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STOCHASTIC DISPLACEMENT SPECTRA FOR A LIQUEFIABLE GROUND TREATED WITH GRANULAR COLUMNS

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Abstract

Elastic response spectra based on the equivalent linear site response method do not account for the nonlinear elasto-plastic deformation and complex inertial interaction of liquefiable ground. The seismic performance of the foundation-structure system on the liquefiable ground largely depends on the ground displacement rather than the ground acceleration, especially for the long-period content of input shaking. The attenuation or amplification of input waves primarily governed by the liquefaction extent of the ground. Moreover, inherent soil variability should be taken into account for the reliable engineering judgment of liquefiable ground deformations and associated displacement response spectra. Dynamic centrifuge tests were performed to investigate the liquefaction-induced effects on the foundation-structure system treated with granular columns. The centrifuge test results are used to validate the numerical modeling scheme which is carried out using the OpenSees framework with PDMY02 elasto-plastic soil constitutive model. Soil variability is implemented with stochastic realizations of overburden and energy-corrected, equivalent clean sand, $(N1)_{60cs}$ values using spatially correlated Gaussian random field. Three-dimensional finite element simulations are performed for the sufficient number of realizations to map the scale of fluctuation of stochastic displacement spectra. The ground densification due to the installation of granular columns is also incorporated during the numerical simulations. The correlation between spectral displacement and average surface settlement of the ground along with the stochastic range of the peak ground displacement is evaluated. Besides, the implications of the longer period content/range of input shaking on the displacement spectra are also investigated.

Keywords: Granular columns, liquefaction, inelastic displacement spectra, numerical simulation, stochastic model

1. Introduction

Liquefaction has caused severe damage to the built environment on shallow foundations such as settlement, tilting, and sinking during many past earthquakes. History has witnessed the devastating consequences of liquefaction all over the world. The surface manifestations of level grounds, lateral spreading, failure of superstructures because of loss of bearing capacity of the ground, differential settlement of foundations, sand boils and slumping, and failure of port facilities are few of the many liquefaction-induced effects. For instance, numerous instances of bearing capacity failure of shallow foundations due to liquefaction were observed in the 1964 Niigata and 1990 Luzon (Philippines) Earthquakes. Most of the damaged buildings were two to four stories founded on shallow foundations and relatively thick and uniform deposits of clean sand [1, 2]. Surprisingly, many of the damaged structures were influenced by liquefaction of thin deposits of silt and silty sand [3, 4] in the 1999 Kocaeli (Turkey) Earthquake. Many researchers [5-7] have also elaborated on the role of the liquefaction to the severe damage of buildings, specifically in the reclaimed land during 2011 off the Pacific coast of Tohoku Earthquake.

Usually, the construction on the liquefiable ground is not recommended unless the appropriate liquefaction mitigation measures are taken at such sites. Liquefaction mitigation by granular columns is one of the pronounced techniques which has been widely used across the continents. The granular columns are used to dissipate the excess pore water pressure generated during the earthquake [6,8] and thus help to mitigate the liquefaction-induced effects on the built environment. Besides, the granular columns densify the surrounding soil during installation and re-distribute the earthquake-induced or pre-existing stresses [9]. The reliable assessment of the performance of granular columns is important as the extent of liquefaction affects the response of the foundation-structure system. Many researchers have found that the pioneering design charts for granular columns developed by Seed and Booker [9] overestimate the performance of gravel drainage [6, 10-13]. Adalier and Elgamal [12] have performed centrifuge experiments to understand the liquefaction mitigation performance of granular columns and associated ground deformations. They concluded that the performance of granular columns depends on their drainage capacity and the densification of the ground during the installation of granular columns is inevitable. Besides, the auxiliary benefits of treating the ground with granular columns are the restriction of shear deformation and offering the containment of the encapsulated soil, and providing stiffening-matrix effects (reducing the stress in adjacent soil) [6-13], though these effects are not well established yet, and more research is needed in this direction.

The response of the foundation-structure system significantly influenced by the extent of the liquefaction during the earthquake. Elastic response spectra based on the equivalent linear site response method do not account for the nonlinear elasto-plastic deformation and complex inertial interaction of liquefiable ground. The seismic performance of the foundation-structure system on the liquefiable ground largely depends on the ground displacement rather than the ground acceleration, especially for the long-period content of input shaking. The attenuation or amplification of input waves primarily governed by the liquefaction extent of the ground. The long-period structures such as long-span bridges, high-rise buildings, and tall isolated structures, possess the sensitivity to displacement rather than force and thus displacement response spectrum is more appropriate to estimate the design seismic demand [14]. Besides, it is important to explore the implications of the longer period content/range on amplified displacement spectra for the better prediction of the response of the foundation-structure system during the earthquake.

The inherent soil variability should be taken into account for the reliable engineering judgment of liquefiable ground deformations. Physical modeling for liquefaction related problems has been widely adopted; although, modeling of inherent soil variability is still not possible. However, this task can be achieved with the help of advanced nonlinear finite element analyses using the well-calibrated sophisticated elasto-plastic soil constitutive models. Results of dynamic centrifuge tests to investigate the liquefaction-induced effects on the foundation-structure system with granular columns are used to validate the numerical modeling scheme which is carried out using the OpenSees framework with PDMY02 elasto-plastic soil constitutive model. The soil variability is implemented with stochastic realizations of overburden and energy-corrected, equivalent clean sand, $(N1)_{60cs}$ values using spatially correlated Gaussian random field.

Three-dimensional finite element simulations are performed for a sufficient number of realizations to map the scale of fluctuation of stochastic displacement spectra. The ground densification due to the installation of granular columns is also incorporated during the numerical simulations. The correlation between spectral displacement and average surface settlement of the ground along with the stochastic range of the peak ground displacement is also evaluated.

2. Physical model

Dynamic centrifuge experiment was carried out to investigate the liquefaction-induced effects on shallow foundation resting on a level deposit of liquefiable Toyoura sand treated with granular columns [13]. The centrifuge model contained two shallow foundations and associated superstructures, namely Buffer Tank (BT) and Flare Stack (FS), imposing an average bearing pressure of 51.2 kPa and 71.2 kPa respectively at 0.8 m below the surface of the model ground in prototype scale as shown in Fig. 1. Dynamic centrifuge test was carried out utilizing the Tokyo Tech Mark III centrifuge facility having a radius of 2.45 m, at a centrifugal acceleration of 40 g (N = 40). The centrifuge test simulated a prototype saturated soil deposit of 10 m depth, having a water table at 1.8 m below the top surface. The model ground was prepared using the Toyoura sand with a target relative density of 50 % by air pluviation method using a sand hopper with a nozzle outlet. After achieving the required level of the model ground with Toyoura sand, Silica no.3 was poured inside all the casings placed before pouring Toyoura sand to create granular columns. More details about the physical modeling of liquefaction-induced effects on the shallow foundation resting on liquefiable Toyoura sand treated with granular columns and the comprehensive interpretation of centrifuge test results can be found Kumar et al. [13].

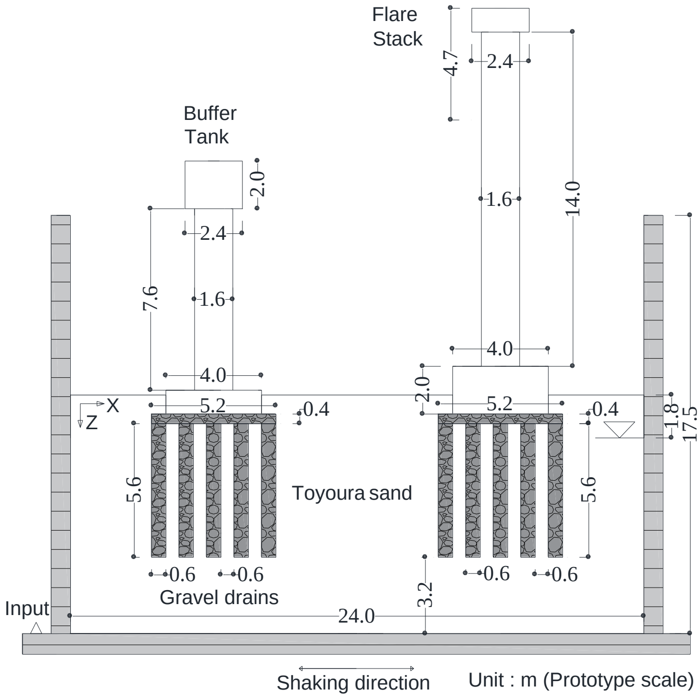

Fig 1. Centrifuge model configuration in the prototype scale

3. Numerical model

Initially, the deterministic analysis is carried out (with D_R = 50 %) before performing the series of stochastic analyses to investigate the stochastic displacement spectra for a liquefiable ground treated with granular columns. Half of the granular column (due to symmetry) under the Buffer Tank and associated model ground (influenced with the drainage of the granular column) is considered for the numerical simulations, as shown in Fig. 2. Numerical simulations are carried out using the framework of OpenSees. Rayleigh damping of 1 % at a frequency of 1 Hz corresponding to the first-mode of a typical nonlinear ground response is used in the analyses. The model ground is modeled using brick u-p (8-node brickUP) elements. The load from the foundation-structure system is modeled as surface pressure for simplicity. The bottom nodes of the model ground are kept fixed in all the degrees of freedom. Displacement time series of Tokachi-Oki ground motion (NS component of recorded shaking at the Hachinohe Port in 1968) is imposed on the bottom nodes of the model ground during the dynamic analyses using the multiple support excitation technique in OpenSees. The side nodes of the model ground are connected using equalDOF to ensure the laminar behavior during the dynamic analyses. All the nodes above the water table are assigned zero pore water pressure. The nodes of the planes (Y= 0 and 0.7 m, see Fig. 2) are fixed against out-of-plane displacement.

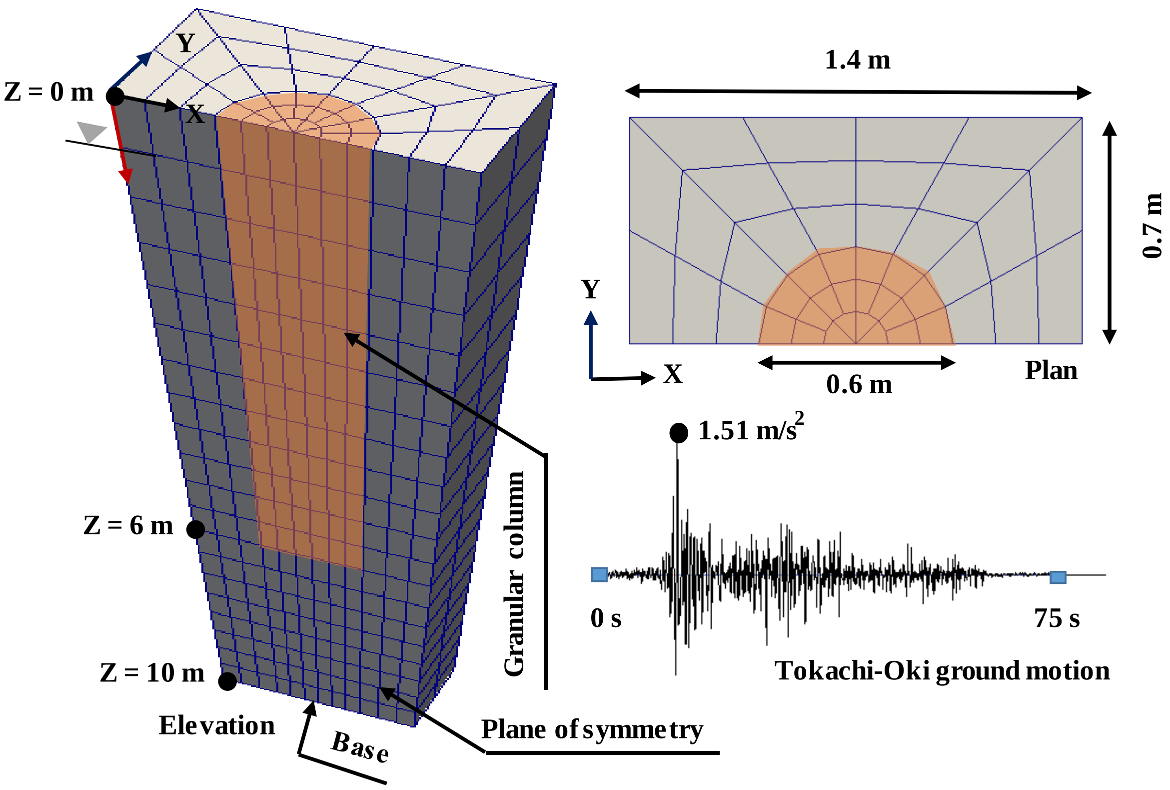

Fig. 2 Three-dimensional (3D) numerical model

PDMY02 soil constitutive model is used to model the ground. The PDMY02 Model is an elasto-plastic soil-liquefaction constitutive model originally developed to simulate the cyclic liquefaction response and the associated accumulation of cyclic shear deformation in clean sand and silt [15]. Within a stress–space plasticity framework, the PDMY02 Model employs a new flow rule and strain–space parameters to simulate the cyclic development and evolution of plastic shear strain. The PDMY02 Model does not account for the critical state soil mechanics framework, and hence the model parameters are calibrated for different apparent relative densities.

The parameters of the PDMY02 Model are calibrated to achieve the single-amplitude shear strain of 3 % in cyclic undrained simple shear loading with zero initial static shear stress ratio on a horizontal plane at a single element level. Laboratory test results from Chiaro et al. [16] are considered as the dynamic behavior of saturated Toyoura sand with a relative density of 50 % at a single element level for the calibration of the

PDMY02 Model's parameters. Figure 3 (a) shows a typical response of calibrated PDMY02 Model for cyclic stress ratio (CSR) = 0.10, D_R = 50 %, and σ'_{vc} = 100 kPa in cyclic undrained simple shear loading with zero initial static shear stress ratio on a horizontal plane. The PDMY02 Model exhibits the ability of shear strain accumulation commonly referred to as cyclic mobility, which is evident from the stress-strain behavior. The stress path is shown in Fig. 3 (b). The vertical effective stress ratio drops down to nearly zero within 15 cycles and triggered large shear strains afterward. Numerically simulated cyclic response at the single element level is obtained after calibrating the parameters of the PDMY02 Model to achieve a similar response as observed in the experiment in terms of cyclic mobility, initial shear modulus, and the accumulation rate of shear strain. Figure 3 (c) shows the shear strain accumulation with the drop in vertical effective stress ratio. Figure 3 (d) shows the CSR curves corresponding to single-amplitude shear strains of 3 % with zero initial static shear stress ratio. The calibrated values of the PDMY02 Model for relative density D_R = 50 % are shown in Table 1.

The aptness of the numerical modeling scheme is also examined through the simulation of the centrifuge test (see Fig. 1) to evaluate the performance at the system level using the calibrated parameters of the PDMY02 Model. Figure 4 shows the time histories of simulated and measured average surface settlement (Z =0, see Fig. 2). The measured average surface settlement is extracted as the average settlement of BT footing from the centrifuge test. The numerical model is able to capture the measured settlement progression until 50 s with good agreement. However, the numerical model exhibited limitations in capturing the measured settlement that took place in the latter part of shaking (after 50 s) and in the post-shaking phase. Besides, the settlement progression after shaking is evident in the case of the measured settlement, whereas; the numerical model does not show such a tendency. The numerical models typically exhibit limitations to capture the settlement caused by partial drainage setup during the shaking and reconsolidation during and after the shaking because of the characteristics of their constitutive formulations [18-20].

4. Stochastic model

The non-uniformity of the ground is mapped using the overburden and energy-corrected, equivalent clean sand, SPT $(N1)_{60cs}$ values as suggested by Montgomery and Boulanger [21]. A series of three-dimensional stochastic dynamic analyses are performed considering the non-uniformity of the model ground and the densification caused during the installation of the granular column in the ground using anisotropic, spatially correlated Gaussian random fields of $(N1)_{60cs}$ values. A Gaussian correlation function is used, and the random field is generated through LU decomposition of the covariance matrix as per Constantine and Wang [22]. For a given $(N1)_{60cs}$ value, the relative density (D_R) is computed as follow:

$$D_R = \text{sqrt}[(N1)_{60cs}/46] \quad (1)$$

The parameters of PDMY02 are calibrated for a wide range of relative densities corresponding to $(N1)_{60cs}$ of 5 (D_R ~ 32 %) to $(N1)_{60cs}$ of 30 (D_R ~ 80 %). All the parameters for different relative densities are calibrated as described in Section 3. Toyoura sand and the granular column are assigned with a uniform permeability value of 0.0002 m/s and 0.006 m/s, respectively. More index properties of the model ground (Toyoura sand) and granular columns can be found in Kumar et al. [13]. The assigned properties for the granular column is corresponding to $(N1)_{60cs}$ of 30 (D_R ~ 80 %) as suggested by Raymajhi et al. [23]. The calibrated values of the granular column are shown in Table 1. The random field of $(N1)_{60cs}$ values with calibrated parameters of the PDMY02 Model are implemented into the OpenSees numerical model with the help of Matlab code. The non-uniformity of the model ground is modeled with a mean $(N1)_{60cs}$ = 11 (D_R ~ 50 %) considering the densification caused during the installation of the granular column as shown in Fig. 5. It is to be noted that the densification in the centrifuge model ground during the installation of the granular columns was ensured to be minimal [13]; however, these effects are inevitable in the site. The densification of the ground due to the installation of the granular column is modeled as per Faris et al. [24] and Hatanaka et al. [25]. A total of fifty numbers of realizations are considered to trace the whole spectrum of the non-uniformity of the ground. The number of realizations is determined based on the convergence of the mean and standard deviation of average surface settlement. They become stable within fifty realizations (Fig. 6), and hence, reliable statistical interpretation of stochastic data can be obtained from the series of nonlinear dynamic numerical simulations. It is to be noted that, more the number of realizations better is the reliability of statistical interpretation. However, the numerical computational expense should be taken into account, while selecting the total number of realizations without compromising with the stability of mean and standard deviation of the primary stochastic outcome. The coefficient of variation (COV = 40 %) and scale of fluctuation (θ_x = 5 m; θ_z = 0.5 m) are considered to model the non-uniformity of the ground according to Phoon and Kulhawy [26] and Montgomery and Boulanger [21].

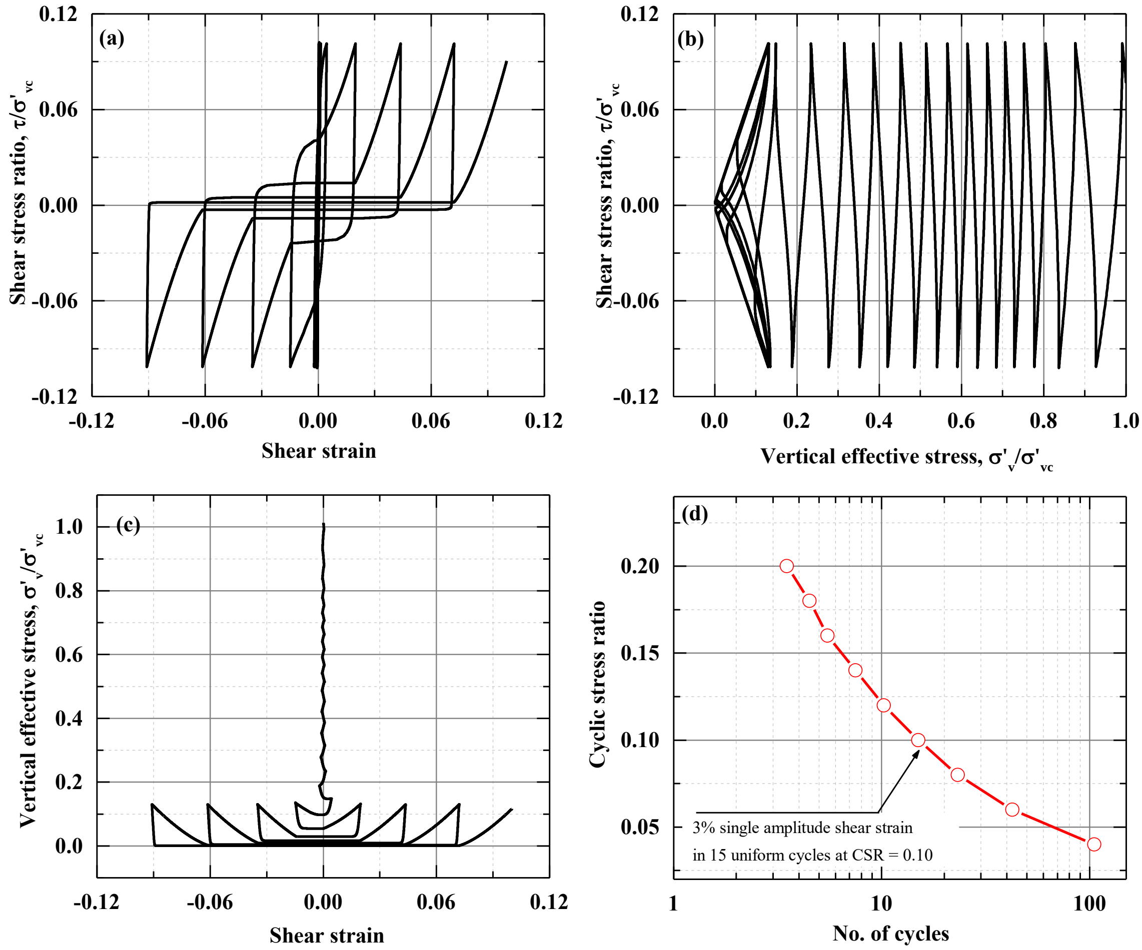

Fig. 3 The response of the calibrated PDMY02 Model at the element level

Table 1. Calibrated parameters for Toyoura sand (D_R ~ 50 %) and granular column

Material/Parameters*	ρ (ton/m^3)	G_{max} (kPa)	B (kPa)	φ	PT_{ang}	C1	C3	D1	D3
Toyoura sand	1.94	3.54E4	8.53E4	33	26	0.065	0.22	0.065	0.20
Granular column	2.14	10.4E4	26.0E4	48	30	0.005	0.0	0.42	0.0

*remaining parameters received default values as per Khosravifar et al. [17]

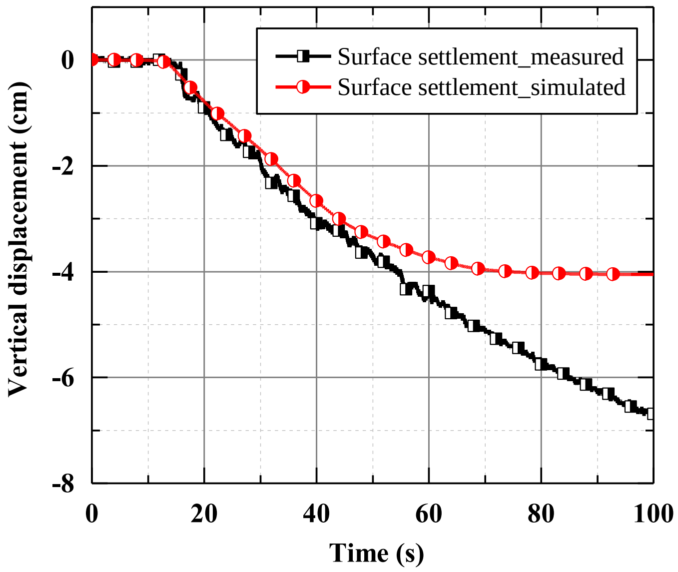

Fig. 4 The Response of the calibrated PDMY02 Model at the system level

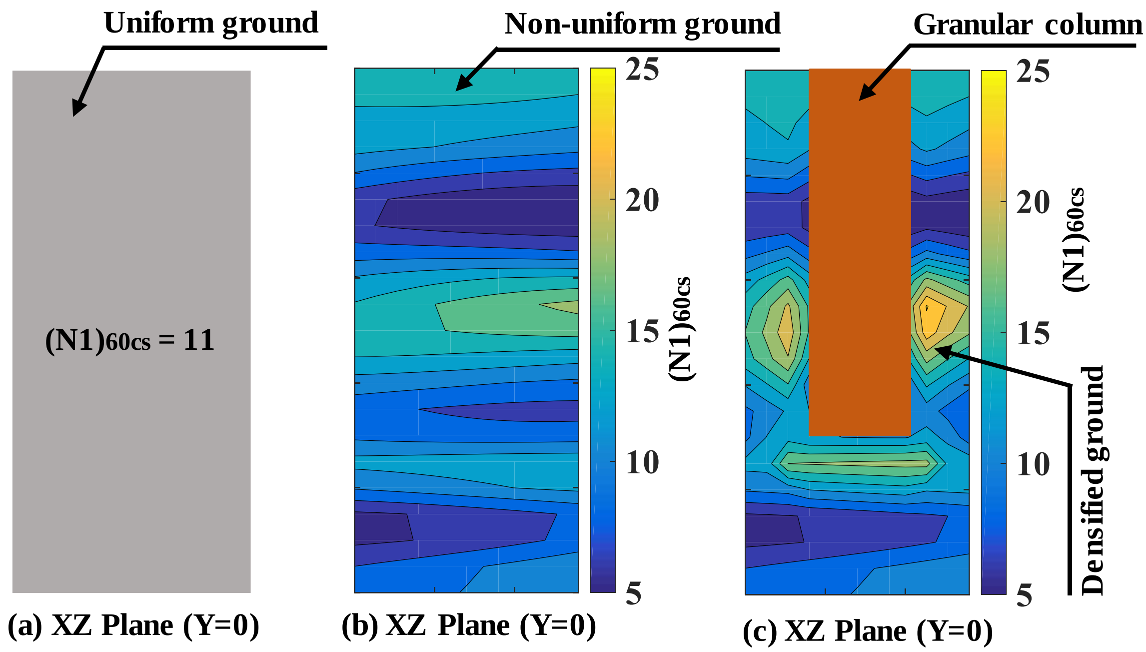

Fig. 5 A typical scenario of the ground condition at XZ Plane (Y= 0; see Fig. 2): (a) uniform ground, (b) non-uniform ground, and (c) non-uniform ground densified during to installation of granular column

5. Results and discussions

Initially, the deterministic analysis (with $D_R \sim 50$ %) is carried out to evaluate the manifested effects of the granular column on the displacement response of the surface of the ground in comparison with the one for applied Tokachi-Oki ground motion at the base before performing the series of stochastic analyses. Figure 7 (a) shows the attenuated surface acceleration time histories in comparison with the applied Tokachi-Oki ground motion at the base of the model ground. Acceleration records at the surface of the ground exhibit that the extent of attenuation in the case of ground with the granular column is significantly lower than what is observed in the case of untreated ground (no granular column). The presence of granular column helps to minimize the stretch of liquefaction and this observation echoes with the general notion that the extent of soil liquefaction results in drastic attenuation of applied seismic motion. A liquefied ground usually filters the high-frequency content of the incident wave and the amplification in the magnitude of low-frequency content of incident wave has a significant impact on spectral displacement of the surface of the ground. The presence of granular column helps to minimize the bound of liquefaction (Fig. 7 (c)) and thus, attenuation in surface acceleration is relatively less in comparison with the untreated ground. However, this attenuation in surface acceleration time histories does not suggest that the maximum displacement of the associated structure is less. For instance, the displacement response spectrum (considering 5 % structural damping) corresponding to the surface acceleration time histories for both treated (with granular column) and untreated ground (no granular column) is more than the one for Tokachi-Oki ground motion at the base for a wide range of periods (shaded zone) as shown in Fig. 7 (b). The presence of the granular column minimizes the extent of liquefaction and it is important to trace the upper and lower bound of the displacement spectra for the associated foundation-structure system.

A series of stochastic analyses are carried out to trace the upper and lower bound of the displacement response spectra on the surface of the non-uniform liquefiable ground treated with the granular column. The densification of the ground during the installation of the granular column is also considered. Figure 8 (a) shows the upper and lower bound of surface spectral displacement in comparison with the one for applied Tokachi-Oki ground motion at the base. The stochastic bound of surface spectral displacement (between red and blue lines) exhibits that the spectral displacement of the surface of the ground is significantly large for a wide range of the periods (0.55 ~ 2.5 s). The effects of this amplified spectral displacement are evaluated in correlation with the ground behavior in terms of average surface settlement (at Z = 0 m, see Fig. 2) of the ground. Figure 8 (b) shows the correlation between average amplification in spectral displacement and average surface settlement. The average amplification in spectral displacement is calculated by taking the ratio of average spectral displacement (for periods of 0.55 -2.5 s, as the amplification in spectral displacement, is evident from Fig. 7 (b)) at the surface to the average spectral displacement at the base. It is evident from Fig. 8 (b) that more is the amplification in spectral displacement, larger is the average surface settlement of the ground. The probability of deviation of stochastic spectral displacement (for a different range of periods) from their respective deterministic values is evaluated and presented in Fig. 9.

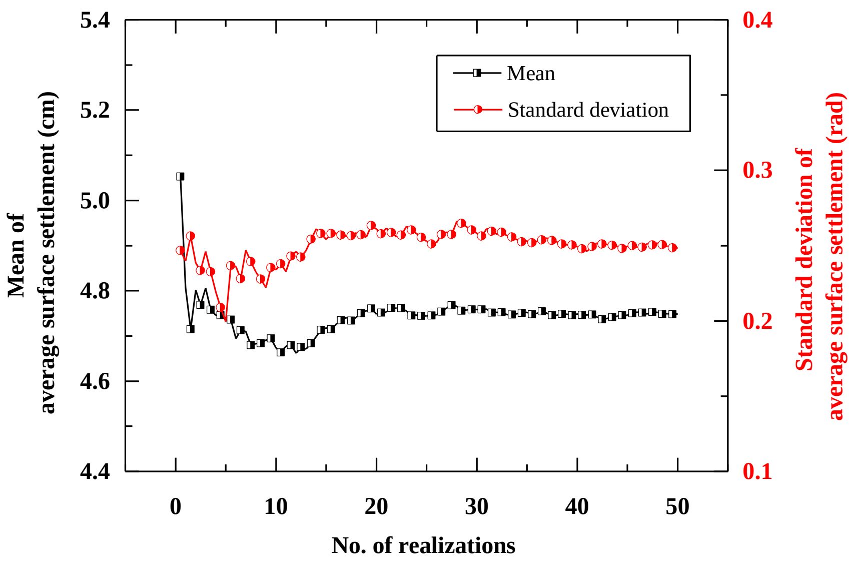

Fig. 6 Convergence check for sufficiency of the number of realizations

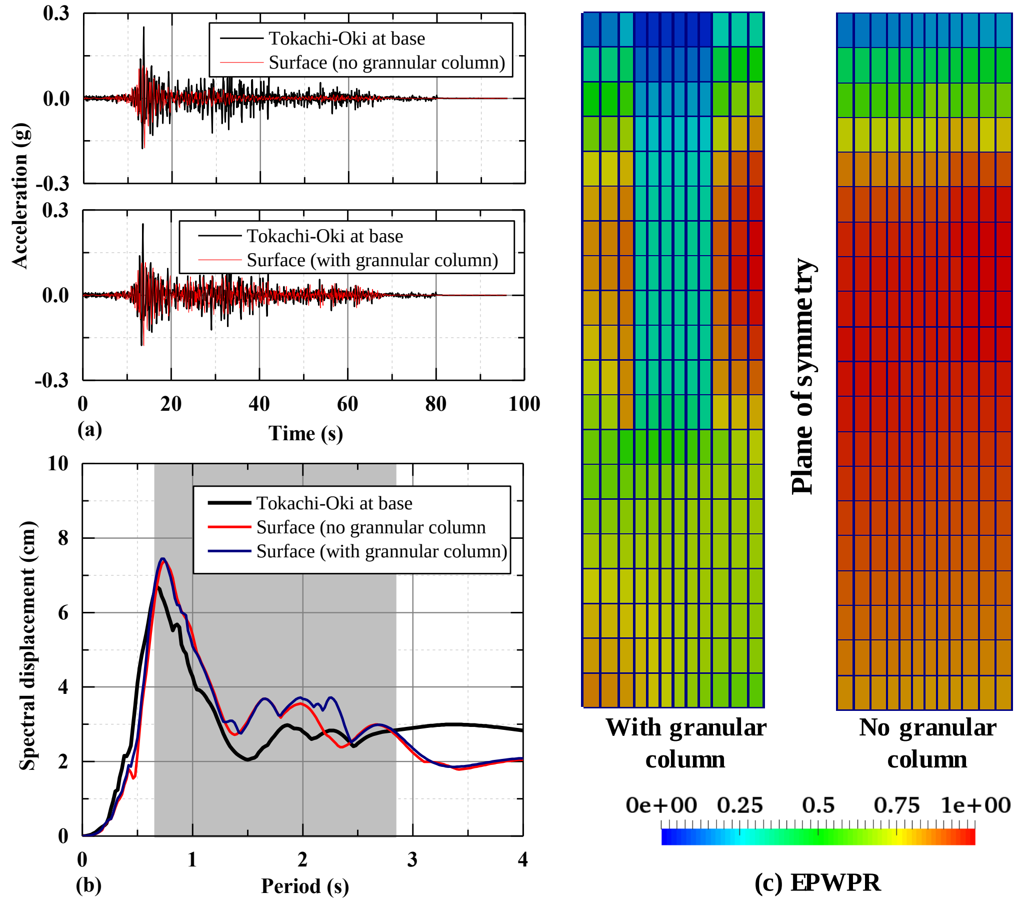

Fig. 7 Deterministic analysis (D_R = 50 %): (a) accelerations at surface (Z= 0 m) in comparison with applied Tokachi-Oki ground motion at base, (b) displacement response spectra (with 5 % damping), and (c) distribution of EPWPR on XZ plane (Y=0) at Time, t = 18 s

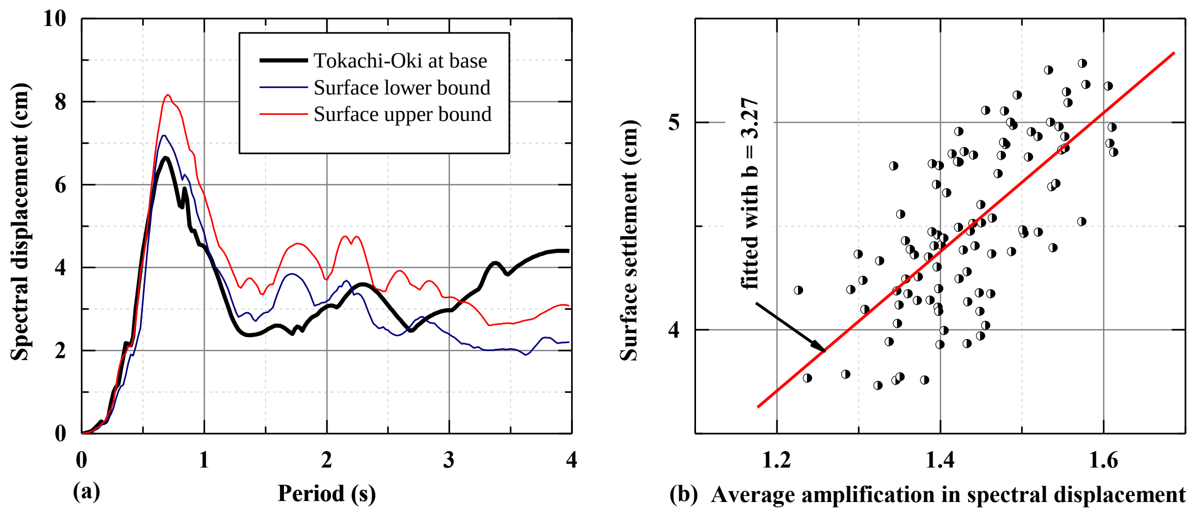

Fig. 8 Stochastic analyses: (a) upper and lower bound of displacement response spectra (with 5 % damping) at the surface (Z= 0 m) and (b) correlation between average surface settlement and average amplification (period T = 0.5 – 3.0 s) of spectral displacement

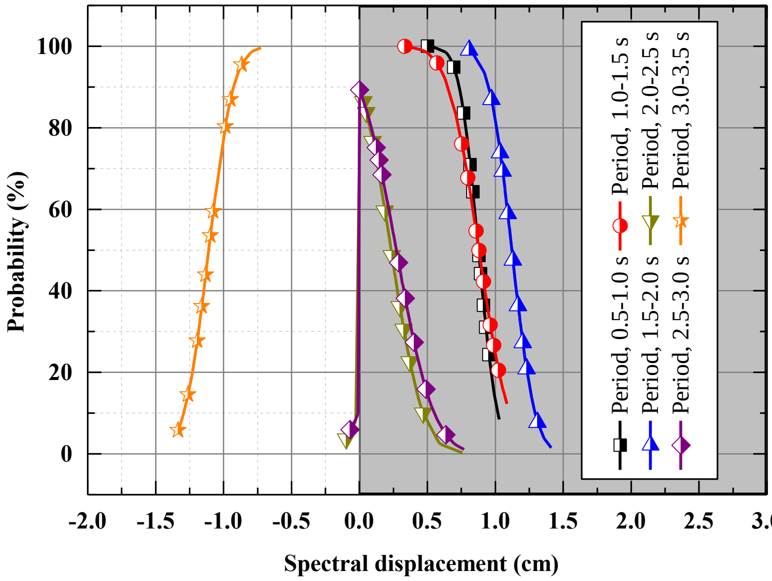

Fig. 9 Probability of deviation of average spectral displacement (for different period range) from their respective deterministic values

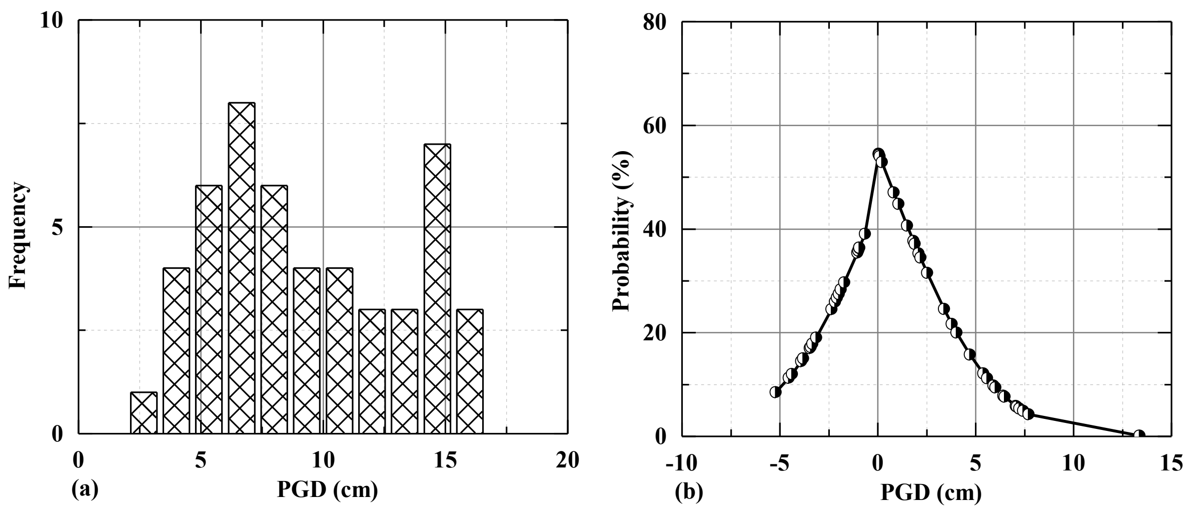

Fig. 10 Peak ground displacement: (a) stochastic distribution and (b) probability of deviation of PGD from the deterministic value of 8.15 cm

The deviations of spectral displacement are considered in the positive side (more than the deterministic value) and the negative side (less than the deterministic value). The spectral displacement being less than the deterministic value is found in the range of 0.75 cm (with 98.14% probability of occurrence) to 1.36 cm (with 5.26% probability of occurrence) for structure periods between 3.0 – 3.5 s. Whereas, the spectral displacement being more than the deterministic value is found in the range of 0.83 cm (with 98.42 % probability of occurrence) 1.46 cm (with 3.56 % probability of occurrence) for structure periods 0.5 – 3.0 s. This wide range of deviation in spectral displacement from their deterministic values emphasizes that the non-uniformity of the ground (traced with the presented stochastic analyses) is important to consider for a better insight of the surface response spectrum. The other important parameter to estimate the seismic demand (displacement-based design) is the peak ground displacement (PGD). The values of PGD may attenuate or amplify depending upon wave propagation of incident ground motion, non-uniformity of the ground, and liquefaction extent of the ground. Furthermore, the densification caused by the installation of granular columns may alter the surface PGD which cannot be traced from deterministic analysis alone. Therefore, the stochastic distribution of surface PGD is evaluated and presented as shown in Fig. 10 (a). The values of surface PGD vary significantly from 3 cm to 18 cm. The surface PGD obtained from deterministic analysis (D_R ~ 50 %) is 8.15 cm. The probability of deviation of stochastic surface PGD from its deterministic value is evaluated and presented in Fig. 10 (b). The surface PGD is found to be less than the deterministic value by 5.12 cm (with 9.12% probability of occurrence) and more than by 13.32 cm (with 1.26% probability of occurrence) having PGD close to the deterministic value (8.15 cm) with the probability of occurrence as high as 58 %.

6. Conclusions

For long period structures such as long-span bridges, high-rise buildings, and tall isolated structures possess the sensitivity to displacement rather than force and thus displacement response spectrum is more appropriate to estimate their seismic demand. The conventional elastic response spectra based on the equivalent linear site response method do not account for the nonlinear elasto-plastic deformation and complex inertial interaction of liquefiable ground. The seismic performance of the foundation-structure system on the liquefiable ground largely depends on the ground displacement rather than the ground surface acceleration, especially for the long-period content of input shaking. Stochastic displacement response spectra for non-uniform liquefiable ground treated with the granular columns are evaluated and presented in this paper. Centrifuge test results are used to validate the numerical modeling scheme which is carried out using the OpenSees framework with PDMY02 elasto-plastic soil constitutive model. Soil variability is implemented with stochastic realizations of $(N1)_{60cs}$ values using spatially correlated Gaussian random field. Three-dimensional finite element simulations are performed for a sufficient number of realizations to map the scale of fluctuation of stochastic displacement spectra. The ground densification due to the installation of granular columns is also incorporated during the numerical simulations. It is found that the displacement response spectrum at the surface for both treated (with the granular column) and the untreated ground is more than the one for applied ground motion at the base for a wide range of periods. The stochastic bound of surface spectral displacement is found to be significantly large for a wide range of periods (0.55 ~ 2.5 s). The amplification in surface spectral displacement with respect to the one for applied shaking is found to be correlated with the average surface settlement of the ground. It is observed that more is the amplification in spectral displacement, larger is the average surface settlement of the ground. Stochastic results also exhibited that the PGD may vary significantly based on the propagation of incident wave and associated liquefaction extent of the ground and should be taken into account for the reliable design seismic displacement demand.

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