DYNAMIC MONITORING OF CULTURAL HERITAGE ASSETS: THE BELL TOWER OF SAN FRANCESCO CHURCH IN PISA (ITALY)

E. Lorenzo[(1)], R. M. Azzara[(2)], G. Chellini[(3)], E. Cieri[(4)], A. De Falco[(5)], C. Resta[(6)], G. Sevieri[(7)]

[(1)] *PhD Student, Department of Civil and Industrial Engineering, University of Pisa, Italy, emanuele.lorenzo@unifi.it*
[(2)] *Researcher, Istituto Nazionale di Geofisica e Vulcanologia, Osservatorio Sismologico di Arezzo, Italy, riccardo.azzara@ingv.it*
[(3)] *PhD, Department of Civil and Industrial Engineering, University of Pisa, Italy, giuseppe.chellini@ing.unipi.it*
[(4)] *Eng., Department of Civil and Industrial Engineering, University of Pisa, Italy, emilianocieri@gmail.com*
[(5)] *PhD Assistant Professor, Department of Energy, Systems, Territory and Constructions Engineering, University of Pisa, Italy, a.defalco@ing.unipi.it*
[(6)] *Researcher, Department of Energy, Systems, Territory and Constructions Engineering, University of Pisa, Italy, carlo.resta@posteo.net*
[(7)] *Research associate, Dept. of Civil, Environmental & Geomatic Eng., University College London, UK, g.sevieri@ucl.ac.uk*

Abstract

The preservation of cultural heritage assets from natural and man-made disasters is paramount for our community because of their architectural and social value. Cultural heritage assets are directly linked to the economy of a region through cultural tourism. They also have a symbolic value for the community: their damage and partial/total collapse may have a huge impact on social cohesion, sustainable development and psychological well-being.

Among natural hazards, earthquakes are the most dangerous for ancient buildings, which are usually built following empirical knowledge and reflecting the tradition of a community. Moreover, modifications over time, local repairs or partial/total reconstructions can even worsen the seismic performance of cultural heritage assets.

In this context, dynamic Structural Health Monitoring systems represent a powerful tool to control the health state of the structure, manage maintenance interventions and optimise retrofitting.

This paper presents the preliminary results of a research work aimed at defining a probabilistic framework for monitoring systems for cultural heritage assets. In particular, in this paper the experimental campaign on the bell tower of the San Francesco church in Pisa is described. The San Francesco bell tower is a unique example of masonry tower, since it rests on the transept walls of the church (12 m above the ground level). For this reason, it represents an interesting case study to test limits and advantages of the dynamic monitoring of masonry towers.

The experimental campaign is aimed at determining the modal characteristics of the structure, i.e., frequencies and mode shapes, by recording the effect of ambient vibrations on the tower. These observations are then elaborated through Operational Modal Analysis techniques to characterise the structural modal behaviour. A finite element model has been finally calibrated by comparing numerical eigenfrequencies and mode shapes with experimental ones. The calibrated model will be used in future steps of the project to understand the complex dynamic behaviour of this unique masonry tower and to assess its seismic fragility.

Keywords: Structural Health Monitoring; masonry tower; structural identification Operational Modal Analysis.

1. Introduction

The preservation of Cultural Heritage (CH) assets is a problem of great complexity. Current approaches are generally intermittent and lacking in methodical strategy, too often applied in emergency situations [1]. Planned and distributed monitoring would conversely allow for more structural control and effective conservation efforts [2].

Numerical models are commonly adopted to assess the structural health of CH assets. However, most of the CH assets are non-engineered masonry constructions, which have been built based on empirical knowledge. Epistemic uncertainties related to the geometry, material parameters and boundary conditions [3] strongly affect the numerical results. For this reason, Finite Element (FE) models may fail to predict the actual structural behaviour. Material test results as well as monitoring data should then be used to calibrate the uncertain model parameters thus improving the accuracy of the results.

The use of dynamic non-destructive characterization techniques, such as ambient vibrations-based modal identification, is widely diffused in this field because such approaches match the well-known *conservation* and *minimum intervention criteria* [4]. The building response under ambient vibrations provides information about its modal characteristics (i.e., natural frequencies and mode shapes) [5]. Experimental modal characteristics allow model parameters to be calibrated, thus improving the estimation of the structural behaviour [6,7].

The present paper focuses on the results of a one-day dynamic monitoring experimental campaign of the bell tower of the San Francesco church in Pisa (Italy). This is the first step towards the definition of a probabilistic framework for Structural Health Monitoring (SHM) systems for CH assets. The experimental campaign on the bell tower was carried out to determine its modal characteristics, i.e., natural frequencies and mode shapes, by recording its response to ambient vibrations. Accelerometers and velocimeters, commonly used for structural control, were employed. Their simultaneous presence allowed us to perform a comparison between the two.

The records were elaborated through Operational Modal Analysis (OMA) techniques to characterize the structural modal behaviour of the tower [5]. Experimental mode shapes and frequencies were used as a source of information for parameter calibration of the FE model, thus reducing the effect of epistemic uncertainties on the final results. A FE model of the structure is shown and the use of a damaging constitutive law for its pre-conditioning of the model is suggested, and its consequences discussed.

2. The San Francesco bell tower

The San Francesco church was built during the second half of the XIII century under the direction of architect Giovanni di Simone [8]. It represents a typical Franciscan church, with sobriety in the use of decorations, absence of frescos and a large nave to host a large number of worshippers. Its dimensions make it the second largest church in the city, after the Cathedral of Pisa: the nave measures 70.5 m x 17.7 m.

Its main peculiarity consists in a *hanging* bell tower, having no foundations of its own. Two of its sides are supported by the north transept's walls, while the others rest on two limestone brick cantilevers embedded within the same transept walls at 12 m above the ground level.

Two possible reasons for this particular structural solution have been pointed out. Firstly, the Franciscan rule at the time prescribed that belfries should not be self-standing towers, in accordance with the Order's modesty principles. Secondly, by the time of construction the nearby leaning bell-tower of the Cathedral of Pisa had already shown the general deformability of soil in Pisa so that the architect's choice may have simply been a precautionary measure [9].

Fig. 1a shows the point cloud of the church; Figg. 1b and 1c show the bell tower and its base resting on the walls of the transept.

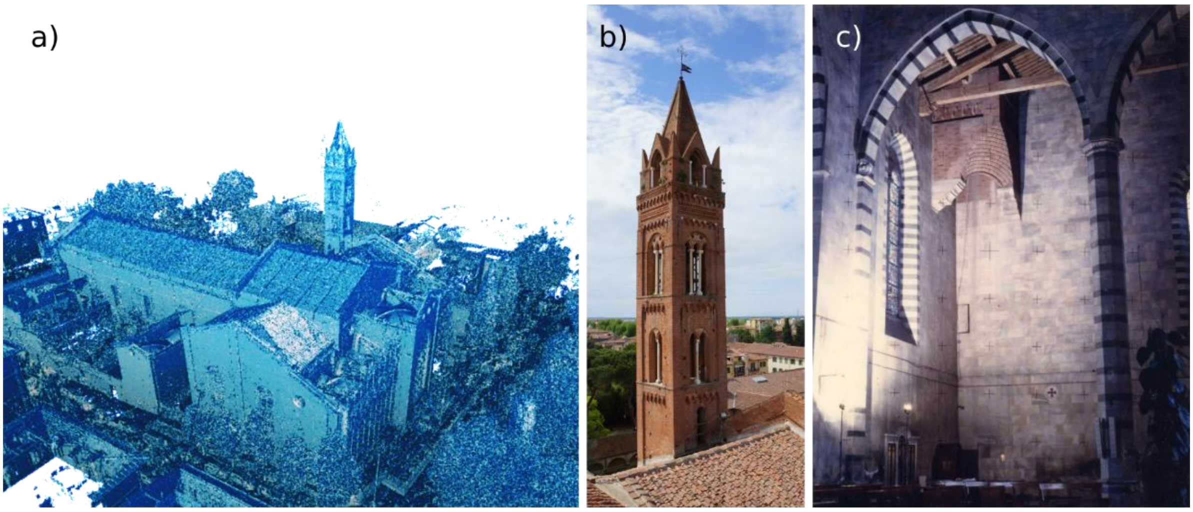

Fig. 1 – a) Point cloud of the church; b) external view of the bell tower; c) internal view of the bell tower base, resting on limestone cantilevers embedded in the transept walls.

3. Experimental campaign

The experimental campaign was carried out on March 30, 2019 and lasted one day. A preliminary survey was performed to assess the safety of structures, particularly the ancient wooden upper floors of the tower, to enable access by operators. A second preliminary phase consisted in the design of instrument positioning and cable management.

3.1 Instrumentation and acquisition

Different sensor typologies were employed for the dynamic identification of the bell tower. Accelerometers and velocimeters were placed in mostly redundant positions in order to perform a comparison between the two. Accelerometers were furnished by the Structural Lab of the University of Pisa, while velocimeters were provided by the *Istituto Nazionale di Geofisica e Vulcanologia* (INGV).

Ten monoaxial capacitive accelerometers (PCB DC capacitive accelerometers – Model 3701G3FA3G) and six piezoelectric accelerometers (PCB ICP accelerometers – Model 393C) were employed, grouped in six biaxial and one triaxial stations; six triaxial velocimeters (SARA – Model SS20), were used [10]. The technical characteristics of the employed sensors are summarized in Table 1.

The transfer function of the signal recorded by velocimeters requires to be deconvolved in order to enlarge the usable band of the device in the low frequency range, as required by experimentation [11]. Battery-powered velocimeters were synchronized through GPS (Fig. 3). Accelerometers were instead connected to an acquisition system (LMS Scadas Mobile) controlled by a dedicated application (*LMS Test.Lab 8* [12]).

Velocimeters and accelerometers were placed at ground level, at the tower base and on the first two floors of the bell tower (i.e., 0.0 m, 12.1 m, 16.7 m and 22.9 m above ground, respectively). On the second floor, two different measurement stations were employed to appreciate torsional modes. A single biaxial accelerometer was placed on the third floor (28.5 m above ground). The degradation level of the wooden floor prevented velocimeters to be installed on the third floor. For the same reasons, no sensors were placed on the fourth floor and at roof level. The layout of sensors is shown in Fig. 2, where accelerometers are indicated with a red square and velocimeters with a blue circle. Another velocimeter (P00a) was placed on the ground outside the church, at a distance of about 20 m from the bell tower.

Six separate 30-minute acquisitions were performed using accelerometers, sampling at 512 Hz. Velocimeters were contemporarily used to perform a four-hour continuous recording, sampling at 200 Hz.

Table 1 – Technical characteristics of the employed sensors.

PCB DC capacitive accelerometers model 3701G3FA3G		
Average sensitivity @ 30 Hz	1000 mV/g	
Average eigenfrequency	540 Hz	
Frequency range (± 5%)	0 to 100 Hz	
PCB ICP piezoelectric accelerometers model 393C		
Average sensitivity @ 100 Hz	1000 mV/g	
Average eigenfrequency	5.3 kHz	
Frequency range (± 5%)	0.025 to 800 Hz	
SARA model SS20 - triaxial velocimeters		
Acquisition system	SL06 – 24 bit	
SS20 sensors	Eigenfrequency	2.0 Hz
	Nominal sensitivity	200 V/m/s
	Usable band	0.1 to 100 Hz

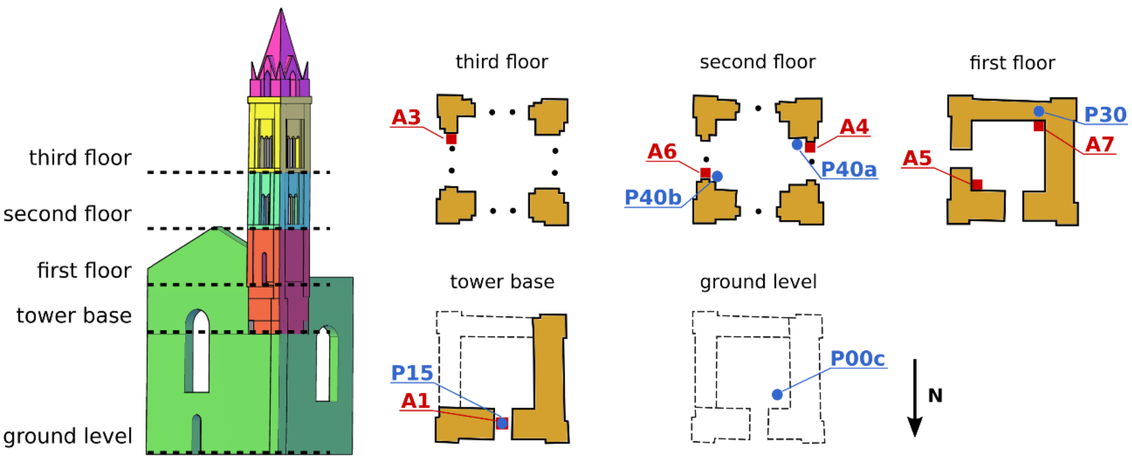

Fig. 2 – Sensor positioning along the tower.

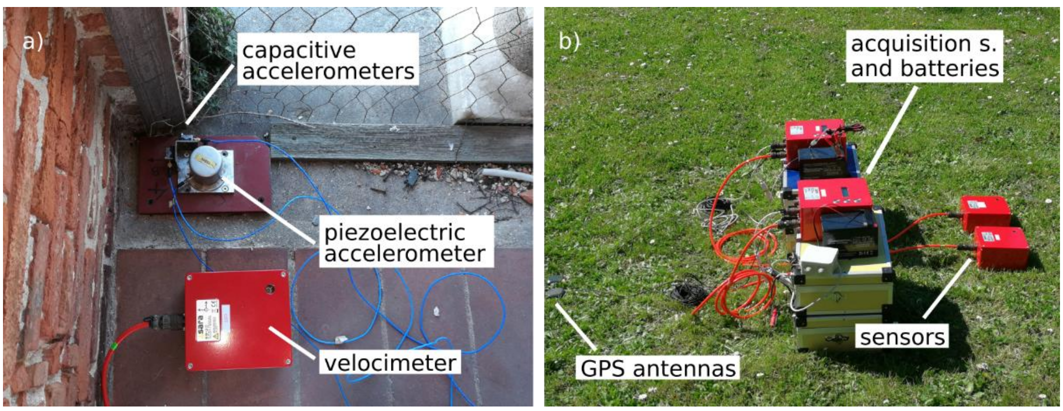

Fig. 3 – Instrumentation: a) sensor positions P40a and A4; b) components of the triaxial velocimeter.

3.2 Data elaboration and discussion

Data from accelerometers and velocimeters were processed using *LMS Test.Lab 8*, through the application of Power Spectrum functions [13], Hanning windowing with 16768 spectral lines and 10% overlap, and manual peak-picking to extract natural frequencies. Modal shapes were obtained by applying the PolyMAX method [14,15] as implemented in the software *LMS Test.Lab 8*.

The two sensors placed in the opposite corners of the second floor allowed torsional motions to be detected. Under the hypothesis that torsional motion can be recognized from the anticorrelated portion of the signal, the frequencies corresponding to torsional modes are investigated by calculating the time-domain difference of horizontal synchronous signals from sensors on the second floor. To enhance the torsional component, the vertical signal from one of the two stations was deconvolved from the obtained signal difference [16] calculating the spectral ratio between the former and the latter

Fig. 4a shows the Power Spectrum function for accelerometer A3, mounted on the third floor of the bell tower, for directions X and Y. Fig. 4b shows the results of signal deconvolution. Figg. 5a and 5b were obtained in the same way from data extracted from velocimeters P40a and P40b.

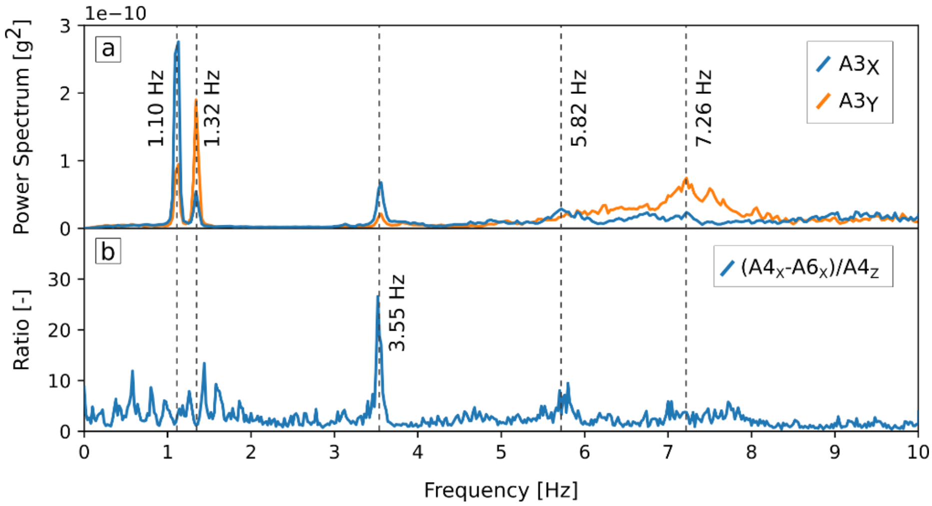

Fig. 4 – Frequency response from accelerometers: a) Power Spectrum of signal from accelerometer A3; b) deconvolution of vertical component from the difference of horizontal signals.

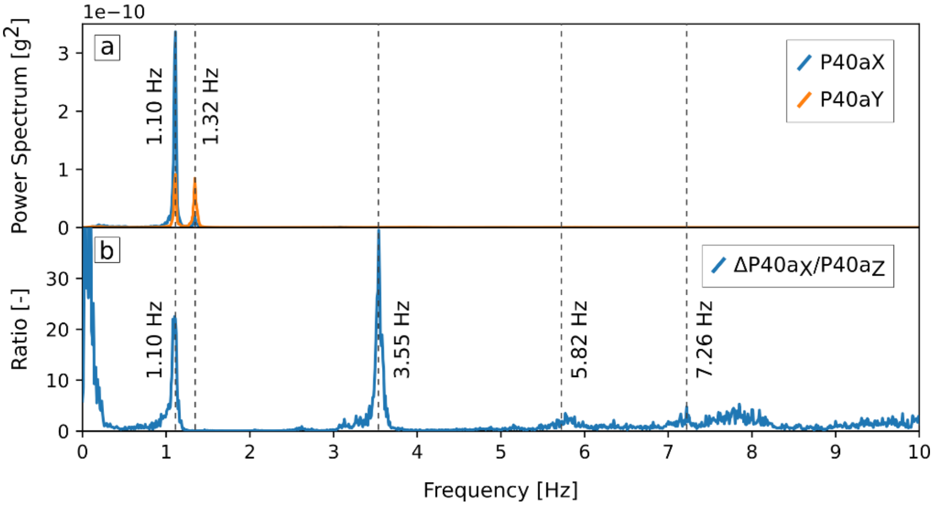

Fig. 5 – Frequency response from velocimeters: a) Power Spectrum of signal from velocimeter P40a; b) deconvolution of vertical component from the difference of horizontal signals.

Both Fig. 4 and Fig. 5 show the clear presence of peaks at frequencies 1.10 Hz, 1.32 Hz and 3.55 Hz. Less defined peaks may be present between 3.55 Hz and 10 Hz. The first two peaks may correspond to the resonant frequencies of the first two flexural modes of the bell tower. The third, which is particularly visible from the Power Spectrum of the anticorrelated portion of the horizontal signal, may correspond to a mostly torsional mode.

The presence of other resonant frequencies in the range 0-10 Hz is confirmed by the stabilization diagram resulting from the application of the PolyMax algorithm, shown in Fig. 6. Frequencies in the range 0 to 10 Hz were investigated and a model order of up to 200 was considered [14].

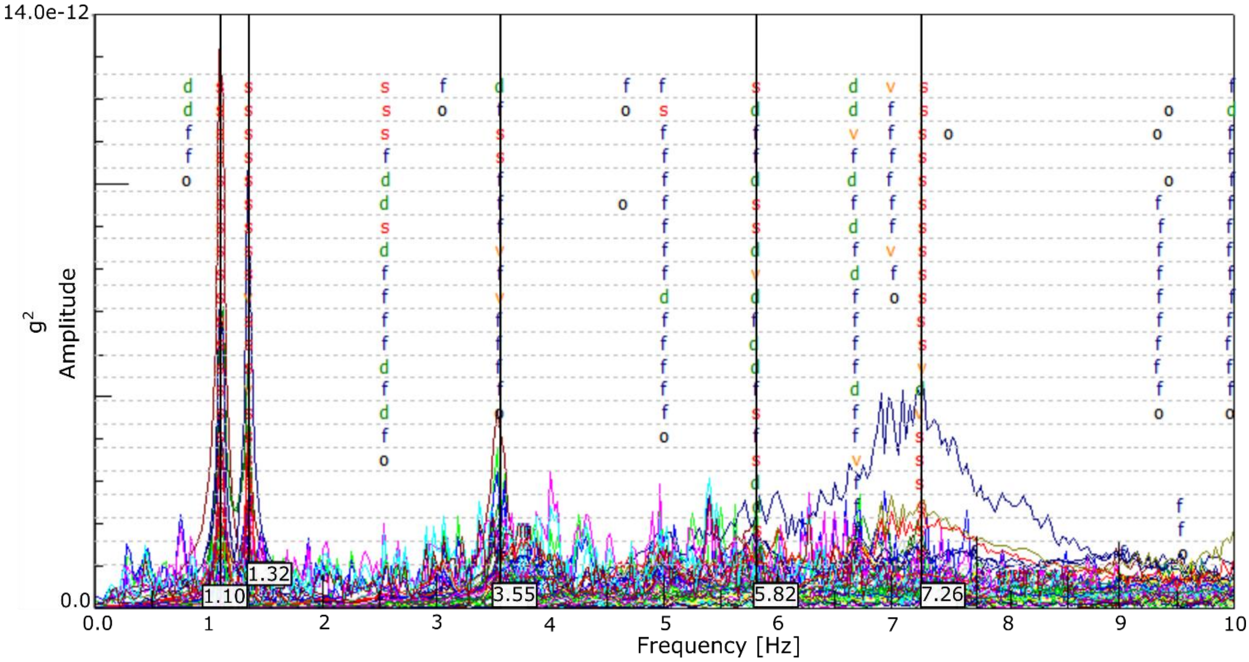

Fig. 6 – Stabilization Diagram from PolyMax algorithm.

References at 5.82 Hz and 7.26 Hz have thus been added to Figg. 4 and 5. In the former, there is a correspondence between these frequencies and wide, low amplitude peaks in the spectrum.

The triaxial velocimeters P00c and P00a, placed at ground level, inside and outside the church respectively, were used to investigate the propagation of vibrations from the bell tower. The vertical signal presents a frequency peak at 1.10 Hz, corresponding to the first identified mode of the bell tower, while horizontal signals show no distinct peaks. This corroborates the hypothesis that the bell tower's vibrations do not propagate to the neighbouring soil [16]. This is probably due to its peculiar features.

Following the construction of the stabilization diagram, modal shapes corresponding to the first three vibrational modes were extracted, once again using *LMS Test.Lab 8*. They are shown in Fig. 7, together with the corresponding modal shapes resulting from a FE model.

The direction of motion of the first two modes was identified by rotating the time-domain signals and computing their FFT, until a separation of the corresponding frequency peaks was obtained. Horizontal particle motion of the two signals (respectively in the 1.05 - 1.20 Hz and in the 1.25 - 1.40 Hz bands) was also employed. It was observed that the first mode is strongly characterized by motion along an axis rotated 30 degrees anticlockwise with respect to the North direction, while the second mode is roughly orthogonal (Fig. 8).

Having employed both velocimeters and accelerometers in mostly redundant positions, a comparison between the two was carried out. Signals from velocimeters need to be pre-processed to obtain information about the band lower than the sensor eigenfrequency. The resulting frequency response shows less noise content in the low frequency band when compared to that of the accelerometers. Looking at Figg. 4 and 5, the frequency peak corresponding to the first torsional mode (3.55 Hz) is absent from the velocimeter's Power Spectrum. This could be due to the positioning of accelerometer A3, which was placed one floor higher up than velocimeter P40.

From a practical perspective, regarding the two specific sensors that were employed, velocimeters allowed for a much quicker set up, being stand-alone systems. Their dependence on GPS signal for synchronicity, however, makes their use much more difficult when indoors. In further steps of this study, the eventual extension of the monitoring network to the church will require a careful planning of sensor positioning in order to optimize the use of external GPS antennas.

4. Finite Element Model

A FE model of the bell tower, comprising a neighbouring portion of the San Francesco church, was realised using the software *Comsol Multiphysics 5.5* [17]. In this preliminary phase, a modal analysis was performed to compare the resulting natural frequencies with the first three obtained from the experimental campaign. Model parameters were, at this stage, obtained from literature.

4.1 Finite Element model of the bell tower

The geometry of the bell tower was defined starting from a point cloud obtained as result of a survey fieldwork with laser scanner (Fig. 1a). Once imported into *AutoCAD 2019* [18], the point cloud was sectioned with horizontal planes at intervals of 0.20 m to obtain reference cross-section plans of the bell-tower. These were joined constructing an interpolating solid model, successively imported in *Comsol Multiphysics.*

Some simplification was applied to the geometry in order to define a mesh that, while maintaining detail, allowed for reasonable computation times. Tetrahedral finite elements with Serendipity quadratic shape function were employed. The resulting mesh is composed of 144796 elements. The maximum dimensions of mesh elements were set at 1.00 m for the church walls, and at 0.25 m for the bell tower. The model is shown in Fig. 7.

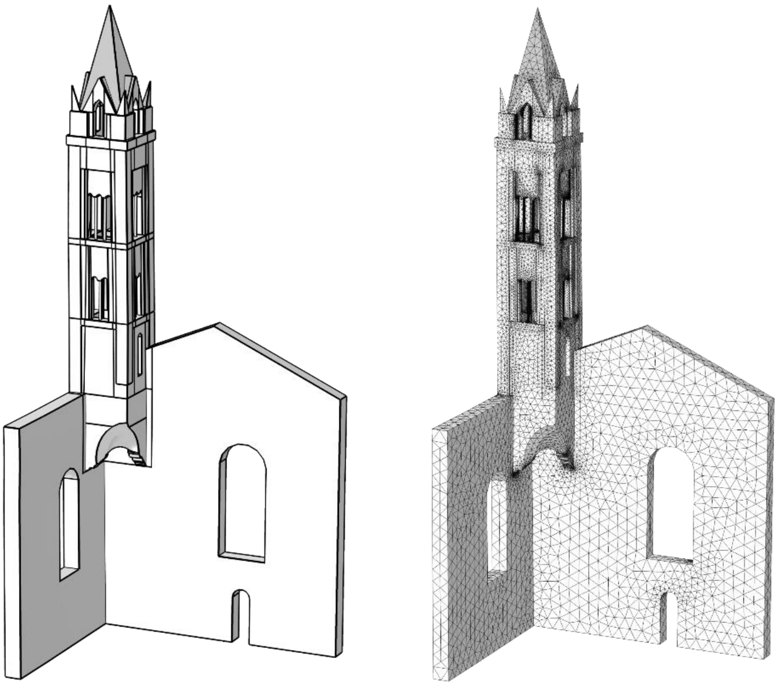

Fig. 7 – Preliminary FE model of the bell tower with the neighbouring portion of the church's walls.

Different values were used for Young's moduli of the church and the bell tower in the linear elastic model. These will be the object of an automated updating procedure in a further research; at this stage, their values were manually estimated following the current Italian building code [19] , and set at 1300 MPa and 1800 MPa for the walls of the transept and bell tower, respectively.

4.2 Modal information from the FE model

A modal analysis of the model was performed. The first three resulting natural frequencies are reported in the following Table 2, together with the values obtained during the experimental campaign.

Analysis of the model produced a large number of vibrational modes., The first three were extracted by comparing the corresponding modal shapes with those derived from experimental data. This can be seen in Fig. 8, where mode shapes from the first three modes are presented. Even though a proper updating process still hasn't been performed, model results show good accordance with experimental data.

Table 2 – First natural frequencies of the bell tower, as resulting from model analysis and experimental measurements.

Mode	Model frequencies [Hz]	Measured frequencies [Hz]
1	1.10	1.10
2	1.34	1.32
3	3.52	3.55

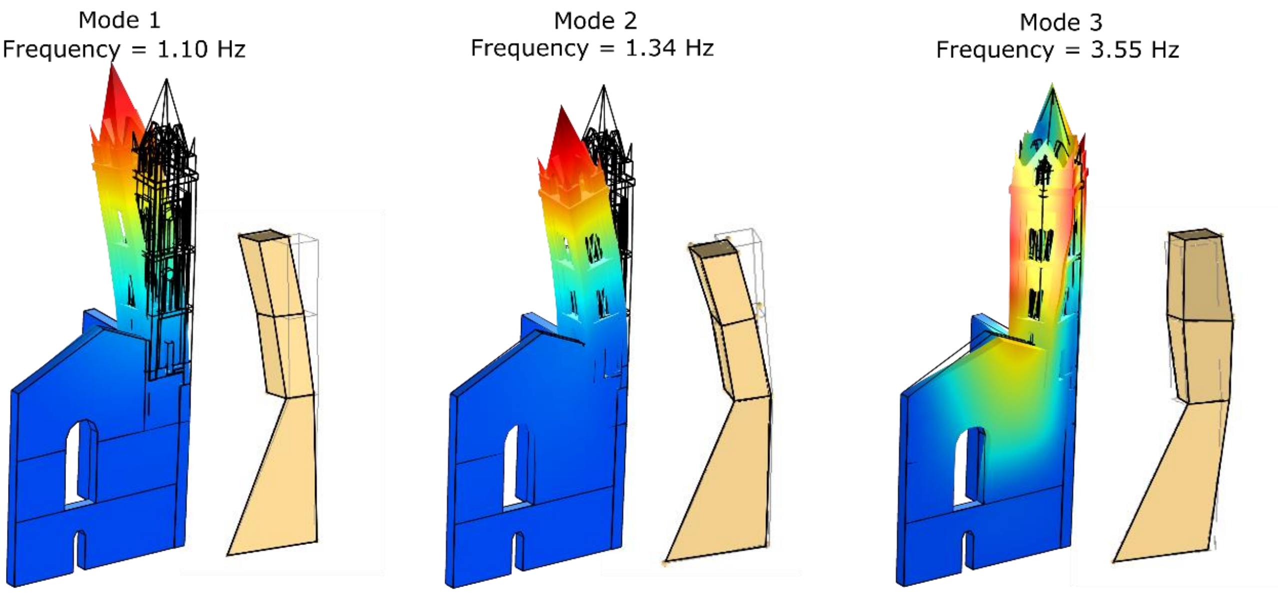

Fig. 8 – Comparison between the first modal shapes obtained by FE model and experimental data.

4.3 Frequency response of a damaged model

The linear elastic model does not necessarily represent the current state of the structure. Self-weight alone is responsible for damage in masonry structures, generally in the form of crack patterns. In our case, crack patterns can be observed on the bell tower walls (Fig.9). The linear elastic model does not explicitly account for damage, so any parameter obtained in an identification process represents also the effects of cracks over the whole structure.

A pre-conditioning of the model, performed using a nonlinear constitutive law for the materials, would allow us to investigate the effects of self-weight damage on the vibrational behaviour of the structure. This was performed through the implementation of Mazars' continuum damage model [20]. Initially developed for concrete, the model uses internal variables to follow the evolution of irreversible strains in the material. The isotropic Mazars' formulation introduces a damage variable d used to modify the value of Young's modulus as follows:

$$E^d = (1-d)E_0. \tag{1}$$

Here, E_0 represents the undamaged value assumed for Young's modulus. Total damage d is defined as a weighted combination of damage due to compression (d_c) and tension (d_t):

$$d = \alpha_t d_t + \alpha_c d_c \tag{2}$$

Weighting coefficients α_t and α_c are defined as functions of the principal values of positive strains ε_t and negative strains ε_c respectively.

$$\varepsilon_{ij}^t = (1-d)C_{ijkl}^{-1}\sigma_{kl}^t \quad and \quad \varepsilon_{ij}^c = (1-d)C_{ijkl}^{-1}\sigma_{kl}^c \tag{3}$$

C_{ijkl}^{-1} are the components of the compliance tensor.

Eq. (4) and Eq. (5) show the expressions of the weighting coefficient α_t and α_c:

$$\alpha_t = \sum_{i=1}^{3}\left(\frac{\langle\varepsilon_i^t\rangle\langle\varepsilon_i\rangle}{\tilde{\varepsilon}^2}\right)^\beta \tag{4}$$

$$\alpha_c = \sum_{i=1}^{3}\left(\frac{\langle\varepsilon_i^c\rangle\langle\varepsilon_i\rangle}{\tilde{\varepsilon}^2}\right)^\beta \tag{5}$$

where the Macaulay brackets indicate replacing negative strains with zeros. $\tilde{\varepsilon}$ is an equivalent strain and it is defined as:

$$\tilde{\varepsilon} = \sqrt{\sum_{i=1}^{3}(\langle\varepsilon_i\rangle_+)^2} \tag{6}$$

Tensile and compressive damage are then given by

$$d_t(\kappa) = 1 - \frac{\kappa_0(1-A_t)}{\kappa} - A_t e^{-B_t(\kappa-\kappa_0)} \tag{7}$$

$$d_c(\kappa) = 1 - \frac{\kappa_0(1-A_c)}{\kappa} - A_c e^{-B_c(\kappa-\kappa_0)} \tag{8}$$

Here A_t, B_t, A_c, B_c, and κ_0 are material parameters. A_t, and A_c influence the residual strength beyond the peak value, and B_t, and B_c govern the peak strength itself and the slope of the softening branch.

κ is the state variable which keeps track of the maximum effective strain, given by $\tilde{\varepsilon}$. Its initial value κ_0 can be defined as a function of maximum tensile strength f_t.

$$\kappa_0 = \frac{f_t}{E_0} \tag{9}$$

Fig. 9 shows the results of the application of self-weight on the stiffnesses of the bell tower modelled using Mazars' continuum damage model for the materials. Blue portions represent a concentration of damage, where residual stiffness is the lowest. For some of these, corresponding cracks in the masonry have been detected.

Modal analysis was performed again on the damaged model, using the same initial values of Young's moduli. Vibrational modes corresponding to the ones measured during the experimental campaigns were once again identified through their mode shapes. The corresponding natural frequencies resulting from the model differ substantially from the ones identified earlier, as can be seen in Table 3.

The damaged model can be more suitable for parameter identification when nonlinear analyses are performed. However, difficulties were encountered in matching eigenfrequencies with experimental data during the updating procedure in this case. This suggests a more thorough investigation, involving a probabilistic updating procedure.

Table 3 – First natural frequencies of the bell tower, as resulting from damaged and undamaged model.

Mode	Damaged model frequencies [Hz]	Undamaged model frequencies [Hz]
1	1.03	1.10
2	1.28	1.32
3	3.03	3.55

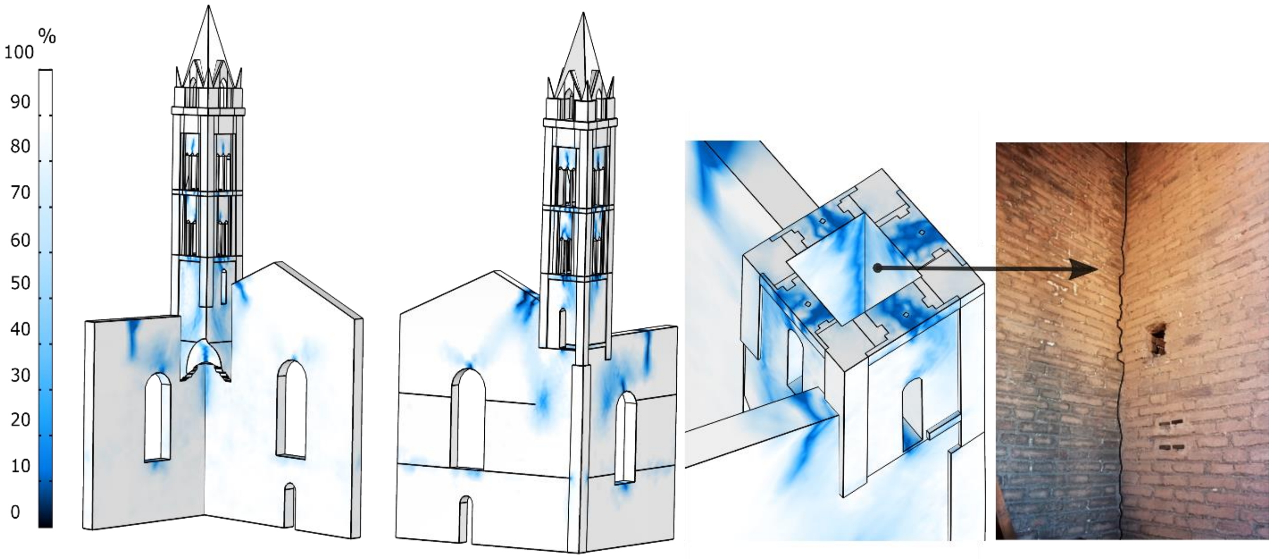

Fig. 9 – Residual stiffness of the damaging model after self-weight application; correspondence of one of the detected cracks with model results.

5. Conclusions

This paper presents the first steps of a study that will focus on the development of a probabilistic framework for the structural health monitoring of cultural heritage assets. The peculiar masonry bell tower of the San Francesco church in Pisa (Italy) was analysed as a case study. A testing campaign was carried out on the bell tower in order to identify its dynamic properties (eigenfrequencies and modal shapes). Both velocimeters and accelerometers were employed, recording the effects of ambient vibration on the structure, and a comparison between the two kinds of sensors was performed. Data showed that velocimeters need a specific calibration in order to record the low frequencies typical of tower structures but produce a generally cleaner signal than the latter. The results were used to calibrate the mechanical properties of a finite element model of the structure. Literature values for the elasticity modulus of bell tower and church produced eigenfrequencies and modal shapes in good agreement with experimental data. In future steps of the research, this model will serve as a reference to quantify the seismic fragility of the structure and to optimize sensor layout for continuous monitoring. As a further development, the use of Mazars' continuum damage model was suggested, in order to investigate the effect of self-weight damage on the vibrational behaviour of the structure. Preliminary results show a promising correspondence with detected damage localization, and modal analysis points to a significant reduction in eigenfrequencies. Further on, a thorough comparison between the damaged and undamaged model will have to be performed to identify the one most suitable for dynamic monitoring.

6. References

[1] Ramos LF, Masciotta M.G, Morais MJ, Azenha M, Ferreira T, Pereira EB et al. (2018): HeritageCARE: Preventive conservation of built cultural heritage in the South-West Europe. In: Van Balen K, e Vandesande A, curatori. *Innovative Built Heritage Models*, 1° ed. CRC Press. pag. 135–48. https://doi.org/10.1201/9781351014793-16

[2] Balageas D, Fritzen CP, Güemes A, curatori. (2006): Structural health monitoring. ISTE, London ; Newport Beach, CA.

[3] De Stefano A, Matta E, Clemente P (2016) Structural health monitoring of historical heritage in Italy: some relevant experiences. *Journal of Civil Structural Health Monitoring*, **6**, 83–106. https://doi.org/10/ggcxfc

[4] Bonazza A, Drdácký M, Hanus C, Maxwell I, Vintzileou E, European Commission et al. (2018): Study on safeguarding cultural heritage from natural and man-made disasters: a comparative analysis of risk management in the EU.

[5] Brincker R, Ventura CE (2015): Introduction to Operational Modal Analysis: Brincker/Introduction to Operational Modal Analysis. John Wiley & Sons, Ltd, Chichester, UK. https://doi.org/10.1002/9781118535141

[6] Sevim, B, Atamturktur S, Altunişik AC, Bayraktar A (2016): Ambient vibration testing and seismic behavior of historical arch bridges under near and far fault ground motions. *Bulletin of Earthquake Engineering*, **14**, 241–59. https://doi.org/10.1007/s10518-015-9810-6

[7] Sabia, D, Aoki T, Cosentini RM, Lancellotta R (2015): Model Updating to Forecast the Dynamic Behavior of the Ghirlandina Tower in Modena, Italy. *Journal of Earthquake Engineering*, **19**, 1–24. https://doi.org/10/ggjdmz

[8] Redi F (1991): Pisa com'era. Archeologia, urbanistica e strutture materiali dell'impianto tardoantico alla città comunale. Liguori Editore.

[9] De Falco A (1996): Il campanile di S. Francesco a Pisa: un esempio di audacia statica del XIII secolo. Degree thesis.

[10] Azzara R, De Falco A, Girardi M, Pellegrini D (2017): Ambient vibration recording on the Maddalena Bridge in Borgo a Mozzano (Italy): data analysis. *Annals of Geophysics*, **60**, 9. https://doi.org/10.4401/ag-7159

[11] Azzara RM, Girardi M, Iafolla V, Lucchesi DM, Padovani C, Pellegrini D (2019): Ambient Vibrations of Age-old Masonry Towers: Results of Long-term Dynamic Monitoring in the Historic Centre of Lucca. *International Journal of Architectural Heritage*, 1–17. https://doi.org/10/ggjdmf

[12] Simcenter Testlab. https://www.plm.automation.siemens.com/global/it/products/simcenter/testlab.html

[13] Brandt A (2011): Noise and vibration analysis: signal analysis and experimental procedures. Wiley, Chichester.

[14] Peeters B, Van der Auweraer H, Guillaume P, Leuridan J (2004): The PolyMAX Frequency-Domain Method: A New Standard for Modal Parameter Estimation? *Shock and Vibration*, **11**, 395–409. https://doi.org/10.1155/2004/523692

[15] Peeters B, Lowet G (2004): A New Procedure for Modal Parameter Estimation. *SOUND AND VIBRATION*, 5.

[16] Azzara RM, Zaccarelli L, Morelli A, Trombetti T, Dellavalle G, Cavaliere A et al. (2014): Seismic monitoring of the Asinelli and Garisenda medieval towers in Bologna (Italy), an instrumental contribution to the engineering

modeling direct to their protection. *Proceedings of the second International Conference on Historical Construction (PROHITECH) 2014*, Instambul.

[17] Comsol. (2019): COMSOL Multiphysics Reference Manual.

[18] Autodesk | 3D Design, Engineering & Construction Software. https://www.autodesk.com/

[19] Consiglio Superiore dei Lavori Pubblici. (gen 21, 2019): Circolare n.7 - 21 gennaio 2019.

[20] Mazars J, Pijaudier-Cabot G (1989): Continuum Damage Theory—Application to Concrete. *Journal of Engineering Mechanics*, **115**, 345–65. https://doi.org/10.1061/(ASCE)0733-9399(1989)115:2(345)