STIFFNESS IDENTIFICATION OF MULTI-STORY BUILDINGS USING DISPLACEMENT RESPONSES DURING EARTHQUAKES

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Abstract

In the earthquake-prone areas, it is of extreme importance to detect any possible damage of a building due to earthquakes. Damage of a building can be detected by observing changes in structural properties, which include modal information (natural frequencies and mode shapes) and structural parameters (stiffnesses and damping ratios). These properties might be derived from dynamic responses of the structure in an unexpected event, e.g., earthquake, or in a scheduled experiment. Moreover, it is more straightforward to look at the structural parameters than the modal information for damage detection purpose.

In this study, we present one time-domain and one frequency-domain algorithms for identification of story stiffnesses of a multi-story building that can be modeled as a shear building analysis model. In particular, we make use of the displacement responses of the building subjected to earthquakes. Shear building analysis model is widely used in practical structural designs for low-rise and middle-rise buildings.

Instead of using velocity or acceleration responses as in conventional system identification methods, displacement responses are concerned in this study. Displacement measurement in high accuracy (up to mm level) and in high sampling rate becomes available, due to the rapidly developing global navigation satellite systems (GNSS).

In the time-domain method, the velocity as well as acceleration responses of a multi-story building are approximated by using its displacement responses by making use of the concept of Newmark-β method. The equations of motion of a multi-story building are written in matrix form, where the damping coefficients as well as story stiffnesses are summarized in the structural parameter vector to be identified. By incorporation of the motion equations at different time together, the structural parameters can then be estimated in the sense of least squares estimation.

In the frequency-domain method, the displacement responses in time domain are transformed into those in frequency domain. The story stiffnesses as well as damping coefficients can then be directly identified. Moreover, the spectrum ratios are used to improve its identification accuracy.

Numerical examples demonstrate that the time-domain can estimate the story stiffnesses in a time series, when that data window containing the responses within a short period is consecutively shifted. The stiffness can be estimated in high accuracy, except for the cases with relatively small displacements, however, it is notable that the method is sensitive to measurement errors which could limit its usage in practical applications.

On the other hand, the frequency-domain has been verified to be robust in considering measurement errors, while maintaining its high efficiency.

Keywords: Stiffness, Shear building, System identification, Displacement, Earthquake.

1. Introduction

In the earthquake-prone areas, it is of extreme importance to detect any possible damage of a building due to earthquakes. Damage of a building can be detected by observing changes in its structural properties, which include modal information (natural frequencies and mode shapes) and structural parameters (stiffnesses and damping ratios) [1, 2]. These properties might be derived from dynamic responses of the structure in an unexpected event, e.g., earthquake, or in a scheduled experiment. Moreover, it is more straightforward to look at the structural parameters than the modal information for damage detection purpose. Derivation of structural properties from responses rely on the system identification (SI) techniques [3, 4].

In this study, we present a time-domain algorithm and a frequency-domain algorithm for identification of story stiffnesses of a multi-story building, which can be modeled as a shear building analysis model. Shear building analysis model is widely used in practical structural designs for low-rise and middle-rise buildings. This relies on the fact that the shear building model can predict the structural responses in high enough accuracy, and furthermore, it is easy to analyze even in dynamics.

Instead of velocity or acceleration, displacement responses are concerned in this study, since displacement measurement is of high accuracy (up to mm level) in high sampling rate (up to 100Hz) due to the rapidly developing global navigation satellite systems (GNSS)[5].

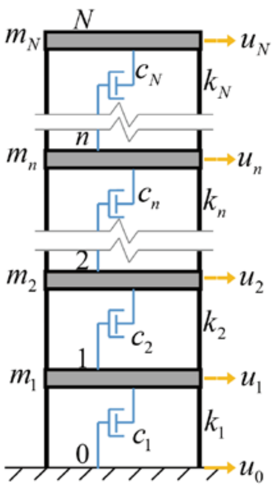

Fig. 1 – Shear building analysis model.

2. Formulation

In this study, we consider a multi-story building analysis model as shown in Figure 1. The building is composed of *N* stories. The floors are numbered from the bottom to the top; i.e., *N*-th floor refers to the roof or top of the building, and 0-th floor refers to the ground floor.

Furthermore, we assume that the building (approximately) satisfies the following two conditions, concerned about deformations of its structural elements (beams, columns, and floors): a) The beams and floors are rigid in bending, such that no bending deformation occurs in beams or floors; b) Axial deformation of the beams and columns, as well as the effect of axial forces on the stiffness of the columns are neglected.

2.1 Time-domain method based on least square prediction

Equation of motions of the shear building, which is subjected to earthquake, can be written in matrix form as follows [6]

$$\mathbf{M}\ddot{\mathbf{u}} + \mathbf{C}\dot{\mathbf{u}} + \mathbf{K}\mathbf{u} = \mathbf{0} \tag{1}$$

where $\mathbf{u}, \dot{\mathbf{u}}, \ddot{\mathbf{u}}$ are respectively the absolute displacement, velocity, and acceleration responses. $\mathbf{M}, \mathbf{C}, \mathbf{K}$ are the mass, viscous damping, and stiffness matrices, respectively. Eq. (1) can be rearranged in another form with respect to the viscous damping vector **c** and story stiffness vector **k**:

$$\mathbf{M}\ddot{\mathbf{u}} + \mathbf{D}\mathbf{c} + \mathbf{F}\mathbf{k} = \mathbf{0} \tag{2}$$

where

$$\begin{aligned} D_{n,n} &= \dot{u}_n - \dot{u}_{n-1} \quad D_{n,n+1} = -(\dot{u}_{n+1} - \dot{u}_n) \\ F_{n,n} &= u_n - u_{n-1} \quad F_{n,n+1} = -(u_{n+1} - u_n) \end{aligned} \tag{3}$$

$$\begin{aligned} D_{N,N} &= \dot{u}_N - \dot{u}_{N-1} \\ F_{N,N} &= u_N - u_{N-1} \end{aligned} \tag{4}$$

Suppose that we have measured and/or estimated all responses, including displacement, velocity, and acceleration at time t_i and t_j at different time. They should (approximately) satisfy

$$\begin{aligned} \mathbf{M}\ddot{\mathbf{u}}_i + \mathbf{D}_i\mathbf{c} + \mathbf{F}_i\mathbf{k} &= \mathbf{e}_i \\ \mathbf{M}\ddot{\mathbf{u}}_j + \mathbf{D}_j\mathbf{c} + \mathbf{F}_j\mathbf{k} &= \mathbf{e}_j \end{aligned} \tag{5}$$

where **e** is the error in estimating the equation of motion, and we assume that the structural parameters **c** and **k** do not change. These equations can be summarized as

$$\mathbf{e} = \begin{pmatrix} \mathbf{e}_i \\ \mathbf{e}_j \end{pmatrix} = \begin{pmatrix} \mathbf{M}\ddot{\mathbf{u}}_i \\ \mathbf{M}\ddot{\mathbf{u}}_j \end{pmatrix} + \begin{pmatrix} \mathbf{D}_i & \mathbf{F}_i \\ \mathbf{D}_j & \mathbf{F}_j \end{pmatrix} \begin{pmatrix} \mathbf{c} \\ \mathbf{k} \end{pmatrix} = \mathbf{g} + \mathbf{A}\mathbf{x} \tag{6}$$

where

$$\mathbf{A} = \begin{pmatrix} \mathbf{D}_i & \mathbf{F}_i \\ \mathbf{D}_j & \mathbf{F}_j \end{pmatrix}, \quad \mathbf{x} = \begin{pmatrix} \mathbf{c} \\ \mathbf{k} \end{pmatrix}, \quad \mathbf{g} = \begin{pmatrix} \mathbf{M}\ddot{\mathbf{u}}_i \\ \mathbf{M}\ddot{\mathbf{u}}_j \end{pmatrix} \tag{7}$$

To minimize the estimation error **e**, we have the following unconstrained optimization problem

$$\text{Minimize } \mathbf{e}^{\mathrm{T}}\mathbf{e} \tag{8}$$

which can be solved as

$$\mathbf{x} = -(\mathbf{A}^{\mathrm{T}}\mathbf{A})^{-1}\mathbf{A}^{\mathrm{T}}\mathbf{g} \tag{9}$$

It is easy to extend the idea to include more responses at different time.

Suppose that we have measured the displacement responses $\mathbf{u}_k$ of a the multi-story building at time t_k. To compute **A** and **g**, we need to approximate the velocity $\dot{\mathbf{u}}_k$ and acceleration responses $\ddot{\mathbf{u}}_k$, for which we make use of the concept of Newmark-β method for linear systems. In the follows, we adopt $\gamma = 1/2, \beta = 1/4$, such that

$$\begin{aligned} \dot{\mathbf{u}}_k &= \frac{1}{2\Delta t}(\mathbf{u}_{k+1} - \mathbf{u}_{k-1}) \\ \ddot{\mathbf{u}}_k &= \frac{1}{(\Delta t)^2}(\mathbf{u}_{k+1} - 2\mathbf{u}_k + \mathbf{u}_{k-1}) \end{aligned} \tag{10}$$

Algorithm 1: Time-domain method for stiffness identification

Step 1: Calculate the velocity $\dot{\mathbf{u}}_k$ and acceleration responses $\ddot{\mathbf{u}}_k$ by using Eq. (10) and the measured displacement responses $\mathbf{u}_k$ (k = 1, 2, …, I) at time t_k . I is the number of measured data.

Step 2: Compute $\mathbf{D}_k$ and $\mathbf{F}_k$ by using Eqs. (3) and (4) associated with the responses $\mathbf{u}_k, \dot{\mathbf{u}}_k, \ddot{\mathbf{u}}_k$ obtained in Step 1.

Step 3: Assemble **A** and **g** by Eq. (7) in the specified time window containing the data in a series of time steps. Calculate the approximate structural parameters **c** and **k** by using Eq. (9).

Step 4: Shift the time window and repeat Step 3.

2.2 Frequency-domain direct method

The authors [7] presented a direct method in frequency-domain for stiffness identification of multi-story buildings. Through the (numerical, laboratory-based, and real) examples using acceleration responses, it has been demonstrated to be a highly accurate, robust, and efficient method for structural parameter identification of shear buildings. Following the same idea, the algorithm making use of displacement responses is given as follows:

Algorithm 2: Direct frequency-domain algorithm

Step 1: Transform the time-domain displacement responses u_j at the j-th (j = 0, 1, …, N) floor into frequency-domain displacement responses U_j with I discrete data.

Step 2: Compute the identification values of story stiffness $k_n(\omega_i)$, for the I different circular frequency ω_i (i = 1, 2, …, I), of the n-th story by using Eq. (11).

$$k_n(\omega_i) = \omega_i^2 \,\mathrm{Re}\left(\frac{\sum_{j=n}^{N} m_j U_j}{U_n - U_{n-1}} \right) \tag{11}$$

Step 3: Estimate value of the story stiffness $\bar{k}_n$ by application of the following weighted sum of the identification values k_n associated with the frequency dependent weights $w(\omega_i)$:

$$\bar{k}_n = \frac{\sum_{i=1}^{I} w(\omega_i) k_n(\omega_i)}{\sum_{i=1}^{I} w(\omega_i)} \tag{12}$$

Since the frequency-domain responses are ready in Step 1, the mean values of their output-to-input spectrum ratios $H(\omega_i)$ defined as follows can be used as the weights:

$$H(\omega_i) = \frac{1}{N} \sum_{j=1}^{N} \frac{\left|\ddot{U}_j(\omega_i)\right|^2}{\left|\ddot{U}_0(\omega_i)\right|^2} \tag{13}$$

3. Numerical Examples

As a numerical example, we consider the four-story building. The parameters related to dynamic analysis are given in Table 1. Responses of the building are derived by carrying time history analysis within linear elasticity, where the El Centro motion as shown in Fig. 2 is applied at the ground floor. Time history analysis is conducted by using Newmark-β method, with β = 1/4 and γ = 1/2. The maximum absolute as well as relative displacement responses of the building are listed in Table 2.

The identification error is defined as follows

$$\varepsilon_{x_n} = 100 \times \frac{\left|\hat{x}_n - \overline{x}_n\right|}{\hat{x}_n} \% \quad (x := k \text{ or } c), \tag{14}$$

Table 1 – Structural parameters for the four-story shear building

Floor/Story	**1**	**2**	**3**	**4**
Mass (10^3kg)	22.0	22.0	22.0	18.0
Stiffness (10^7N/m)	3.8	3.2	2.4	2.0
Damping (10^4N/m/s)	3.0	3.5	4.0	4.5

Table 2 – Maximum absolute and relative displacement responses

Floor	**0**	**1**	**2**	**3**	**4**
Absolute (m)	0.2570	0.2595	0.2706	0.3011	0.3160
Relative (m)	0.0	0.0638	0.0733	0.1129	0.1224

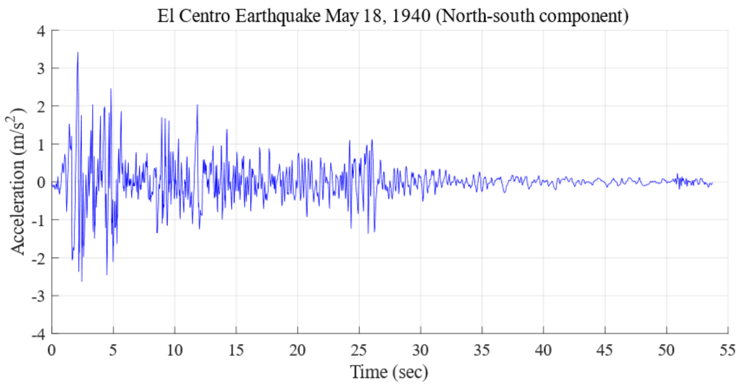

Fig. 2 – El Centro ground motion.

3.1 Implementation of time-domain Algorithm 1

Case 1: without measurement error

As the first case, measurement errors are not considered. Width of the time window is set to 3 seconds, including 150 data, and the window is shifted every 30 data. The identification errors for story stiffness as well as damping coefficients by using Algorithm 1 in each time window are plotted in Fig. 3.

The errors of stiffness identification range from -1.5% to -2.5%, and those of damping identification are much larger.

Case 2: with measurement error

For case 2, we consider randomly distributed measurement errors in displacement responses ranging within ± 0.005m. The maximum measurement error is 1.95% of the maximum absolution displacement of the ground, and 7.84% of the maximum relative displacement response at the first floor.

By implementation of Algorithm 1 with the randomly noised measurements, the identification errors for each story with 100 trials are plotted in Fig. 4, where the red line shows the average identification error in each time window.

It can be observed from the figures that the stiffnesses are overestimated in most cases. The identification errors are higher than the maximum measurement errors, especially for upper stories.

3.2 Implementation of frequency-domain Algorithm 2

Case 3: without measurement error

In this case for Algorithm 2, measurement errors are not considered. The identification values for story stiffness as well as damping coefficients are presented in Fig. 5. It is notable in the figures that there exist ‘plateaus’ near the peak spectrum ratio, especially for the stiffness identification. The maximum identification error for story stiffness is 0.12%, while that for the damping ratio is 8.55%.

Case 4: with measurement error

In this case we consider the same measurement errors as in Case 2. Table 3 summarizes the mean, maximum, as well as variation of measurement errors for stiffness identification with 100 times random measurement errors.

4. Conclusions

With the vision in rapid development of displacement measurement, for example by using global navigation satellite systems (GNSS), we consider the system identification problem of multi-story buildings by using displacement responses.

Two different methods were presented in this paper: the time-domain method and the frequency-domain method. Both of these two methods do not need iterative computations, such that they are of high efficiency. Numerical examples show that the time-domain method is sensitive to measurement errors, while the frequency-domain method has much higher robustness.

Future studies will include experiments for purpose of verification of these two methods in practical applications.

5. Acknowledgements

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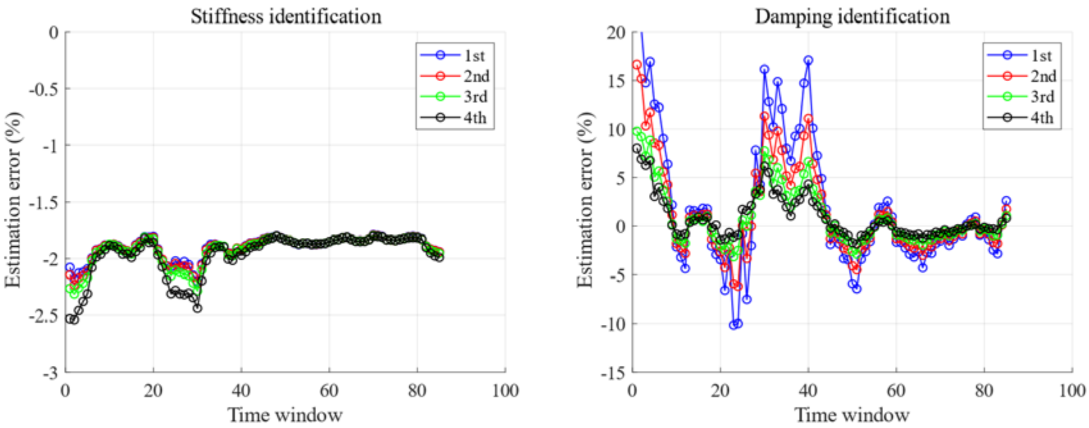
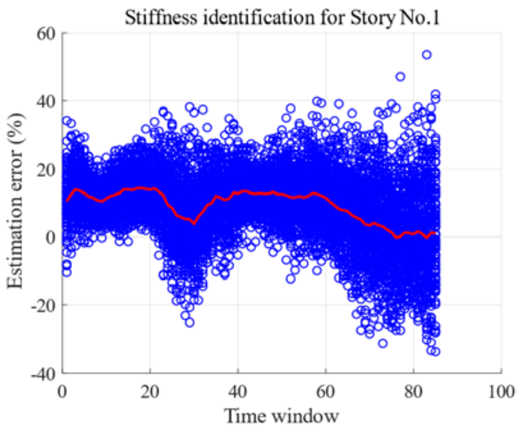
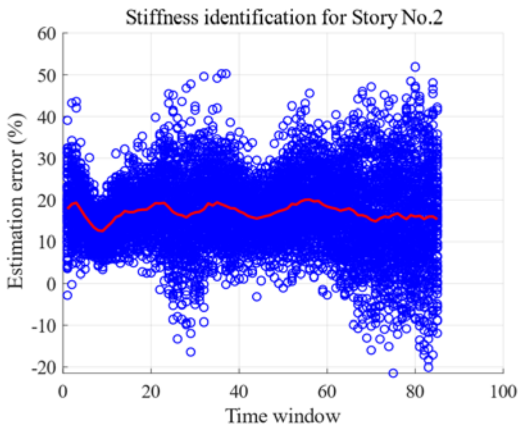


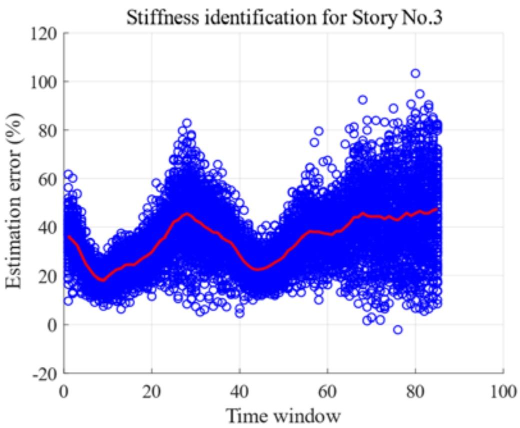
Fig. 3 – Identification errors of story stiffness and damping coefficient by the time-domain Algorithm 1 without measurement errors for the four-story shear building (Case 1).



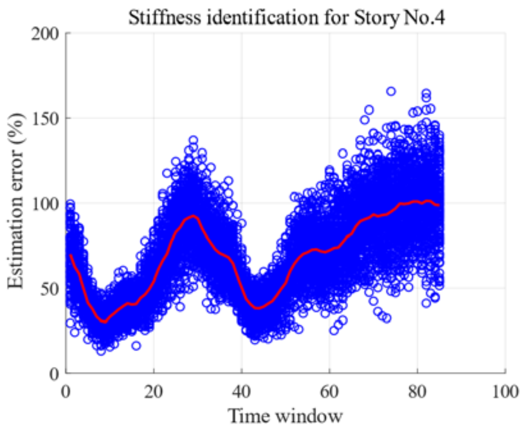
(a) 1st Story



(b) 2nd Story

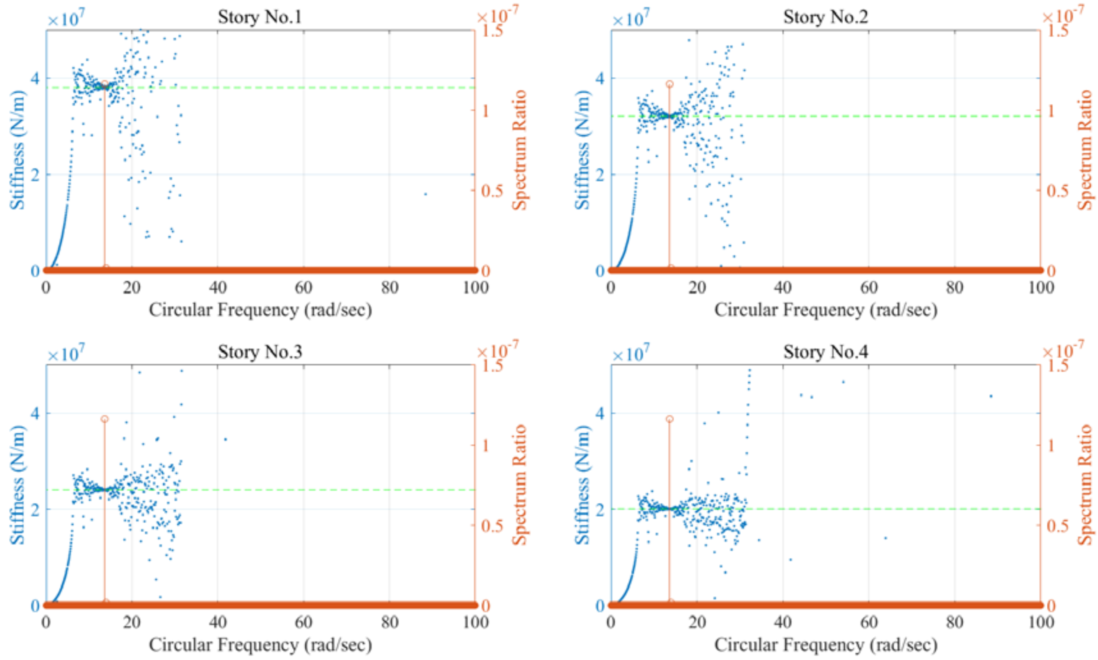


(c) 3rd Story

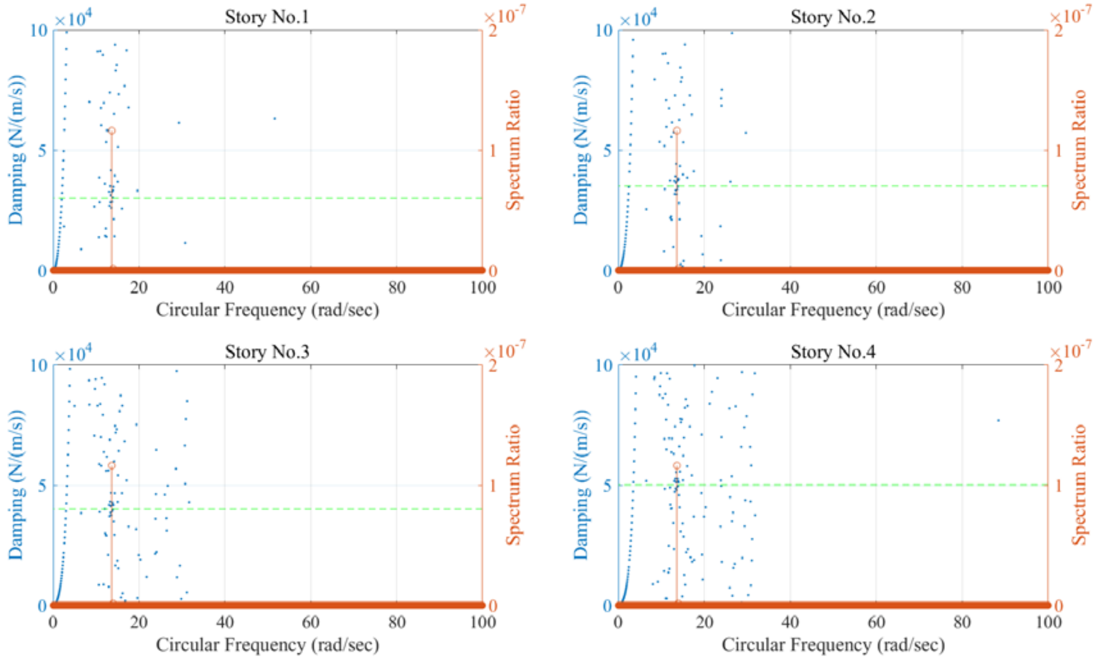


(b) 4th Story

Fig. 4 – Errors in stiffness identification by the time-domain Algorithm 1 with uniformly distributed measurement errors for the four-story shear building (Case 2).



(a) Stiffness identification



(b) Damping identification

Fig. 5 – Identification of story stiffness and damping coefficient by the frequency-domain Algorithm 2 without measurement errors for the four-story shear building (Case 3).

Table 3 – Estimation errors for story stiffness of the four-story building by using the frequency-domain Algorithm 2 (Case 4)

Story	**1**	**2**	**3**	**4**	
Mean	1.0080	1.0803	1.1690	1.7513	%
Maximum	10.6084	25.4412	17.0036	25.9635	%
Variation	2.8163	7.9030	7.3201	13.6745	%

6. References

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