17th World Conference on Earthquake Engineering, 17WCEE
Sendai, Japan - September 13th to 18th 2020

HETEROGENEOUS MULTI-SENSOR DATA FUSION FOR POST-EARTHQUAKE STRUCTURAL HEALTH ASSESSMENT

S. Adarsh [(1)], K. Prajapat[(2)], S. Ray-Chaudhuri[(3)*]

[(1)] *Ph.D. Candidate, Dept. of Civil Eng., Indian Institute of Technology Kanpur, India, E-mail: adarshss@iitk.ac.in*
[(2)] *Visiting Asst. Professor, Dept. of Civil, Constr. and Env. Eng., Iowa State University, Iowa, USA, Email: kantaiitk@gmail.com*
[(3)] *Professor, Dept. of Civil Eng., Indian Institute of Technology Kanpur, India, E-mail: samitrc@iitk.ac.in*

Abstract

This study explores the implementation of Bayesian finite element model updating, by combining operational acceleration and strain data from a sparse array of sensors, to detect the damage which would have incurred in a structure during an earthquake. A small-scale model of four-story, single bay, shear building model was fabricated. The building was instrumented with accelerations at floor levels and strain gauges at the columns, closer to the floor. At first, the story stiffness of the model was evaluated using static pull tests. The model was then mounted on a miniature shake table and was subjected to white noise base excitation to simulate ambient conditions. Both displacement and strain modal parameters were recovered from simulated ambient vibration data measured using data from the accelerometers and strain gauges. Further, stochastic subspace algorithm (SSID) was used to identify the natural frequencies, displacement mode shape amplitudes, and strain mode shape amplitudes. The identified modal parameters of the healthy structure are then used to update an analytical model of the system, which had been developed based on the physical geometric measurements of different elements and compared with experimentally obtained stiffness of the elements. Further, a damage scenario is simulated by loosening of the bolts in the first floor. The modal parameters of the damaged model are then identified and used to again update the analytical model. Markov Chain Monte Carlo (MCMC) simulation using the Metropolis-Hastings algorithm is used for generation of posterior distributions of the unknown stiffens parameters. The results of the study demonstrate that utilization of strain data for model updating significantly improve the updating results. Further studies are however needed to understand sensitivity of sensors and their locations on damage identification.

Keywords: Post-earthquake condition assessment; Multi-sensor data fusion; Bayesian FEMU; OMA

1. Introduction

1.1 Background

In the context of civil infrastructure, the importance of structural health monitoring (SHM) in life safety and providing long term economic benefit is well accepted. Among the various schemes of structural condition assessment, which have evolved over the years, vibration-based [1,2] methods have gained significant attention of the researchers. This is because the vibration-based techniques are found to be far superior than visual inspection [3], non-destructive [4] and static based methods [5,6]. Vibration based SHM schemes can be used to mitigate issues such as subjectivity of different inspectors and inaccessibility of damaged locations. Further, damages, which are not visible to the naked eye, can also be easily dealt in a superior fashion using vibration-based techniques. Also, in the aftermath of a damaging event such as a strong earthquake, a decision to vacate or reoccupy a building can be taken utilizing operational data in the context of vibration-based health assessment.

Over the past few decades, there had been a significant development in sensor [7] technologies, both wired and wireless [8] ones, and the field of SHM has benefited immensely from this advancement. It is now well-known that each type of sensor and data have their own advantages and disadvantages. For example, displacement mode shapes can easily be recovered through acceleration data. However, these mode shapes are

* ***Corresponding Author: Professor, Dept. of Civil Eng., Indian Institute of Technology Kanpur, India** samitrc@iitk.ac.in*

often less sensitive to some particular damage in comparison to the strain mode shapes, which are evaluated through strain data as damage are often very localized [9]. Further, from acceleration data alone, it is not feasible to detect permanent and quasi-static deformations, which may be easily inferred from strain data. Acceleration measuring devices such as force balance and piezo-electric accelerometers, which are generally used in operational modal analysis [10,11], usually produce data with significantly high signal to noise ratio. But the data from foil-based strain gauges [12] can easily get noisy depending on many factors. Installation of foil-based strain gauges can be more laborious as compared to accelerometers. Certain locations in a structure such as near to the base of a building, strain readings are usually higher compared to the acceleration reading. But free ends usually provide high acceleration readings and less strain readings. Hence, a combination of data from various sensors is envisioned to provide better accuracy of an SHM scheme.

Multi-sensor data fusion (MSDF) [13,14] is the technique in which data from various sensors are combined rather than using only one type of data. Installing a dense array of sensors, which covers all the important degrees of freedom of a structure, is highly uneconomical considering the cost of sensors, wiring, data acquisition system, labour, maintenance etc. The large amount of data from such a dense array of sensors also increase the storage and computational burden on host machines. Thus, most MSDF methods aim to extract quality information from a sparse array of heterogeneous sensors. MSDF can be used (i) to avoid numerical integration and differentiation which otherwise would introduce error, (ii) reconstruction of responses at locations that are not instrumented, and (iii) for considering the effects of environmental factors such as changes in temperature. There are a few studies where displacement and velocity measurements are combined [15], while a few other papers considered fusing displacement and acceleration data to extract meaningful information [16, 17]. Research studies focusing on combining data from (i) acceleration and strain sensors [18,19,20] and strain, acceleration and displacement sensors [21] are also available.

Over the last few decades Bayesian finite element model updating (FEMU) [22,23,24,25] has emerged as one of the most powerful tools for model-based damage detection. The Bayesian framework of probability offers an effective way to quantify various uncertainties, and to combine useful information from both mathematical models and experimental data. Although Bayesian inference can also be used for model class selection, in the present work, it is assumed that the mathematical model of the problem is known a priori and only the prior distributions of the mathematical model are updated by using the various experimentally identified modal parameters and simulation.

1.2 Motivation and Scope

Majority of studies which had been done in Bayesian FEMU in the context of damage detection, focuses only on displacement modal parameters. The purpose of this study is to include both displacement and strain modal parameters from strategic locations for better model updating. For this purpose, a small-scale model of four-story, single bay, shear building model was fabricated. Accelerometers were installed at the floor levels while strain gauges were installed at the columns, closer to the floor. At first, the story stiffness of the model was evaluated using static pull tests. The model was then mounted on a miniature shake table and was subjected to white noise base excitation to simulate ambient conditions. Both displacement and strain modal parameters were recovered from simulated ambient vibration data measured using data from the accelerometers and strain gauges. Further, stochastic subspace algorithm (SSID) was used to identify the natural frequencies, displacement mode shape amplitudes, and strain mode shape amplitudes. The identified modal parameters of the healthy structure are then used to update an analytical model of the system, which had been developed based on the physical geometric measurements of different elements and compared with experimentally obtained stiffness of the elements. Further, a damage scenario is simulated by loosening of the bolts in the first floor. The modal parameters of the damaged model are then identified and used to again update the analytical model. Markov Chain Monte Carlo (MCMC) simulation using the Metropolis-Hastings algorithm is used for generation of posterior distributions of the unknown stiffens parameters.

2. Strain Modal Analysis

This section reviews the formulation of strain modal analysis, where it is assumed that a finite element model exists. If $\boldsymbol{\varepsilon}^e$ is the vector containing elemental strains, then it is related to global displacement vector $\boldsymbol{u}$ as follows:

$$\boldsymbol{\varepsilon}^e = Au \tag{1}$$

where **A** is the matrix, which maps global displacements to elemental strains. If N is the total number of elements and n is the total degrees of freedom of the model, $\boldsymbol{\varepsilon}^e$ will have dimensions Nx1 and $\boldsymbol{u}$ will have dimensions nx1.

Now, the dynamic equilibrium equation for a multi-degree of freedom system is given as

$$\boldsymbol{M\ddot{u}}(\boldsymbol{t}) + \boldsymbol{C\dot{u}}(\boldsymbol{t}) + \boldsymbol{Ku}(\boldsymbol{t}) = \boldsymbol{f}(\boldsymbol{t}) \tag{2}$$

In Eq. 2, $\boldsymbol{M}$, $\boldsymbol{C}$, and $\boldsymbol{K}$ are the mass, damping and stiffness matrices, respectively, and $\boldsymbol{f(t)}$ is the forcing vector. The modal decomposition of the responses of such a system in terms of real displacement mode shape is given as

$$\boldsymbol{u}(\boldsymbol{t}) = \boldsymbol{\Phi q}(\boldsymbol{t}) \tag{3}$$

where $\boldsymbol{\phi}$ is the displacement mode shape matrix and $\boldsymbol{q(t)}$ is the vector of the modal coordinates. From Eqs.1 and 3, one can write,

$$\boldsymbol{\varepsilon}^{\boldsymbol{e}} = \boldsymbol{A\Phi q}(\boldsymbol{t}) \tag{4}$$

The strain mode shape matrix $\boldsymbol{\Psi}^{\boldsymbol{e}}$ is defined from Eq.4 as follows:

$$\boldsymbol{\Psi}^{\boldsymbol{e}} = \boldsymbol{A\Phi} \tag{5}$$

In a similar manner, the strain frequency response function (FRF) $\boldsymbol{H}^{\varepsilon}(\boldsymbol{\omega})$ can be derived from the displacement FRF $\boldsymbol{H}(\boldsymbol{\omega})$ as

$$\boldsymbol{H}^{\varepsilon}(\boldsymbol{\omega}) = \boldsymbol{CH}(\boldsymbol{\omega}) \tag{6}$$

3. Bayesian Finite Element Model Updating

Bayesian model updating is a statistical tool to minimize the error in the outcomes of a physical process from its mathematical model. The following relation is employed for Bayesian model updating schemes:

$$Posterior\ distribution \propto Likelihood \times Prior\ distribution \tag{7}$$

Prior distribution for any model parameter is determined based on the available knowledge about the parameter. The term likelihood is a probabilistic function which relates the plausibility of getting the observed outcome of the physical process for a given value of a parameter of the mathematical model. Finally, the term posterior distribution gives a more favorable distribution for the model parameters by operating the likelihood function on the prior and thus, refining the prior distribution to posterior.

Mathematically, if the outcome of the mathematical model is expresses as $\underline{x}(i;\underline{a})$ for some model class with model parameter vector $\underline{a}$ and system input Z_i for $i = 1,2...n$ time steps for each time step then the experimental/real time outcome of the physical process can be related to $\underline{x}(i;\underline{a})$ as:

$$\underline{y}(i) = \underline{x}(i;\underline{a}) + \underline{e}(i;\underline{a}) \qquad i = 1,2,...n \tag{8}$$

Eq. 8 comprises of a deterministic part $\underline{x}(i;\underline{a})$ and a random part $\underline{e}(\boldsymbol{i};\underline{\boldsymbol{a}})$ (error or difference of mathematical and experimental/real time outcome). The random part $\underline{e}(i;\underline{a})$ can be represented with some probabilistic distribution. If $\underline{e}(1), \underline{e}(2), ...\underline{e}(n)$ represent the error distribution functions for each time step, then the error probability model for n time steps can be represented as some function of these error distribution functions.

$$p(E^n \mid \underline{\sigma}) = h_n(\underline{e}(1),...,\underline{e}(n);\underline{\sigma}) \tag{9}$$

where $\underline{\sigma}$ is a vector of variances of $\underline{e}(1),\underline{e}(2),...\underline{e}(n)$. Therefore, the total uncertain parameter vector comprises of the uncertain model parameters vector $\underline{a}$ and the uncertain variances vector $\underline{\sigma}$ and can be represented with a vector $\underline{\alpha}$ as shown in Eq. 10.

$$\underline{\alpha} = [\underline{a}^T, \underline{\sigma}^T]^T \tag{10}$$

Given the error probability model as defined in Eq. 9, the outcome probability model conditioned on the parameter vector $\underline{\alpha}$ and input Z^n can be defined as

$$p(Y^n \mid \underline{\alpha}, Z^n) = f_n(\underline{y}(1),...,\underline{y}(n);\underline{\alpha}, Z^n)$$

$$= h_n(\underline{y}(1) - \underline{x}(1;a),...,\underline{y}(n) - \underline{x}(n;a);\sigma) \tag{11}$$

if $\pi(\underline{\alpha})$ represents the prior joint distribution of unknown parameters, then the joint probability of outcome probability model with parameter vector $\underline{\alpha}$ conditioned on input Z^n can be written as

$$p(Y^n \underline{\alpha} \mid Z^n) = f_n(\underline{y}(1),...,\underline{y}(n);\underline{\alpha}, Z^n)\pi(\underline{\alpha}) \tag{12}$$

Integrating Eq. (12) over the space S_α of parameter vector $\underline{\alpha}$ the marginal distribution of outcome probability model conditioned on input Z^n can be given as

$$p(Y^n \mid Z^n) = \int_{S_{\underline{\alpha}}} f_n(Y^n;\underline{\alpha}, Z^n)\pi(\underline{\alpha})d\underline{\alpha} \tag{13}$$

Now, if D represents a data set of input Z^m and output Y^m for ‘m’ time steps, then mathematically the relation $Posterior\ distribution \propto Likelihood \times Prior\ distribution$ can be expressed as

$$p(\underline{\alpha} \mid Y^m) = kp(Y^m \mid \underline{\alpha}, Z^m)\pi(\underline{\alpha})$$

$$= kf_m(Y^m;\underline{\alpha}, Z^m)\pi(\underline{\alpha}) \tag{14}$$

where

$$k^{-1} = p(Y^m \mid Z^m) \tag{15}$$

Thus, with the information of the prior distributions and experimental outcome, one can obtain a more favorable posterior distribution of unknown parameters employing Bayesian approach. In this work, the well-accepted Gaussian likelihood for frequency and mode shape components are used. The details of the formulation can be found in Prajapat and Ray-Chaudhuri (25). It may be noted here that, with the increased number of unknown parameters, the dimensionality of parameter space $S_{\underline{\alpha}}$ increases. This makes the analytical evaluation of Eq. 14 difficult. Therefore, in this work Markov Chain Monte Carlo (MCMC) simulation is employed to draw samples from a higher dimensional joint distribution. A brief discussion on Metropolis-Hasting sampling algorithm is presented in the next section, which is used in this work to draw samples under MCMC environment.

3.1 Metropolis-Hasting Algorithm

A Markov chain is said to be reached stationary distribution when it satisfies the following equation:

$$\omega(x)q(x,y) = \omega(y)q(y,x) \tag{16}$$

In Eq. (16), ω is the target distribution and q represents the transitional probability distribution function which gives the transition probability from one realization of random value to other realization. In case when the exact transition probability function is not known, the transition probability function (often known as proposal distribution) has to be assumed. In such case, Eq (16) becomes

$$\omega(x)q(x,y) > \omega(y)q(y,x) \tag{17}$$

Metropolis-Hasting sampling algorithm provides a way to reach the stationary distribution in Markov chain by introducing a probability of move $\alpha(x,y)$ as given by the following relation:

$$\alpha(x,y) = \min\left[\frac{\omega(y)q(y,x)}{\omega(x)q(x,y)},1\right], \quad \omega(x)q(x,y) > 0$$

$$= 1 \qquad \text{otherwise} \tag{18}$$

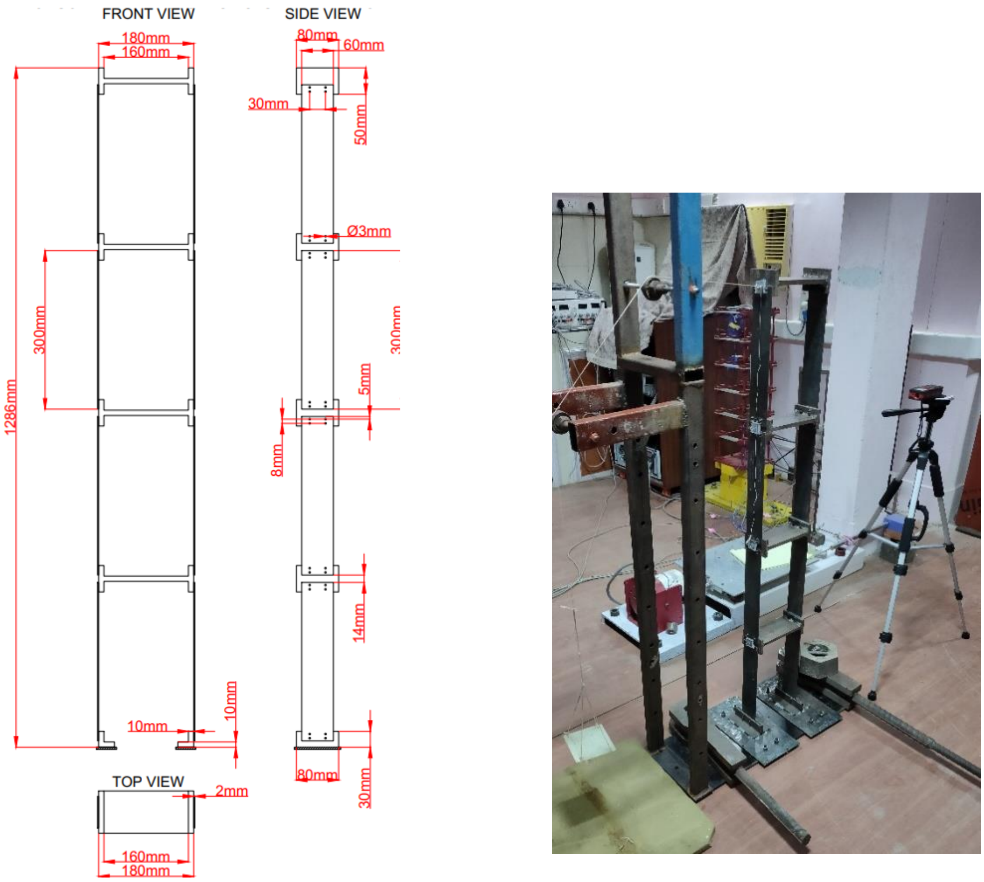
So, the proposed sample from the proposal distribution is accepted with a probability $\alpha(x,y)$ to achieve the target distribution ω.

3. Experimental Investigation

3.1 Experimental Set-up and Instrumentation

The set up consisted of a single-bay, four-storey shear building, made from mild steel plates of thickness 2mm and width 30mm. Total height of the building was 1286mm. To simulate floor masses, 14mm thick steel plates were used. Bolted connections were used for ease of assembling and removal of various elements. A schematic diagram showing relevant dimensions are provided in Fig. 1. To characterize the story stiffness of the model, static pull test was carried out. In this test, each floor was pulled laterally using a loading pan and pulley arrangement and the corresponding floor displacements were measured using a non-contact laser distance meter. The purpose of using the non-contact device instead of conventional linear variable differential transformers (LVDTs) for measuring the displacement of the frame was to make sure that the stiffness of the frame was not affected by measurement device. The loading was done for each story and the load was increased by adding different weight till significant displacement was noted. By using this procedure, the load verses deformation curve was generated, for each combination of loading and measurements point. Based on this, the flexibility matrix was evaluated. The flexibility matrix was then utilized to evaluate the story stiffness corresponding to each floor. This information of story stiffness was then used later to check the accuracy of model updating.

For shake table tests, the model was instrumented with 4 piezo-electric accelerometers (PCB 393B04) and 8 foil-type stain gauges (TML FLA-3-350-11) as shown in Fig. 2. Thus, each floor had one accelerometer, and the columns at one side had 2 strain gauges installed on these, one at each end. Further, to capture input motion to have an idea of base excitation intensity, one accelerometer and one LVDT were used as can be seen from Fig. 2. NI PXIe-1062Q chassis was used for data acquisition and NITB-43330 8 channel bridge input card was used for strain gauges. The input ground motion was generated on Keysight 33500B series signal generator and was amplified by Beak power amplifier BAA 1000. The amplified signal was given to a linear electromagnetic shaker (www.dataphysics.com), which in turn was connected to a miniature shake table through floor mounting bolts.



(a) (b)

Fig.1: (a) Schematic diagram showing various dimensions of the model and (b) Experimental set-up of static pull test for stiffness evaluation

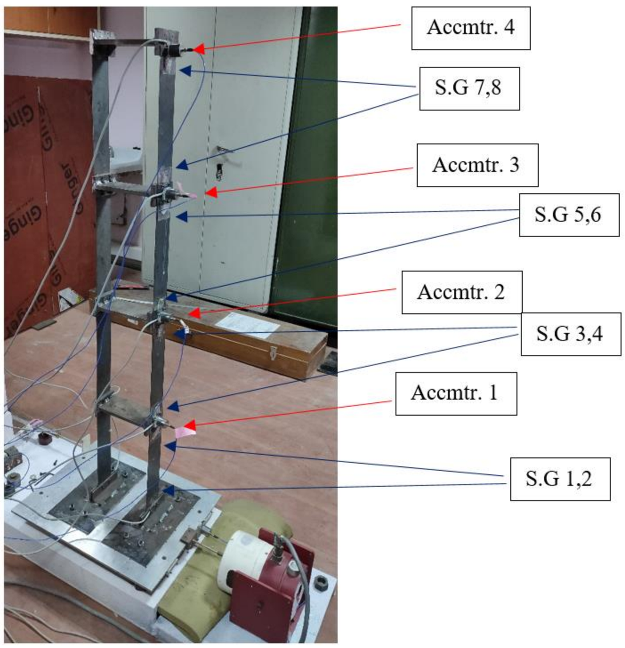
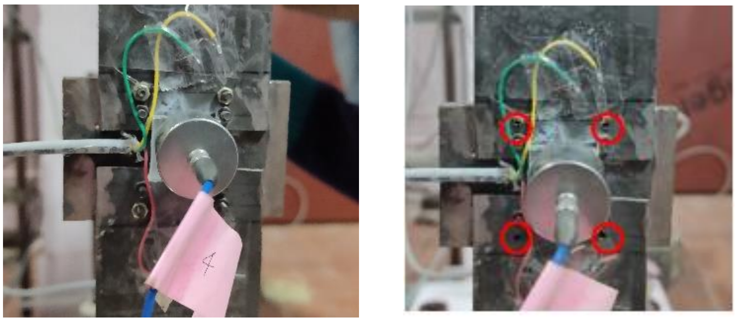


Fig. 2: Picture of model on shake table showing sensor details

3.2 Experimental Procedure and Damage Scenario

Once the model was fixed on the shake table, a band-limited white noise (upper cut 40 Hz) was given as input to simulate ambient excitation. The data from all sensors, i.e., accelerometers and strain gauges were recording for approximately eight minutes. This test was repeated several times to get an idea of repeatability.

Further, in order to study the case of damage detection, a damage scenario was simulated by loosening four bolts of the first floor out of total eight bolts as shown in Fig. 3. One can note from Fig. 3a that all eight bolts are present, while in Fig. 3b only 4 bolts are loosened as shown by red circles. After the removal of the bolts, tests were conducted in the same manner as that of the undamaged case. Several trials were also made for this case and all strain and acceleration data were recorded.



(a) (b)

Fig. 3: Picture depicting: (a) Undamaged state and (b) Damage case with loosened bolts

4. Results and Discussion

4.1 Experimental Results

For system identification, well-known stochastic subspace identification (SSID) approach was used. The results were verified using stabilization diagram. Table 1 provides the natural frequencies obtained for three trials using both acceleration and strain data for the undamaged case. Mode shapes obtained from experiments are shown Table 2 using both acceleration and strain data. One can note that estimates of natural frequencies are quite close using both strain and acceleration data. However, displacement mode shapes obtained from the acceleration data are different than those from the strain mode shapes. As mentioned earlier, the both data have unique advantages in terms of identifying damage. The results for the damage case are not shown here to save space.

Table 1: Natural frequencies obtained from experiment for undamaged case

Trials	Mode 1 (Hz)	Mode 2 (Hz)	Mode 3 (Hz)	Mode 4 (Hz)
	Acceleration data			
Trial 1	2.8431	11.3448	16.7702	20.5879
Trial 2	2.8406	11.3477	16.7699	20.5869
Trial 3	2.8608	11.3200	16.7647	20.5976
	Strain data			
Trial 1	2.8443	11.3533	16.7692	20.5883
Trial 2	2.8355	11.3344	16.7711	20.5873
Trial 3	2.8448	11.3444	16.7524	20.5966

Table 2: Mode shapes obtained from experimental data (undamaged case)

	Floor	Mode 1	Mode 2	Mode 3	Mode 4
		Acceleration Data			
Trial 1	4	1.0000	1.0000	1.0000	1.0000
	3	0.9135	0.0443	-1.0859	-1.9489
	2	0.7467	-1.0228	-0.6017	1.8948
	1	0.5380	-0.8796	1.5259	-1.2326
Trial 2	4	1.0000	1.0000	1.0000	1.0000
	3	0.9140	0.0405	-1.0863	-1.9485
	2	0.7483	-1.0223	-0.6006	1.8932
	1	0.5424	-0.8767	1.5243	-1.2288
Trial 3	4	1.0000	1.0000	1.0000	1.0000
	3	0.9122	0.0405	-1.0852	-1.9499
	2	0.7467	-1.0170	-0.6027	1.9016
	1	0.5381	-0.8827	1.5227	-1.2476
		Strain data			
Trial 1	4	1.0000	1.0000	1.0000	1.0000
	3	1.9735	1.0024	-0.2110	-1.1416
	2	2.5188	-0.1420	-0.8232	0.7690
	1	3.5299	-1.1148	0.7351	-0.4038
Trial 2	4	1.0000	1.0000	1.0000	1.0000
	3	1.9728	1.0055	-0.2113	-1.1416
	2	2.5210	-0.1316	-0.8241	0.7705
	1	3.5344	-1.1116	0.7344	-0.4037
Trial 3	4	1.0000	1.0000	1.0000	1.0000
	3	1.9718	1.0038	-0.2064	-1.1430
	2	2.5210	-0.1346	-0.8267	0.7767
	1	3.5320	-1.1112	0.7259	-0.4135

4.1 Analytical Model

An analytical model of the building was developed as a base model for the undamaged case. The modeling was done considering shear building assumptions and considering the properties of mild steel and utilizing geometric dimensions to get the stiffness matrices. The mass matrix was formed by weighing the individual components and using a consistent mass matrix for the connecting column elements. Since, the mass of the columns was not very low as compared to the floor masses, the consistent mass matrix was used instead of tributary mass commonly used for shear building. It was found that the assumption of consistent mass matrix provided better match with the experimental observed results of modal parameters.

4.2 Updating results

For model updating, the base model was considered, and the experimental data are used for forming likelihood functions. All story stiffness coefficients are considered as unknown. Exponential non-informative distribution with mean some arbitrary mean is taken as the prior distribution and Gamma distribution is chosen for the proposal distribution. Markov chain is run for a total of 1000 simulation for each unknown parameter. Modal data (Frequencies and mode shapes of four modes) from the known structure are considered. Metropolis-Hasting algorithm is adopted to simulate the posterior distribution. A burning period of 200 and a thinning parameter of 5 are used in Markov chain to get the final posterior distribution. Two sets of simulations are conducted. One with using modal parameters from only the acceleration sensors and the other by using modal data from both acceleration and strain data. The results are shown in Table 3. Experimentally evaluated stiffness values for the undamaged case (first column) are also provided in this table. One can notice that by using both strain data and acceleration data, the results improved noticeably as one can notice from the absolute deviation values. It may also be noted that, since the experimental stiffness coefficients were not evaluated for the damaged case, the absolute deviation in this case couldn't be determined.

Table 3: Evaluated unknown story stiffness parameters

	Only acceleration data		Both acceleration and strain data	
Unknown story stiffness parameters with actual value (kN/m)	Mean	Absolute deviation (%)	Mean	Absolute deviation (%)
Undamaged case				
$k_1 = 7.715$	8.815	14.26	7.997	3.66
$k_2 = 6.732$	7.223	7.29	6.773	0.60
$k_3 = 10.404$	9.120	12.34	10.05	3.42
$k_4 = 10.326$	9.236	10.56	9.963	3.52
Damaged case				
k_1	8.253	-	8.341	-
k_2	6.839	-	6.780	-
k_3	8.953	-	8.963	-
k_4	9.178	-	9.180	-

5. Conclusions

This work demonstrates how operational acceleration and strain data from a sparse array of sensors can improve the model updating results in the framework of Bayesian finite element model updating. Experiments were conducted by fabricating a small-scale model of four-story, single bay, shear building model. The building was instrumented with accelerations at floor levels and strain gauges at the columns, closer to the floor. Actual stiffness of the model was evaluated using static pull tests and compared with the model updating results. Ambient excitation to the model was simulated using white noise base excitation on a miniature shake table. Using stochastic subspace algorithm (SSID), natural frequencies, displacement and strain mode shapes were evaluated. These results were then used for finite element model updating of the base model. It is found from this study that both strain and acceleration data provide similar identified frequencies. Further, it is noted that strain mode shapes are considered along with displacement mode shapes derived from the acceleration data, the updating results significantly. Hence, it may be concluded that the implementation of Bayesian FEMU by using natural frequencies, displacement mode shapes and strain mode shaped identified from OMA can lead to a more efficient SHM scheme. Future studies are however needed to understand sensitivity of sensors and their locations on damage identification.

6. Acknowledgements

The authors would like to thank all Structural engineering Laboratory staff of IIT Kanpur for the enthusiasm and dedication they had shown during the fabrication, instrumentation and testing of the specimen. The first author would also like to thank MHRD (Govt. of India) for funding his Ph.D work.

7. References

[1] Doebling SW, Farrar CR, Prime MB. A summary review of vibration-based damage identification methods. Shock and vibration digest. 1998 Mar 1;30(2):91-105.

[2] Farrar CR, Worden K. Structural health monitoring: a machine learning perspective. John Wiley & Sons; 2012 Nov 19.

[3] Mizuno Y, Abe M, Fujino Y, Abe M. Development of interactive support system for visual inspection of bridges. In Health Monitoring and Management of Civil Infrastructure Systems 2001 Aug 3 (Vol. 4337, pp. 155-166). International Society for Optics and Photonics.

[4] P. J. Shull, Nondestructive evaluation: theory, techniques, and applications, CRC press, 2002.

[5] Jenkins CH, Kjerengtroen L, Oestensen H. Sensitivity of parameter changes in structural damage detection. Shock and Vibration. 1997;4(1):27-37.

[6] Zhao L, Shenton III HW. Structural damage detection using best approximated dead load redistribution. Structural Health Monitoring. 2005 Dec;4(4):319-39.

[7] Fardeen J. Handbook of modern sensors: physics, designs, and applications Springer Science & Business Media 314.

[8] Nagayama T, Spencer Jr BF. Structural health monitoring using smart sensors. Newmark Structural Engineering Laboratory. University of Illinois at Urbana-Champaign.; 2007.[9] Yam LY, Leung TP, Li DB, Xue KZ. Theoretical and experimental study of modal strain analysis. Journal of Sound and Vibration. 1996 Mar 28;191(2):251-60.

[10] Brincker R, Ventura C. Introduction to operational modal analysis. John Wiley & Sons; 2015 Sep 8.

[11] Rainieri C, Fabbrocino G. Operational modal analysis of civil engineering structures. Springer, New York. 2014; 142:143.

[12] Murray WM, Miller WR. The bonded electrical resistance strain gage: an introduction. Oxford University Press; 1992 Apr 30.

[13] Wu RT, Jahanshahi MR. Data fusion approaches for structural health monitoring and system identification: past, present, and future. Structural Health Monitoring. 2018 Sep 24:1475921718798769.

[14] J. Esteban, A. Starr, R. Willetts, P. Hannah, P. Bryanston-Cross, A review of data fusion models and architectures: towards engineering guidelines, Neural Computing & Applications 14 (2005) 273–281

[15] Smyth A, Wu M. Multi-rate Kalman filtering for the data fusion of displacement and acceleration response measurements in dynamic system monitoring. Mechanical Systems and Signal Processing. 2007 Feb 1;21(2):706-23.

[16] Roberts GW, Meng X, Dodson AH. Integrating a global positioning system and accelerometers to monitor the deflection of bridges. Journal of Surveying Engineering. 2004 May;130(2):65-72.

[17] Chatzi EN, Smyth AW. The unscented Kalman filter and particle filter methods for nonlinear structural system identification with non-collocated heterogeneous sensing. Structural Control and Health Monitoring:

The Official Journal of the International Association for Structural Control and Monitoring and of the European Association for the Control of Structures. 2009 Feb;16(1):99-123.

[18] Law SS, Li XY, Zhu XQ, Chan SL. Structural damage detection from wavelet packet sensitivity. Engineering structures. 2005 Aug 1;27(9):1339-48.

[19] Soman R, Kyriakides M, Onoufriou T, Ostachowicz W. Numerical evaluation of multi-metric data fusion based structural health monitoring of long span bridge structures. Structure and Infrastructure Engineering. 2018 Jun 3;14(6):673-84.

[20] Sim SH, Spencer Jr BF, Nagayama T. Multimetric sensing for structural damage detection. Journal of engineering mechanics. 2011 Jan;137(1):22-30.

[21] Zhang CD, Xu YL. Structural damage identification via multi-type sensors and response reconstruction. Structural Health Monitoring. 2016 Nov;15(6):715-29.

[22] Beck JL, Au SK. Bayesian updating of structural models and reliability using Markov chain Monte Carlo simulation. Journal of engineering mechanics. 2002 Apr;128(4):380-91.

[23] Yuen KV. Bayesian methods for structural dynamics and civil engineering. John Wiley & Sons; 2010 Feb 22.

[24] Vanik MW, Beck JL, Au S. Bayesian probabilistic approach to structural health monitoring. Journal of Engineering Mechanics. 2000 Jul;126(7):738-45.

[25] Prajapat K, Ray-Chaudhuri S. Prediction error variances in Bayesian model updating employing data sensitivity. ASCE Journal of Engineering Mechanics. 2016 Dec 1;142(12):04016096.

[26] Peeters B, De Roeck G. Reference-based stochastic subspace identification for output-only modal analysis. Mechanical systems and signal processing. 1999 Nov 1;13(6):855-78.

[27] Maia NM, e Silva JM. Theoretical and experimental modal analysis. Research Studies Press; 1997 Oct.

[28] Fu ZF, He J. Modal analysis. Elsevier; 2001 Sep 4.

[29] Bernasconi O, Ewins DJ. Modal strain/stress fields. International Journal of Analytical and Experimental Modal Analysis. 1989 Apr; 4:68-76.

