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A MATERIAL HOMOGENSATION APPROACH FOR PUSHOVER ANALYSIS OF MASONRY STRUCTURES

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Abstract

Masonry is a composite material consisting of units (brick or stones) and mortar at head and bed joints, characterised by an overall anisotropic behaviour. Various approaches have been proposed for modeling and analysis of masonry, including both detailed and discretised (micro-modeling) and equivalent homogenisation (macro-modeling) techniques. Since the elimination of inherent uncertainties regarding material properties and failure criteria is difficult, even for the case of elastic analysis, a detailed framework should be developed for an accurate, but computationally efficient assessment of masonry structures. Based on the above, the scope of the present research paper is to propose a comprehensive framework for inelastic analysis of masonry structures modelled using an equivalenthomogenised orthotropic material. The proposed two-step homogenisation procedure is analytically described for the estimation of the properties of masonry structures. The properties of the finally computed orthotropic homogenised material are used as input for analysis and the equivalent stresses are calculated. The decomposition of homogenised stresses to the respective composite materials is described, along with the failure criteria adopted for each component. Furthermore, a step-by-step adaptive methodology for the inelastic pushover analysis of masonry structures is proposed, using updated properties of the homogenised material. The methodology proposed is applied to a specific masonry wall configuration and the results are compared to available experimental and numerical results in literature.

Keywords: masonry; homogenisation; inelastic pushover; failure criteria

1. Introduction

Masonry is a composite material, consisting of units (stones or bricks) and mortar at head and bed joints. In general, the arrangement of units is irregular; therefore, rational assumptions should be made for the modeling and analysis of masonry structures.

Several methods have been proposed for the structural modeling of masonry structures, having a wide range of accuracy and computational efficiency. The use of finite elements, macroelements and equivalent frame elements has been proposed in the literature and applied for the modeling and inelastic pushover analysis of masonry structures, while the efficiency of each modeling approach and the relative inelastic analysis results have been compared and evaluated. In line with the different modeling approaches, varying failure criteria and/or inelastic plastic hinge moment-rotation curves have been proposed in the literature.

The most popular, until recently, modeling approach for the inelastic pushover analysis of masonry structures, is the use of equivalent frame elements with inelastic behaviour at beam ends. In particular, both equivalent frame elements with lumped plasticity (concentrated plastic hinges) [1], [2] and distributed plasticity (fiber model) [3] have been proposed for the modeling of piers and spandrels. Furthermore, a Simplified Analysis Method (SAM) has been proposed [4], where the nonlinearity is considered using three types of plastic hinges, namely shear hinges (V type), bending hinges (M type) and rocking hinges (PM type). For the modeling of inelastic behavior of piers according to SAM, shear hinges are considered in the middle of the deformable part and rocking hinges at the member's ends while, for the modeling of the inelastic behavior of spandrels, shear hinges are considered in the middle and bending hinges at member's ends. It should be noted that based on literature evaluation and conclusions, the aforementioned simplified approaches for the modeling of masonry structures are, in many cases, proven unable to predict different types of failure mechanisms, since they are related to the type of structure and/or loading. To this end, the inelastic modeling of masonry structures with macroelements has been recently proposed [5], [6], along with strut-and-tie modeling, in order to be able to predict critical failure modes.

The most detailed and efficient method for the modeling of masonry structures is the use of finite elements, considering pertinent inelastic constitutive models for the unit (stone or brick) and mortar material behaviour. Several methods available in literature propose micro-modeling, with discrete elements for units and mortar [7], [8], while the use of combined or homogenized material [9] for masonry structures is also proposed. Regarding nonlinear analysis for horizontal actions, the use of non-linear step-by-step procedures using changing shape elements has been proposed [10], however, the model update at every step renders the procedure difficult to implement, and with a high computational cost.

Based on literature recommendations, masonry with two sets of mortar joints (bed and head joints) can be represented by an equivalent homogenous anisotropic material, under the assumption that no slippage between the mortar layers and the brick units occurs, while the head joints are considered continuous [9]. A homogeneous material is proposed for masonry, while the properties of the equivalent material are calculated on the basis of strain energy equality of the initial material (consisting of units and mortar) and the homogenized material. The homogenisation procedure is based on the methodologies developed for the estimation of elastic moduli of stratified rock mass [11], [12] and the equivalent material can be used for elastic analysis of masonry structures. Based on elastic analysis results and the relevant distribution of stresses, deconvolution to stresses at brick units and mortars can be implemented, assessing the failure of brick, head joint and bed joint by comparing the resulting stresses with known material strengths. Therefore, the selection of properties of the constituent materials and the relevant failure criteria [13], are the most critical issues for the reliable application of the homogenisation procedure. It should be noted that several methodologies have been proposed for the homogenisation of masonry structures, with the most recent presented by Nino & Luongo (2019) [14] and proposed for elastic or inelastic step-by-step analysis (with elastic steps). Furthermore, the homogenisation procedure has been incorporated and implemented in popular and widely used software like MIDAS® [15]. Finally, it should be noted that, until recently, there are no guidelines or detailed methodologies available in the literature for the step-by-step inelastic pushover analysis of masonry structures, assuming homogenized material properties.

In line with the above, the scope of the present paper is to apply finite element modeling for the analysis of masonry structures considering an equivalent homogenized material on the basis of literature recommendations and to introduce a step-by-step methodology for inelastic static (pushover) analysis of masonry structures. The proposed procedure is iterative; elastic analysis is performed at each step, stresses of the homogenized masonry material are calculated and a deconvolution methodology is applied to estimate stresses at component (unit and mortars) level. Component stresses are then compared to material strengths, based on the failure criteria adopted, and the model is updated at the next step using the homogenized material properties that are estimated excluding the components that have exceeded their capacity. A MATLAB-based [16] software is developed for the application of the proposed methodology, using SAP2000 [17] platform for analysis of the finite element masonry model. The proposed step-by-step inelastic pushover analysis is described, and a detailed flowchart with the distinct steps is presented. Finally, the proposed methodology is applied to a solid shear wall experimentally tested to horizontal loading and the results, in terms of P-d curves, are compared. Therefore, the methodology is evaluated and the results are discussed.

2. Homogenisation procedure for orthotropic masonry material

2.1 General principles of the homogenisation procedure

According to the homogenisation procedure proposed by Pante et al. [9], the anisotropic elastic properties of the equivalent material are estimated considering the properties of the constituent materials, namely bed (horizontal) joints (JH), head (vertical) joints (JV) and stone/brick units (BR). Based on analysis results, the homogenized stresses are calculated, and failure criteria should be checked. Since failure modes and criteria vary for units and mortar, the deconvolution of homogenized stresses to the stresses developed at the constituent materials is essential in order to apply specific criteria (different for units and mortar) and to evaluate potential failure modes.

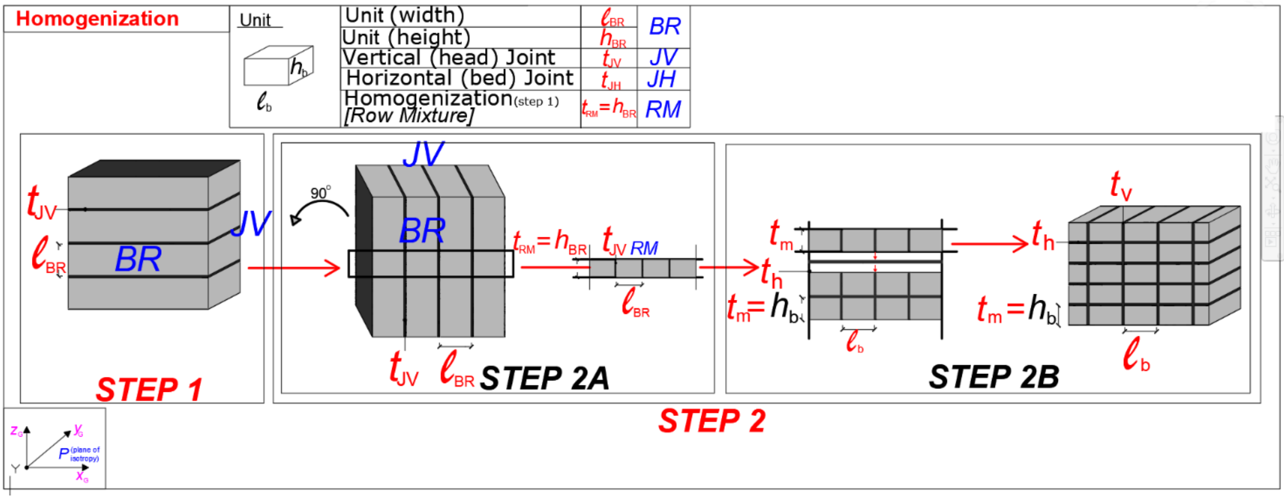


Fig. 1 – Outline of the homogenisation procedure

The homogenisation procedure for the estimation of anisotropic elastic properties of the equivalent material is outlined in Fig.1 and has two distinct steps:

Step 1: A stratified material is considered, consisting of a stacked system with two alternating horizontal isotropic layers, namely one layer with brick/stone unit properties (BR) and one layer with head (vertical) joint properties (JV). Based on the strain energy equality of the initial, two-layered (consisting of units and JV mortar) and the equivalent materials, the homogenisation procedure is followed to estimate the material properties of the first step (Raw Mixture - RM). It should be noted that the material of the first step is a cross anisotropic material having one plane of isotropy [10].

Step 2: The step 2 has two individual sub-steps. At first (step 2A), a 90° rotation of the homogenized material of step 1 (RM) is performed (Fig. 1, Step 2A). It should be noted that due to the rotation, the –initialy horizontal- mortar material is now located vertically (as head joint), therefore, the consideration of the horizontal mortar properties at step 1 as JV is explained. Subsequently (Step 2B), a horizontal layer with bed joint (JH) mortar properties is considered and the homogenisation procedure of step 1 is repeated. Therefore, at step 2, two different layers, namely the RM (cross anisotropic material) and the mortar (isotropic material) are combined in order to calculate the equivalent orthotropic material. Applying the homogenisation procedure of the second step, the properties of the final, homogenized masonry material are estimated based on all constituent materials properties and the aforementioned reasonable assumptions.

As already stated, the scope of the homogenisation procedure is not only to estimate the properties of the homogenized material that will be used for analysis and calculation of homogenized strains and stresses, but to form a deconvolution methodology as well, in order to be able to calculate the developed stresses at the constituent materials and check them against specific failure criteria.

2.2 Details of the homogenisation and deconvolution procedure

The three distinct steps forming the proposed homogenisation and deconvolution procedure are presented in Fig.2. As already mentioned, the properties of the equivalent orthotropic material are estimated in steps 1 and 2, and are used as input material properties of shell elements in order to perform a F.E. analysis and calculate the homogenised stresses ($\sigma_{,HOM}$). At step 3, a deconvolution procedure is applied (in two steps) in order to estimate the stresses at the constituent materials (units and mortar), based on $\sigma_{,HOM}$. The theoretical background and documentation of the proposed methodology is presented in detail in [9], [11], [12].

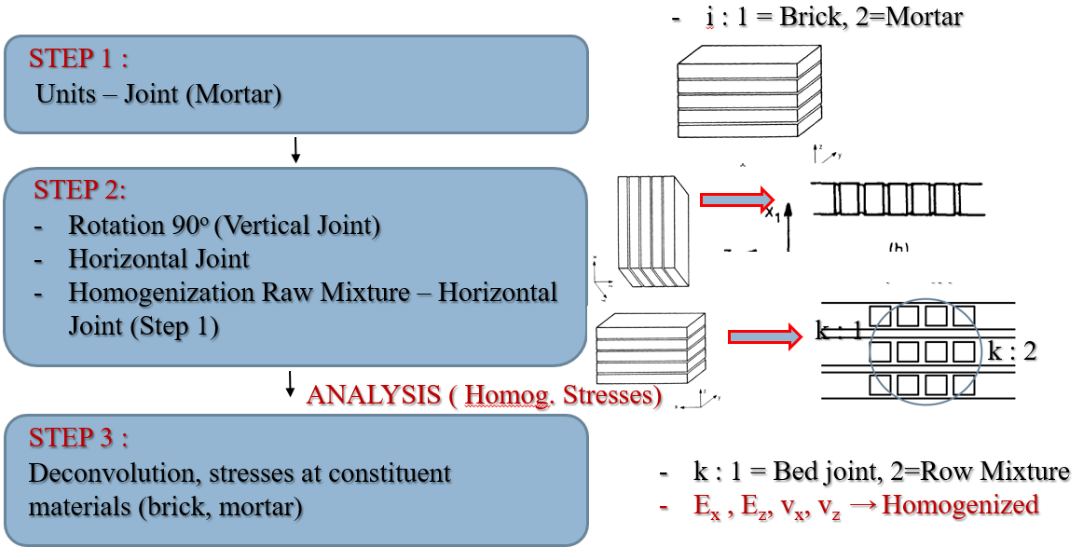


Fig. 2 – Flowchart for the homogenisation and the deconvolution procedure

The homogenisation and deconvolution procedure is based upon two basic assumptions [9], namely that there is no slippage between the mortar layers and the brick units and the assumption of continuous head joints, ignoring any possible irregular arrangement. At first sight, the second assumption seems to restrict the application range of the homogenisation procedure, however it has been shown that the assumption of continuous vertical joints instead of staggered joints does not have a significant effect on the stress state of the constituent materials, at least for the case of elastic analysis [14].

The theoretical background for the homogenisation of masonry material is outlined in brief below:

Step 1: *Homogenisation of units (BR) and vertical joint mortar (JV) -horizontally arranged before rotation- to a stratified cross anisotropic material (RM)*

The constituent masonry materials, namely units (stone or brick) and mortar, are isotropic materials. For the estimation of the properties of the homogenised material, the assumption of an ideal volume consisting of a cross anisotropic material that can describe the properties of the stacked material with two alternating isotropic layers, is made. This cross anisotropic material has a plane of isotropy and a vertical (perpendicular) direction with elastic properties (E, ν) different from those within the isotropy plane. Assuming that the layers of stacked material are arranged within the X-Y plane (Fig. 1), this is defined as the plane of isotropy. The estimation of the properties of the equivalent homogenised material (Raw Mixture – RM) is based on the strain energy equality concept, therefore stresses and strains of the stacked material of the first step are calculated on the basis of Eqs. (1)-(4).

$$\overline{\sigma_x} = \frac{1}{V} \cdot \left[\int_{V_{BR}} \sigma_{x,BR} dV + \int_{V_{JV}} \sigma_{x,JV} dV \right], \overline{\varepsilon_x} = \frac{1}{V} \cdot \left[\int_{V_{BR}} \varepsilon_{x,BR} dV + \int_{V_{JV}} \varepsilon_{x,JV} dV \right] \quad (1)$$

$$U_{re} = \frac{1}{2} \cdot \int [(\sigma_{x,BR} \cdot \varepsilon_{x,BR} + \sigma_{y,BR} \cdot \varepsilon_{y,BR} + ... + \tau_{yz,BR} \cdot \gamma_{yz,BR}) + (\sigma_{x,JV} \cdot \varepsilon_{x,JV} + \sigma_{y,JV} \cdot \varepsilon_{y,JV} + ... + \tau_{yz,JV} \cdot \gamma_{yz,JV})] dV \quad (2)$$

$$U_e = \frac{1}{2} \cdot \left(\overline{\sigma_x} \cdot \overline{\varepsilon_x} + \overline{\sigma_y} \cdot \overline{\varepsilon_y} + ... + \overline{\tau_{yz}} \cdot \overline{\gamma_{yz}} \right) dV \quad (3)$$

$$U_{re} = U_e \quad (4)$$

Based on the energy equality rule and the plane of isotropy of the cross anisotropic material described above, the stress-strain relationship is presented in Eq. (5), based on the properties of the homogenised material of step one (the method for the derivation of the elastic properties of RM which are shown in Eq. (5) are given in [10]).

$$\begin{bmatrix} \varepsilon_{XX,RM} \\ \varepsilon_{YY,RM} \\ \varepsilon_{ZZ,RM} \\ \gamma_{YZ,RM} \\ \gamma_{XZ,RM} \\ \gamma_{XY,RM} \end{bmatrix} = \begin{bmatrix} (1/E_{X,RM}) & (-v_{YX,RM}/E_{Y,RM}) & (-v_{ZX,RM}/E_{Z,RM}) & 0 & 0 & 0 \\ (-v_{XY,RM}/E_{X,RM}) & (1/E_{Y,RM}) & (-v_{ZY,RM}/E_{Z,RM}) & 0 & 0 & 0 \\ (-v_{XZ,RM}/E_{X,RM}) & (-v_{YZ,RM}/E_{Y,RM}) & (1/E_{Z,RM}) & 0 & 0 & 0 \\ 0 & 0 & 0 & (1/G_{YZ,RM}) & 0 & 0 \\ 0 & 0 & 0 & 0 & (1/G_{XZ,RM}) & 0 \\ 0 & 0 & 0 & 0 & 0 & (1/G_{XY,RM}) \end{bmatrix} \cdot \begin{bmatrix} \sigma_{XX,RM} \\ \sigma_{YY,RM} \\ \sigma_{ZZ,RM} \\ \tau_{YZ,RM} \\ \tau_{XZ,RM} \\ \tau_{XY,RM} \end{bmatrix} \quad (5)$$

Based on Eqs. (1)-(4), it is concluded that the homogenization procedure is applicable to masonry with irregular unit/vertical joint arrangement, due to the fact that the methodology proposed is based on strain energy equality within a volume of $V=1m^3$ and the volumetric ratio of constituent materials and not on the schematic arrangement.

Step 2: *Homogenisation of stratified material (RM) and horizontal joint (JH)*

At the beginning of step 2 α 90° rotation of the stacked, Raw Mixture (RM) material of step 1 is performed (Fig. 1). Then, new layers of horizontal mortar are introduced, forming a two-layer material that consists of the equivalent RM material of step 1 (units (BR) and mortar of vertical joints (JV)) and horizontally arranged layers of mortar (JH) (Fig.1-Step 2B). The homogenization procedure, as described above at Step 1, is repeated at step 2 and the properties of the final, orthotropic equivalent masonry material are also calculated on the basis of the strain energy equality (Eqs. (1)-(4)). The stress-strain relationship for the final homogenized orthotropic material is presented in Eq. (6).

$$\begin{bmatrix} \varepsilon_{XX,HOM} \\ \varepsilon_{YY,HOM} \\ \varepsilon_{ZZ,HOM} \\ \gamma_{YZ,HOM} \\ \gamma_{XZ,HOM} \\ \gamma_{XY,HOM} \end{bmatrix} = \begin{bmatrix} (1/E_{X,HOM}) & (-v_{YX,HOM}/E_{Y,HOM}) & (-v_{ZX,HOM}/E_{Z,HOM}) & 0 & 0 & 0 \\ (-v_{XY,HOM}/E_{X,HOM}) & (1/E_{Y,HOM}) & (-v_{ZY,HOM}/E_{Z,HOM}) & 0 & 0 & 0 \\ (-v_{XZ,HOM}/E_{X,HOM}) & (-v_{YZ,HOM}/E_{Y,HOM}) & (1/E_{Z,HOM}) & 0 & 0 & 0 \\ 0 & 0 & 0 & (1/G_{YZ,HOM}) & 0 & 0 \\ 0 & 0 & 0 & 0 & (1/G_{XZ,HOM}) & 0 \\ 0 & 0 & 0 & 0 & 0 & (1/G_{XY,HOM}) \end{bmatrix} \cdot \begin{bmatrix} \sigma_{XX,HOM} \\ \sigma_{YY,HOM} \\ \sigma_{ZZ,HOM} \\ \tau_{YZ,HOM} \\ \tau_{XZ,HOM} \\ \tau_{XY,HOM} \end{bmatrix} \quad (6)$$

Step 3: *Deconvolution of stresses (analysis results) of the homogenised material to the stresses of the constituent materials (unit and mortar stresses)*

The properties of the homogenized, orthotropic material calculated on the basis of the two-step homogenisation procedure are used as input material properties of shell elements for the F.E. analysis. Subsequently, analysis is performed and the stresses of the homogenized material are obtained ($\sigma_{,HOM}$). In order to correlate the stresses of the homogenised material to the stresses of constituent materials, namely stone or brick units (BR) and mortar of horizontal (JH) and vertical (JV) joints, the two-step deconvolution procedure described in [9] is proposed. The two steps of the deconvolution procedure are similar to the steps of the homogenisation. Therefore, the stresses at the RM and the mortar of the horizontal joint (JH) are calculated at the first deconvolution step. At the second deconvolution step, stresses of RM are subsequently correlated to stresses at stone or brick units (BR) and mortar of the vertical joints (JV). The matrices for the application of the two-step deconvolution procedure are presented in Eqs. (7), (8). More specifically, the matrices for the first step of the deconvolution procedure i.e. the matrices which relate the stresses in RM and JH with the stresses in the homogenized material are:

$$\begin{bmatrix}\sigma_{XX,RM}\\ \sigma_{YY,RM}\\ \sigma_{ZZ,RM}\\ \tau_{YZ,RM}\\ \tau_{XZ,RM}\\ \tau_{XY,RM}\end{bmatrix}=\begin{bmatrix}\alpha_{11,RM} & \alpha_{12,RM} & \alpha_{13,RM} & 0 & 0 & 0\\ \alpha_{21,RM} & \alpha_{22,RM} & \alpha_{23,RM} & 0 & 0 & 0\\ 0 & 0 & 1 & 0 & 0 & 0\\ 0 & 0 & 0 & 1 & 0 & 0\\ 0 & 0 & 0 & 0 & 1 & 0\\ 0 & 0 & 0 & 0 & 0 & \alpha_{66,RM}\end{bmatrix}\cdot\begin{bmatrix}\sigma_{XX,HOM}\\ \sigma_{YY,HOM}\\ \sigma_{ZZ,HOM}\\ \tau_{YZ,HOM}\\ \tau_{XZ,HOM}\\ \tau_{XY,HOM}\end{bmatrix}\quad \begin{bmatrix}\sigma_{XX,JH}\\ \sigma_{YY,JH}\\ \sigma_{ZZ,JH}\\ \tau_{YZ,JH}\\ \tau_{XZ,JH}\\ \tau_{XY,JH}\end{bmatrix}=\begin{bmatrix}\alpha_{11,JH} & \alpha_{12,JH} & \alpha_{13,JH} & 0 & 0 & 0\\ \alpha_{21,JH} & \alpha_{22,JH} & \alpha_{23,JH} & 0 & 0 & 0\\ 0 & 0 & 1 & 0 & 0 & 0\\ 0 & 0 & 0 & 1 & 0 & 0\\ 0 & 0 & 0 & 0 & 1 & 0\\ 0 & 0 & 0 & 0 & 0 & \alpha_{66,JH}\end{bmatrix}\cdot\begin{bmatrix}\sigma_{XX,HOM}\\ \sigma_{YY,HOM}\\ \sigma_{ZZ,HOM}\\ \tau_{YZ,HOM}\\ \tau_{XZ,HOM}\\ \tau_{XY,HOM}\end{bmatrix} \quad (7)$$

The corresponding matrices for the second step, i.e. the matrices which relate the stresses in BR and JV with the stresses in the RM are:

$$\begin{bmatrix}\sigma_{XX,BR}\\ \sigma_{YY,BR}\\ \sigma_{ZZ,BR}\\ \tau_{YZ,BR}\\ \tau_{XZ,BR}\\ \tau_{XY,BR}\end{bmatrix}=\begin{bmatrix}1 & 0 & 0 & 0 & 0 & 0\\ \alpha_{23,BR} & \alpha_{22,BR} & \alpha_{21,BR} & 0 & 0 & 0\\ \alpha_{13,BR} & \alpha_{12,BR} & \alpha_{11,BR} & 0 & 0 & 0\\ 0 & 0 & 0 & \alpha_{66,BR} & 0 & 0\\ 0 & 0 & 0 & 0 & 1 & 0\\ 0 & 0 & 0 & 0 & 0 & 1\end{bmatrix}\cdot\begin{bmatrix}\sigma_{XX,RM}\\ \sigma_{YY,RM}\\ \sigma_{ZZ,RM}\\ \tau_{YZ,RM}\\ \tau_{XZ,RM}\\ \tau_{XY,RM}\end{bmatrix}\quad \begin{bmatrix}\sigma_{XX,JV}\\ \sigma_{YY,JV}\\ \sigma_{ZZ,JV}\\ \tau_{YZ,JV}\\ \tau_{XZ,JV}\\ \tau_{XY,JV}\end{bmatrix}=\begin{bmatrix}1 & 0 & 0 & 0 & 0 & 0\\ \alpha_{23,JV} & \alpha_{22,JV} & \alpha_{21,JV} & 0 & 0 & 0\\ \alpha_{13,JV} & \alpha_{12,JV} & \alpha_{11,JV} & 0 & 0 & 0\\ 0 & 0 & 0 & \alpha_{66,JV} & 0 & 0\\ 0 & 0 & 0 & 0 & 1 & 0\\ 0 & 0 & 0 & 0 & 0 & 1\end{bmatrix}\cdot\begin{bmatrix}\sigma_{XX,RM}\\ \sigma_{YY,RM}\\ \sigma_{ZZ,RM}\\ \tau_{YZ,RM}\\ \tau_{XZ,RM}\\ \tau_{XY,RM}\end{bmatrix} \quad (8)$$

It must be stressed that the transformation coefficients α in the matrices of Eqs. (7), (8) are calculated on the basis of a methodology which is described in references [10], [13].

3. Methodology for pushover analysis of masonry structures using the homogenization procedure

The proposed methodology is based on iteration procedures which are illustrated in Fig. 3 and Fig. 4. The main features of this methodology are:

- The horizontal loads for pushover analysis are calculated on the basis of the distribution of the structure's mass. Thus, horizontal forces are introduced at the nodes of the F.E. model, proportional to their masses. Therefore, the inertial discretization of structure and the calculation of the lumped masses at its nodes are essential.
- The non-linear behavior of the masonry is approached by means of the progressive exclusion of the masonry's components (bricks and mortar) that exceed their tensile strength. Thus, after each failure a new homogenization procedure is required in order to calculate the new equivalent materials for the 2D-shell elements.

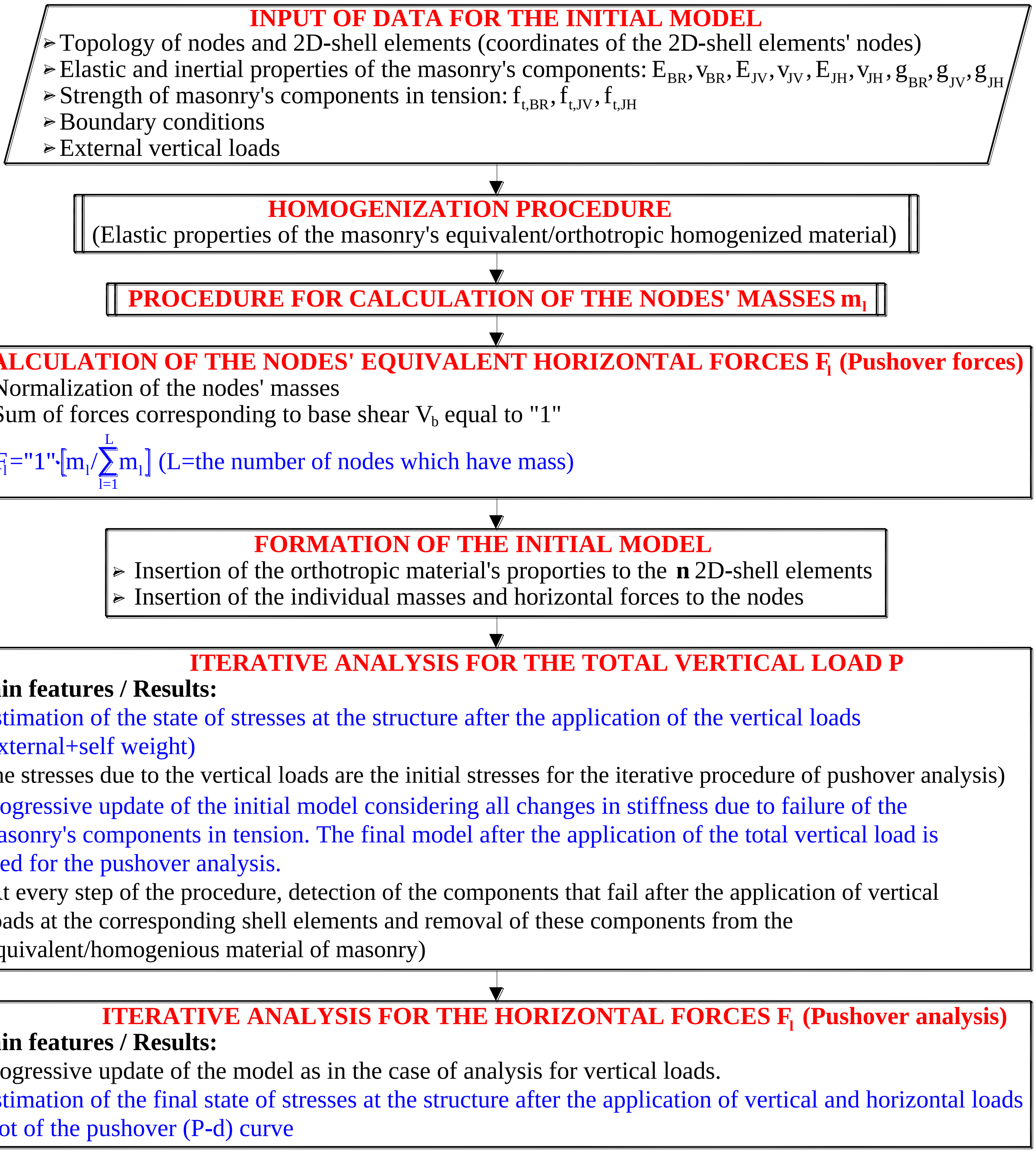


Fig. 3 – Outline of the main steps of the proposed methodology

The steps of the iterative procedures for the application of vertical and horizontal (pushover) loads, are presented in detail in Fig.4. The basic aspects of these procedures are summarized below:

- The deconvolution procedure allows for the discrete check of exceeding (or not) each of the masonry components' strength in tension, instead of checking the exceedance of the composite material's tensile strength that requires several assumptions. Therefore, both the failure mode and the critical components are defined and checked explicitly.
- The check of exceeding (or not) of each of the masonry components' strength in tension is performed using the principal stresses. Therefore, a more accurate approach of the stress state of the masonry is utilized.

ITERATIVE ANALYSIS FOR THE TOTAL VERTICAL LOAD P

(a) Analysis → Tensors of homogenized stresses $\boldsymbol{\sigma}_{HOMj,i}$ (j=1÷**n**) [The first step (i=1) concerns the initial model]

(b) Formation & initialization of "Damage matrices" $Dm_{j,i}=[F_{BR}\ F_{JV}\ F_{JH}]$ F_p=1/0 → Failure/No Failure of component p

(c) For each one of the **n** shell elements:

(i) Deconvolution of the tensors of homogenized stresses: $\boldsymbol{\sigma}_{HOMj,i} \rightarrow \boldsymbol{\sigma}_{BR,j,i}, \boldsymbol{\sigma}_{JV,j,i}, \boldsymbol{\sigma}_{JH,j,i}$

(ii) "Rotation" of tensors to their principal directions: $\boldsymbol{\sigma}_{BR,j,i}, \boldsymbol{\sigma}_{JV,j,i}, \boldsymbol{\sigma}_{JH,j,i} \rightarrow \boldsymbol{\sigma}_{I\text{-}II,BR,j,i}, \boldsymbol{\sigma}_{I\text{-}II,JV,j,i}, \boldsymbol{\sigma}_{I\text{-}II,JH,j,i}$

(iii) Iterative procedure (with k loops, k=1÷k_j) for the detection of the minimum value of multiplier of tensors (parameter $\lambda_{minj,i}$) which leads to the first failure of one of the masonry's components in tension:

$\sigma_{t,BR,j,i}, \sigma_{t,JV,j,i}, \sigma_{t,JH,j,i} \rightarrow$ if $\boxed{\lambda_{j,k,i}\cdot\sigma_{t,BR,j,i}=f_{t,BR} \text{ or } \lambda_{j,k,i}\cdot\sigma_{t,JV,j,1}=f_{JV} \text{ or } \lambda_{j,k,i}\cdot\sigma_{t,JH,j,i}=f_{JH}} \rightarrow \boxed{\lambda_{minj,i}=\lambda_{j,k,i}}$

(d) Detection of the minimum $\lambda_{minj,i} \rightarrow \boxed{\lambda_i}$ (Detection of the component that fails and the corresponding shell element)

➤If $\lambda_i<1$ → **(e1)** The masonry's component that fails is removed from the corresponding shell element

(e2) New homogenization procedure for the estimation of this element's equivalent material properties

(e3) Update of the structural model

(e4) Update of the $Dm_{j,i}$ of the element with component failure (i.e. the shell element which extracts the λ_i) → Replacement of 0 with 1 for the masonry's component that fails

(e5) Multiplication of the **n** tensors $\boldsymbol{\sigma}_{HOMj,i}$ with the parameter $\lambda_i \rightarrow \boldsymbol{\sigma}_{final,j,i}=\lambda_i\cdot\boldsymbol{\sigma}_{HOMj,i}$

(e6) Storage of tensors $\boldsymbol{\sigma}_{final,j,i}$ for utilization as initial stresses in the step i+1 [i.e. the tensors $\boldsymbol{\sigma}_{final,j,i}$ will be added to the tensors $\boldsymbol{\sigma}_{HOMj,i+1}$ of analysis of the next iteration's step (i+1)]

$$\boxed{\text{Generally: } \boldsymbol{\sigma}_{initial,j,i}=\boldsymbol{\sigma}_{final,j,i-1}=\sum_{i=1}^{i-1}(\lambda_i\cdot\boldsymbol{\sigma}_{HOMj,i}) \quad [\text{For } i>1, \text{ while for } i=1: \boldsymbol{\sigma}_{initial,j,1}=\mathbf{0}]}$$

(f) Iteration of steps **(a), (b), (c), (e1)-(e6)** [Step i+1]

➤If $\lambda_i>1$ → **(e1)** The analysis for vertical load P is terminated.

(e2) The stored tensors $\boldsymbol{\sigma}_{final,j,i-1}$ of the previous analysis (which extracts $\lambda_i<1$) is used as initial stresses for analysis for horizontal forces. The same holds for the model AND "Damage matrices" $Dm_{j,i-1}$.

(a)

ITERATIVE ANALYSIS FOR THE HORIZONTAL FORCES F_i (Pushover analysis)

(A) Definition of a control node for the monitoring of the reference horizontal displacement of structure → Node **c**

(B) Step (i_H) → Analysis for the load pattern of the equivalent normalized forces F_i of model's nodes (V_b="1")

• For i_H=1 the analysis concerns the updated model which arises from the last step of analysis for the vertical load P

• From this analysis the following are exported → The tensors of homogenized stresses: $\boldsymbol{\sigma}^{(F_i)}_{HOMj,i_H}$ (j=1÷**n**)

→ The horizontal displacement of the node **c**: u_{c,i_H}

(C) Performance of steps similar to **(c), (d)** → Extraction of the value of the multiplier λ_{H,i_H}

(D) Post-proccesing of the results of analysis i_H → Calculation of the part of:

→ The tensors of homogenized stresses: $d\boldsymbol{\sigma}^{(F_i)}_{final,j,i_H}=\lambda_{H,i_H}\cdot\boldsymbol{\sigma}^{(F_i)}_{HOMj,i_H}$ (j=1÷**n**)

→ The horizontal displacement of node **c**: $du_{c,i_H}=\lambda_{H,i_H}\cdot u_{c,i_H}$

→ The base shear: $dV_{b,i_H}=\lambda_{H,i_H}\cdot\text{"1"}=\lambda_{H,i_H}$

(E) Calculation AND storage of the total hor. displacement of node **c** after all steps until step i_H: $u^{(tot)}_{c,i_H}=\sum_{i_H=1}^{i_H}(\lambda_{H,i_H}\cdot du_{c,i_H})$

➤If $u^{(tot)}_{c,i_H}<u_{c,max\text{-}permited}$ ($u_{c,max\text{-}permited}$=pre-defined maximum allowed horizontal displacement of the control node **c**)

(F1) Performance of steps similar to **(e1)-(e4)** [For case $\lambda_i<1$]

(F2) Calculation AND storage of the total base shear after all steps until step i_H: $V^{(tot)}_{b,i_H}=\sum_{i_H=1}^{i_H}(\lambda_{H,i_H})$

(F3) Calculation AND storage of the total stresses after all steps until step i_H: $\boldsymbol{\sigma}^{(F_i)}_{final,j,1}=\sum_{i=1}^{i_H}(\lambda_{H,i_H}\cdot\boldsymbol{\sigma}^{(F_i)}_{HOMj,i_H})+\sum_{i=1}^{i_{tot,P}}(\lambda_i\cdot\boldsymbol{\sigma}^{(P)}_{HOMj,i})$

The values $u^{(tot)}_{c,i_H}$ and $V^{(tot)}_{b,i_H}$ constitute the coordinates of the point of pushover curve which corresponds to step i_H

(G) Iteration of steps **(B), (C), (D), (E), (F1)-(F3)** [Step i_H+1]

➤If $u^{(tot)}_{c,i_H}>u_{c,max\text{-}permited}$ → The analysis is terminated and the last point of the pushover curve is the point of step i_H-1

(b)

Fig. 4 – Outline of the iterative procedure for (a) the vertical and (b) the horizontal (pushover) loads

4. Examples and comparison with experimental results

In the current section the proposed methodology is applied and tested using as reference the results of the experimental tests that were carried out by Raijmakers and Vermeltfoort (as described in [18]) and the numerical tests of Lourenço & Rots [18]. In the framework of these tests the shear wall illustrated in Fig. 5 was one of the specimens. This wall was firstly subjected to a vertical load q (uniformly distributed), and then to a monotonically increased horizontal force F_H. The horizontal force was increased under top displacement control until failure. The data of the specimen are briefly presented in Fig. 5. It must be noted that the strength of masonry's components in tension was estimated equal to 2,0MPa for bricks and the tension strengths of the horizontal and vertical mortar were assumed equal to 0.25MPa and 0.16MPa respectively

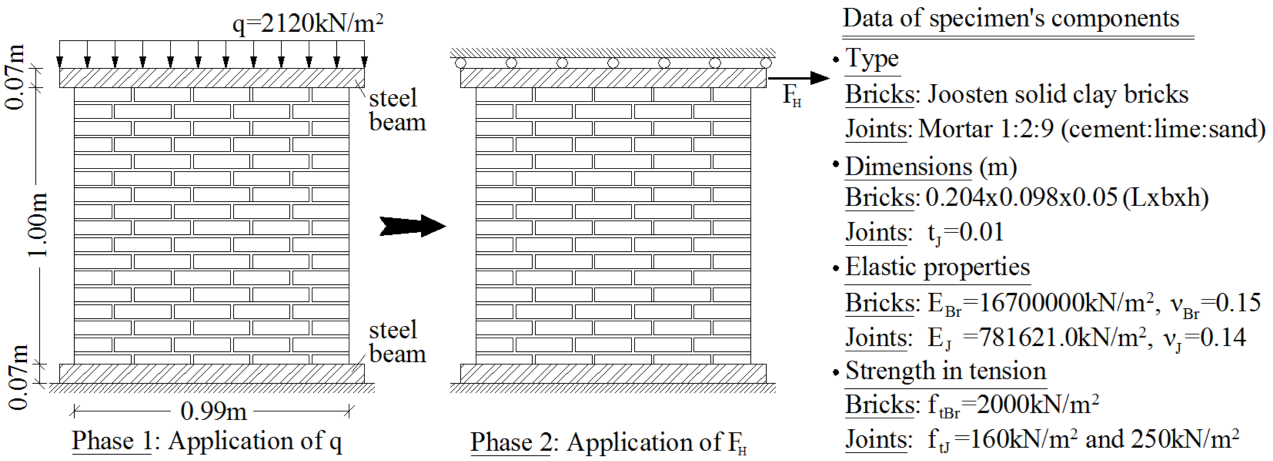


Fig. 5 – The data of the wall specimen utilized as reference of the proposed method's results

Regarding the FE modeling of the specimen, it must be noted that the shear wall was modeled using 4-noded shell elements in a uniform mesh. The rationale of the selection of the shell elements' dimensions is illustrated in Fig 5. As it can be concluded from the figure, the modeling of the shear wall was based on the consideration of "units" i.e. elements which include one brick and the half of the thickness of the surrounding (vertical and horizontal) joints. This assumption is valid since the homogenization technique is used.

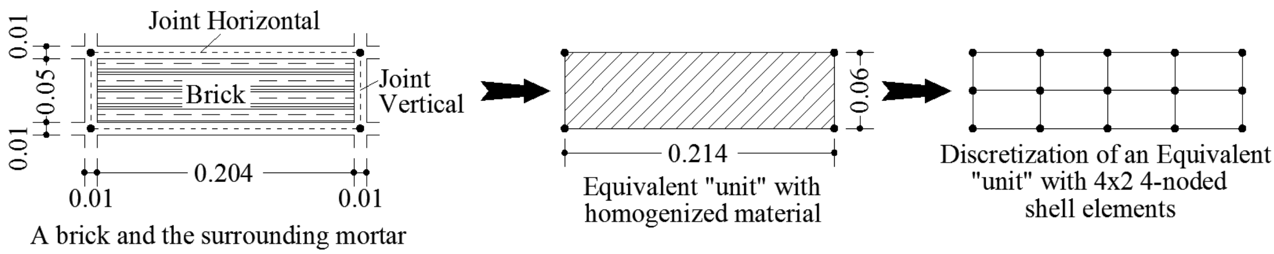


Fig. 6 – The utilized technique for the discretization of the analyzed shear wall

The modeling of the sequence of the specimen's loading (Phase 1 and Phase 2, Fig. 4) was carried out using the Open Application Programming Interface (OAPI) features, which permit the data exchange between Matlab [16] and SAP2000 [17]. More specifically, a code in Matlab platform was developed, implementing the flow chart of Figs. 3 and 4. To this end, the code modifies the wall's FE model on the basis of the stress field at each step of loading and calls SAP2000 for the solution of the updated model. Thus, the sequence (modification)-(solution)-(re-modification) of the wall's FE model finally leads to the extraction of the Load-Displacement (pushover) curve. As depicted in Figures 7 and 8, both pushover curves and failure modes are in satisfactory agreement, considering both the different assumptions and the dependence of results on the modeling assumptions (mesh, etc).

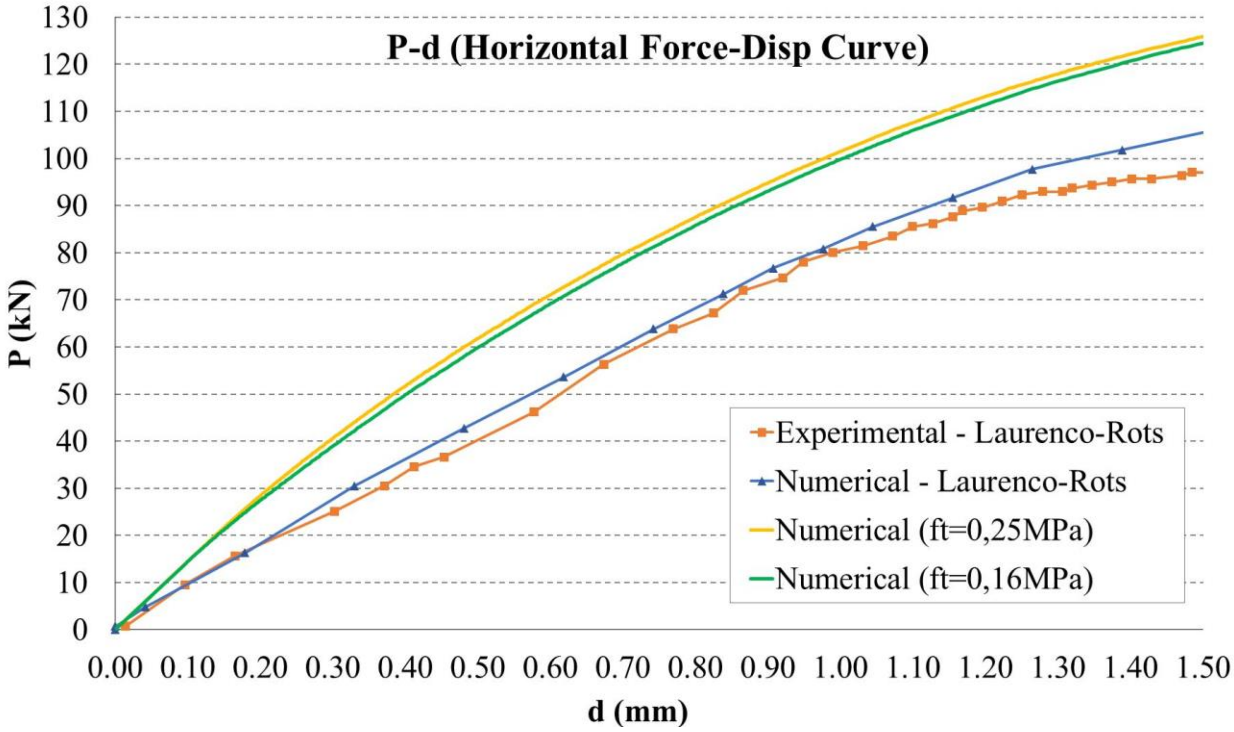
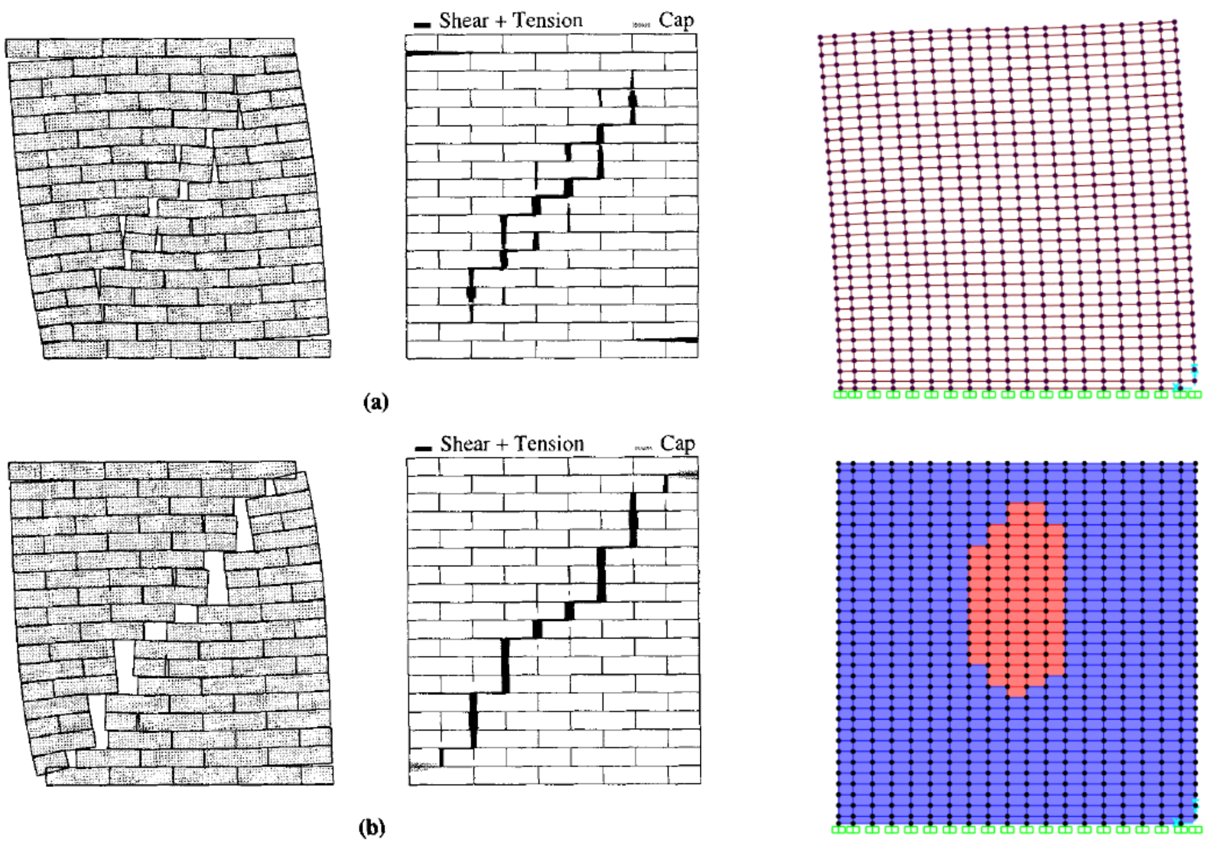


Fig. 7 – Pushover Curves according to the proposed methodology and Lourenço and Rots



Failure Modes (Lourenco & Rots [18]) Displ. & Failure Modes (Proposed model)

Fig. 8 – Failure modes and displacement shape according to the proposed methodology and Lourenco-Rots

5. Conclusions

A new methodology for the inelastic static (pushover) analysis of structures is proposed herein and applied using the homogenization technique for masonry modeling. Furthermore, a software code is developed and the methodology proposed is applied for the pushover analysis of a previously experimentally tested solid wall. The most important findings from the application of the methodology are summarized below:

- The homogenization technique can be easily applied for the modeling of masonry structures, developing a software code. The scope of the homogenization procedure is not only to estimate the properties of the homogenized material that will be used for analysis and the calculation of homogenized strains and stresses (in a 2-step procedure), but also to form a deconvolution methodology as well, in order to be able to calculate the principle stresses developed at each constituent material and check them with specific failure criteria.
- The deconvolution procedure allows for the discrete check against exceeding (or not) each of the masonry components' strength in tension, instead of checking the exceedance of the composite material's tensile strength that requires several assumptions. Therefore, both the failure mode and the critical components are defined and checked explicitly.
- The non-linear behavior of the masonry is approached by means of the progressive failure of the masonry's components (bricks and mortar) that exceed their tensile strength. Thus, after each failure a new homogenization procedure is required in order to calculate the equivalent material for the 2D-shell elements.
- Inelastic pushover curves for a solid wall, the estimated failure modes and their evolution are in satisfactory agreement with existing experimental results, considering both the different assumptions and the dependence of results on the modeling assumptions.

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