

# DESIGN OF NOVEL VIBRATING BARRIERS FOR THE SEISMIC PROTECTION OF THE MESSINA CATHEDRAL

P. Cacciola<sup>(1)</sup>, I. Caliò<sup>(2)</sup>, N. Catania,<sup>(1,2)</sup>, N. Fiorini<sup>(3)</sup>, P. Morris<sup>(1)</sup>, G. Occhipinti<sup>(4)</sup>, D. Spina<sup>(3)</sup>, A. Tombari<sup>(1)</sup>

<sup>(1)</sup> School of Environment and Technology University of Brighton, UK, {p.cacciola, a.tombari, n.catania, p.e.morris}@brighton.ac.uk

<sup>(2)</sup> Department of Civil Engineering and Architecture, University of Catania, Italy, icalio@dica.unict.it

<sup>(3)</sup> Department of Civil Protection - Seismic Risk Office, Italy {noemi.fiorini, daniele.spina}@protezionecivile.it

(4) Institute of Environmental Geology and Geoengineering, (CNR) Rome, Italy, giuseppe.occhipinti@igag.cnr.it

## Abstract

Heritage buildings are amongst the most vulnerable structures affected by natural hazards due to their inherent historic nature with regards to materials and construction techniques. Although traditional strengthening and vibration control strategies are technically sound, they might face serious challenges such as the compatibility of materials between new and historic construction and the requirements of the structural intervention. For heritage structures, these challenges are a clear barrier to the seismic protection, and the use of such techniques is therefore limited. In this context, a novel non-invasive passive control device called the Vibrating Barrier (ViBa), has been recently proposed. The ViBa is a large-scale oscillating mass-spring-damper unit contained in the ground and tuned to mitigate the motion of surrounding structures under earthquake-induced ground motion, without being directly in contact to them, through a structure-soil-structure interaction mechanism. The effectiveness of the ViBa device to protect various idealized structures and clusters of buildings has been proved in previous contribution co-authored by the first and last author. In this contribution, a real existing structure is investigated: the Messina Cathedral. The Cathedral was originally built during Norman time (XI century), and was destroyed by the Messina earthquake in 1908. The Cathedral was rebuilt shortly after by adopting a mixed masonry-reinforced concrete structure that for the bell tower was realized following the innovative confined masonry typology recommended by a structural code just after the Messina earthquake. Currently, the Cathedral bell tower hosts the largest and most complex mechanical and astronomical clock in the world. The design of the Vibrating Barrier for this real case scenario has required the following steps: i) the structural identification of the structure and the development of a consistent FE model; ii) the definition of a realistic ground motion model due to the high seismicity of the area and iii) the calibration of the ViBa's unknown mechanical parameters to minimize the dynamic response. Ambient vibration tests have been performed and a permanent monitoring system has been recently installed in the bell tower by the Department of Civil Protection (Seismic Risk Office) allowing the calibration of a reliable FE model including soilstructure interaction effects. A stochastic approach has been used to determine a pertinent ground motion model for determining the probability of exceedance of a selected response parameter consistent with the Response Spectrum at the site. Multiple ViBas have been designed to protect both the bell tower and the Cathedral. A novel Vibrating Barrier has been also designed to control both translational and rocking behavior of the bell tower. Numerical results are presented to show the effectiveness of the ViBa technology to cope with complex real case scenarios offering a novel viable strategy to reduce the seismic risk of existing structures, from future earthquakes, without altering their heritage value.

Keywords: Vibrating Barrier (ViBa), Messina Cathedral, structure-soil-structure interaction, vibration control, rocking



## 1. Introduction

The design and the construction material of historic structures in seismic prone areas are generally not adapted to withstand earthquakes and decay occurring over the years. In addition, climate change further modifies the seismic vulnerability of historic areas and current models and technologies do not take into account this effect. Many historic areas have also been affected by past earthquakes making them even more vulnerable. The most recent earthquakes, such as the ones occurring in Central Apennines (Italy) during the 2016-2017 seismic sequence, responsible for the collapse of the Cathedral of S. Benedetto at Norcia, are clear examples of the need for prompt and collective action to protect heritage structures from further natural disaster-induced damage. Historic structures are vital to our understanding of how the cultural, artistic, and technical skills of humanity have developed over time. Up to now the problem of protecting existing structures by seismic action has been managed using retrofitting strategies directly applied to the construction. Apart from few attempts to protect existing structures the use of vibration control devices is still restricted to new buildings and/or constructions. One main reason is that the introduction of control devices in existing structures is too invasive, costly and requires the demolishing of some structural and/or non-structural components. For heritage structures clearly such technologies cannot be applied and therefore rarely seismic protection actions are taken to protect such artistic treasure. Bearing in mind the global necessity to protect existing structures from earthquakes and the limitation of current technologies the novel vibrating barrier (ViBa) control strategy has been recently proposed [1]. The ViBa device is a massive structure, hosted in the soil, calibrated for protecting structures by absorbing a significant part of the ground motion input energy. As a difference with other technologies buried in the soil (i.e. trenches, piles, seismic metamaterials) that are focused on surface waves only (see. [2,3]) the ViBa is designed to absorb seismic body waves (i.e. shear waves). In its simplest configuration it is made by a mass-spring system buried in the soil and able to vibrate. The concept is based on the generally known structure-soil-structure interaction (SSSI) and on the findings of the first works of Warburton et al. [4] and Luco and Contesse [5]. Up to now the ViBa has been applied to various case studies. Specifically, Cacciola et al. [6] investigated the potential of ViBa for the seismic protection of monopiled structures, Tombari et al. [7] considered ViBa to mitigate seismic risk of a nuclear reactor, and Tombari et al. [8] explored the efficiency of the ViBa to protect a cluster of buildings.

This paper investigates the effect of the possible application of a ViBa on a realistic model of an existing heritage structure: the Messina Cathedral and its bell tower (Figure 1 a). The Cathedral was originally built in the Norman time (XI century) and was destroyed by the Messina earthquake in 1908. The Cathedral was rebuilt in mixed reinforced concrete-masonry just after the earthquake and again damaged during the World War II. Only the crypt (Figure 1b) and parts of the façade and apses still preserves their original magnificence. The bell tower was also rebuilt in mixed reinforced concrete-masonry and inaugurated in 1933. The bell tower hosts the largest and most complex mechanical and astronomical clock in the world.



Fig. 1 – Messina Cathedral a) and its crypt b)



Both the Cathedral and the bell tower clearly possess inherent modelling difficulties due to the composite structural typology and their historic evolution. In this paper, a linear finite element formulation is used for modelling the Cathedral, the bell tower and the soil. The bell tower eigenproperties have been identified through the measurement of environmental noise and have been used to calibrate the basic FE model structural parameters. The design of the ViBas has been then performed considering various configurations. Moreover, due to the identified rocking behavior of the bell tower, a novel device able to control both translation and base rotation has been also proposed. Results show how the proposed ViBas technology, if correctly calibrated, can lead to a relevant reduction of the structural response providing a seismic risk mitigation of the heritage structure without altering the original structural layout.

## 2. Messina Cathedral numerical model

The Messina Cathedral numerical model has been implemented by considering the detailed geometry reported in the original technical drawing and reports made available by the Archdioceses of Messina-Lipari and S.Lucia del Mela. The coupled soil-structure FE model of the Cathedral – Bell tower and the soil underneath has been then implemented in SAP2000. Figure 2 shows a 3D view of the model. The soil deposit is characterized by a shear wave velocity of Vs =300 m/s (ground type C) and a depth of about 60m. The significant size assumed for the soil model has been determined to avoid boundary effects on the free field ground motion. The main structure of the Cathedral has been modelled by using solid 3D elements for the foundation, shell 2D elements for the walls and the roofs, and beam 1D elements for the roof structural parts and the columns of the main nave and the crypt. Mechanical parameters of the materials used in the model have also set using original technical documents (see e.g. [9]). Both the Cathedral and the bell tower are therefore mixed reinforced concrete – masonry structures. The Cathedral, moreover, preserves part of the ancient stone masonry structure of the crypt and part of the main façade.



Fig. 2 – FE of the Messina Cathedral and Bell Tower

### 2.1 Structural identification of the bell tower

Experimental measurement of environmental noise has been performed using 30 unidirectional ICP piezoelectric accelerometers with a measurement range of  $\pm$  0.5 g and a resolution of 10<sup>-6</sup> g in the bandwidth 1Hz-200Hz. Four LMS SCADAS XS data acquisition systems have been employed to manage the signals. The transducers have been located on the floor next to the columns (See Figure 3). Each of them measures two acceleration signals in perpendicular directions. Vertical component has been also measured to capture rocking behaviour of the tower. Two set of couple of sensors for each floor have been used considering various setups. One typical example is reported in Figure 4.



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Fig. 3 - Accelerometers used in the environmental noise measurement



Fig. 4 – Example of measurement set-up

The Cathedral was rebuilt shortly after the Messina earthquake by adopting a mixed masonry-reinforced concrete structure that for the bell tower was realized following the innovative confined masonry strategy recommended by a structural code just after the Messina earthquake [10]. Preliminary measurement highlighted the needs to consider the external masonry walls as structural elements in the FE model and confirmed a rocking behaviour of the tower. The FE model of the tower developed in this paper is, therefore, an upgraded version of the one presented in [9]. Moreover, a permanent monitoring system has been also installed following the configuration presented in Figure 5. The monitoring system is a GeoSig Digital Sensor System composed of 2 biaxial and 3 triaxial digital force-balance accelerometers with a measurement range of  $\pm 2$  g and a resolution of 0.8 x 10<sup>-6</sup> g in the bandwidth 1Hz-200Hz. The triaxial accelerometer placed on the ground, connected to the others by an ethernet cable, in endowed with a 24-bit AC converter and an UMTS router for data transmission.

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Fig. 5 – Map of the permanent monitoring system installed in the Messina Cathedral Bell Tower.

## 2.2. Modal Analysis

Mechanical properties of the bell tower have been determined through a basic model updating procedure to minimize the difference between the numerical and experimental modal data. Only the masonry mechanical parameters have been updated in the optimization process as the other properties have been retrieved by original technical documents. Those mechanical properties have been then extended to the Cathedral's masonry. On the other hand, the ancient stone masonry, in absence of experimental values, has been set using literature values. Modal analysis has been then performed on the FE model. Table 1 shows the comparison between experimental and numerical modal frequencies.

Mode	Experimental Frequencies (Hz)	Numerical Frequencies (Hz)
1 (Bell Tower)	1.46	1.38
2 (Bell Tower)	1.49	1.43
3 (Cathedral)	N/A	1.91
4 (Cathedral)	N/A	2.61
5 (Cathedral)	N/A	2.76

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Mode shapes of the Cathedral and the bell tower are also reported in Figures 6. The first 2 mode shapes concern the bell tower only. Both mode shapes and natural frequencies are in good agreement with the experimental results. Modes 3-5 refer to the Cathedral, not yet validated by experimental tests.

## **3. Design of the Vibrating Barriers**

The Vibrating Barrier (ViBa) is a large-scale oscillating mass-spring-damper unit buried in the ground and tuned/designed to protect surrounding structures without being directly in contact to them but exploiting a structure-soil-structure interaction mechanism. In order to design the Vibrating Barrier, therefore, the first step is to identify potential locations where the ViBa might be buried. For the Messina Cathedral the location of the ViBas has been selected considering the space available and the numerically evaluated eigen properties of Cathedral. For protecting the bell tower, it was selected the area available between the tower and the Cathedral. Figure 7 shows the ViBa locations considered in this paper. The depth of the ViBa foundations has been selected at the same level of the Cathedral foundation in order to increase the structure soil-structure mechanism. Clearly different set-up and number of ViBas can be adopted. The challenges involved in an optimization process aimed to find the best location and the minimum number of ViBa to be adopted are related to the fact that do not exist nowadays explicit solution for the structure-soil-structure interaction between two foundations of general shapes, as a consequence, numerical investigations are required. In order to protect the Cathedral four unidirectional ViBas have been located along the principal directions (X and Y). Therefore, four Vibas are designed for the main Cathedral structure and an additional ViBa is specifically designed to control the Bell Tower in both the principal directions, as reported in Figure 7. The calibration of the ViBas' springs is done through simplified models (see e.g. [1] and [9]). Figures 8 and 9 show the lumped discrete models used for the Cathedral and the Bell Tower respectively.



Fig. 6 -Mode shapes 1 to 5 of the Messina Cathedral and Bell Tower a) 3D view; b) top view



Fig. 7 –ViBas location (rectangles)) and control points (circles)

Note that the preliminary design of the device is done through the simplest model accounting for the structuresoil-structure interaction mechanism. Specifically, the discrete model used for the Cathedral is 4DoFs system accounting four translational displacements only, while a 6DoF system has been developed for the coupled ViBa-bell tower system. The latter includes rotational degrees of freedom of both the foundation of the Tower and the ViBa and furthermore takes into account the rotational and translational coupling of soil stiffness and the structures. The superstructures of both the Cathedral and the Bell Tower is a SDOF system with mass given by the participation mass of the fundamental mode in the direction of the input and stiffness calibrated using the pertinent modal frequency. The foundation masses are taken directly by the FE model.



Fig.8 Discrete model for the coupled Cathedral - ViBa system

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Fig. 9 - Discrete model for the coupled bell tower - ViBa system

The soil springs have been determined through the direct stiffness method. Once defined the simplified numerical models for both the bell tower and the Cathedral coupled with the ViBa devices it can be seen that unknowns of the problem are the ViBas masses  $m_{ViBa}$ , and stiffness  $\tilde{k}_{ViBa}$ . The optimization procedure used for the design of the ViBas is based on the minimization of the 50% fractile of the absolute displacement of the SDOF system. Considering the ground motion at the bedrock zero-mean Gaussian stationary process, due to the linearity of the system, the response of the coupled system will be also zero mean and Gaussian. Therefore, it is fully defined by the response power spectral density matrix, that is:

$$\boldsymbol{G}_{\boldsymbol{U}\boldsymbol{U}}(\omega) = \boldsymbol{H}(\omega) \cdot \boldsymbol{H}^{*}(\omega) \cdot \boldsymbol{G}_{\boldsymbol{u}\boldsymbol{g}}(\omega) \tag{1}$$

where \* is the complex conjugate transpose,  $G_{UU}(\omega)$  is the power spectral density function of the absolute response displacement,  $G_{ug}(\omega)$  is the power spectral density function of the ground displacement process and  $H(\omega)$  is the transfer function given by

$$H(\omega) = \widetilde{K}_{dyn}(\omega)^{-1} \cdot Q \tag{2}$$

with

$$\widetilde{K}_{dyn}(\omega) = \widetilde{K} - \omega^2 M \tag{3}$$

where  $\tilde{K}$  and M are the complex stiffness and the mass matrix of the coupled stiffness, respectively and Q is given by

$$\boldsymbol{Q} = \widetilde{\boldsymbol{K}}\boldsymbol{\tau} \tag{4}$$

 $\tau$  being the incidence vector. Note that complex stiffness matrix  $\tilde{K}$  includes damping effects. The power spectral density matrix of the response lists both the response of the ViBa and the structure to be protected, then it can be used to minimize the maximum response of the structure by calibrating the ViBa structural parameters. The fractile of order *p* of the distribution of the maxima of the absolute displacement of a specified degree of freedom of the structure has been selected as the parameter to be minimized. Clearly different



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response parameters can be used (e.g. base shear, bending moment, etc.). Specifically, the top displacement of the SDOF superstructure is selected to be minimized (see Figures 8 and 9), that is

$$X_U = \eta_U \ (T_s, p) \cdot \sqrt{\lambda_{0,U}} \tag{5}$$

where  $\eta_U$  ( $T_s$ , p) is a peak factor depending on the order of the fractile p and the time of the observing window  $T_s$ , while  $\lambda_{0,U}$  is the zero-order response spectral moment. The peak factor is computed by the equation defined by Vanmarcke [11]:

$$\eta_U (T_s, p) = \sqrt{2ln \left\{ 2N_U \left[ 1 - exp \left[ -\delta_U^{-1.2} \sqrt{\pi ln(2N_U)} \right] \right] \right\}}$$
(6)

with

$$N_U = \frac{T_S}{-2\pi \ln p} \sqrt{\frac{\lambda_{2,U}}{\lambda_{0,U}}}$$
(7)

and

$$\delta_U = \sqrt{1 - \frac{\lambda_{1,U}^2}{\lambda_{0,U} \lambda_{2,U}}}$$
(8)

where the response spectral moment  $\lambda_{i,U}$  are computed with the following equation:

$$\lambda_{i,U} = \int_0^{+\infty} \omega^i G_{U \ U} \ (\omega) d\omega \tag{9}$$

 $G_{U \ U}(\omega)$  being the power spectral density of the absolute displacement of structure,  $T_S = 10$ s is the time observing window taken identical to the duration of the strong motion phase and p=0.5 represents the 50% fractile (i.e. the median value). The optimization process has as a target to determine the optimal ViBa's stiffness of the ViBa for a selected mass so to reduce the 50% fractile of the absolute maximum displacement  $X_U$ :

$$\min\{X_U \ (\alpha)\}, \ \alpha = \{k_{ViBa}\} \in \mathbb{R}_0^+$$
 (10)

Constant damping has been assumed equal to 5% for all the modes.

#### 4. Numerical Results

The seismic input is modelled as quasi-stationary zero mean Gaussian process compatible with the Pseudo Acceleration Response Spectrum (RSA) defined by Italian Code (NTC2008) at the site where the Cathedral is built. In this regard the response-spectrum-compatible power spectral density function  $G_{\ddot{u}_g}(\omega)$  has been determined using the procedure proposed by Cacciola et al.[12]. Figure 10 shows the response-spectrum-compatible power spectral density function between the average response spectrum (100 samples) and the target one defined by the NTC2008 to satisfy the response spectrum compatibility criteria.



Fig 10: Power Spectral Density function a) and comparison of target and simulated response spectra b) for the Messina Cathedral site.

The simulated ground motion time histories have been deconvoluted and applied to the bedrock in both the principal directions as two independent analyses. Only the results for the Y direction are presented in this paper. The masses of the three ViBas acting in the Y direction is set as  $40.39 \times 10^6$  kg with stiffness provided in Table 2.

Stiffness	x10 <sup>9</sup> N/m
k <sub>V1</sub>	15.80
k <sub>V2</sub>	17.08
kv3	9.11

Table 2 -- ViBas stiffness

The ViBa pertinent to the bell tower has mass 1070900 kg and stiffness in both X and Y directions. Note that, the masses used in this paper are ideal and assigned to reach a reduction of above 20% to the selected response parameters. Those masses can be strongly reduced for practical purposes by incorporating an inerter within the ViBa device as shown by Cacciola et al.[13]. The analyses have been conducted first on the simplified model first to reach an optimal configuration of the ViBa. After the optimal ViBa parameters have been identified they have been implemented in the large Finite Element model. Figures 11a-b-c-d show the result of the comparison between the response power spectral density functions of the accelerations determined at the selected points (see Figure 7) by a Monte Carlo study performed on the large FE model. The curves have been determined by averaging the responses to 100 samples for both the cases with and without the ViBas. As it can been seen from all the Figures there is an evident reduction of both the overall area and the response peak, due to the presence of the ViBas. Randomly selected samples of the response acceleration trajectories are also reported in Figure 12 for the same selected points. As it can be seen also for those figures there is an evident reduction of the response figures there is an evident reduction of the response acceleration trajectories are also reported in Figure 12 for the same selected points. As it can be seen also for those figures there is an evident reduction of the response figures there is an evident reduction of the selected figures there is an evident reduction of both the selected figures there is an evident reduction of the response acceleration trajectories are also reported in Figure 12 for the same selected points. As it can be seen also for those figures there is an evident reduction of the response in all the points.

### 5. Concluding Remarks

In this paper the design of the Vibrating Barrier for the Messina Cathedral and its bell tower has been addresses. A FE model has been first determined using data from original drawing and technical reports. The model has been calibrated using environmental noise measurement undertaken on the Bell Tower. The importance of the external masonry walls on the stiffness characteristics of the bell tower has been pointed out. The eigenproperties of the numerical model pertinent to the bell tower are in good agreement with those determined with the experimental results. Potential ViBas' locations have been identified and a simplified procedure able



to readily determine the unknown design parameter has been proposed. Specifically, the structures are modelled as single degrees of freedom having frequency identical to a selected mode and mass provided by the pertinent mass participation.



Fig. 11 – Power spectral densities of response accelerations at the selected control in presence of the ViBas (solid line) and without the ViBas (dotted lines): a) Point A, b) Point B, c) Point C and d) Point D.



Fig. 12 –Response accelerations at the selected control in presence of the ViBas (solid line) and without the ViBas (dotted lines) : a) Point A, b) Point B, c) Point C and d) Point D.



The parameters of the lumped model are determined through the direct stiffness methods for simplicity sake. Clearly more refined interaction models can be defined but for this particular study the simplified models provided predesign parameters with a high level of accuracy. Moreover due to the rocking mechanism manifested in the fundamental modes of the bell tower the traditional ViBa device have been upgraded for accounting for the rotation of the base foundations. Reductions of above 20% the response determined in the full scale FE model have been observed in all the control points. It is noted the masses of the devices are too large to be implemented in a real case scenario and need to be considered for academic purposes. However the adoption of the inerters in the ViBa when such large masses need to be implemented can make those solutions feasible. The paper wanted to offer an alternative strategy to the seismic protection of heritage structures through a non-invasive approach, from the results presented it seems that the ViBa can offer a promising alternative whereas traditional and well consolidated techniques cannot be applied.

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