed at the base. A 20,000 MPa of Young's Modulus and 0.3 of Poisson's Ratio are used for the standard concrete and steel sections of both beams and columns.

Table 1 – Mode shape and period from modal analyses

		CSMIP No.58506	CSMIP No. 57562	CSMIP No. 24370	CSMIP No. 24385	CSMIP No. 24601
1 <sup>st</sup> - mode	Period (s)	0.694	0.689	1.407	0.711	1.164
	movement	Y-dir. translation	Y-dir. translation	X-dir. translation	torsional	X-dir. translation
2 <sup>nd</sup> - mode	Period (s)	0.615	0.667	1.383	0.665	1.066
	movement	X-dir. translation	X-dir. translation	Y-dir. translation	X-dir. translation	torsional
3 <sup>rd</sup> - mode	Period (s)	0.449	0.567	0.904	0.576	1.025
	movement	torsional	torsional	torsional	Y-dir. translation	Y-dir. translation

The best analytical results are chosen and the values of the modal properties from those analytical results and those from the recorded are compared in Table 1. The hysteretic seismic responses from the recorded and analytical results are compared in Figure 4, where most of the analytical results simulate the recorded results very well especially the shear forces in the X and Y direction.

For Richmond 3-story structure, the drift in the X-direction and the torsional moment from the recorded motions are very large compared to those of the analytical results in Figure 4-i. Similarly, for the LA 17-story structure in Figure 4-v, the drift in the X- and Y-direction and the torsional moments from the recorded motions are significantly larger than those of the analytical results. The reason for these discrepancies is thought to be the inaccuracy of the recorded data. The hysteretic curves of torsional moment and deformation from analytical results are far from those of the recorded results in the Burbank 6-story structure in Figure 4-iii(c). The points of the maximum edge-frame drift ( $\delta_{edge}$ ) with the corresponding drift at the center ( $\delta_{cent}$ ), rotational deformation ( $\theta_i$ ) and torsional moments ( $T_{total}$ ) are denoted blue solid circles in X-direction with red solid circles in the Y-direction. Although analytical results simulate the recorded reasonably well, it can be noted for the cases of Richmond 3-story and LA 17-story structures that large discrepancy exists between the analytical and the recorded because of the inherent inelastic behaviors in the recorded motions as noted in Figures 4-i and v. The authors cannot identify the reason of inelastic behavior in the recorded motions to date.

The time histories of edge-frame drifts in the X-direction and Y-direction from the recorded motion and dynamic analysis with the shifted CM (CM1) are given for the San Jose 3-story and the Burbank 6-story structures in Figures 5-i and 5-ii, respectively. The diamond and square markers represent the peak edge-frame drifts at the ground story only when exceeding one-half of the maximum positive and negative drifts denoted with solid diamond and solid rectangle for the two edge frames.

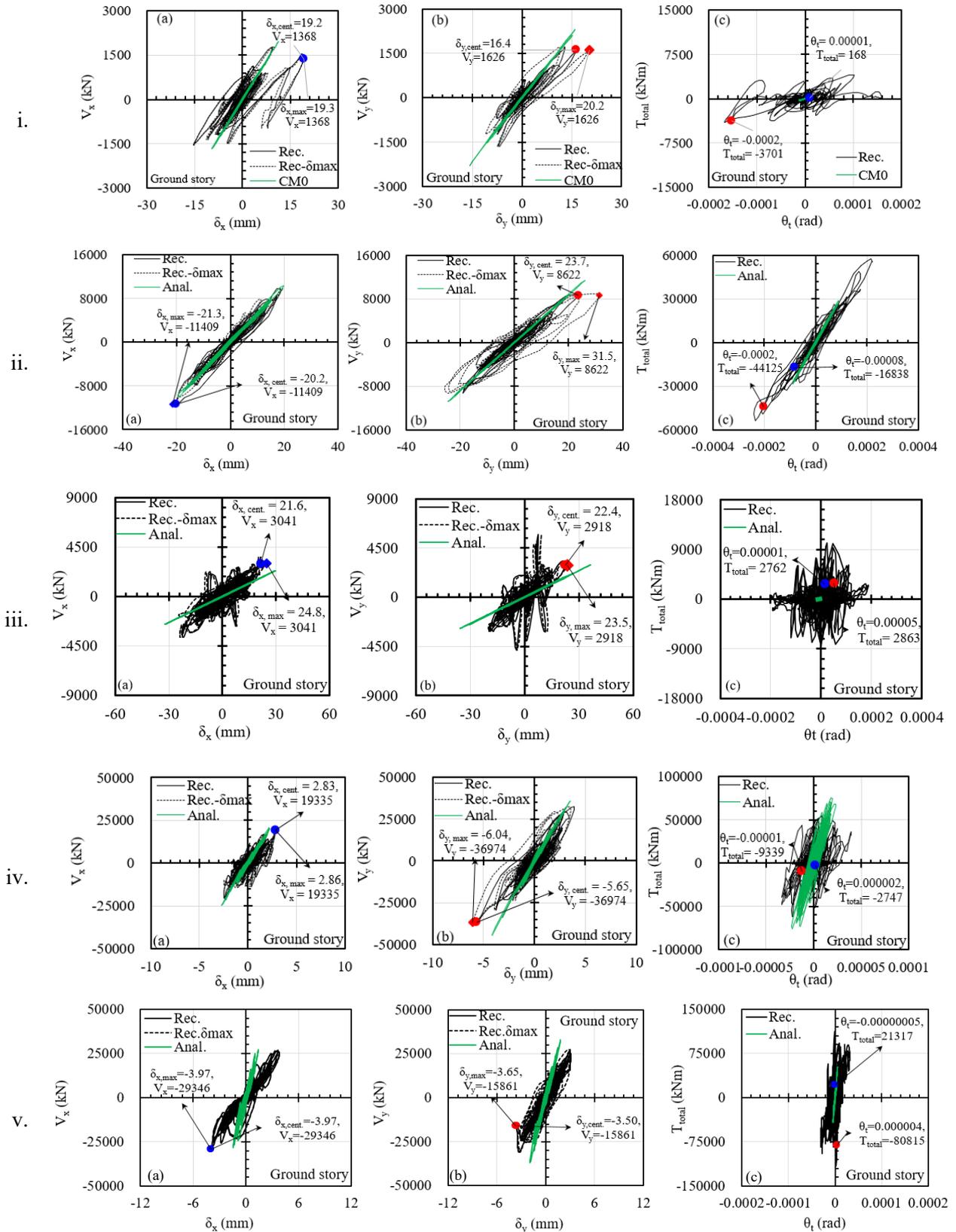
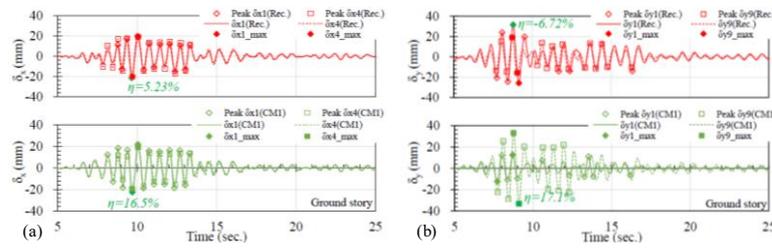


Figure 4. Hysteretic relations between (a)  $V_x$ - $\delta_x$ , (b)  $V_y$ - $\delta_y$  and (c)  $T_{total}$ - $\theta_t$ , for: i) Richmond 3-story, ii) San Jose 3-story, iii) Burbank 6-story, iv) Burbank 10-story and v) LA 17-story.



i)



ii)

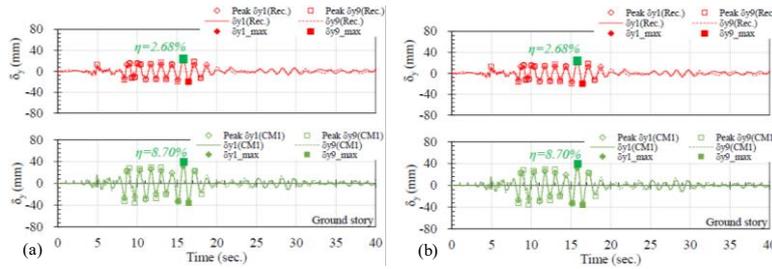


Figure 5. Time history of edge-frame drifts ( $\delta_{edge}$ ) in the (a) X-direction and (b) Y-direction for: i) San Jose 3-story and ii) Burbank 6-story

#### 4. Estimation of maximum $\delta_{edge}$ with $\eta_{ellip}$ by ellipsoidal bounding method

Lee and Hwang [4] have suggested that instead of using a specific value of eccentricity as a design parameter, the demand in torsion can be determined in the direct relationship with the base or story shear represented as an ellipse constructed with the maximum points in its principal axes located by two adjacent dominant modal spectral values. This approach can provide a simple but transparent conceptual design tool. In this study, the maximum edge-frame drift and the corresponding total resistance eccentricity are estimated using the ellipsoidal boundaries of forces and deformations relations.

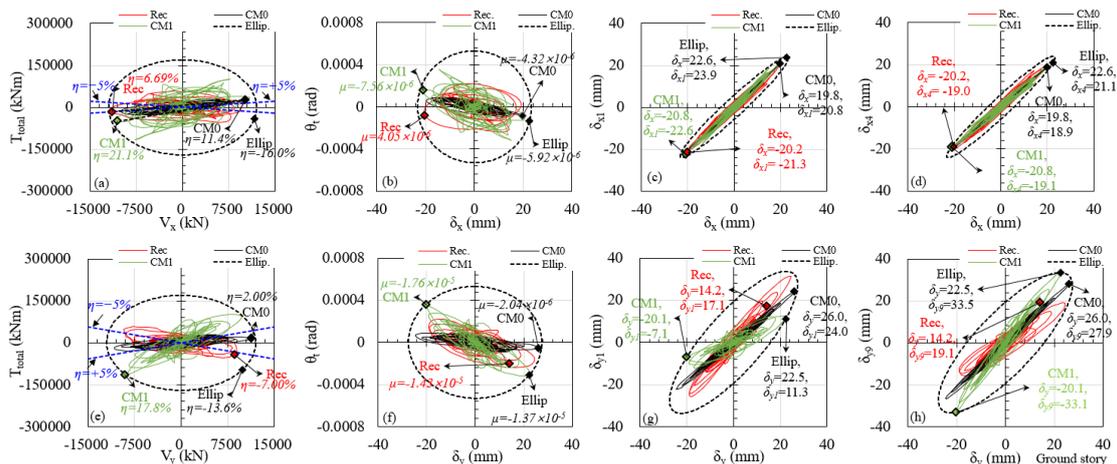


Figure 6. Elliptical bounding of (a)  $T_{total}-V_x$ ; (b)  $\theta_t-\delta_x$ ; (c)  $\delta_{x1}-\delta_x$ ; (d)  $\delta_{x4}-\delta_x$ ; (e)  $T_{total}-V_y$ ; (f)  $\theta_t-\delta_y$ ; (g)  $\delta_{y1}-\delta_y$ ; and (h)  $\delta_{y3}-\delta_y$  for San Jose 3-story.

Since the maximum values of recorded motions are exceptionally large in one building structure (Richmond 3-story), the elliptical boundaries are constructed to bound only the maximum responses of the dynamic analyses with different locations of CM (CM0~CM4). The response histories of  $T_{total}-V_x$ ,  $\theta_t-\delta_x$ ,  $\delta_{edge}-\delta_x$ ,  $T_{total}-V_y$ ,  $\theta_t-\delta_y$ ,  $\delta_{edge}-\delta_y$ , bounded by elliptical boundaries, are given in Figures 6 and 7 for the San Jose



3-story and Burbank 6-story structures, respectively. These figures overlap the response histories of recorded motions (red), dynamic analysis with the CM at the idealized location, CM0, (black), and dynamic analysis with the shifted CM, CM1, (green).

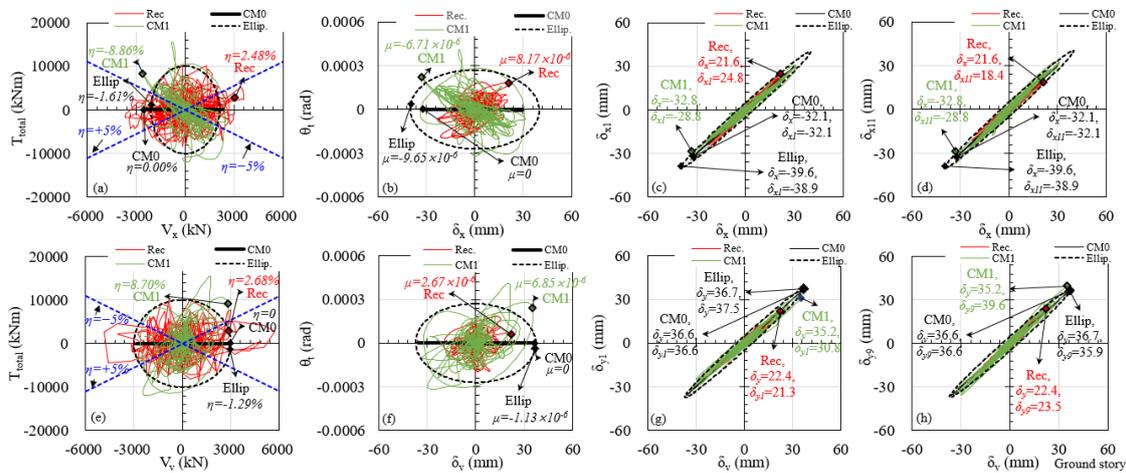


Figure 7. Elliptical bounding of (a)  $T_{total}$ - $V_x$ ; (b)  $\theta_t$ - $\delta_x$ ; (c)  $\delta_{x1}$ - $\delta_x$ ; (d)  $\delta_{x4}$ - $\delta_x$ ; (e)  $T_{total}$ - $V_y$ ; (f)  $\theta_t$ - $\delta_y$ ; (g)  $\delta_{y1}$ - $\delta_y$ ; and (h)  $\delta_{y9}$ - $\delta_y$  for Burbank 6-story.

For the San Jose 3-story structure, the elliptical boundaries of  $\delta_{x1}$ - $\delta_x$  and  $\delta_{x4}$ - $\delta_x$  in Figure 6 (c) and (d) have a large ratio of the major axis to the minor axis in the ellipsoid compared to those of  $\delta_{y1}$ - $\delta_y$  and  $\delta_{y9}$ - $\delta_y$  in Figure 6 (g) and (h). The reason is that the code degree of torsion irregularity in the Y-direction, 1.17, is larger than that of in the X-direction, 1.07, (Table 2). Likewise, for the Burbank 6-story structure, the elliptical boundaries of  $\delta_{x1}$ - $\delta_x$ ,  $\delta_{x4}$ - $\delta_x$ ,  $\delta_{y1}$ - $\delta_y$ , and  $\delta_{y9}$ - $\delta_y$  in Figures 7(c), (d), (g) and (h), respectively, have a very narrow shape because the code degree of torsion irregularity in both X and Y direction is very near to 1. From this, it can be found that the shape of the elliptical boundaries is affected by the degree of code torsion irregularity.

For San Jose 3-story and Burbank 6-story structures, the distributions of responses of  $\delta_{edge}$ ,  $\theta_t$ ,  $V_x$ ,  $V_y$ , and  $T_{total}$  at the instants of peak edge-frame drifts in Figure 5 are given with respect to  $\eta_y$  and  $\eta_x$  in Figures 8 and 9, respectively, where the dashed curves are derived from the elliptical boundaries in Figures 6 and 7. For San Jose 3-story structure, the dashed curves from the elliptical bounding method reasonably predict the maximum responses from the recorded motions and dynamic analyses with the shifted CMs in Figure 8. For Burbank 6-story structure, although all the maximum responses of  $\delta_x$ ,  $\delta_y$ ,  $\theta_t$ ,  $V_x$ ,  $V_y$ , and  $T_{total}$  are not included within the elliptical boundaries, the maximum edge-frame drifts appear to be almost same in Figures 7 and 9. For San Jose 3-story and Burbank 6-story structures, the trends of  $V_x$  and  $V_y$  at the peak points show bell shape whereas the trends of  $T_{total}$  reveal the shape of an hourglass with zero  $T_{total}$  as the neck in Figures 8 and 9. The solid circles in Figures 8 and 9 represent the maximum values of peak edge-frame drifts and the corresponding forces from the results of the recorded motions, those of the dynamic analyses with different locations of the CM and those of the elliptical bounding method.

The degree of torsional irregularity is checked as the ratio of the maximum edge-frame drift to the central drift in the excitation direction ( $\delta_{max}/\delta_{cent}$ ) when the structure is subjected to the design shear force applied at the location shifted by the accidental eccentricity. In Table 2, the values of degree of torsional irregularity in the X-direction and Y-direction are given for the five buildings, and it is interesting to note that all five buildings are torsionally regular in the X-direction with the values of  $\delta_{max}/\delta_{cent} < 1.2$  (limit of torsional irregularity given by the code), although the Burbank 10-story and L.A. 17-story structures are torsionally irregular in the Y-direction with the values of  $\delta_{max}/\delta_{cent}$  being greater than 1.2.

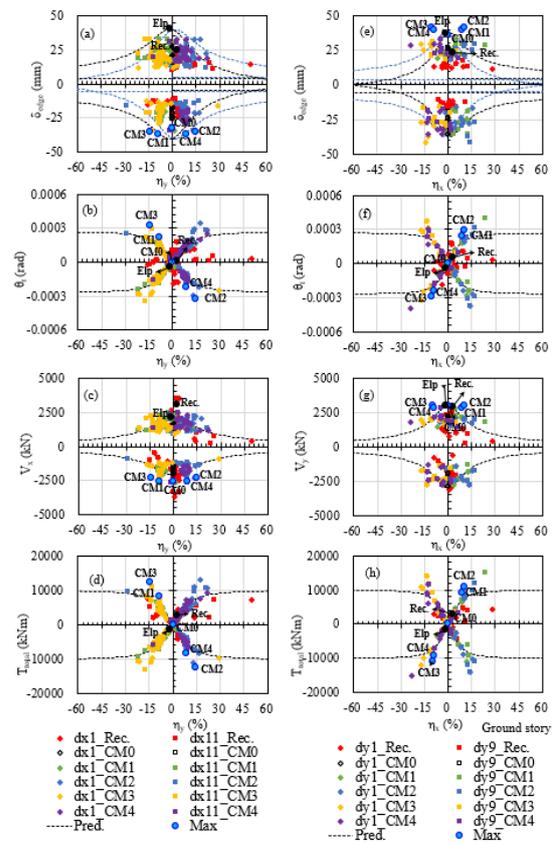
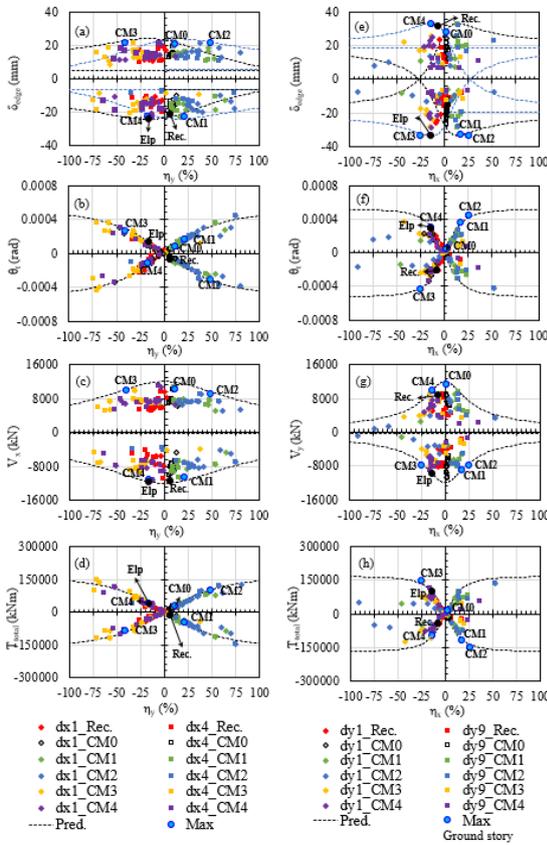


Figure 8. Distributions of critical responses of (a)  $\delta_{edge}-\eta_y$ ; (b)  $\theta_r-\eta_y$ ; (c)  $V_x-\eta_y$ ; (d)  $T_{total}-\eta_y$ ; (e)  $\delta_{edge}-\eta_x$ ; (f)  $\theta_r-\eta_x$ ; (g)  $V_y-\eta_x$ ; and (h)  $T_{total}-\eta_x$  at instants of peak  $\delta_{edge}$  compared to prediction equations for **San Jose 3-story**.

Figure 9. Distributions of critical responses of (a)  $\delta_{edge}-\eta_y$ ; (b)  $\theta_r-\eta_y$ ; (c)  $V_x-\eta_y$ ; (d)  $T_{total}-\eta_y$ ; (e)  $\delta_{edge}-\eta_x$ ; (f)  $\theta_r-\eta_x$ ; (g)  $V_y-\eta_x$ ; and (h)  $T_{total}-\eta_x$  at instants of peak  $\delta_{edge}$  compared to prediction equations for **Burbank 6-story**.

Table 2. Degree of torsional irregularity according to the code for the first story

$\delta_{max}/\delta_{cent}$	Richmond 3-Story	San Jose 3-Story	Burbank 6-Story	Burbank 10-Story	L.A. 17-Story
<b>X - dir.</b>	1.03	1.02	1.00	1.05	1.06
<b>Y - dir.</b>	1.09	1.17	1.00	1.45	1.28

The values of the maximum edge-frame drifts and the corresponding drift ratios ( $\delta_{max}/\delta_{cent}$ ) from the recorded motions, the elliptical bounding method and the dynamic analyses with the shifted CMs are compared for five building structures in Figure 10. Although the elliptical bounding method predicts in general reasonably the value of the maximum edge-frame drift obtained from the recorded motion and the dynamic analyses, it can be noted, however, for the cases of Richmond 3-story and LA 17-story structures that the values of the maximum edge-frame drifts in virtually inelastic hysteretic curves from the recorded motions are significantly larger than those from elastic predictions in Figures 4-i and 4-v.

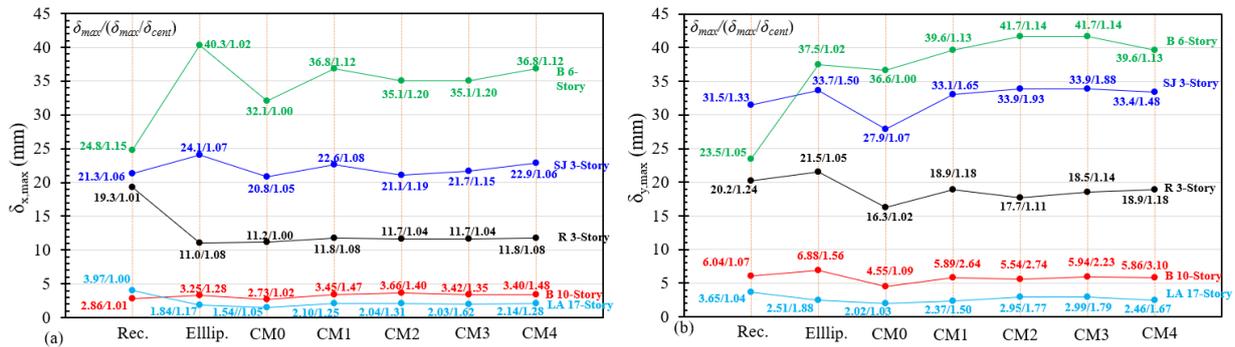


Figure 10. Maximum edge-frame drift ( $\delta_{max}$ ) and ratio of maximum edge-frame drift to central drift ( $\delta_{max}/\delta_{cent}$ ) at the 1st story of five buildings in the: (a) X-direction and (b) Y-direction

As it is mention earlier, the true accidental eccentricity,  $\eta_{true}$ , is defined by the difference between the total eccentricity from recorded response,  $\eta_{rec}$ , and that obtained from dynamic analyses with the CM at the idealized location,  $\eta_{CM0}$ , while dynamic accidental eccentricity,  $\eta_{dyn}$ , is determine as the difference between the value of  $\eta_{CM0}$  and the maximum value among  $\eta_{CM1}$ ,  $\eta_{CM2}$ ,  $\eta_{CM3}$ , and  $\eta_{CM4}$ , while the value of  $\eta_{ellip}$  is obtained from the proposed ellipsoidal bounding method. The values of  $\eta_{true}$ ,  $\eta_{dyn}$ , and  $\eta_{ellip}$  in the X-direction and Y-direction are given in Table 3 for five building structures. The range of dynamic accidental eccentricity,  $\eta_{dyn}$ , is 9.41%~27.0% for torsionally regular structures and 18.58%~67.3% for torsionally irregular structures, whereas that of the true accidental eccentricity,  $\eta_{true}$ , is 0.30%~9.00% for torsionally regular structures and 1.21%~7.43% for torsionally irregular structures. The ellipsoidal bounding method provides the range of  $\eta_{ellip}$  is 1.20%~16.3% for torsionally regular structures and 8.47%~32.0% for torsionally irregular structures.

Table 3.  $\eta$  (%) at the maximum value of edge-frame drift for the first story

Structures	$ \eta_{rec.} $	$ \eta_{CM0} $	$ \eta_{CM1-4} $	$\eta_{true}$ = $ \eta_{rec.}  \sim^*  \eta_{CM0} $	$\eta_{dyn}$ = $ \eta_{CM1-4}  \sim^*  \eta_{CM0} $	$ \eta_{ellip} $
<b>X-dir.</b>						
Richmond 3-Story	0.50	0.20	12.8	0.3	12.6	4.10
San Jose 3-Story	6.69	11.4	38.4	4.71	27.0	16.0
Burbank 6-Story	2.48	0.00	14.4	2.48	14.4	1.61
Burbank 10-Story	0.62	8.05	61.2	7.43	53.2	32.0
L.A. 17-Story	2.42	4.70	72.0	2.28	67.3	20.4
<b>Y-dir.</b>						
Richmond 3-Story	4.53	0.15	9.26	4.38	9.11	1.20
San Jose 3-Story	7.00	2.00	24.8	5.00	22.8	13.6
Burbank 6-Story	2.68	0.00	10.1	2.68	10.1	1.29
Burbank 10-Story	0.38	0.83	27.6	0.45	26.8	8.47
L.A. 17-Story	7.37	0.28	18.3	7.09	18.0	17.5

\*The difference between the recorded and the analytical (CM0)



## 5. Summary and Conclusions

### 5.1. Summary

The accidental torsion is intended to account for building torsion arising from the discrepancies between the idealized distribution of the mass, stiffness, and strength in analysis and the true distributions at the time of an earthquake; torsional vibrations induced by a rotational motion of the building base; and other sources of torsion not considered explicitly in analysis. Seismic codes require to take into consideration this accidental torsion by following one of two design approaches: (1) the use of accidental eccentricity,  $\eta_{static} = \beta = e_d/b$ , in an equivalent lateral force (static) procedure, (2) the use of dynamic analysis, where the center of mass (CM) at each story is shifted from its idealized location in each direction by a distance equal to  $e_a = \beta b$  with the code-specified value of  $\beta$  being 5%.

Since the primary interest in torsion design is to control the maximum edge-frame drift and the accidental torsional moment in the static procedure is defined with the total resistance eccentricity,  $\eta_y = T_{total}/V_x$  (rather than the resistance eccentricity  $e_y = T_x/V_x$ ), the values of  $\eta_y$  at the instant of the maximum edge-frame drift are used to determine the accidental eccentricity such that; the true accidental eccentricity,  $\eta_{true}$ , is the difference between the value of  $\eta_{rec}$  from the recorded motions and that of  $\eta_{CM0}$  calculated from the dynamic analysis with the CM at the idealized location (CM0), while the values of the dynamic total accidental eccentricity,  $\eta_{dyn}$ , is determined as the difference between the value of  $\eta_{CM0}$  and the maximum value among  $\eta_{CM1}$ ,  $\eta_{CM2}$ ,  $\eta_{CM3}$ , and  $\eta_{CM4}$  which are obtained by dynamic analyses with shifted locations of the CM, while the value of  $\eta_{ellip}$  is obtained from the proposed ellipsoidal bounding method.

The purpose of this study is to identify the true accidental torsion by examining the torsional behaviors of five building structures recorded in California Strong Motion Instrumentation Program (CSMIP) and to evaluate the amount of the dynamic accidental torsion caused by the dynamic procedure, and then, to propose an ellipsoidal bounding method to estimate the maximum edge-frame drift and the corresponding design accidental torsion which bounds at least all the maximum edge-frame drifts from the dynamic analyses with shifted CMs.

### 5.2. Conclusion

- (1) The values of the maximum edge-frame drifts obtained from the recorded data, those calculated by dynamic analyses with different locations of CM (CM0, CM1, CM2, CM3, and CM4), and those predicted by the ellipsoidal bounding method for the five CSMIP structures, appear to be overall similar.
- (2) The range of dynamic accidental eccentricity,  $\eta_{dyn}$ , is 9.11%~27.0% for torsionally regular structures and 18.0%~67.3% for torsionally irregular structures, whereas that of the true accidental eccentricity,  $\eta_{true}$ , is 0.30%~5.00% for torsionally regular structures and 0.45%~7.43% for torsionally irregular structures, with the code-specified static accidental eccentricity,  $\eta_{static} = \beta$ , being 5%.
- (3) The ellipsoidal bounding method provides the range of  $\eta_{ellip}$  is 1.20%~16.0% for torsionally regular structures and 8.47%~32.0% for torsionally irregular structures with the values of the maximum edge-frame drifts bounding reasonably those from dynamic analyses.

## 6. References

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