



## VALIDATION OF NUMERICAL PREDICTIONS OF LATERAL SPREADING BASED ON A LARGE CENTRIFUGE-MODELS DATABASE

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### Abstract

During the last decades, important efforts and developments in the computational modeling of soil liquefaction have been developed; however, results of numerical simulations have still important discrepancies with physical models; therefore, exercises of verification and validation (V&V) are required to enhance the reliability of numerical models for liquefaction.

LEAP (Liquefaction Experiments and Analysis Projects) is a joint project that pursues verification, validation and uncertainty quantification of numerical procedures for predicting the effects of liquefaction, based on centrifuge experiments. One of the main goals of two past LEAP exercises (LEAP-UCD-2017 and LEAP-ASIA-2019), was the investigation of the dynamic behavior of a submerged, uniform-density, 20m long, 4 m deep and 5 degrees sloping deposit of Ottawa F-65 sand. In these exercises 48 different models were tested in 10 different facilities around the world, placing special emphasis on the determination of the median values and variability under different relative densities (50% - 80%), and peak ground accelerations (0.1g – 0.35g); additionally, twenty-three Torsional Hollow Cylinder Shear Tests were carried out, aiming to study the mechanical characteristics of Ottawa F-65 Sand.

The main objective of this paper is to validate and evaluate the capabilities of the “Strain Space Multiple Mechanism Model” to simulate the lateral spreading phenomena under a diverse range of densities and input motions, taking as a base the LEAP Database. In this model (which incorporates a new stress-dilatancy relationship), the behavior of granular materials is idealized on the basis of a multitude of virtual simple shear mechanism oriented in arbitrary directions; it is worth to mention that the model has been widely used as an effective stress model to simulate the dynamic behavior of sands.

In order to develop the validation exercise, initially, the consistency of the LEAP centrifuge models was evaluated in order to estimate the median response of the ground and its associated variability. Following this step, the numerical modeling process was started by the calibration of the model parameters, based on the results of the Torsional Shear Tests; in this process, due to the high amount of model parameters, the input variability for the simulations was defined based on potential variations of the simulated Liquefaction Resistance Curve (LRC), rather than individual changes in the parameters. Taking as a basis the calibrated parameters, several numerical models were developed under four different densities, and a wide range of peak acceleration values, aiming to replicate the centrifuge models; in this process, the variability of the numerical model was estimated by propagating the variability established in the previous process (i.e. calibration of parameters process) by means of a Monte Carlo simulations. As a final step, the outcomes of the numerical and physical models were compared, taking into account the median values and its associated variability.

*Keywords: Validation, LEAP, Liquefaction, Torsional Shear Tests, Lateral Spreading*



## 1. Introduction

During the last decades, important efforts and developments in computational modeling of geo-materials have contributed to increasing the accuracy of prediction of the dynamic response of soil systems; due to its catastrophic consequences, special emphasis has been pointed to the liquefaction induced ground failures. However, despite the efforts, results of numerical modeling have discrepancies with results obtained in physical models; so, exercises of verification and validation (V&V) are required to enhance the reliability of numerical models for liquefaction prediction. In this sense, a new international collaborative project (named “LEAP”) was developed, aiming a re-assessment of the reliability of modern numerical techniques in the analysis of liquefaction related problems<sup>1)</sup>.

In order to generate a reliable database for the development of current and future V&V processes of liquefaction models, two main exercises were developed in the LEAP Framework (LEAP-UCD-2017<sup>2)</sup> and LEAP-ASIA-2019<sup>3)</sup>); focusing on the study of the dynamic behavior of a saturated sloping ground deposit of Ottawa F-65 Sand. As part of these exercises, forty-eight centrifuge models were developed in ten different centrifuge facilities; and also, twenty-three Torsional Hollow Cylinder Shear Tests were carried out<sup>4)</sup>, aiming to study the mechanical characteristics of Ottawa F-65 Sand.

The main objective of this paper is to validate and evaluate the capabilities of the “Strain Space Multiple Mechanism Model”<sup>5)</sup> to simulate the lateral spreading phenomena under a diverse range of densities and input motions, taking as a base, the LEAP Database. Figure 1 shows a schematic diagram of the main steps followed in the validation process. Initially, the consistency of the centrifuge models was evaluated in order to estimate the median response of the ground and its associated variability. Following this step, the numerical modeling process was started by the calibration of the model parameters, based on the results of the Torsional Shear Tests; in this process, due to the high amount of model parameters, the variability was defined based on potential variations of the simulated Liquefaction Resistance Curve (LRC). Taking as a basis the calibrated parameters, several numerical models were developed under four different densities, and a wide range of peak acceleration values, aiming to replicate the centrifuge models; in this process, the variability of the numerical model was estimated by propagating the variability established in the previous process (i.e. calibration of parameters process) by means of a Monte Carlo simulations. As a final step, the outcomes of the numerical and physical models were compared, taking into account the median values and its associated variability.

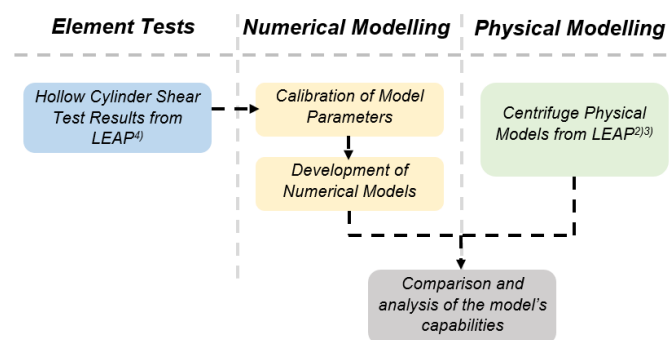


Fig. 1 – Schematic diagram of the main steps of the validation exercise

## 2. Physical Modeling

### 2.1 Model Specifications

The standard sand selected for the LEAP Exercises was Ottawa F-65; this can be described as a clean, poorly graded, whole grain silica sand, with less than 0.5% fines by mass. In order to keep consistency and reduce



the variability related to potential variations in the material, the sand was provided to all the facilities by UC Davis prior to the development of the models<sup>6</sup>.

As described by Kutter et. al<sup>6</sup>, a uniform-density, 20 m long, 4 m deep at center, and 5 degrees sloping deposit of Ottawa F-65 Sand inside a rigid container, was specified for the LEAP Exercises. Figure 2 shows the geometry, dimensions, and instrumentation of the target models (in model scale), applicable to the tests developed at Kyoto University.

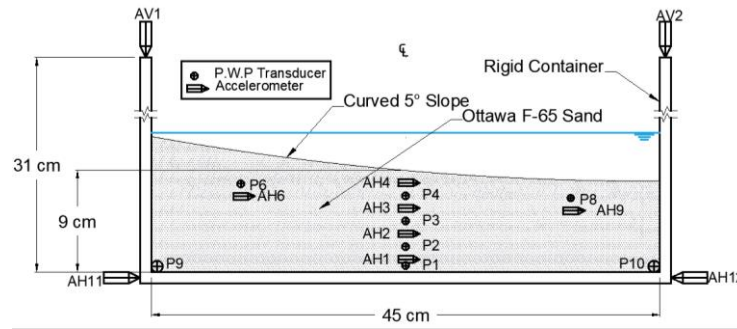


Fig. 2 – Model Dimensions – Model Scale

The target input motion consisted of a ramped sinusoidal 1 Hz wave (Figure 3); however, due to the presence of high-frequency vibrations in the achieved motions, and taking into account that higher frequency components have some but relatively small effect on the behavior of the model, the project (as a first approximation) used the concept of Effective PGA ( $PGA_{eff}$ )<sup>2</sup>.

$$PGA_{eff} = PGA_{1Hz} + 0.5 * PGA_{hf} \quad (1)$$

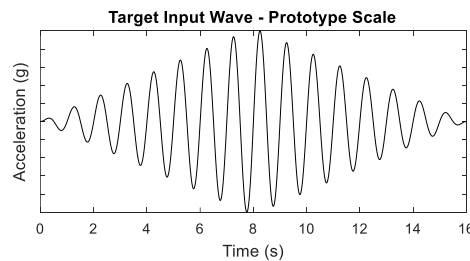


Fig. 3 – Specified Ramped Sine Wave

Where: “ $PGA_{1Hz}$ ” represents the PGA of the 1 Hz component of the achieved motion, and “ $PGA_{hf}$ ” represents the higher frequency components of the ground motion.

Additionally, it has been reported that small errors in the density estimation process, have a significant influence in the computation of the Relative Density<sup>7</sup>; in order to reduce the uncertainties in the estimation of this parameter, Cone Penetration Tests (CPT) were developed in most of the tests, with a new 6 mm Mini-CPT<sup>8</sup>; this method, although providing an indirect measurement (i.e. tip resistance “ $qc$ ”), has proven to be reliable in the estimation of the uniformity of the ground and its associated dry density<sup>2</sup>.

## 2.2 Correlation and Variability Estimation

As mentioned in the previous section, as part of LEAP Exercises, forty-eight tests were developed in ten different centrifuge facilities all over the world, aiming to study the dynamic response of a saturated sloping ground under different relative densities ( $Dr$ ), and Peak Ground Accelerations (PGA).

Based on the LEAP-UCD-2017 results, Kutter et. al<sup>2</sup> found that, for the lateral spreading phenomena, the final surface displacements ( $U_x$ ) are primarily a function of the intensity of shaking and the relative



density of the sand, by developing a nonlinear correlation between  $U_x$ ,  $PGA_{eff}$ , and  $Dr$  (estimated through CPT measurements). As part of this paper, the aforementioned correlation was updated by including the LEAP-ASIA-2019 results<sup>9)</sup> (See Figure 4); aiming that this correlation will serve as the basis for the validation process.

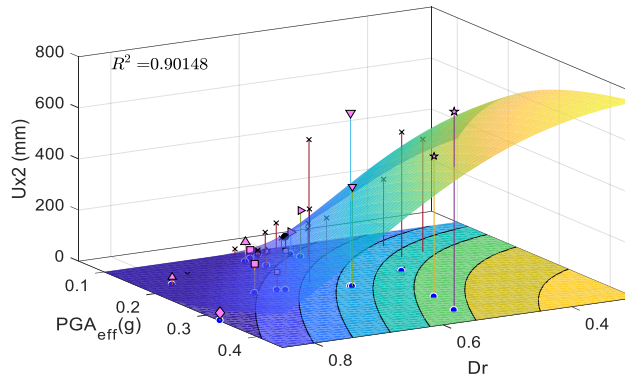


Fig. 4 – Updated correlation between  $U_x$ ,  $Dr$ , and  $PGA$

In addition to estimating the median value of the correlation, and as an attempt to estimate the variability in the tests, by assuming that the displacement values can be represented as a random variable that follows a Gaussian distribution, the upper and lower bounds for a 95% probability were estimated as well (see Figure 5).

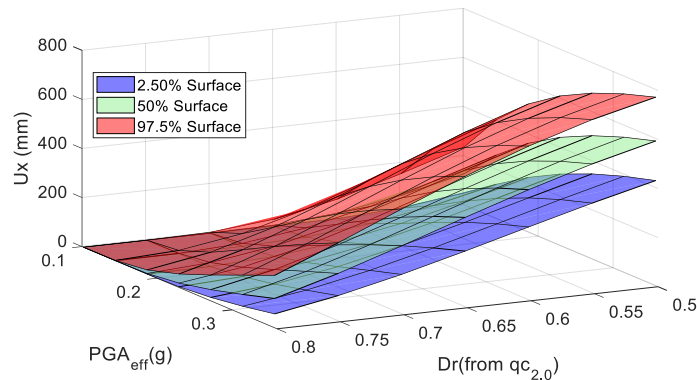


Fig. 5 – Updated correlation, comparison between the median trend (50% Surface) and the Upper (97.5% Surface) / Lower (2.5%) Bounds for a 95% probability

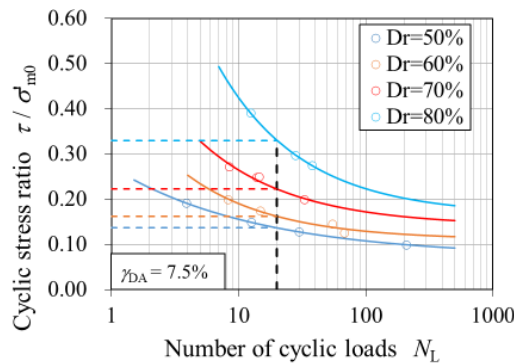


Fig. 6 – LRC of Ottawa F-65 Sand for  $\gamma_{DA}=7.5\%$ <sup>4)</sup>



### 3. Element Tests

In order to study the mechanical properties of Ottawa F-65 Sand, Vargas et. al<sup>4)</sup> conducted a series of Hollow Cylinder Dynamic Torsional Shear Tests for four different relative densities ( $D_r=50\%$ ,  $60\%$ ,  $70\%$ , and  $80\%$ ), under a wide range of Cyclic Stress Ratio (CSR) values. Figure 6 shows the achieved liquefaction resistance curves (LRC) for  $\gamma_{DA}=7.5\%$ .

### 4. Numerical Modeling

#### 4.1 Numerical Model

As stated in Section 1, the main objective of this paper is the validation of the capabilities of the “Strain Space Multiple Mechanism Model” to simulate the lateral spreading phenomena. In this model, the behavior of granular materials is idealized on the basis of a multitude of virtual simple shear mechanism oriented in arbitrary directions<sup>5)</sup>. The original version of the model was proposed by Iai et.al<sup>10)</sup> in 1992 as a strain-space model for cyclic mobility; and, in 2011, the model was updated in order to incorporate a new stress-dilatancy relationship. The updated model was implemented in a commercial finite element software, called “FLIP ROSE”; it is important to mention that this software has been widely used for evaluating the seismic performance of soil-structure systems in the research and engineering practice fields.

For a plane-strain application of the model, and by assuming an isotropic texture of the material, the strain space multiple mechanism model has 17 primary parameters for the analysis of liquefaction; among them, five specify the volumetric mechanism, three specify the shear mechanism, and nine controls the dilatancy.

#### 4.2 Element Test Simulations

Based on a parametric study, the seventeen parameters of the model (for each density under study) were estimated following an iterative procedure, taking as a primary reference the LRC for  $\gamma_{DA}=7.5\%$ . As can be seen in Figure 7, a good agreement between the measured and computed values of the LRC was obtained. Additionally, as an example, Figures 8 shows the detailed results of the element-tests simulations for  $D_r=70\%$  ( $CSR= 0.247$ ), including the records of the time history of shear strain development, the time history of EPWP ratio development, the stress-strain response, and the stress path.

A well-known approach for the model uncertainty quantification<sup>11)</sup>, corresponds to model the uncertainties by defining the model parameters as random variables (i.e. define the uncertainty of each parameter), and propagate the uncertainty through the model. This approach seems to be reasonable and effective for cases in which the model parameters can be independently estimated and the correlation between them can be established. Unfortunately, regardless that each model parameter has a physical interpretation, the current state of the art of the liquefaction modeling, and the soil element testing, does not allow an independent estimation of all the parameters; basically, due to the large number of required parameters. In this sense, and as mentioned in the previous paragraph, the estimation of model’s parameters is usually developed in an iterative fashion, aiming to simulate a specific LRC; this framework of parameter estimation allows that, for a specific LRC, more than one combination of parameters can be obtained. It is important to mention that, as part of the parametric analysis developed for the parameter estimation, a good similarity was found between the results of the numerical models (that simulates lateral spreading) carried out by using two different sets of parameters that simulate a similar LRC, suggesting that, the numerical modeling results (for lateral spreading) depend on the simulated LRC, rather than on a specific set of parameters. In addition to that, up to date, several well-established constitutive models that simulate liquefaction are available, each of them defining a different set of parameters, however, for almost all of them, is a common practice to compare the measured and simulated LRC as a metric to evaluate the parameter estimation.

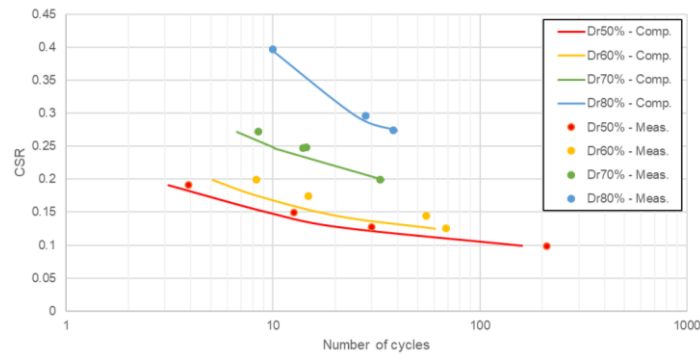


Fig. 7 – Measured<sup>4)</sup> and Computed Liquefaction Resistance Curve for  $\gamma_{DA}=7.5\%$ .

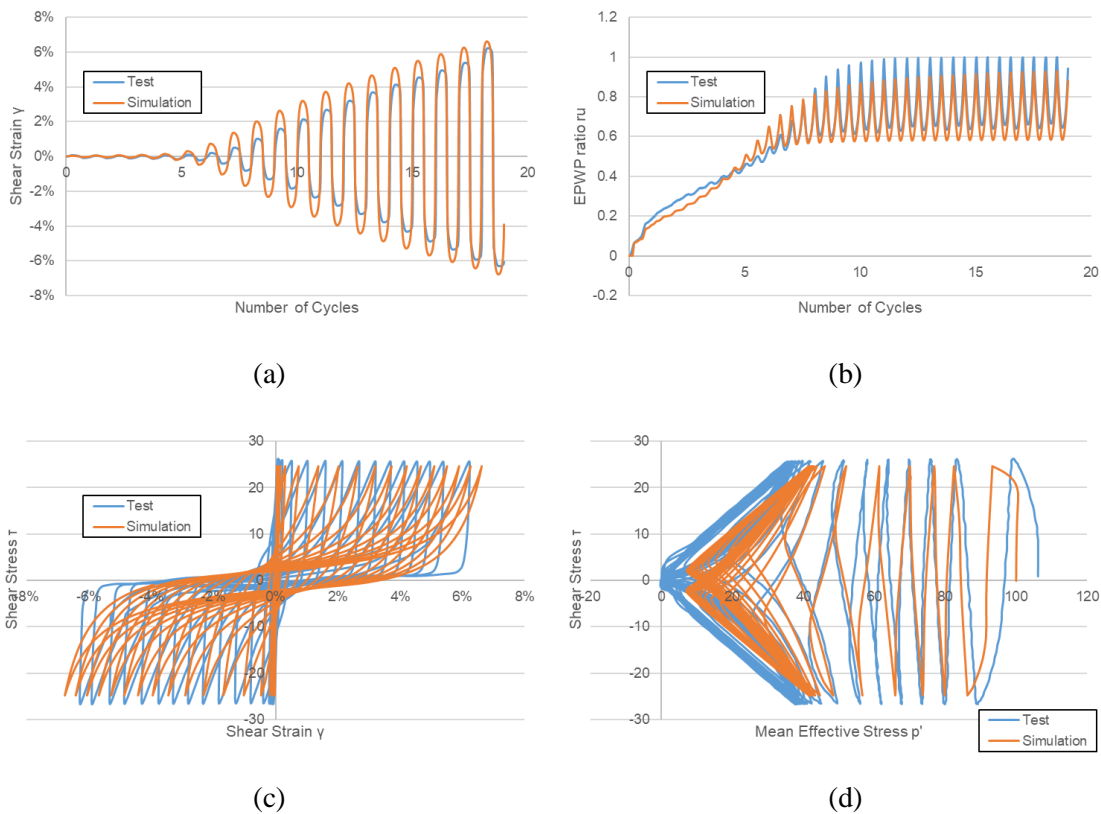


Fig. 8 – Numerical Simulation of Element Tests  $Dr=70\%$ ,  $CSR=0.247$  (a) Time history of shear strain development, (b) Time history of EPWP ratio development, (c) Stress-strain response, (d) Stress path

Based on the above, in this paper, the input variability of the model is proposed to be quantified as changes in the simulated LRC, rather than individual changes in the parameters. In order to express the variations in the simulated LRC, several realizations were developed by changing the parameters in a random fashion; in this process, it has been found that six of them have a positive correlation with the LRC (i.e. an increase of the parameter leads to an increase in the simulated resistance), four have a negative correlation (i.e. an increase of the parameter leads to a decrease in the simulated resistance), and five have a small effect. If we assume that the estimated parameters (Figure 7) correspond to the median values (i.e. 50% percentile), it has been found that the “upper probability bound”, can be obtained in a single simulation by increasing the parameters that keep a positive correlation (with the LRC), and reducing the parameters that keep a negative correlation (and conversely for the “lower probability bound”). Due to the complex



characteristics of the soil, its geological conditions, and the quality of data exploration, the LRC's variability will change from site to site; hence, a unique variability scenario cannot be determined. In this sense, three different variability-scenarios were considered.

The LRC's upper/lower bounds, for each variability scenario, were estimated by following the methodology explained in the previous paragraph; the three variability-scenarios consist of increasing/decreasing the parameter's values by 1% (Scenario 1), 5% (Scenario 2) and 10% (Scenario 3), respectively. Additionally, since the variability scenarios were chosen arbitrarily, it has been assumed that the upper and lower bounds correspond to 2.5% and 97.5% percentiles, respectively.

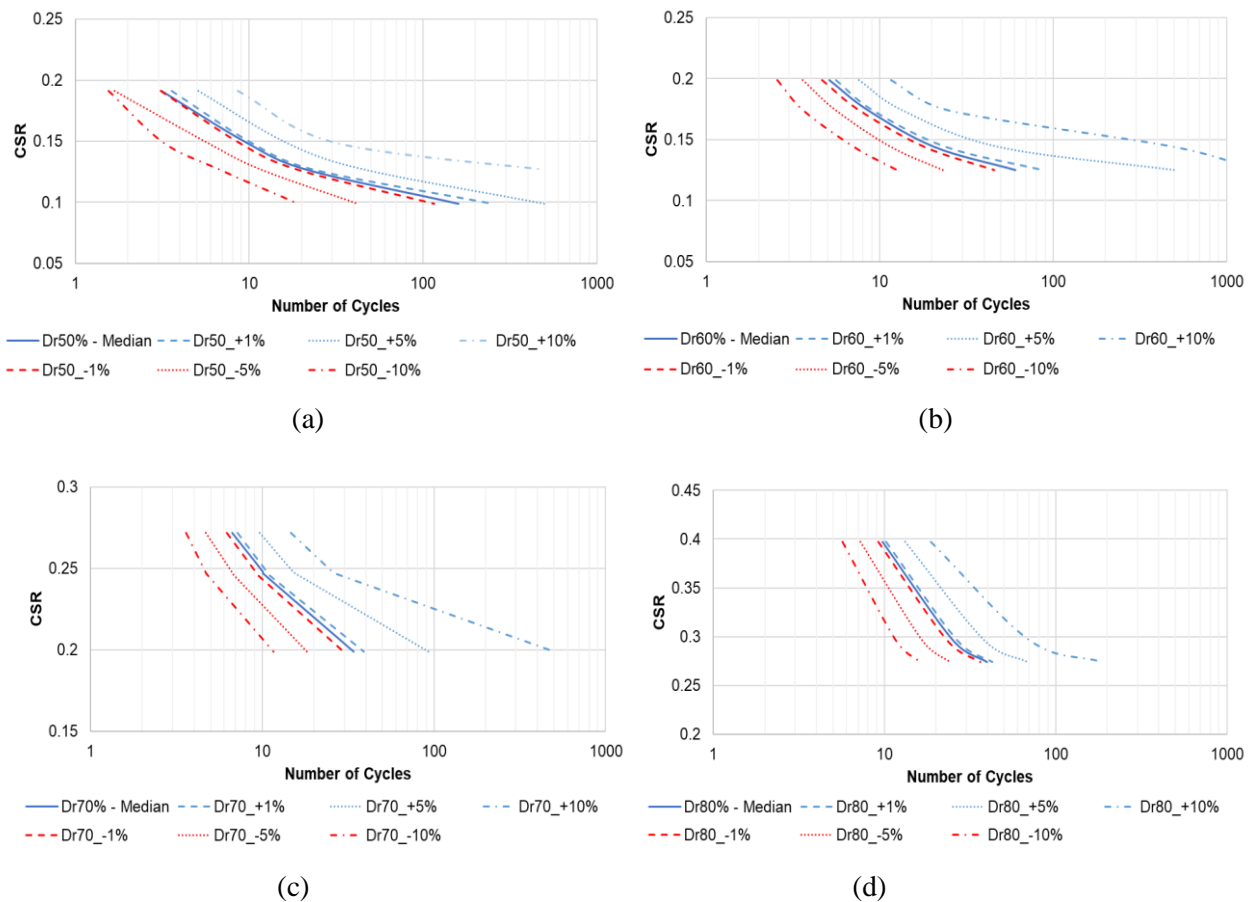


Fig. 9 – Simulated LRC's - Comparison between the median trend (50%) and the Upper (97.5%) / Lower (2.5%) Bounds for a 95% probability (a) Dr=50%, (b) Dr=60%, (c) Dr=70%, (d) Dr=80%

### 4.3 Physical Model Simulations

Based on the characteristics of the models, and using the calibrated parameters, the analysis was carried out under 2-D plane-strain conditions, aiming to simulate the models in prototype scale. Figure 10 shows the mesh and the boundary conditions used in the analysis; as seen in the figure, 384 4-node quadrilateral elements (including the pore water elements) were used. In order to replicate the boundary condition of the experiments (rigid boxes were used in all the experiments), horizontal and vertical displacements were fixed at the base boundary, meanwhile, only vertical displacements were allowed at the lateral boundaries; additionally, the lateral and base boundaries were set to be impermeable, and the pore water pressure at the ground surface was specified to represent a hydrostatic condition.

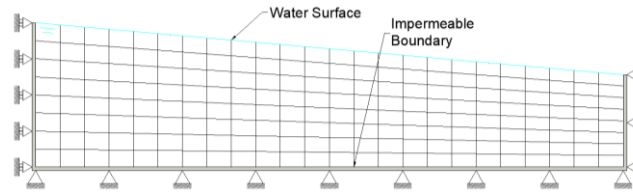


Fig. 10 – FEM Mesh and Boundary Conditions

First, a self-weight analysis was carried out, to obtain the initial stress distribution before shaking; after finishing this step, a dynamic response analysis was performed for 130 s, considering pore water flow migration (during and after the shaking). The numerical time integration was done by the SSpj method, using a time step of  $\Delta t=0.002$  s. In the dynamic simulation, as suggested by Ueda & Iai<sup>12)</sup>, Rayleigh damping ( $\alpha=0.0$ ,  $\beta=0.00032$ ), was used to ensure the stability of the numerical solution process.

## 5. Comparison between Physical and Numerical Models

The updated correlation (Figure 5) was used as the base for the validation process; by using the model presented in Section 4.3, and based on the parameters estimated on Section 4.2, several numerical models were developed in order to replicate the combinations of  $D_r$ - $PGA_{eff}$  for which the correlation was developed (i.e.  $50\% < D_r < 80\%$ , and  $0.1g < PGA_{eff} < 0.35g$ ). Since the uncertainty of the LRC was defined in section 4.2, it was propagated through the model by means of Monte-Carlo simulations, in order to obtain the median trend and the variability of the computed displacements.

Figures 11, 12, 13 and 14 show a comparison between the computed displacements (obtained by the numerical simulations) and the results obtained in the estimated correlation (Figure 5), for  $D_r= 50\%$ ,  $60\%$ ,  $70\%$ , and  $80\%$ , respectively. It is important to mention that, the figures include a comparison between the mean values (i.e. 50% probability) and the upper/lower boundaries for a 95% probability, for the three scenarios of variability defined in Section 4.2.

The main outcomes observed in Figures 11, 12, 13 and 14, it can be summarized as follows:

- As for the median response, a good agreement between the computed displacements and the estimated correlation can be seen; it suggests that if the model parameters are calibrated based on high-quality laboratory tests, a good agreement can be obtained for PGA values lesser than 0.25g.
- For a 95% of probability, the confidence bounds of the computed displacements (considering the three different variability scenarios), are located between the confidence bounds of the experimental outcomes (with few exceptions); validating the model for the three variability conditions considered in this exercise, for PGA values lesser than 0.25g.
- It was found that, for a given density, the slope of the curve Displacement-PGA is smaller in the computed displacements than in the estimated correlations (for both median response and confidence bounds); this means that, for a given model, the increase in the displacements produced by an increase in the PGA values, is smaller in the simulations than in the physical models; this may cause a potential underestimation of the deformations, especially for  $PGA > 0.25g$ .
- Important variations were found between the experimental and numerical results for PGA values higher than 0.25g; this may be explained due to the fact, that few physical models were developed in this range of accelerations, and also, due to its magnitude, the induced CSR values cannot be replicated in the element tests (due to the instability of the sample); so, additional research efforts would be required to explore the validity of the “Strain Space Multiple Mechanism Model” in these range of accelerations.



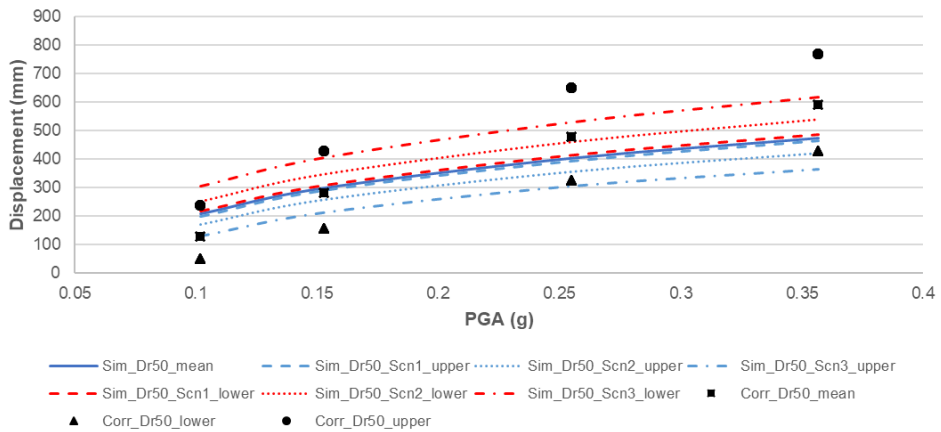


Fig. 11 – Comparison between the displacements obtained in physical and numerical models, for Dr = 50%

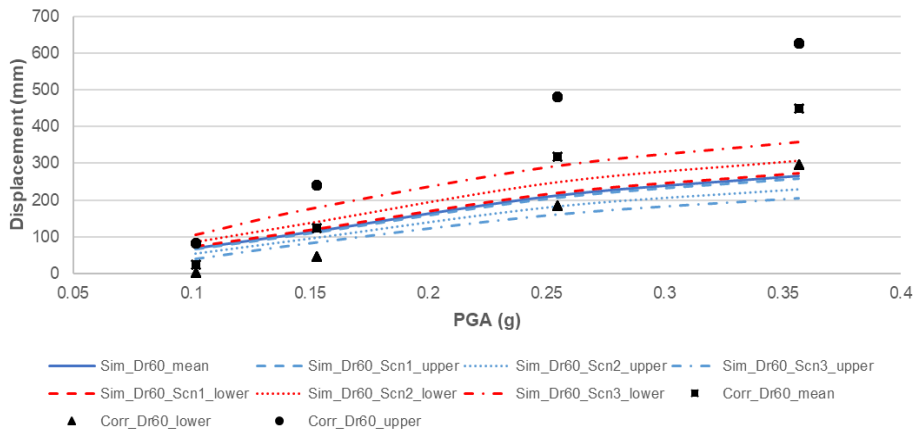


Fig. 12 – Comparison between the displacements obtained in physical and numerical models, for Dr = 60%

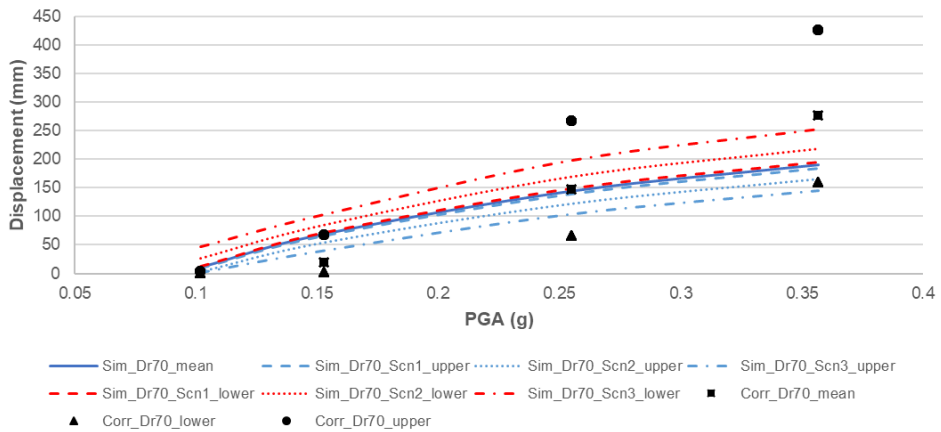


Fig. 13 – Comparison between the displacements obtained in physical and numerical models, for Dr = 70%

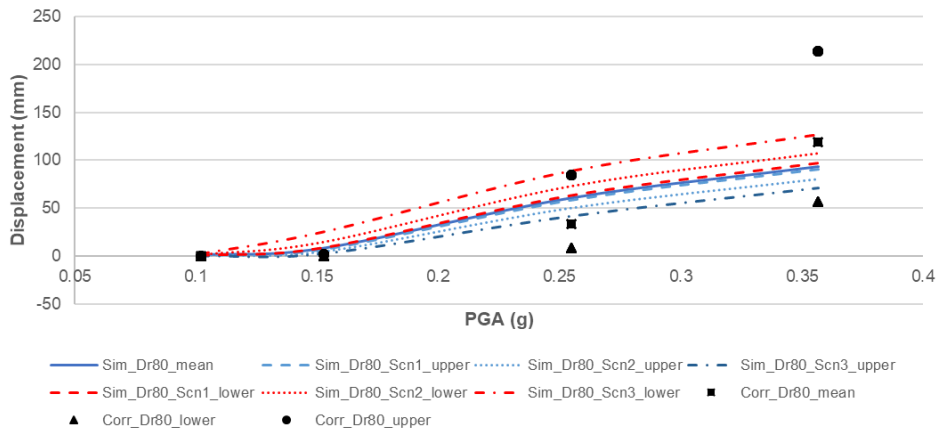


Fig. 14 – Comparison between the displacements obtained in physical and numerical models, for  $Dr = 80\%$

## 6. Conclusions

A validation exercise was developed in order to explore the capabilities of the “Strain Space Multiple Mechanism Model” to simulate the lateral spreading phenomena under a diverse range of densities and input motions, based on a large and reliable database of centrifuge models and element tests; for which:

- The updated correlation, based on LEAP Tests, provides a reliable characterization of the response (i.e. median response and its variability) of the sloping sand deposit under a wide range of densities ( $Dr50\% - Dr 80\%$ ), and peak accelerations ( $0.1g - 0.35g$ ). In this sense, they can be used as a reliable database in the development of current and future V&V processes of liquefaction models.
- It has been shown that, for liquefaction-related topics, an alternative to express the input variability in the numerical modeling might be to express it in terms of the variability in the computed liquefaction resistance curve, rather than individual changes in the parameters.
- Due to the complex characteristics of the soil, its geological conditions, and the quality of data exploration, the LRC’s variability will change from site to site; hence, a unique variability scenario cannot be determined. In this sense, three different variability-scenarios were considered in this paper.
- In the comparison between the computed displacements, and the results obtained in the estimated correlation (based on a forty-eight centrifuge tests), the model has shown, a good performance among the tested densities ( $Dr= 50, 60, 70$  and  $80\%$ ); the main outcomes of this analysis can be summarized as follows:
  - For the median response, it has been shown that if the model parameters are calibrated based on high-quality laboratory tests, a good agreement can be obtained for PGA values lesser than  $0.25g$ .
  - For a 95% of probability, the confidence bounds of the computed displacements (considering the three different variability scenarios), are located between the confidence bounds of the experimental outcomes (with few exceptions); validating the model for the three variability conditions considered in this exercise, for PGA values lesser than  $0.25g$ .
  - It was found that, for a given density, the slope of the curve Displacement-PGA is smaller in the computed displacements than in the estimated correlations (for both median response and confidence bounds).
  - Important variations were found between the experimental and numerical results for PGA values higher than  $0.25g$ ; so, additional research efforts would be required to explore the validity of the “Strain Space Multiple Mechanism Model” in these range of accelerations.



## Acknowledgements

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