17th World Conference on Earthquake Engineering, 17WCEE
Sendai, Japan - September 13th to 18th 2020

COMPARISON OF CYCLE COUNTING METHODS FOR POTENTIAL LIQUEFACTION AND STRUCTURAL FATIGUE ASSESSMENT

R. Stelzer[1], B. Carlton[2], S. Mazzoni[3]

[1] *Project Engineer, Norwegian Geotechnical Institute, ryan.stelzer@ngi.no*
[2] *Project Engineer, Norwegian Geotechnical Institute, brian.carlton@ngi.no*
[3] *University of California, Los Angeles, smazzoni@ucla.edu*

Abstract

Most of the current methods to assess earthquake shaking are based on laboratory testing performed with uniform cycles of shaking that measure the number of cycles to failure for a given amplitude. As a result, researchers have proposed ground motion prediction equations (GMPE) to estimate an equivalent number of cycles with a uniform amplitude, n_{eq}, based on simplified ground motion parameters. Developing these GMPEs generally involves estimating the equivalent number of uniform cycles at different amplitudes through a cycle counting method (CCM), converting these cycles to n_{eq} using a weighting factor curve (WFC), then performing regression analyses to develop a simplified model. Consequently, the resultant GMPE is dependent on the chosen CCM and WFC.

This paper examines the effect of peak counting, level-crossing, mean-crossing, and rainflow counting cycle counting methods applied to 5,840 two component ground motions from the NGA-West2 database using a single WFC to calculate n_{eq}. The lowest n_{eq} consistently resulted from the rainflow cycle counting method. Relative to rainflow, the lowest to highest CCM are mean-crossing (7% higher), level-crossing (21% higher), and peak counting (35% higher).

The effect on n_{eq} of truncating the acceleration time series at 5% of accumulated Arias intensity and 75% or 95% of accumulated Arias intensity was also investigated within the same CCM. When compared with the entire acceleration time series, truncating the seismic record on average reduced n_{eq} to 95% (when truncated from 5% to 95% of accumulated Arias intensity) and to 83% (when truncated from 5% to 75% of accumulated Arias intensity).

Finally, this paper presents equations to convert n_{eq} calculated using one CCM to n_{eq} for a different CCM. This will allow researchers and practitioners to adjust GMPEs for n_{eq} based on one CCM to another to account for epistemic uncertainty and maintain flexibility in their approach.

Keywords: ground motion prediction equations; soil liquefaction; cycle counting methods

1. Introduction

Earthquakes are a global phenomenon that can have devastating impacts on property and people. Earthquake shaking can cause pore pressure build up and liquefaction in cohesionless soils, strength and stiffness degradation in cohesive soils, and fatigue damage in structures. Most of the current methods to assess these issues are based on laboratory testing performed with uniform cycles of shaking that measure the number of cycles to failure for a given amplitude. As a result, researchers have proposed ground motion prediction equations (GMPE) to estimate an equivalent number of cycles with a uniform amplitude, n_{eq}, based on simplified ground motion parameters [1, 2, 3, 4]. The methodology for developing these GMPEs generally involves estimating the equivalent number of uniform cycles at different amplitudes through a cycle counting method (CCM), converting these to n_{eq} using a weighting factor curve (WFC), then performing regression analyses to develop a simplified model. Therefore, GMPEs developed for n_{eq} that follow this methodology are inherently dependent on the chosen CCM and WFC.

This paper examines the effect of cycle counting method on the estimated number of equivalent cycles. We evaluate four different CCM: peak counting, level-crossing, mean-crossing, and rainflow counting [5]. We use over 5,840 two component ground motions from the NGA-West2 database [6], and the WFC of De Alba et al. [7] to calculate n_{eq}. In addition, we investigate the effect on n_{eq} of truncating the acceleration time series at 5% of accumulated Arias intensity, I_A, and 75% or 95% of accumulated Arias intensity [8] compared with including the entire acceleration time series in the CCM analysis. The following sections provide a brief background on the different CCM, the methodology employed in this study, and the results.

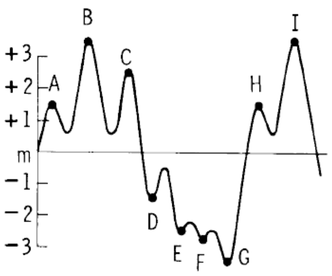
2. Cycle Counting Methods

Cycle counting is a method used to convert a continuous time series of irregular loads into a discrete number of load cycles, where the precise definition of a cycle corresponds to the counting method applied. We investigated four different CCM in this study: peak counting, level-crossing, mean-crossing, and rainflow counting. We followed the definition of these counting methods described in the American Society for Testing and Materials (ASTM) standard E 1049 – 85 [5]. The following sections give a brief description of each of the cycle counting methods.

2.1 Peak Counting

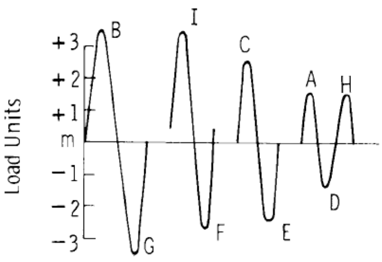
The peak counting method is illustrated in Fig. 1 with a brief description provided below.

- <u>Count peaks and valleys</u> – Identify all peaks and valleys relative to a reference load. Count all peaks at different loading levels and count all valleys at different loading levels.
- <u>Create load cycles</u> – Construct load cycles by combining a peak with a valley. The most damaging cycle is created first by combining the highest peak with the lowest valley. This is followed by the second most damaging cycle combining the second highest peak and second lowest valley. This is continued until all counted peaks and valleys are combined into cycles.



Peak	Counts
+3.5	2
+2.5	1
+1.5	2
−1.5	1
−2.5	1
−2.7	1
−3.5	1

(a) Counting of peaks in time series



Range (units)	Cycle Counts
7	1
6.2	1
5	1
3	1.5

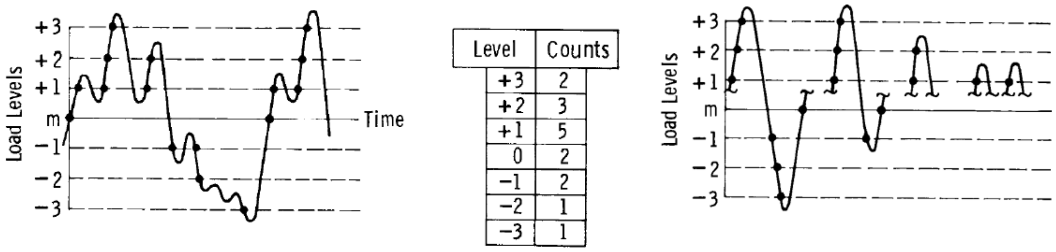
(b) Cycles created from peak counting

Fig. 1 – Peak counting method from ASTM E 1049 – 85 [5]

2.2 Level-crossing

The level-crossing counting method is illustrated in Fig. 2 with a brief description provided below.

- Count levels crossed – Identify load levels of interest above a reference load. Count all positive sloped portions of the load at the load levels above the reference load, then count all negative sloped portions of the load at the load levels below the reference load.
- Create load cycles – Construct load cycles by combining positive and negative levels crossed. The most damaging cycle is created first by combining the most levels crossed above with the most levels crossed below the reference load. This is followed by the second most damaging cycle combining the second most levels crossed above with the second most levels crossed below the reference load. This is continued until all crossed levels are converted to cycles.



Level	Counts
+3	2
+2	3
+1	5
0	2
−1	2
−2	1
−3	1

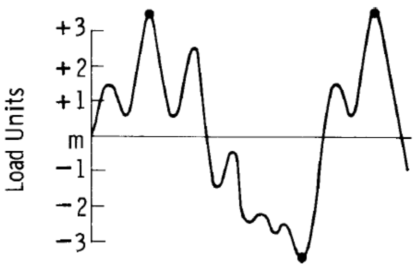
Range (levels)	Cycle Counts
7	1
6	0
5	1
4	0
3	0
2	1
1	2

(a) Counting of levels in time series (b) Cycles created from crossed levels

Fig. 2 – Level-crossing method from ASTM E 1049 – 85 [5]

2.3 Mean-crossing

Mean-crossing or mean-crossing peak counting (also called peak counting excluding non-zero crossing peaks by Hancock and Bommer [9]), is a variation of the peak counting method where the largest peak or valley is counted between two crossings of the mean as shown in Fig. 3, instead of all of the peaks or valleys as in the peak counting method (Fig. 1). This method reduces the number of small amplitude loads considered but does not eliminate them, as small amplitude loads about the mean are still counted.

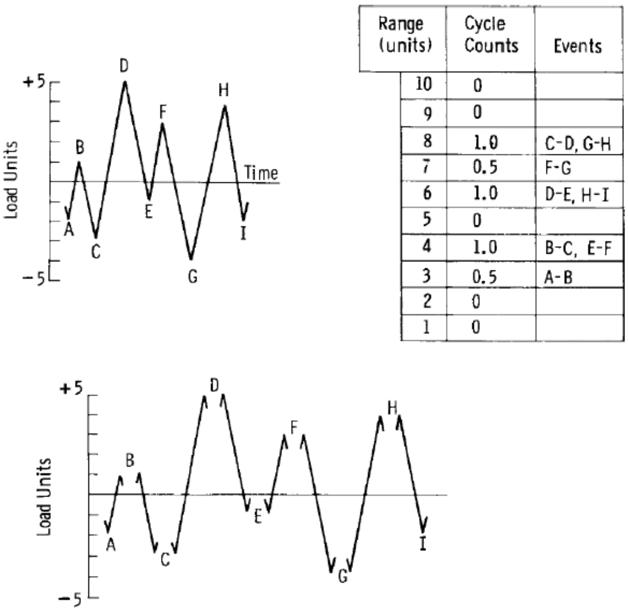


Peak	Counts
+3.5	2
−3.5	1

Fig. 3 – Mean-crossing peak counting [5]

2.4 Rainflow Counting

Range counting methods consider the difference between successive valley and peak loads (positive range) or peak and valley loads (negative range) as shown in Fig. 4. Rainflow counting is a type of range counting method, which also accounts for cycle load reversals on a broader scale. Its name derives from how water would flow if a load history was rotated vertical.



Range (units)	Cycle Counts	Events
10	0	
9	0	
8	1.0	C-D, G-H
7	0.5	F-G
6	1.0	D-E, H-I
5	0	
4	1.0	B-C, E-F
3	0.5	A-B
2	0	
1	0	

Fig. 4 – Simple range counting method [5]

3. Methodology

The following sections detail the methodology followed in this research for comparing n_{eq} values from the four cycle counting methods. First we compiled a ground motion database. Next, we applied the four cycle counting methods to estimate the number of cycles at different amplitudes. Then we applied a weighting factor to calculate the equivalent number of cycles with a uniform amplitude (n_{eq}). Each cycle counting method was also applied to each acceleration time series truncated to 5% to 95% and 5% to 75% of accumulated Arias intensity. As a result, each ground motion record has 12 different cycle counting values.

3.1 Ground motion database

Ground motions for this research came from the PEER NGA-West2 database [6]. The NGA-West2 database includes worldwide ground-motion data recorded from shallow crustal earthquakes in active tectonic regimes. The database used contained 15,464 entries, with records dating from 1966 to 2011.

Data filters were applied to ensure all records had epicentral, hypocentral, Joyner-Boore, and closest distance measures, which was done for future consideration of using the data for regression analysis. The entire dataset was filtered to include earthquakes of moment magnitude equal to or greater than 5, within 500 km of the epicenter. The final dataset contained 5,840 two component ground motions.

3.2 Weighting Factor Curve

A weighting factor curve assigns the relative damage generated by each shaking cycle of an earthquake and enables comparison between various seismic records. Most weighting curves are based on the Palmgren-Miner cumulative damage hypothesis [10,11], which is based on fatigue analysis of metals. Some common weighting

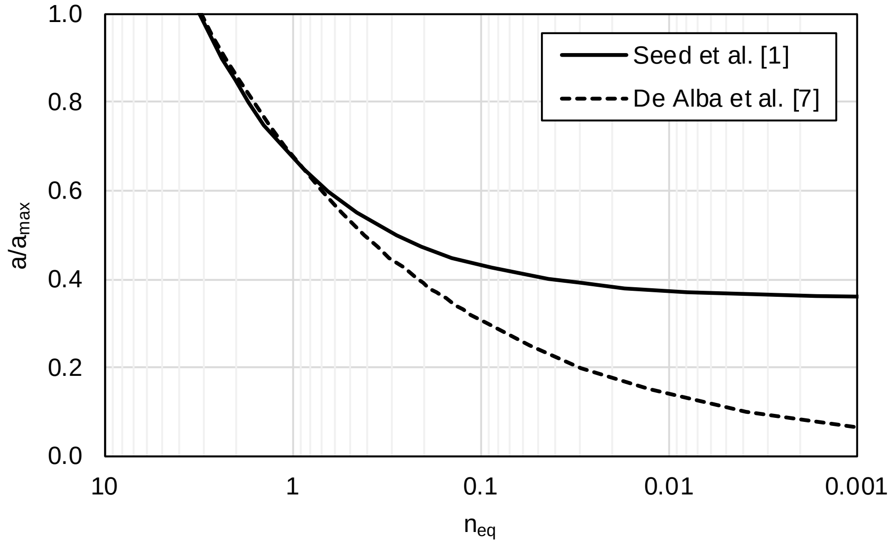


Fig. 5 – Weighting factor curves to produce the number of equivalent cycles at 65% maximum amplitude, equations from Castiglia and Santucci de Magistris [4]

factor curves are those of De Alba et al. [7], Seed et al. [1], and Malhotra [12]. The weighting curve proposed by Seed et al. [1] was one of the first developed for liquefaction analysis of soils. It ignores cycles with $a/a_{max} < 0.36$, as they deemed these low amplitude cycles to be negligible for liquefaction analyses. We wanted to include small amplitude cycles to better understand the difference between the cycle counting methods and the duration cutoffs rather than explicitly exclude them with weighting factors. As a result, we used the WFC by De Alba et al. [7]. This curve covers a more complete range of normalized accelerations as shown in Fig. 5. An alternative method that does not require a CCM or WFC is an energy based approach as used by Green and Terri [13], however this requires information of the site profile.

3.3 Accounting for both components of a ground motion

Seed et al. [1] presented their equation to predict the number of cycles based on regression for individual horizontal components of motion. This implies that mutually perpendicular components of motion are independent (i.e., do not influence one another) and that predicting n_{eq} based on the strongest horizontal component of ground motion is sufficiently conservative for liquefaction assessment.

Work by Liu et al. [14] utilized vector sum normalization whereby both horizontal components were combined and then normalized to the maximum acceleration. As pointed out by Hancock and Bommer [9], using a vector sum eliminates negative amplitudes, such that portions of cycle reversal could be missed. The loss of the original motion's sign (i.e., relative direction) also precludes the use of range-counting methods (i.e., from peak to valley) as this would give only half the cycle amplitude.

The energy based approach of Green and Terri [13] accounts for both components of a ground motion by adding the energy dissipated in the soil from both components. They argue that since energy is a scalar quantity, the calculations can be decoupled and then summed together.

In this paper, we consider both components by summing counted cycles from both horizontal components. Treating the components separately was considered unrealistic since an earthquake will never cause uniaxial shaking. We realize that simply summing horizontal components together does not robustly account for multidirectional shaking. However, it was regarded as a simple and straightforward method for comparison of the different cycle counting methods that included both components of earthquake shaking. If we had considered only one component of motion, and shaking occurred predominantly along one measurement axis, then selection bias could be introduced. Peak ground acceleration corresponding to the horizontal component with the largest value was used for normalizing cycles from both components of motion. Shaking in the vertical direction was not considered.

4. Results

Values of n_{eq} computed by various CCM were plotted with respect to other CCM as shown in Fig. 6. Each point in the scatter plots corresponds to a single seismic record, scatter plot points are coloured such that overlapping data points with low density are plotted in blue, while areas of high density are plotted in red. The line along the diagonal is a parity line with a slope of 1:1, seismic records plotting on this line have the same n_{eq} for each CCM. For this dataset, rainflow CCM produces the lowest n_{eq}. This is apparent from the first column of graphs in Fig. 6, where all data points plot above the parity line. For example, if a seismic record had rainflow n_{eq} of 50, then all other CCM n_{eq} values are greater than 50, and plot above the parity line.

Mean-crossing predicts n_{eq} closest to rainflow n_{eq}, while level-crossing and peak counting are considerably higher. Relative to rainflow, the lowest to highest CCM are mean-crossing (7% higher), level-crossing (21% higher), and peak counting (35% higher). Peak crossing is consistently higher than rainflow and mean-crossing but is more distributed about the parity line with respect to level-crossing. These results are similar to those of Hancock and Bommer [9], who found that mean crossing and rainflow predicted similar values of n_{eq}, whereas peak counting predicted larger values of n_{eq}.

To assess any dependency cycle count values may have with respect to the level of shaking per seismic event, n_{eq} was normalized by various CCM. This is presented in Fig. 7 where values along the vertical ordinate are normalized by the CCM on the horizontal abscissa. A non-linear curve was fit to normalized n_{eq} values of the form shown in Eq. (1), whereby $n_{eq,1}$ represents n_{eq} from one CCM normalized to n_{eq} from a second CCM, $n_{eq,2}$. The coefficients for the equation fit to normalized n_{eq} are shown in Fig. 7, along with their p-value (where 0.05 = 5%). It is recommended to consider n_{eq} for when the majority of normalized values are greater than 1. For example, to predict peak counting n_{eq} relative to rainflow n_{eq} use coefficients a = 1.4064 and b = -0.01, and not a = 0.9324 and b = -0.0618.

$$n_{eq,1}/n_{eq,2} = a \cdot n_{eq,2}^{b} \quad (1)$$

When fitting a trend line through the data points, coefficients with a p-value for the b coefficient greater than 0.05 (i.e., not significant on a 5% significance level) indicate that the normalized number of cycles from one method is independent of the number of cycles of the other counting method. This can be seen for peak counting n_{eq} normalized to mean-crossing n_{eq} where the p-value of b is 43.98%, so b can be taken as 0, such that normalized peak counting n_{eq} is considered a constant 1.2937 for all mean-crossing n_{eq}.

The final task was to compare the influence of seismic record truncation by significant duration. For this task, each CCM n_{eq} at 5% to 95%, and 5% to 75% accumulated Arias intensity was normalized by its own n_{eq} at full duration and plotted against an arbitrary record number along the horizontal abscissa. These results are shown in Fig. 8 to Fig. 11. Truncating the seismic record to 5% to 95% accumulated Arias intensity generally reduced the average n_{eq} to 95% for all CCM except level-crossing, which was 93%. Truncating the seismic record to 5% to 75% accumulated Arias intensity reduced n_{eq} by 82% to 85%. This shows that filtering and ground motion processing can have an effect on the estimated n_{eq} for all cycle counting methods.

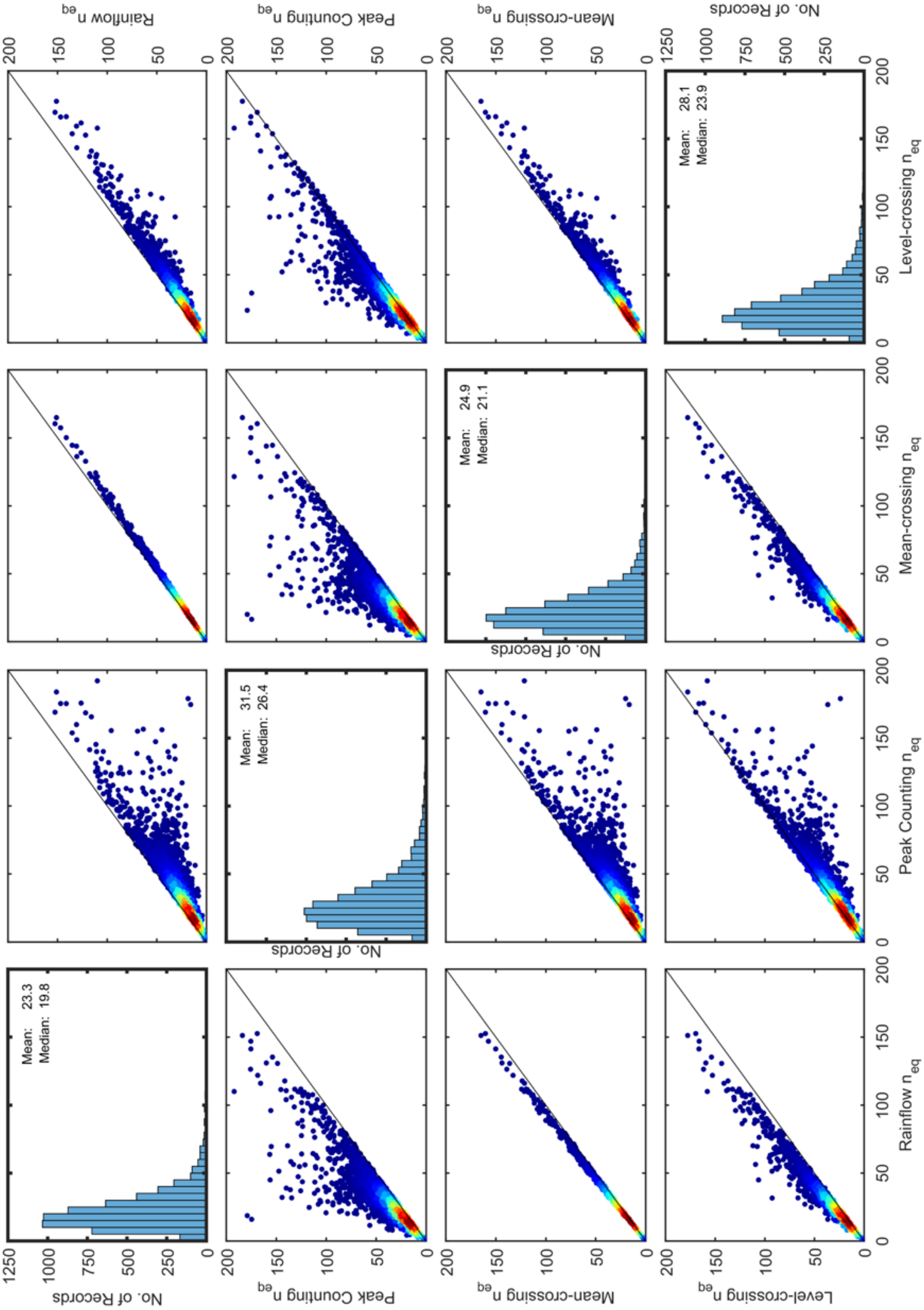


Fig. 6 – Comparison of n_{eq} from various CCM, with histograms of n_{eq} for each CCM plotted along the diagonal (colour indicates the density of data points, blue = low density, red = high density)

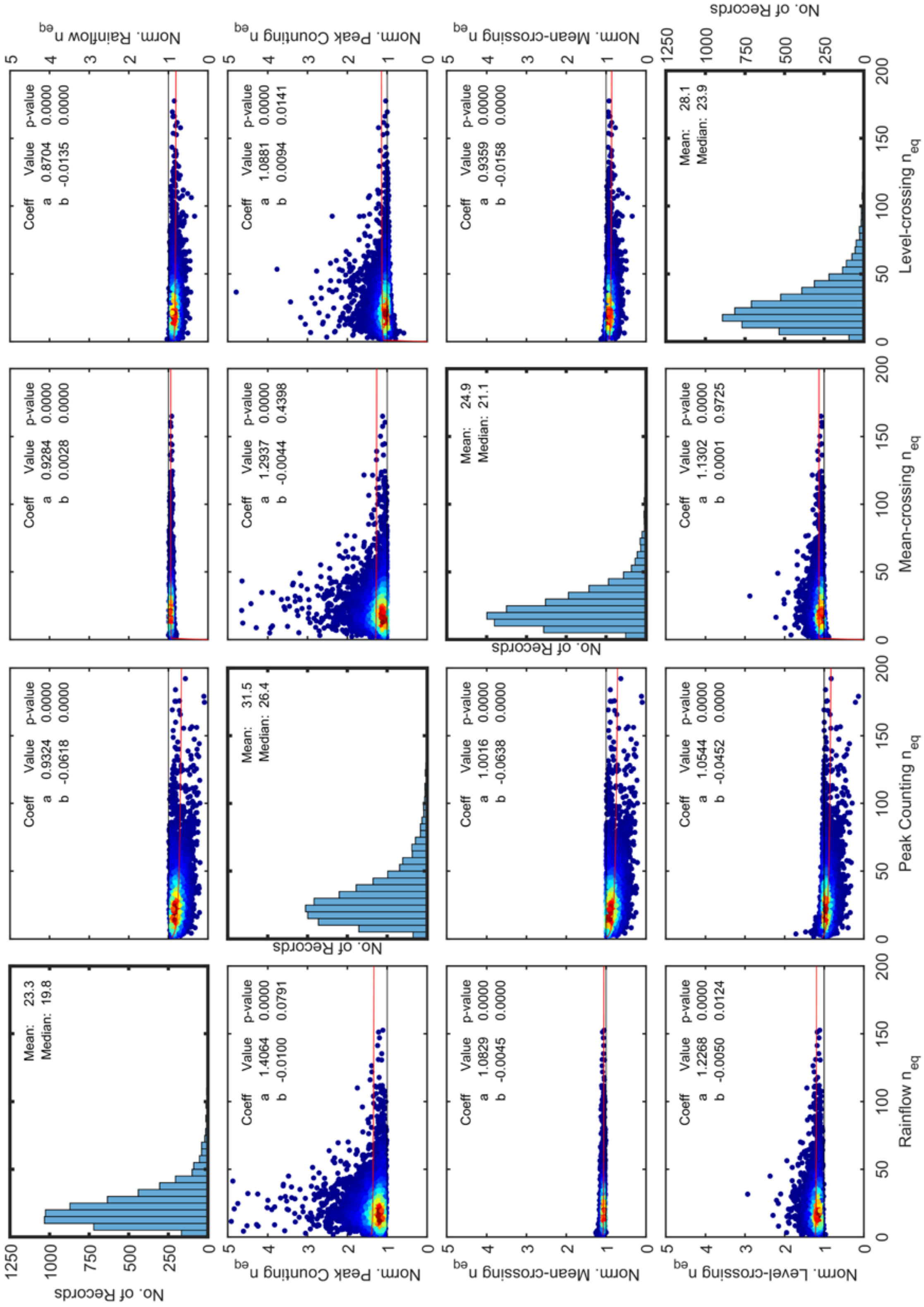


Fig. 7 – Comparison of normalized n_{eq} from various CCM, with histograms of n_{eq} for each CCM plotted along the diagonal (colour indicates the density of data points, blue = low density, red = high density)

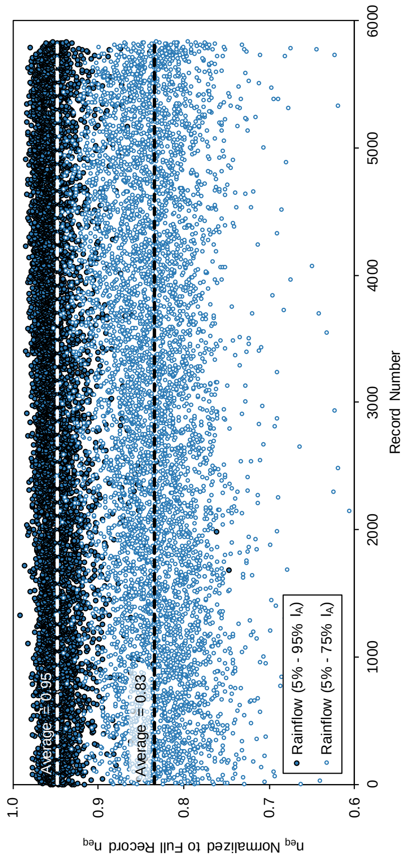


Fig. 8 – Rainflow n_{eq} normalized to various durations

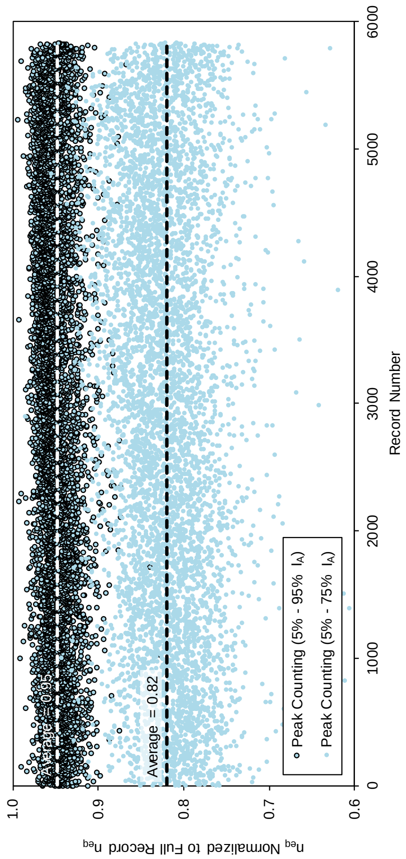


Fig. 9 – Peak counting n_{eq} normalized to various durations

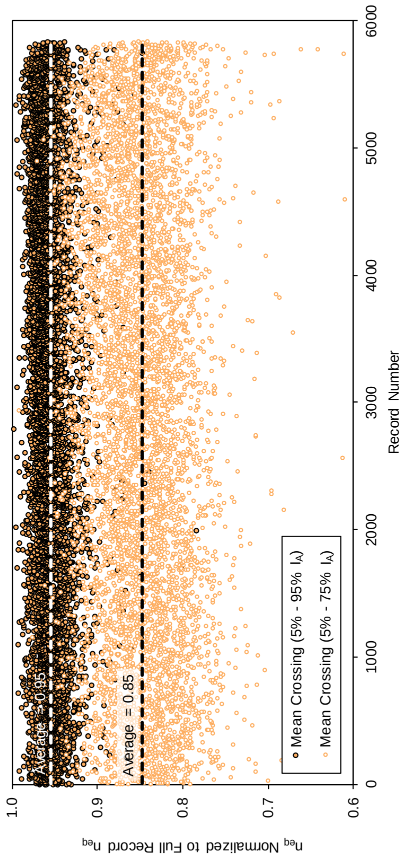


Fig. 10 – Mean-crossing n_{eq} normalized to various durations

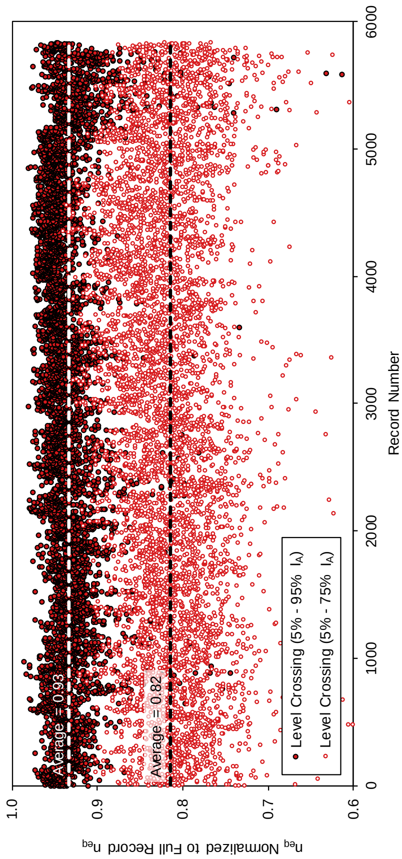
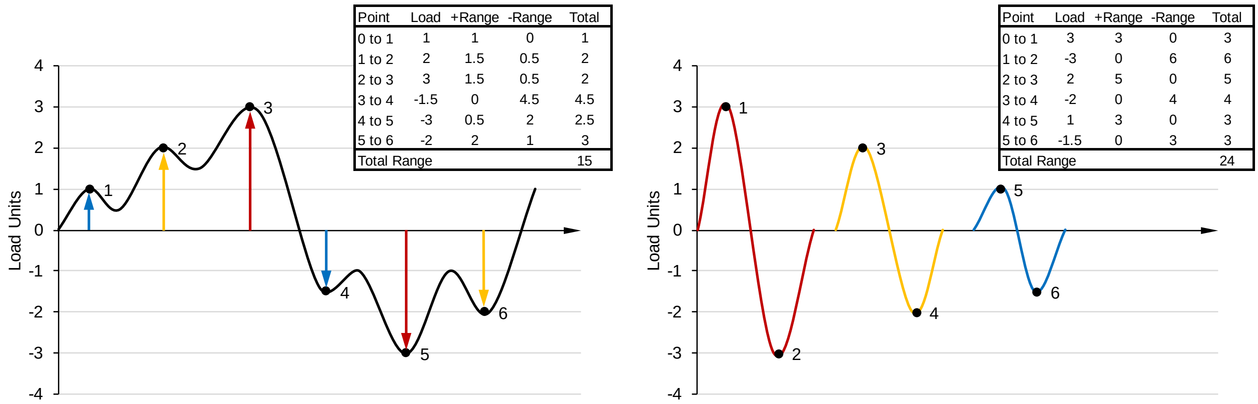


Fig. 11 – Level-crossing n_{eq} normalized to various durations

5. Discussion

One reason the peak counting method might predict larger values of n_{eq} than the rainflow counting method is that the peak counting method assumes that non-zero crossing peaks begin at zero when peaks are assembled into full cycles, when they might only be local peaks (see Fig. 12). If a seismic accelerogram has several load reversals about the central axis (0 g) then the difference in the number of cycles relative to range counting methods is likely minimal. Level-crossing predicts fewer cycles than peak counting because unless the cycle peak corresponds exactly with a load level (as shown in Fig. 2), then a portion of that cycle below the next level is excluded from counting. If enough discrete load levels are considered, then the number of cycles from level-crossing should approach that of peak counting. With the mean-crossing peak counting method, adjusting the reference to the mean compensates for asymmetry (i.e., one sided acceleration pulses) in the time series data such that this method more closely approximates the rainflow counting method.



Point	Load	+Range	-Range	Total
0 to 1	1	1	0	1
1 to 2	2	1.5	0.5	2
2 to 3	3	1.5	0.5	2
3 to 4	-1.5	0	4.5	4.5
4 to 5	-3	0.5	2	2.5
5 to 6	-2	2	1	3
Total Range				15

Point	Load	+Range	-Range	Total
0 to 1	3	3	0	3
1 to 2	-3	0	6	6
2 to 3	2	5	0	5
3 to 4	-2	0	4	4
4 to 5	1	3	0	3
5 to 6	-1.5	0	3	3
Total Range				24

Fig. 12 – Difference between range (left) and peak (right) cycle counting methods

6. Conclusions

In this paper we evaluated the effect of four different cycle counting methods on the equivalent number of cycles with a uniform amplitude (n_{eq}). The four cycle counting methods are peak counting, level-crossing, mean-crossing, and rainflow counting. On average, peak counting produced the highest n_{eq}, followed by level-crossing, mean-counting, and finally rainflow counting. When normalized to rainflow counted cycles, on average the other CCMs are higher by 35%, 21%, and 7% respectively. The results show that the chosen CCM can have a significant effect on the estimate of n_{eq}.

For all counting methods, when truncating the time series data at 5% of accumulated Arias intensity and 75% or 95% of accumulated Arias intensity, n_{eq} is 83% and 95% on average of what it is for when the entire acceleration time series is used, respectively. This shows that filtering and ground motion processing can have an effect on the estimated n_{eq}, regardless of cycle counting method.

This work provides a quantitative assessment of how various CCMs used to develop n_{eq} GMPEs may differ, however, it did not evaluate if any one counting method is more sufficient than others. If, in the future, one counting method were determined to be best suited for a certain type of analysis, then a correction based on the equations developed in this study could potentially be applied to methods using other CCMs. In addition, the proposed equations allow researchers and practitioners to adjust GMPEs for n_{eq} based on one CCM to another to account for epistemic uncertainty and maintain flexibility in their approach.

7. References

[1] Seed, H. B., Idriss, I. M., Makdisi, F., and Banerjee, N. (1975). Representation of irregular stress time histories by equivalent uniform stress series in liquefaction analysis. *Technical Report EERC 75-29*, Earthquake Engineering Research Center, College of Engineering, University of California, Berkeley, California, USA.

[2] Biondi, G., Cascone, E., and Maugeri, M. (2004). Number of uniform stress cycles equivalent to seismic loading. *In Proceedings of 11th International Conference on Soil Dynamics and Earthquake Engineering and 3rd International Conference on Earthquake Geotechnical Engineering*, Vol. 2, International Society of Soil Mechanics and Geotechnical Engineering, 705–712.

[3] Stafford, P. J. and Bommer, J. J. (2009). Empirical equations for the prediction of the equivalent number of cycles of earthquake ground motion. *Soil Dynamics and Earthquake Engineering*, 29(11):1425–1436.

[4] Castiglia, M. and Santucci de Magistris, F. (2018). Prediction of the number of equivalent cycles for earthquake motion. *Bulletin of Earthquake Engineering*, 16(9):3571–3603.

[5] ASTM (1997). Standard practices for cycle counting in fatigue analysis. Standard E 1049 – 85 (Reapproved 1997), American Society for Testing and Materials, West Conshohocken, PA, USA.

[6] Ancheta, T. D., Darragh, R. B., Stewart, J. P., Silva, W. J., Chiou, B. S. J., Wooddell, K. E., Graves, R. W., Kottke, A. R., Boore, D. M., Kishida, T., and Donahue, J. L. (2013). PEER NGA-West2 Database. *Technical Report PEER 2013/03*, Pacific Earthquake Engineering Research Center.

[7] De Alba, P., Seed, H.B., Chan, C.K. (1976) Sand liquefaction in large scale-scale simple shear test. *Journal of the Geotechnical Engineering Division* 102(9):909–927.

[8] Arias, A. (1970). A measure of earthquake intensity. In Hansen, R., editor, *Seismic Design for Nuclear Power Plants*, pages 438–483. MIT Press.

[9] Hancock, J. and Bommer, J. J. (2005). The effective number of cycles of earthquake ground motion. *Earthquake Engineering and Structural Dynamics*, 34(6):637–664.

[10] Palmgren, A. (1924). Die lebensdauer von kugellagern (Life length of roller bearings). *Zeitschrift des Vereins Deutscher Ingenieure*, 68(14):339–341 (in German).

[11] Miner, M. A. (1945). Cumulative damage in fatigue. *Journal of Applied Mechanics*, 12(3):A159–A164.

[12] Malhotra, P.K. (2002), Cyclic-demand spectrum. *Earthquake Engineering and Structural Dynamics*, 31: 1441–1457.

[13] Green, R. A. and Terri, G. A. (2005). Number of equivalent cycles concept for liquefaction evaluations–revisited. *Journal of Geotechnical and Geoenvironmental Engineering*, 131(4):477–488.

[14] Liu, A. H., Stewart, J. P., Abrahamson, N. A., and Moriwaki, Y. (2001). Equivalent number of uniform stress cycles for soil liquefaction analysis. *Journal of Geotechnical and Geoenvironmental Engineering*, 127(12):1017–1026.

