



Variation tendencies and general rules for critical curve of liquefaction evaluation

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Abstract

The relationship between the liquefaction critical curve and the depth of the soil layer is the basic characterization of the liquefaction evaluation methods, but the comparison outcomes show that the critical curves of the existing liquefaction evaluation methods have different performance modes and even qualitative opposite. In this paper, bases on the Seed-Idriss model, deduces the theoretical solution of the effect of soil depth on the liquefaction potential, and puts forward the general relationship between the depth of the soil layer and the liquefaction critical curve, meanwhile obtains the variant trend and general rules of liquefaction critical curves. The results show that the rate of horizontal shear stress increase is faster than the rate of soil layer resistance of liquefaction by the add to soil depth, as well as the liquefaction potential and liquefaction threshold have a positive correlation with the depth of soil layer; the critical curve increases nonlinearly with soil depth, in the shallow layer tends to drastic meanwhile the deep layer tends to be gentle for the change of liquefaction potential; it is qualitative error and need to correct that the liquefaction critical curve has a decreasing relationship with the soil depth on CPT liquefaction evaluation of the national standards. The results of this paper can provide theoretical basis and reference for the development of liquefaction evaluation methods. The research outcome can be referred for future revision of national and professional standards.

Keywords: liquefaction critical curve, soil layer depth, liquefaction evaluation, cone penetrate test, standard penetrate test

1. Introduction

Earthquake disaster prevention plays an important role in the new concept of disaster prevention, mitigation and relief, which focuses on the transformation from post-disaster relief to pre-disaster prevention. The core task of earthquake effect prevention in engineering sites is to distinguish liquefaction of sites [1-3]. The reasonableness of liquefaction judgment method restricts the cost of engineering, and reliability affects the earthquake disaster prevention. It has been highly valued by experts and scholars at home and abroad [4,5]. It is an important research field of geotechnical engineering [6,7].

There are two main types of liquefaction discrimination methods [8]. One is indoor soil dynamic test, but the reliability of indoor test method is greatly questioned because of the disturbance caused by sampling method inevitably in the process of sampling. The other is direct in-situ test to evaluate the liquefaction resistance of sand layer in site, which is the mainstream of current liquefaction discrimination methods. In China, the liquefaction discrimination formula is basically composed of three parameters, namely, the burial condition of sand layer (groundwater level and depth of sand layer), the measured indexes (SPT of standard penetration test, CPT of cone penetration test and V_s of shear wave velocity test), and the seismic intensity



encountered by the site. It has been widely accepted by engineers and technicians. The liquefaction discrimination method with Chinese characteristics has been formed.

According to the existing understanding of geomechanics, the effective stress and total stress of groundwater level will increase by the growth of sand depth, and the anti-liquefaction ability of soil will be enhanced; the horizontal seismic shear stress of soil will increase with the rise of burial depth, at the same time, and the seismic stress of soil will also be enhanced. It is not known whether the liquefaction potential of soil will be enhanced or weakened under the combined action of the two factors. That is to say, the more general question is how the depth of sand layer affects the liquefaction potential of sand layer. Given the answer to this question, it is very helpful to understand the influence of burial conditions of sand layer on liquefaction more clearly and to judge the practicability and reliability of the existing liquefaction discrimination formula.

Based on the investigation of liquefaction in Bachu earthquake and the comparison with the existing liquefaction discrimination formulas, this paper presents the basic relationship between the buried depth of sand layer and the critical liquefaction curve for liquefaction discrimination. Based on Seed-Idriss model, the formula for calculating the influence of buried depth of sand layer on liquefaction potential is deduced, and the variation pattern and general law of the critical liquefaction discrimination curve with depth are analyzed. The method for liquefaction discrimination is correct. Development provides theoretical basis and guidance.

2. Comparison of existing liquefaction discrimination formulas

The serious liquefaction of sand in the Bachu earthquake is the most significant earthquake in the past 30 years of China after the Tangshan earthquake in 1976.

After the earthquake, SPT and CPT in-situ tests were carried out simultaneously on 40 liquefaction sites in the Bachu earthquake liquefaction site which was the first time in-situ test of liquefaction sites.

According to the survey data, the liquefaction discrimination formula based on SPT and CPT in Xinjiang region [9](abbreviated as Xinjiang SPT formula and Xinjiang CPT formula) is given :

$$N_{cr} = N_0[0.8 - 0.02d_w + 0.08d_s] \quad (1)$$

$$q_c = q_0[0.9 - 0.1d_w + 0.1d_s] \quad (2)$$

where, d_w is the depth of the water table (m); d_s is the depth of the saturated sand layer that needs to be determined whether it is liquefied (m); N_{cr} and q_{cr} are the liquefaction discriminant and cone tip resistance thresholds; N_0 and q_0 are the reference values of the standard and cone tip resistance; At 7 degrees, 8 degrees, and 9 degrees, N_0 (hits) are 13, 15 and 19, respectively, and q_0 (MPa) are 4.8, 5.8, and 7.4, respectively. The linear model of formula (1) is the same as the formula of the early seismic design code of China.

It should be noted that for equations (1) and (2), the coefficients in front of the influence of the sand depth influence term d_s are 0.08 and 0.1, respectively, and the values are different but all positive. This shows that the formulas (1) and (2) represent the liquefaction discriminant formula constructed by the two parameters of SPT and CPT, but are consistent in reflecting the influence of the buried depth of the sand layer on the possibility of liquefaction. Taking d_w as 2 m as an example, the variation of N_{cr} and q_{cr} with the buried depth of the sand layer is shown in Fig. 2. It shows that the critical value of the penetration threshold and the cone tip resistance increase with the increase of the buried depth of the sand layer.

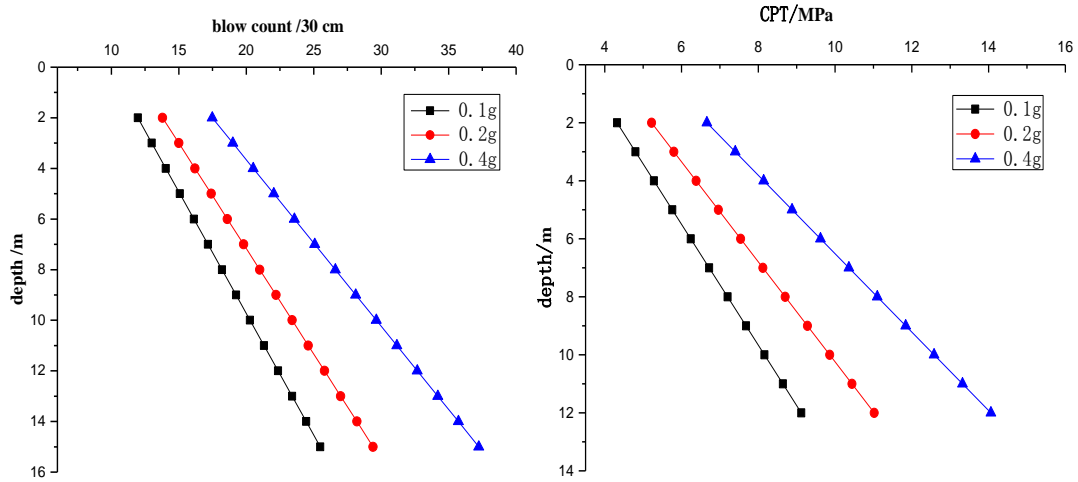


Fig. 2 liquefied critical value in the Xinjiang liquefied evaluation

3. Comparison of existing formulas

The liquefaction critical value calculation formula with the groundwater level, sand depth, SPT or CPT as the direct variable structure is the basic form of the liquefaction discriminant formula in several major seismic codes in China. It appears in the Code for Seismic Design of Buildings (GB50011-2010)[10].

The liquefaction discriminant formula expressed by SPT in the construction regulation is:

$$N_{cr} = N_0 \beta [\ln(0.6d_s + 1.5) - 0.1d_w] \tag{3}$$

where, β is the adjustment factor associated with seismic grouping varying between 0.80-1.05. Taking d_w as 2 m as an example, the variation of N_{cr} with the buried depth of the sand layer is shown in Fig. 3.

The iron regulation adopts the SPT discriminant formula. Although the expression is different from the construction regulation, the variation of N_{cr} with the buried depth of the sand layer is basically the same as that of Fig. 3.

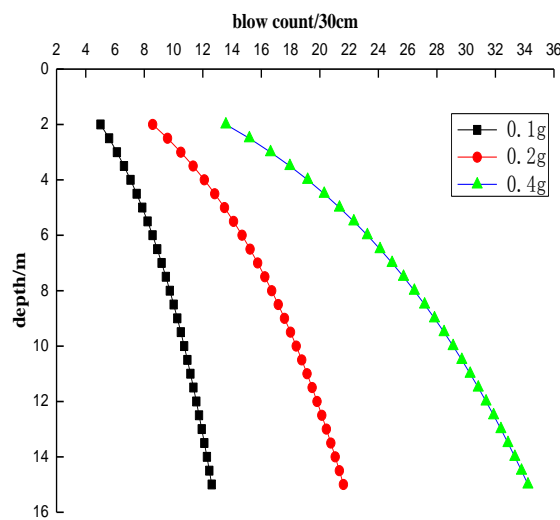


Fig. 3 Liquefied critical value in the standard of SPT



The rock regulation adopts the liquefaction discriminant formula of CPT expression (GB50011-2001)[11], which is expressed as:

$$q_{ccr} = q_{co} \alpha_w \alpha_u \alpha_p \quad (4)$$

$$\alpha_w = 1 - 0.065(d_w - 2) \quad (5)$$

$$\alpha_u = 1 - 0.05(d_u - 2) \quad (6)$$

where, q_{ccr} is the cone tip resistance threshold; q_{co} is the cone tip resistance reference value; α_p is the correction factor related to soil properties, and $\alpha_p = 1$ for sand. Assuming that the average thickness of the liquefied soil layer is 3 m, $\alpha_u = 1 - 0.05(d_s - 3.5)$, and d_w takes 2 m as an example, the change of the critical cone tip resistance with the buried depth of the sand layer is shown in Fig. 4.

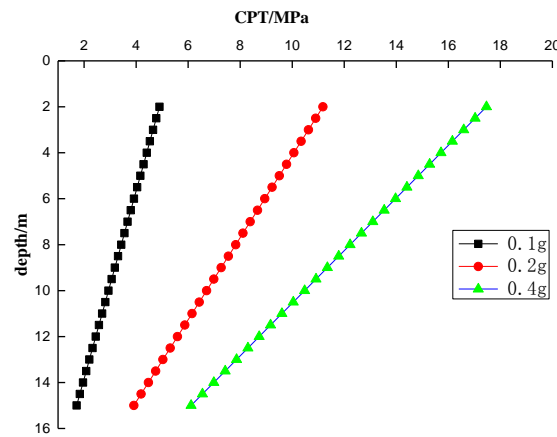


Fig. 4 liquefied critical value in the standard of CPT

It should be noted that the coefficient of the influence of the sand layer depth of the formula (3) is positive, which is qualitatively the same as the formulas (1) and (2); the coefficient of the influence of the sand layer depth of the formula (4) is negative. It is qualitatively opposite to the formulas (1)-(3). This means that the critical value of the liquefaction discriminant or cone tip resistance in the construction and Xinjiang formula increases with the buried depth of the sand layer (Figure 2 and 3), while the critical value of the taper tip resistance increases with the buried depth of the sand layer. Instead, it decreases (Figure 4). At the same time, it should be noted that the critical value of the liquefaction discriminant formula in Xinjiang shows a linear change with the depth of the soil layer, which is the same as the early mode of the SPT liquefaction discrimination method in China, and the critical value of the SPT liquefaction discrimination method with the depth of the soil layer. The presentation of nonlinear changes, the pattern is different.

The essence of these two problems is the problem of the liquefaction potential changing with the depth of the soil layer and the groundwater level. Generally speaking, for a given groundwater level, the deeper the depth of the sand layer is, the more effective stress and total stress are increased. At this time, the soil liquefaction resistance will be enhanced; but at the same time, the soil layer will be increased with the depth of the burial. The seismic shear stress will also increase, and the seismic stress experienced by the soil will also increase. The change of the liquefaction potential with the depth of the soil layer is the result of the combined action of the two. With the increase of the buried depth of the sand layer, the increase and decrease of the liquefaction potential of the sand layer and the basic form, that is, the liquefaction critical curve with the depth change pattern and law, become a problem that needs to be solved theoretically.



4. Theoretical solution to the influence of sand depth on liquefaction potential

The Seed-Idriss model for predicting soil liquefaction potential [12] has been widely recognized at home and abroad. Based on this, the theoretical solution to the influence of groundwater level and soil depth on liquefaction potential is derived.

In the Seed-Idriss model, the maximum horizontal shear stress amplitude of the soil layer is:

$$\tau_m = \gamma_d \frac{a_{\max}}{g} \sigma_\gamma = \gamma_d \frac{a_{\max}}{g} \sum_{i=1}^n \gamma_i h_i \quad (7)$$

where, γ_d is the stress reduction factor ; γ_i is the bulk density of each layer of soil ; h_i is thickness for each soil layer ; a_{\max} is the peak acceleration of the surface ; g is the acceleration of gravity.

The maximum amplitude of the equivalent horizontal seismic shear stress:

$$\tau_{eq} = 0.65\tau_m = 0.65\gamma_d \frac{a_{\max}}{g} \sigma_\gamma = 0.65\gamma_d \frac{a_{\max}}{g} \sum_{i=1}^n \gamma_i h_i \quad (8)$$

According to the Seed-Idriss model, the minimum horizontal seismic shear stress of a saturated sand body can be liquefied:

$$\tau_d = C_\gamma \frac{\sigma_{a,d}}{2\sigma_3} \sigma_\gamma' \quad (9)$$

Where, $\sigma_{a,d}$ is the dynamic stress ; σ_3 is the effective normal stress on the 45° surface of the dynamic triaxial test ; C_γ is the correction factor related to the relative density of sand ; σ_γ' is the vertical effective stress acting on the soil, and its expression is:

$$\sigma_\gamma' = \sum_{i=1}^n \gamma_i' h_i \quad (10)$$

Where, at the water level, γ_i' is the natural gravity of the soil; at the water level, γ_i' is the effective gravity of the soil, expressed as $\gamma_i' = \gamma_{sat} - \gamma_w$; γ_w is the severity of water.

Further, the liquefaction stress ratio is expressed as:

$$\frac{\sigma_{a,d}}{2\sigma_3} = \frac{D_r}{50} \left[\frac{\sigma_{a,d}}{2\sigma_3} \right]_{50} \sigma_\gamma' \quad (11)$$

Where, $\left[\frac{\sigma_{a,d}}{2\sigma_3} \right]_{50}$ is the liquefaction stress ratio at 50% relative density. From equations (9) and (10), the horizontal seismic shear stress required to liquefy saturated sand is:

$$\tau_d = C_\gamma \frac{D_r}{50} \left[\frac{\sigma_{a,d}}{2\sigma_3} \right]_{50} \sigma_\gamma' \quad (12)$$

According to the Seed-Idriss model, when $\tau_{eq} > \tau_d$, that is, when the horizontal seismic shear stress of the saturated sand body is greater than the minimum horizontal seismic shear stress required for the liquefaction of the saturated sand body, the soil layer is judged to be the liquefied region. Conversely, the soil layer is judged to be a non-liquefied area.



5. Theoretical solution to the relationship between sand depth and liquefaction potential

In the Seed-Idriss model, the critical state can be expressed as $\tau_{eq} / \tau_d = 1$, and the liquefaction potential is further expressed by τ_{eq} / τ_d in this paper, that is:

$$f = \tau_{eq} / \tau_d \quad (13)$$

The larger f is, the larger the liquefaction potential is, and the easier the soil is to liquefy, and vice versa.

In this paper, the semi-space homogeneous soil generalization model shown in Fig. 5 is taken as the research object, and the relationship between the buried depth of the sand layer and the liquefaction potential is theoretically derived [13]. In Fig. 5, h_w represents the buried depth of the groundwater level, and h_s represents the buried depth of the sand layer.

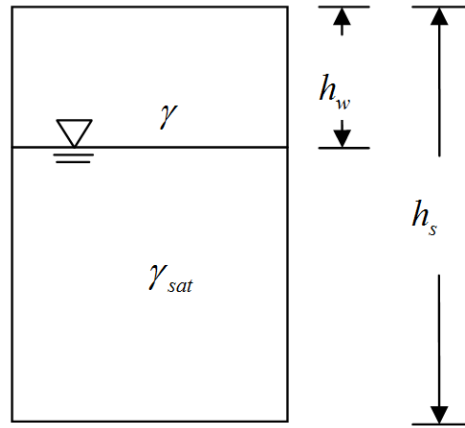


Fig.5 Generalized model of homogeneous soil layer in elastic semi-space

In Figure 5, the total stress σ_γ on the soil layer at h_s can be expressed as:

$$\begin{aligned} \sigma_\gamma &= \gamma h_w + \gamma_{sat} (h_s - h_w) \\ &= (\gamma - \gamma_{sat}) h_w + \gamma_{sat} h_s \end{aligned} \quad (14)$$

The effective stress σ'_γ overlying the soil layer at h_s can be expressed as:

$$\begin{aligned} \sigma'_\gamma &= \gamma h_w + \gamma' (h_s - h_w) \\ &= (\gamma - \gamma') h_w + \gamma' h_s \end{aligned} \quad (15)$$

From equations (13) and (9), there are:

$$f = \frac{\tau_{eq}}{\tau_d} = \frac{0.65 \gamma_d \frac{a_{max}}{g}}{C_\gamma \frac{D_r}{50} \left[\frac{\sigma_{a,d}}{2\sigma_3} \right]_{50}} \frac{\sigma_\gamma}{\sigma'_\gamma} \quad (16)$$

Let $\frac{0.65 \gamma_d \frac{a_{max}}{g}}{C_\gamma \frac{D_r}{50} \left[\frac{\sigma_{a,d}}{2\sigma_3} \right]_{50}}$ be k , then equation (15) can be written as :



$$f = \frac{\tau_{eq}}{\tau_d} = k \frac{\sigma_\gamma}{\sigma'_\gamma} = k \frac{\sum_{i=1}^n \gamma_i h_i}{\sum_{i=1}^n \gamma'_i h_i} \quad (17)$$

For the half-space soil generalization model of Fig. 5, it is assumed that it is homogeneous, that is, sand layers with the same compactness and the same liquefaction stress ratio in the depth direction, and for the soil within 12m depth, the stress reduction factor γ_d is between 1.0 and 0.85, the value of k can be approximated as a constant. In this way, the liquefaction potential $f(h_s)$ expression can be simplified to:

$$\begin{aligned} f(h_s) &= \frac{\tau_{eq}}{\tau_d} = k \frac{\gamma h_w + \gamma_{sat} (h_s - h_w)}{\gamma h_w + \gamma' (h_s - h_w)} \\ &= k \frac{(\gamma - \gamma_{sat}) h_w + \gamma_{sat} h_s}{(\gamma - \gamma') h_w + \gamma' h_s} \end{aligned} \quad (18)$$

For derivation of burial depth of sand layer in equation (17), there is:

$$\begin{aligned} f'(h_s) &= k \frac{\gamma_{sat} \gamma h_w - \gamma \gamma' h_w}{[(\gamma - \gamma') h_w + \gamma' h_s]^2} \\ &= k \frac{(\gamma_{sat} - \gamma') \gamma h_w}{[(\gamma - \gamma') h_w + \gamma' h_s]^2} \end{aligned} \quad (19)$$

Since $\gamma_{sat} - \gamma' > 0$, then from equation (18), there is $f'(h_s) > 0$, that is, $f(h_s)$ is a monotonically increasing function, which means that for any fixed value of the groundwater level, the burial depth of the saturated sand layer becomes larger, and the liquefaction potential becomes larger. That is to say, the liquefaction critical value is positively changed with the buried depth of the saturated sand layer. If the liquefaction potential is a function of the buried depth, the coefficient in front of the buried depth of the sand layer should be positive. Some liquefaction discriminant formulas proposed in the paper are consistent with China's standard CPT liquefaction discriminant formula. It can be seen that China's standard CPT liquefaction discriminant formula has qualitative errors and needs to be corrected.

It can be seen from equation (18) that the liquefaction potential is a monotonically increasing function of the buried depth, and the liquefaction potential of the sand layer in a deeper buried depth in a site is larger. It can be inferred that the rate of increase of soil liquefaction resistance increases with the burial depth of the sand layer is less than the rate of increase of horizontal seismic stress after the burial depth of the sand layer increases, and the liquefaction potential of the soil will eventually increase under the combined action of the two.

6. CONCLUSIONS

Based on the actual seismic liquefaction survey data and the comparison of different methods, the problems of the CPT liquefaction discriminant formula in China are found. Based on the Seed-Idriss model, the theoretical solution to the influence of the buried depth of the sand layer on the liquefaction potential is derived. The general rule of the influence of the buried depth of the sand layer on the liquefaction potential and the liquefaction critical curve is obtained. The main conclusion is:

(1) Theoretical analysis shows that with the increase of sand depth, horizontal seismic shear stress and soil liquefaction resistance increase simultaneously, but the former increases the rate more than the latter, and the liquefaction potential and the critical value are positively correlated with the sand depth.



(2) The liquefaction critical curve and the depth of the sand layer are nonlinearly increasing. The liquefaction potential and the threshold of the saturated sand layer in the shallow burial are increasing sharply, and the deep burial tends to be gentle; in the same depth range, when the groundwater level is deep and the liquefaction potential of the saturated sand layer and the critical value of the liquefaction increase are more severe.

(3) The liquefaction critical curve of the CPT liquefaction discriminant formula in China has a decreasing relationship with the depth of the sand layer. There is a qualitative error and needs to be corrected. In other existing liquefaction discriminant formulas, the liquefaction critical curve has an increasing relationship with the depth of the sand layer, and the qualitative is correct, but part of it is linear incremental relationship, the model needs to be improved.

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8. References

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