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POTENTIAL MECHANISM AND LOAD CARRYING CAPACOTY OF BRACE FRAMES AFTER PLASTIC FAILURE MODE FORMATION

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Abstract

One of the popular limit design methods on framed structure is to utilize the lower bound theorem in limit analysis. In past studies, the second author has suggested a limit analysis method considering the large deformation after the failure mechanism formation (2nd limit analysis). It is generally known that when a horizontal load is applied to a frame with a K-type brace, the plastic hinge is formed at the joint of beam and brace. This results the failure mode which the vertical deformation of beam. However, few studies have examined the effects of this depression on the entire framework. Therefore, this study examines how the frame of this beam affects, herein, the collapse mechanism and the ultimate strength are compared.

First, a single-story frame subjected to horizontal load is analyzed with limit analysis. Here, the frame type is considered with parameters as moment-resisting frame, K-type brace and one-side brace type. It is assumed that the compression braces have disappeared in order to make the frame that has achieved the stable proof stress after buckling capacity. The 2nd limit analysis is performed on the frame model that takes into account the deformation of the model that has reached the stable strength after buckling.

In the same way, the analysis of multi-span model and multi-story model are also performed. The multi-span model is a single-story model with simple frames attached to both sides. And the multi-story model is a multi-span model with two layers

This study confirms that the reduced capacity of the compression braces results in a significant reduction in the load carrying capacity of the frame for horizontal loads, but it is confirmed that the influence of the subduction of the beam by the tension brace on the vertical frame load is less than the effect of the horizontal load on the vertical frame.

In the multi-span model, it is considered that the load carrying capacity against the horizontal load decreases as in the simple frame. However, it was found that there was no effect of the vertical load on the model considering the subduction of the beam.

Similarly, the load carrying capacity for the horizontal load is significantly reduced in the multilayer model, but it is considered that the load carrying capacity for the vertical load is not affected by the subduction of the beam.

Keywords: Redundancy, Brace frame, Limit analysis considering large deformation, Load carrying capacity

1. Introduction

In recent years, seismic design often uses bracing in the frame. It is known that the K-type brace used in such a structure sinks under the influence of the brace when a horizontal load such as an earthquake is applied. It is also known that after the brace reaches the stable strength after buckling, the strength is greatly reduced. The effects of this subduction and after buckling braces have not been well studied.

Therefore, we consider using limit analysis based on the lower theorem. Previous study [1] have suggested a 2nd limit analysis. This is an analysis method that uses the limit analysis that takes deformation into account once the collapse mechanism has been formed, to further extract the load carrying capacity of the frame due to the remaining capacity of the frame.

The analytical method can estimate the potential resistant mechanism of frames under large deformation, however, on the previous studies, the vertical load carrying capacity was only studied. Herein, the brace frame is analyzed on the ultimate state after buckling of compressive brace member formation.

2. Theoretical Description of Limit Analysis of Frames

2.1 Limit Analysis

The "Compact Procedure" proposed by Livesiey [2] is limit analysis using linear programming based on the lower bound theorem.

$$\text{Maximaize } \lambda \text{ (Load factor)} \tag{1}$$

$$\text{Subject to } \lambda\{P_0\} = [C]\{M\} \text{ (Equilibrium equation)} \tag{2}$$

$$|M_j| \leq M_{Pj} \text{ (Plastic condition)} \tag{3}$$

Where, $\{P_0\}$ is external load vector, $[C]$ is connectivity matrix, $\{M\}$ is internal force vector, and M_{Pj} is member resistance.

2.2 Deformation of Equilibrium Equation due to Disappearance of Members

In the case where a member with a frame is lost (2), (3), the column of the connection matrix and the variable of the member vector related to the corresponding member are removed from the equilibrium equation, and a limit analysis may be performed.

$$\lambda_d \left\{ P \right\} = \begin{bmatrix} C_{11} & \cdots & \cancel{C_{1i}} & \cdots & C_{1m} \\ \vdots & \cdots & \vdots & \cdots & \vdots \\ C_{n1} & \cdots & \cancel{C_{ni}} & \cdots & C_{nm} \end{bmatrix} \begin{Bmatrix} M_1 \\ \vdots \\ \cancel{M_i} \\ \vdots \\ M_m \end{Bmatrix}$$

Where, λ_d is load factor after damage

2.3 Calculation of Deformation just before Collapse

Calculation of deformation just before collapse is valid under the following assumptions.

(a) Moment-rotation angle relationship of the plasticized part is a perfect elastic-plastic type.

(b) Load is proportional load of concentrated load.

(c) Plastic hinge keeps rotating in the same direction.

(d) Bending moment distribution at collapse is known

$$\{\tau\}_i = \frac{L_i}{6E_iI_i}\begin{bmatrix} 2 & -1 \\ -1 & 2 \end{bmatrix}\{M - C\}_i \tag{4}$$

Where, $\{M\}_i$ is end moment of member i, $\{\tau\}_i$ is angle of deflection, E_i is modulus of direct elasticity, I_i is polar moment of inertia of area, L_i is length of member, C is the load term.

Bending moment when collapsed the angle of deflection $\{\theta_{t0}\}$ is obtained from the equation (5) from the element distribution $\{M\}$. In the collapse mechanism obtained by the Compact Procedure method, let the plastic rotational angle is $\{\theta_p\}$ and the nodal displacement of the hinge is $\{\delta_p\}$. Considering the displacement as a rigid body, the angle of deflection other than the hinge part are set to 0.

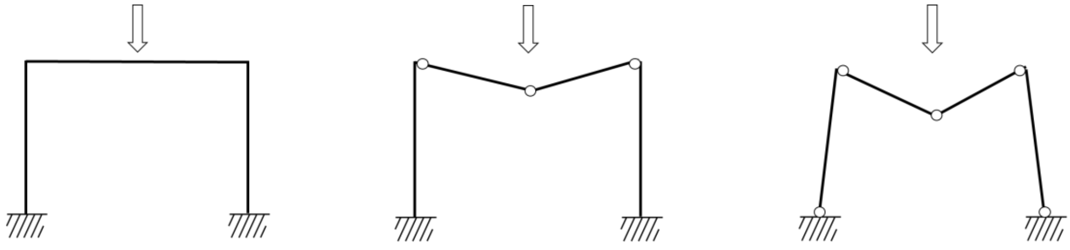
When the nodal displacement for which displacement is to be calculated is 0, the rotational angle and the nodal displacement are $\{\theta_t\}$ and $\{\delta_t\}$. The rotational angle and displacement at the time of collapse are positive when they match the direction of the moment. Equation 6 represents the rotational angle $\{\theta\}$ and the displacement $\{\delta\}$ when any deformation occurs.

$$\{\theta\} = \{\theta_t\} + \alpha\{\theta_p\}, \ \{\delta\} = \{\delta_t\} + \alpha\{\delta_p\} \tag{5}$$

The true deformation of the collapse mechanism can be selected from the deformations in which the rotational angle of an arbitrary plastic hinge is set to 0 and the virtual work made by the load is the largest. This is the plastic hinge formed immediately before the collapse at the point where the rotational angle becomes 0 when α becomes the maximum in equation (6). $\{\theta\}$ and $\{\delta\}$ are the deformations immediately before the collapse.

2.4 2nd Limit Analysis

This is the ultimate behavior of load carrying mechanism considering the contact movement after plastic collapse. This is to calculate the deformation at plastic collapse formation using the result of limit analysis by the matrix method, and to carry out the plastic analysis again.



(a) model (b) Limit Analysis (c) 2nd Limit Analysis

Fig. 1—Conceptual Diagram of 2nd Limit Analysis

3. Study on Single-story Braced Frame

3.1 Outline of analytical study

The single-story braced frame model as shown in Fig.2 is analyzed by limit analysis. Herein, the load carrying capacities before and after the brace is damaged and the load carrying capacity obtained by limit analysis are discussed. And to consider the ultimate state of brace frame after the buckling occurrence, the compressive brace is removed because the strength is assumed to be lost. Fig.3 shows a frame model that reaches a stable proof stress after buckling and considering large deformation.

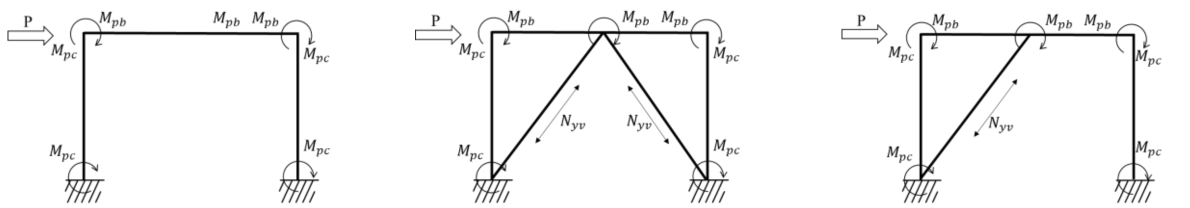


Fig. 2－Single-story Limit Analysis Model

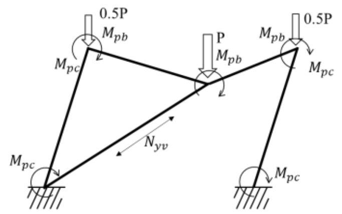


Fig. 3－Single-story 2nd Limit Analysis Model

Where, referring to the reference [3], horizontal load (Ph) is 761 kN, vertical load (Pv) is 48.6 kN (frame end is 24.3 kN), full plastic strength of column is Mp = 723 kN · m, and yield tensile strength of column is Ny = 5139 kN, full plastic strength of beam is Mp = 592 kN · m and Ny = 3095 kN, yield tensile strength of brace is Ny = 1260 kN.

3.2 Analysis result of single-story frame

Fig. 4 shows the expected collapse mechanism of the frame when a horizontal load is applied, and Figs. 5 (a) and (b) show the expected mechanism when the vertical load is applied to the frame models of Figs. 2 (b) and (c). Fig. 5 (c) shows the expected collapse mechanism of Fig 3. Table 1 summarizes the results of this analysis.

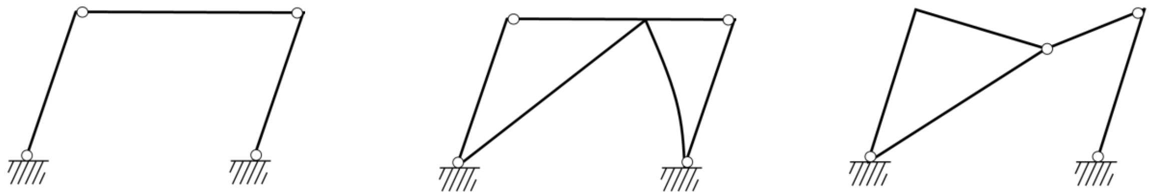
When a horizontal load is applied, in Figs. 4 (a) and (b), a plastic hinge is formed at the beam end and the column base, and a collapse mechanism is formed. When the brace is inserted, the brace resist against compression and tension, respectively, and the stable proof stress greatly increases.

However, when it reached the stable proof stress after buckling and the compression strength of brace is lost after buckling, a resistant mechanism such as truss is formed by the tension side brace, the column and the beam. And a plastic hinge is formed at the center of the beam, and a collapse mechanism is formed. As a result, it is thought that the beam sinks. Fig. 5 and Table 1 show that when a vertical load is applied, when a K-type brace is inserted as compared with a simple moment-resisting frame, the brace is effective for compression and the resistant strength is improved. However, it was confirmed that the one-side brace type model had no brace effect and is almost the same as the simple moment-resisting frame.

From the above results, it is considered that the effect of the brace on the subduction of the beam due to the effect of the brace has a little effect on the frame because the effect of the compression side brace is lost.

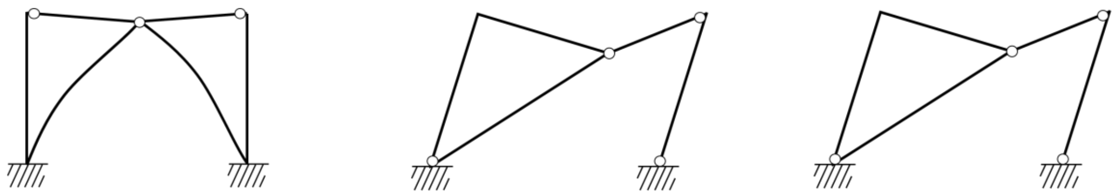
Table 1－Analysis result of single-story frame

λ	Simple Frame	K-type brace	one-side brace type
Horizontal Load	3.29.E−04	7.19.E−01	4.77.E−04
Vertical Load	1.62.E−02	39.4	2.62.E−02
2nd Limit Analysis	−	−	2.40.E−02



(a) Simple Frame (b) K-type brace (c) one-side brace type

Fig. 4－Failure Mode of Single-story Braced Frame



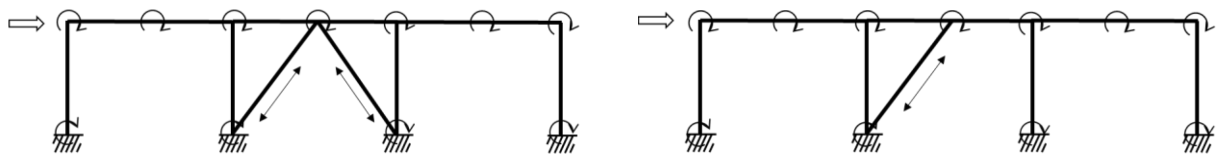
(a) K-type brace (b) one-side brace type (c) 2nd limit analysis

Fig. 5－Failure Modes of Vertical Load and Failure Modes of 2nd Limit Analysis

4. Study on Multi-span Braced Frame

4.1 Outline of analytical study

Here, multis span frame with K-type brace as shown in Fig. 6 is analyzed. The moment-resisting frames are added on the both side of braced frame. And it is clear in Chapter 6 that the brace is inserted to improve the load carrying capacity in the comparative study of the simple frame and the frame with the brace. For this reason, Chapter 4 does not analyze it. Fig. 7 shows 2nd limit analysis model.



(a) K-type brace (b) one-side brace type

Fig. 6－Multi-span Limit Analysis Model

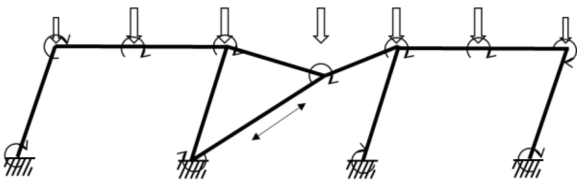


Fig. 7－Multi-span 2nd Limit Analysis Model

4.2 Analysis result of multi-span frame

Fig. 8 shows an analysis result when a horizontal load is applied, and Figs. 9(a) and (b) show an analysis result when a vertical load is applied. Fig. 9 (c) shows an analysis result expected in the 2nd limit analysis. And table 2 summarizes each load factor.

When a horizontal load is applied, the K-type brace model becomes a beam collapse mode like a single-story model and forms a collapse mechanism. On the other hand, in the one-side brace type model, a truss is formed by the tension braces, columns and beams, and a plastic hinge is formed at the center of the beam. It is expected that the beam will fall and the load factor will be deteriorated significantly.

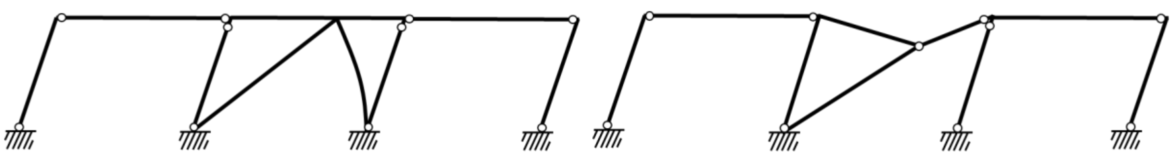
When a vertical load is applied, the result differs from that expected for a single-story model. The results for the K-type brace frame are expected to be the same as those expected for the single-story model. However, in the one-side brace type model, plastic hinges are formed on the beams and column bases due to the formation of the truss frame, here as the results expected in the single-story model are compared with those in the multi-span model. In the multi-span model, the central frame is supported by the frames on both sides, and it becomes a beam collapse mode, and forms a collapse mechanism. Therefore, the load factor does not decrease.

Fig. 9(c) shows the result of applying a vertical load to the frame deformed by the horizontal load. As a result of this analysis, the load factor is not expected to decrease. In other words, it is thought that the beam does not affect the frame.

In this analysis, the model taking into account the deformation does not consider the damage to the frame when subjected to a horizontal load, so the location of the plastic hinge is different. However, the analysis is performed taking into account the deformation of the column, and if the deformation is considered to cause almost no damage to the column base, this result is considered appropriate.

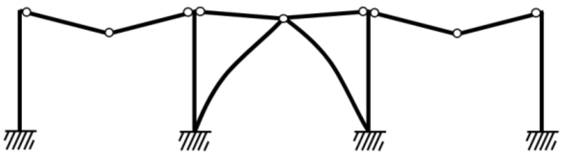
Table 2－Analysis result of multi-span frame

λ	K-type brace	one-side brace type
Horizontal Load	2.95.E-01	3.84.E-04
Vertical Load	1.62.E-02	1.62.E-02
2nd Limit Analysis	-	1.62.E-02

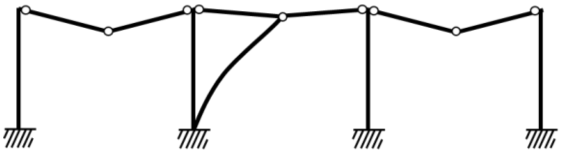


(a) K-type brace (b) one-side brace type

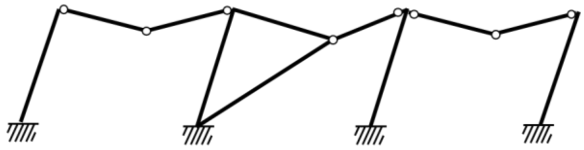
Fig. 8－Failure Mode of Multi-span Braced Frame



(a) K-type brace



(b) one-side brace type



(c) 2nd limit analysis

Fig. 9－Failure Modes of 2nd Limit Analysis of Multi-span

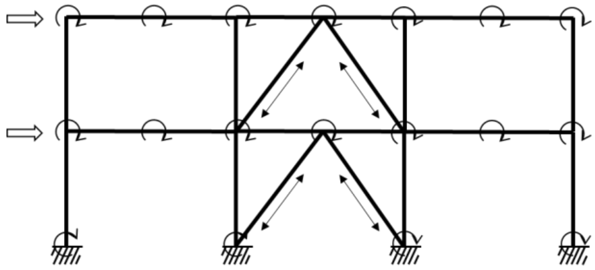
5. Study on Multi-story Braced Frame

5.1 Outline of analytical study

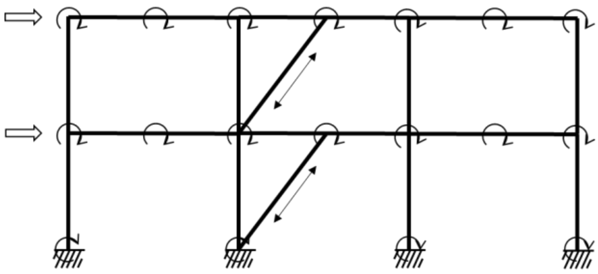
Here, the multi-story frame of K-type brace as shown in Fig. 10 is analyzed. This model confirms the effect on the frame by the depression of the beam when the model in Chapter 4 is made into two stories and there are multiple-story. Fig. 11 shows a 2nd limit analysis.

In this analysis, in Reference [3], the horizontal external force increases as the floor becomes higher, leading to a whole collapse. However, in this analysis, the horizontal external force of the second layer is different from that of the Reference [3] because the only single-layer collapse occurred. This difference in the analysis results is considered to be due to the use of the compact procedure method in this analysis method. The original design had a horizontal load of 1028 kN, but this time the whole collapsed at 761 kN.

Fig. 11 shows a 2nd limit analysis. In this analysis, the vertical load is applied and the reduction of the load coefficient is examined after considering the deformation at the time of whole collapse. Also, this time, as in Chapter 4, the analysis was started from a state where damage to the frame due to horizontal load was not considered.



(a) K-type brace



(b) one-side brace type

Fig. 10－Multi-story Limit Analysis Model

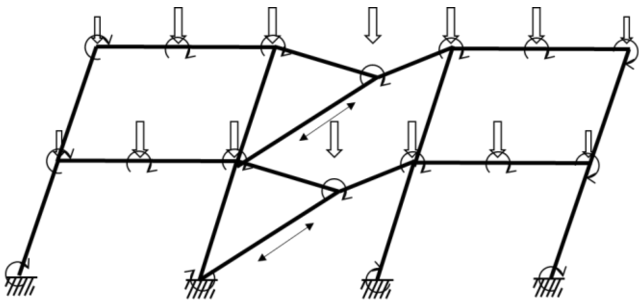


Fig. 11－Multi-story 2nd Limit Analysis Model

5.2 Analysis result of multi-story frame

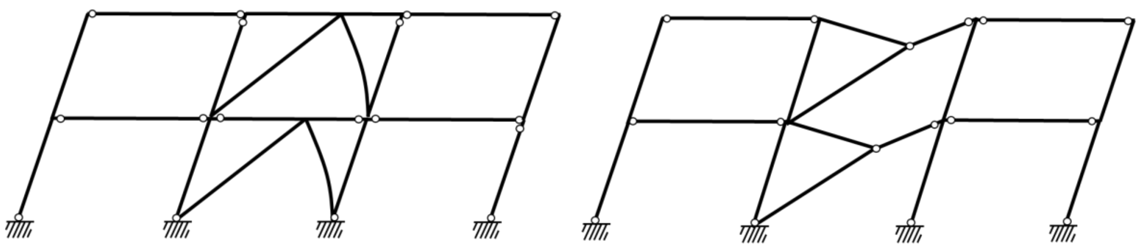
Table 3 shows the load factor obtained by this analysis. Fig. 12 shows the collapse mechanism when a horizontal load is applied. This result is the same as the multi-span collapse mechanism. In the case of the K-type brace frame, the collapse mechanism becomes a beam collapse mode and the whole collapse mechanism. However, it can be confirmed that, in the one-side brace type model, a shearing force is generated by the tension of the brace at the center of the beam and a plastic hinge is formed, so that the load carrying capacity is significantly reduced.

Figs. 13 (a) and (b) show the expected collapse mechanism when a vertical load is applied to each frame, and Fig. 13 (c) shows the expected collapse mechanism based on the results of 2nd limit analysis. Also in this result, there is no change in the load factor as in Chapter 4. Thus, it is considered that the reduction in the stable proof stress after buckling of the compression brace does not affect the vertical load.

Similarly, the results obtained by using 2nd limit analysis show no load carrying capacity reduction. Also, this analysis does not take into account the damage to column bases and beams due to horizontal load, as in Chapter 4, so the collapse mechanism is different from the model under horizontal load. However, the collapse mechanism under vertical load is considered to be almost no loading on the column base because the beam is all collapse mode. Also, from the analysis results of the multi-layer, it is considered that the depression of the beam does not affect the frame even if it becomes multiple layers.

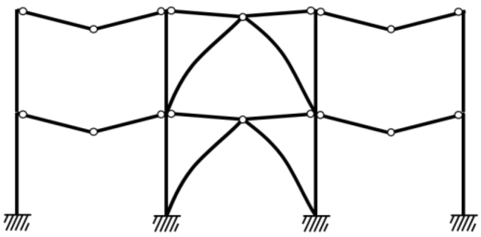
Table 3－Analysis result of multi-story frame

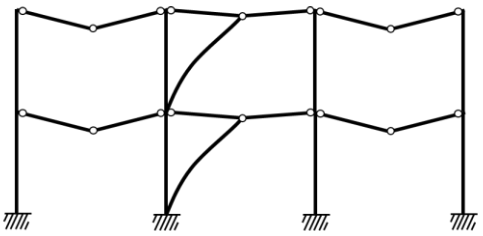
λ	K-type brace	one-side brace type
Horizontal Load	2.95.E−01	3.39.E−04
Vertical Load	1.62.E−02	1.62.E−02
2nd Limit Analysis	−	1.62.E−02



(a) K-type brace (b) one-side brace type

Fig. 12－Failure Mode of Multi-story Braced Frame


(a) K-type brace


(b) one-side brace type

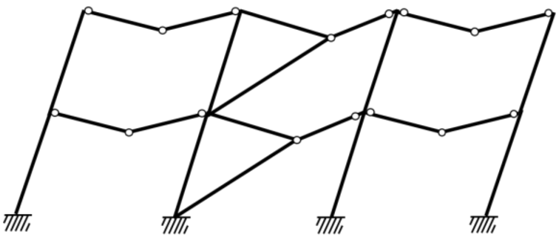

(c) 2nd limit analysis

Fig. 13－Failure Modes of 2nd Limit Analysis of Multi-story

6. Comparison by change of frame

For each frame, consider as a reduction rate. In the single-story model, the one-side brace type model when a horizontal load is applied shows a marked decrease in the load carrying capacity. Also, when a vertical load is applied, the decrease is about 8%.

In the multi-span model, the load carrying capacity under horizontal load is significantly reduced, but the rate of decrease is smaller than that in the single-story model. In the case of a vertical load, there is no decrease in the load carrying capacity unlike the single-story model.

In the multi-story model, the load carrying capacity under a horizontal load is significantly reduced. Also, the multi-story model is lower than the multi-span model. This is probably because the number of beams affected by the braces increased. At the time of vertical load, it does not decrease as in the multi-span model. That is, it is considered that the depression of the beam due to the brace does not affect the entire frame when another frame exists on the same layer.

7. Conclusions

In this study, we examined the effect of compression brace on stable proof stress after buckling and beam depression on the whole frame.

As a result, when the compression brace reaches the stable proof stress after buckling, the shear force is generated at the center of the beam by the tension brace, and the beam falls. This results in a significant decrease in the load carrying capacity for horizontal loads. However, in the case where a frame other than the beam in which the depression has occurred in the vertical load is present in the same layer, it is considered that the collapse is suppressed by the other frames and the load carrying capacity does not decrease.

If an earthquake occurs, it will be deformed more than this analysis. Therefore, in the future, considering the deformation of the frame given by the earthquake motion, we plan to study the effect of the beam depression on the whole frame.

8. Acknowledgement

Professor Takumi Ito and Assistant Professor Kenjiro Mori provided much advice and guidance regarding this research. Thank you.

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