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PROBABILISTIC LIQUEFACTION HAZARD ANALYSIS (PLHA) REVISITED

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Abstract

Methods commonly used in professional practice to assess liquefaction susceptibility have the aim of determining the safety factor against liquefaction, or the probability of liquefaction, along a soil profile whose relevant characteristics are known, for an earthquake of known characteristics.

Results of conventional analyses allow the estimation of liquefaction probability under just one, given event, usually known as the Maximum Credible Earthquake (MCE). Therefore, in general, it is impossible to know how frequently the liquefaction will take place, since there is just a vague link between the MCE and its frequency of occurrence, and because it is evident that there are many earthquakes, besides the MCE, which could contribute with a non-negligible liquefaction probability.

To solve these problems, we will go back to the Probabilistic Liquefaction Hazard Analysis (PLHA), which has been used in the past, and we will update it in a contemporary framework, mainly taken from Probabilistic Seismic Hazard Analysis (PSHA). We will indicate the way in which a rigorous PLHA can be made in order to know how frequently the liquefaction will occur, and how this analysis can be seamlessly included into a modern PSHA code, namely R-CRISIS. Also, we will propose preliminary soil characteristics acceptance criteria, coherent with those commonly used in other areas of Earthquake Engineering. Finally, we will propose some approximations that, in certain conditions, allow to have reasonable estimations of the annual frequencies of liquefaction occurrence in a soil profile without having to carry out a full PLHA.

Liquefaction; PLHA; PSHA; R-CRISIS; Maximum Credible Earthquake

1. Introduction

Soil liquefaction is a phenomenon that happens when, due to earthquake-induced ground motions, soil pore pressure raises in such a way that the constitutive particles of the soil lose contact with each other, so the soil behaves as a liquid after losing shear strength. This usually happens in uniform, saturated sands. Liquefaction has severe consequences for constructions undergoing the phenomenon, so an adequate geotechnical design aims for liquefaction not taking place too frequently.

The problem of predicting if liquefaction would take place in a soil profile with known characteristics, caused by a known earthquake, has been studied since long time ago [1,2]. The most commonly used methods in professional practice aim at computing the safety factor against liquefaction, or the probability that the soil liquefies, given that an earthquake of known characteristics occurred. These methods are deterministic in nature, so they can give reasonable ideas of the liquefaction potential under an individual earthquake -usually, the Maximum Credible Earthquake, MCE-, but they give only vague hints as to how frequently liquefaction will occur, since there is just a vague link between the MCE and its frequency of occurrence, and because it is clear that there are many earthquakes, besides the MCE, which could contribute with a non-negligible liquefaction probability. Therefore, the design/review process usually must be made with weak probabilistic basis.

To solve these problems, we will revisit the Probabilistic Liquefaction Hazard Analysis (PLHA), an approach that has been used in the past in order to compute probabilistic estimations of the liquefaction potential, including: 1) the effects of multiple (generally thousands of) earthquakes, with different magnitudes and locations, that occur with known annual rates; 2) the fact that these earthquakes produce ground motions that can be predicted only with large uncertainties; and 3) the modifications that the seismic waves suffer due to the soil itself, that is, the consideration of site effects.

We will present the PLHA in an updated framework, inherited from Probabilistic Seismic Hazard Analysis (PSHA), in which the seismic environment is described in terms of *events* rather than in terms of multiple integrals; although numerically equal, the event-based approach allows for an easier understanding of the interaction between the conventional liquefaction methods and the probabilistic treatment. As we will see, results from PLHA allow knowing how frequently the liquefaction will occur and the proposal of acceptance criteria for soil properties, coherent with those commonly used in other areas of Earthquake Engineering. Finally, we will show that PLHA can be seamlessly included into a modern PSHA code, namely R-CRISIS.

2. Traditional methods for liquefaction potential evaluation

There is a wide variety of methods to assess liquefaction potential under individual earthquakes, generally based on some *in situ* exploration technique [3–5]. It is not the purpose of this paper to go deeply into these methods but, rather, to illustrate how they can be integrated into the probabilistic framework. Therefore, without loss of generality, we will restrict ourselves to methods that are variants of that of Seed and Idriss [2]. These methods, commonly used in professional practice, focus on computing the safety factor against liquefaction, *Fs*, by comparing two quantities: the Cyclic Stress Ratio (*CSR*) and the Cyclic Resistance Ratio (*CRR*):

$$Fs = \frac{CRR}{CSR} \tag{1}$$

As we will see later, the three quantities of Eq. (1) depend on the depth of the soil section under analysis. However, we have omitted this dependency to simplify the notation. In the definition of the safety factor, it can be appreciated that *CRR* should be a measure of the soil strength whereas *CSR* should be a measure of the size of the seismic action. According to Seed and Idriss [6], the first parameter can be defined in the following way:

$$CRR = CRR_{7.5}MSF \tag{2}$$

where $CRR_{7.5}$ is the cyclic resistance standardized to earthquakes of magnitude, M, equal to 7.5, while MSF is a correction factor for other magnitudes, given by [5]

$$MSF = \left(\frac{7.5}{M}\right)^{2.56} \tag{3}$$

It is worth noting that there are several other equations, similar to Eq. (3), proposed by other authors (see [7] for a comprehensive review). We have chosen this particular one just for illustration purposes. Following the classic approach of Seed and Idriss [2], CSR is defined as follows:

$$CSR = \frac{\tau_{av}}{\sigma'_{vo}} = \frac{0.65a_{max}}{g}\frac{\sigma_{vo}}{\sigma'_{vo}}r_d \tag{4}$$

where a_{max} is the peak ground acceleration (PGA) at the ground surface, g is the acceleration of gravity, σ_{vo} and σ'_{vo} are the total and effective vertical stresses at the depth of analysis, respectively, and r_d is the stress-reduction coefficient, which depends on the soil profile characteristics and on the characteristics of the ground motion (amplitude and frequency) at the depth of analysis. The latter coefficient can be determined from a site response analysis, or it can be approximated, for example, using the average curve reported by Seed and Idriss [2].

Finally, several authors have tried to adapt these deterministic methods to a probabilistic framework and have proposed empirical expressions to compute the probability of liquefaction under a given event. For instance, Ku *et al.* [8] have proposed the following expression:

$$P_L = \frac{1}{1 + \left(\frac{Fs}{0.9}\right)^{6.3}} \tag{5}$$

where P_L is the probability of experiencing liquefaction given that the earthquake characterized by its a_{max} and M values, occurred.

According to deterministic methods, a cross section of the profile will be acceptable under the design earthquake if the safety factor is larger than a conventionally accepted number, or if the liquefaction probability is smaller than a value deemed small enough. However, even if the liquefaction probability under the design earthquake is known, it is not clear how often the liquefaction would occur. A possible, yet incorrect, assumption is that the liquefaction would occur with probability P_L every time the design earthquake takes place. In turn, the design earthquake is assumed, also incorrectly, to have a return period equal to the return period of the a_{max} used in Eq. (4). The first assumption is incorrect because liquefaction can occur, with different probabilities, with many earthquakes, even those smaller than the design event, and not only under the design event. The second assumption is incorrect because the design earthquake is defined not just with its a_{max} value but also with its magnitude; in consequence, the return period of the design event is undefined. This, in our opinion, poses a very difficult problem because, clearly, design decisions would be very different if the liquefaction is expected to occur once every 100 years or once every 1000 years. The problem has no solution under the deterministic framework and the answer must be sought on the probabilistic approaches.

In what follows we will present the way of having probabilistic indications of the liquefaction potential, by integrating the conventional liquefaction analyses into the general framework of PSHA. Even when this integration is not new, probabilistic methods are not common in the professional practice, but in our opinion, they should, so it is worth recalling the possibility of using them. Further, we will present the integration with PSHA using an *event-based approach*, which makes it easier to visualize the integration process.

3. Probabilistic liquefaction hazard analysis (PLHA)

We will follow the formalism of PSHA [9,10], but expressing the hazard computation in terms of individual events, rather than in terms of multiple integrals, allowing a better understanding of PLHA. Thus, we will compute the annual frequency of occurrence of liquefaction at a given depth z, $\nu_L(z)$, in the following way:

$$\nu_L(z) = \sum_{i=1}^{N} \Pr(Liquefaction\ at\ depth\ z|Event\ i)Fa_i \tag{6}$$

where N is the total number of events that constitute the *stochastic catalog of earthquakes*, Pr*(Liquefaction at depth z | Event i)* is the probability of experiencing liquefaction at depth z given that i^{th} event took place, and Fa_i is the annual frequency of occurrence of the i^{th} event The stochastic catalog of earthquakes is, conceptually, a set that contains all possible events that can take place in the future, characterized by many parameters – e.g. its magnitude, location, the characteristics of its rupture area- and, very importantly its annual frequency of occurrence Fa_i within a stationary occurrence process. This catalog of stochastic events is a common output of contemporary seismic hazard models. Therefore, Eq. (6), which is a form of the Total Probability Theorem, tells us that the annual frequency with which liquefaction will occur is the weighted average of the probabilities under all possible earthquakes that will take place in the future, weighted by the annual frequencies of occurrence of each of the events.

As mentioned, an individual earthquake is characterized by several parameters, $\boldsymbol{\theta}$, among which we find its magnitude, its hypocentral location, the rupture area and the orientation of the rupture plane, among others. Therefore, the term Pr*(Liquefaction at depth z | Event i)* requires calculating the probability of liquefaction under an event defined by a set of parameters $\boldsymbol{\theta}$ and not, as in Eqs. 1-5, by an event defined by its a_{max} and M values. It is customary (and very reasonable) in PSHA to model a_{max}, given an event, as a random variable, in order to account for the uncertainty in Ground Motion Prediction Models (GMPM), in view of which Pr*(Liquefaction at depth z | Event i)* must be computed in the following way:

$$\Pr(Liquefaction\ at\ depth\ z|Event\ i) = \int_0^{\infty} \Pr(Liquefaction\ at\ depth\ z|a_{max}, M)\, p(a_{max}|\theta_i)da_{max} \tag{7}$$

where p($a_{max}|\theta_i$) is the probability density function (pdf) of a_{max} given the parameters $|\theta_i$ that characterize this event, that is, its magnitude, location, etc.; this pdf is usually furnished by the GMPM being used and, very importantly, a soil response analysis, since a_{max} is the PGA at the surface of the soil deposit whose liquefaction potential is under analysis; we will go back to this aspect later. On the other hand, Pr*(Liquefaction at depth z | a_{max},M)* is precisely the probability given in Eq. (5), that is, the probability obtained from conventional deterministic liquefaction analyses. In consequence, Eqs. 5-7 illustrate the connection between conventional liquefaction analysis methods and PSHA; these equations can be used to compute the annual frequency of occurrence of liquefaction, not under a single event, but under a complex seismic environment characterized by the stochastic earthquake catalog and a set of GMPM.

In the following section, we will illustrate these concepts with an example. We will show how probabilistic results look like and how they can be used to formulate acceptance criteria for a soil profile.

4. PLHA example

The example we present corresponds to an industrial site in southern Mexico. Abundant soil characterization was made for this project and we will take one of the soil profiles as an example for the present paper. In the following lines, we will give information about the seismic environment of the site, relevant soil characteristics and the way in which site amplification was handled for these calculations; this includes both amplification due to site effects and r_d factor calculations (see Eq. (4)). At the end of this chapter, we will show the results of the PLHA and discuss the implications in terms of acceptance criteria.

4.1 Seismic environment

The site of study is located in southern Mexico, by the coast of the Gulf of Mexico. Most of the project site is composed of soil deposits susceptible to liquefaction. To provide a general idea of the seismic environment of the site, we include in Fig. 1 the hazard curve for the PGA at rock. As we mentioned earlier, the PGA that matters for determining the liquefaction probability is that at the surface of the soil deposit, that is, the acceleration after inclusion of site effects. We will discuss this in 4.3.

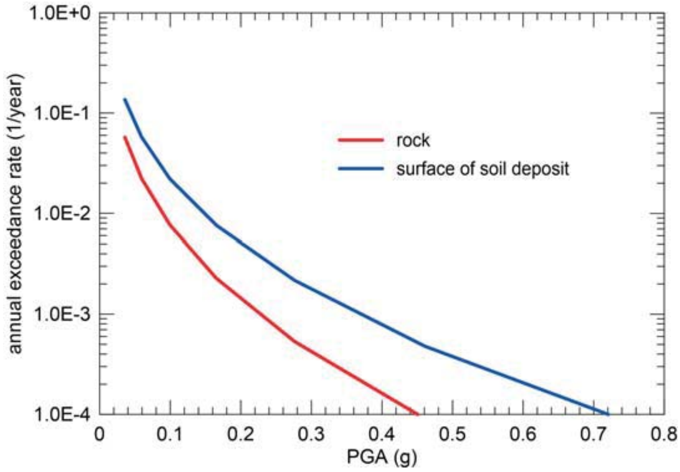

Fig. 1 – Annual exceedance rate of peak ground acceleration (PGA) both at rock and at the surface of the soil deposit, in the site of study

4.2 Soil profile

A comprehensive field investigation was performed at the site under study, including cone penetration tests, standard penetration tests, and the determination of wave propagation velocities using the suspension logging technique. Results of these field tests are given in Fig. 2a and 2b. An idealized stratigraphy is also depicted in Fig. 2a, comprised by: 1) a very loose to medium sandy layer, 2) a soft clay layer, 3) a medium sandy layer interspersed with clay, and 4) a very stiff deep clay layer. The upper sandy layer is identified as particularly susceptible to liquefaction. The estimated initial stresses are given in Fig. 2c; the water table was registered at 0.4 m deep, and hydrostatic conditions were assumed.

For the site response analyses, described in the following section, the initial stiffness properties were derived from the wave velocity measurements. The simplified profile, used for these calculations, is also shown in Fig. 2b. It is important to mention that the full profile considered for the site response analyses has a total depth of 119.2 m, where the half-space is assumed to be, with shear wave velocities higher than 700 m/s. In Fig. 2 only the portion relevant to the liquefaction analyses is shown. Nonlinearities were introduced by means of stiffness degradation and damping relationships. For clayey soils, the hyperbolic model proposed by Darendeli and Stokoe [11] was employed, which takes into account the level of confinement, the plasticity index, and the overconsolidation ratio. In the case of sandy layers, the curves put forward by Seed and Idriss [12] were adopted.

For the evaluation of liquefaction potential, the method by Robertson and Cabal [13] was employed, based on the cone penetration test results. The computed values of the $CRR_{7.5}$ (Eq. 2), according to the site stratigraphy and soil's properties, are given in Fig. 2d.

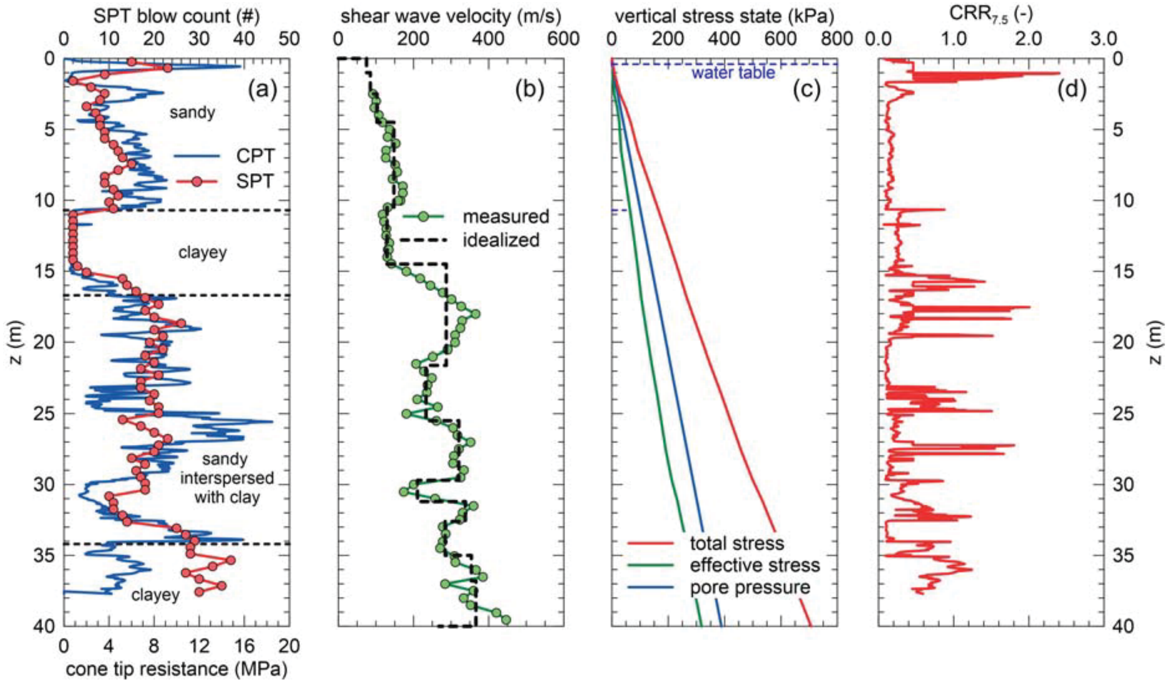

Fig. 2 – Main characteristics of the soil profile regarding liquefaction potential

4.3 Site amplification characteristics

We will assume that amplifications of seismic waves in the soil deposit take place mainly due to the vertical propagation of SH waves. As customary, the soil exhibits stiffness and damping that vary with the level of deformation, so non-linear soil effects are present; relationships employed were quoted in the previous section. Reasoning in terms of the event approach that we have been using so far, in principle, each one of the events would have its own PGA amplification value. In other words, the ratio between PGA at the surface of the deposit and PGA at the base rock would change from event to event due to differences in motion amplitude and frequency content. In principle, one could make several site-response analyses for each one of the events contained in the stochastic catalog. Several analyses would be needed for each event since, in contemporary PSHA, the amplitude and frequency content of the corresponding ground motions are regarded as uncertain, so this uncertainty should be included, perhaps in the form of Monte Carlo simulations. In our example, the stochastic catalog is comprised of about 30,000 events, and, assuming a moderate number of simulations for each event, say 20, this would imply something in the order of 1.2 million different site-response different analyses. In view of the available computer power today, this can certainly be done. However, the effort is considerable, and perhaps disproportionate with the goal of this paper, which is to illustrate the main concepts of PLHA. Therefore, we will follow a simplified approach, easier to use in professional practice, which usually gives results accurate enough.

In this approach, we do the following steps:

1) We first carry out a conventional PSHA for the site, at rock, aimed to obtaining Uniform Hazard Spectra (UHS) for different return periods; Fig. 3 shows these spectra for 5 different return periods.

2) Then, we perform a site-response analysis using as input motion the UHS associated to each of the return periods chosen for analysis. From each of the responses obtained, we compute amplification factors for a_{max} (and other periods if one wishes), which we index to the a_{max} in rock that has the corresponding return period; Fig. 3 also shows the amplification factors for a_{max} obtained in this way. Additionally, the site-response analyses allow computation of the r_d factors, shown in Fig. 4, which we also index to the PGA at rock that has the corresponding return period. As in the case of PGA amplification factors (see Fig. 3), we interpolate for arbitrary rock PGA values.

3) We now perform PLHA including the site effects described by the amplification factor of Fig. 3. In this new analysis, for each event, PGA at the surface of the soil deposit will be computed using the amplification of Fig. 3 which, in turn, is a function of rock PGA alone.

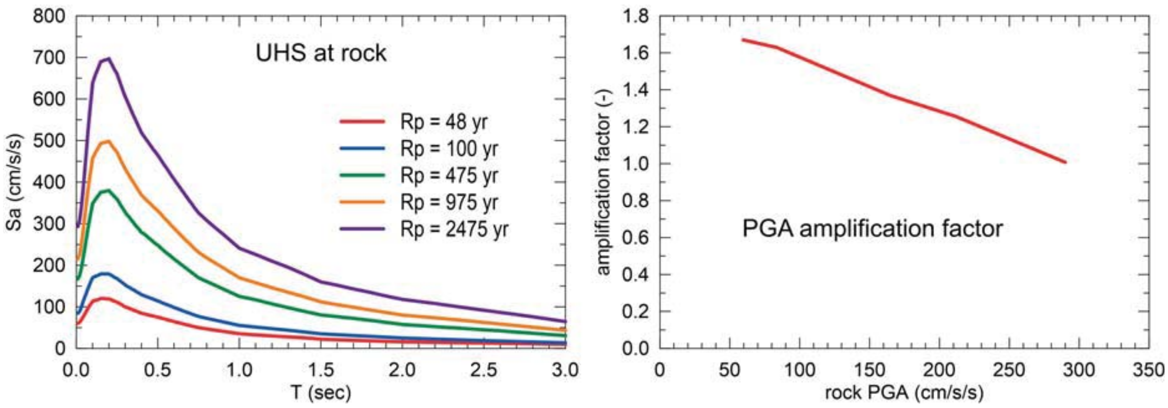

Fig. 3 – Left-hand frame: Uniform Hazard Spectra at rock, for several return periods, Rp. Right-hand frame: PGA Amplification Factor as a function of PGA at rock.

4.4 Results

Results were obtained with program R-CRISIS (http://www.r-crisis.com/), [14] whose newer version has PLHA capabilities. The main results of PLHA are presented in Fig. 5. There, we show three hazard measures, all related among them. Frame a) shows the exceedance rate of liquefaction, $\nu_L(z)$ –see Eq.(6)- that is, the expected number of times per year in which liquefaction will take place at level z of the soil profile; b) the return period, $Rp(z)$, of the liquefaction, computed as the inverse of the exceedance rate; and c) the probability of having at least one event with liquefaction in the next 50 years, Pe(z,50). Fig. 5 shows also the acceptance criteria we propose to decide whether the soil profile is safe enough, or not.

We have directly imported the acceptance criteria from common practices in PSHA. We will say that a section of the soil profile is acceptable if the exceedance rate is smaller than a given number -0.005/year in our case-, or if the return period of the liquefaction is larger than 1/0.002 = 500 yr, or if the exceedance probability in 50 years is smaller than 0.095. Assuming Poisson occurrences, these numbers are related in the following way:

$$Rp(z) = 1/\nu_L(z) \tag{8}$$

$$Pe(z, 50) = 1 - \exp[-50\nu_L(z)] \tag{9}$$

These three numbers are linked among them, so two of them are redundant; we just show the three possibilities because of the diverse preferences different analysts might have.

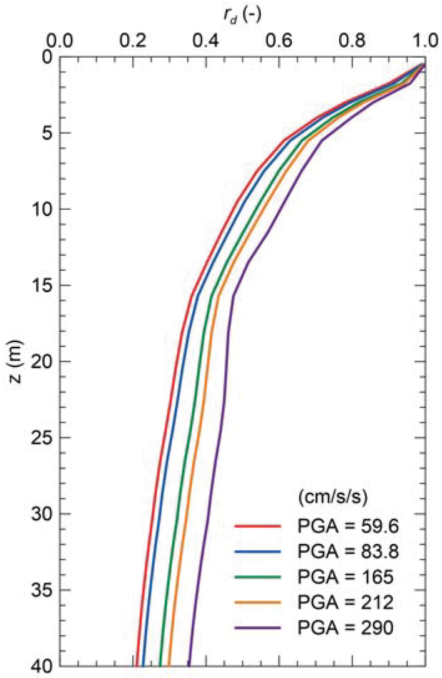

Fig. 4 – r_d factors for different rock PGA values, which correspond to different return periods. In the PLHA, the r_d factors employed in each event depends solely on the rock PGA. Interpolation is used for arbitrary values of rock PGA

In Fig. 5 it can be observed that there is a zone of the profile, at depths around 2m and between 3 and 5m (and also other parts of the profile around z=10 m) in which the exceedance rate is greater than 0.002, the return period is smaller than 500-yr, and the exceedance probability is larger than 0.1. This means, of course, that these portions of the profile are unacceptable, if one agrees with the 500-yr return period chosen for this example as acceptance criteria. We have now clearer indications of how often liquefaction will occur, based on the analysis and an adequate aggregation of the contributions to this hazard of all the earthquakes that, according to our model, will take place in the future. In our opinion, this gives a clearer picture of the safety of the soil profile than that obtained with a conventional deterministic analysis under the MCE.

5. Conclusions

We have presented how conventional liquefaction analysis techniques can be integrated into the frame of probabilistic hazard analysis, in order to obtain hazard measures that are more comprehensive –since they include the effects of many earthquakes that could occur in the future- and that propagate more reasonably the inherent uncertainties.

In view of the formulation of PSHA that is more common these days, that is, the formulation by events, probabilistic liquefaction analysis can be easily included in contemporary computer codes, as we have made with R-CRISIS, and can be more easily understood.

Probabilistic methods are not common in professional liquefaction analysis. Therefore, in the authors' experience, there are not universally accepted acceptance criteria, nor have probabilistic methods been calibrated against the results of more conventional studies; these are open questions. But in our view, bringing probabilistic methods to the attention of the liquefaction experts, along with the existence of computer codes that allow PLHA, will favor the development of probabilistic acceptance criteria that will, very likely, yield to better geotechnical design.

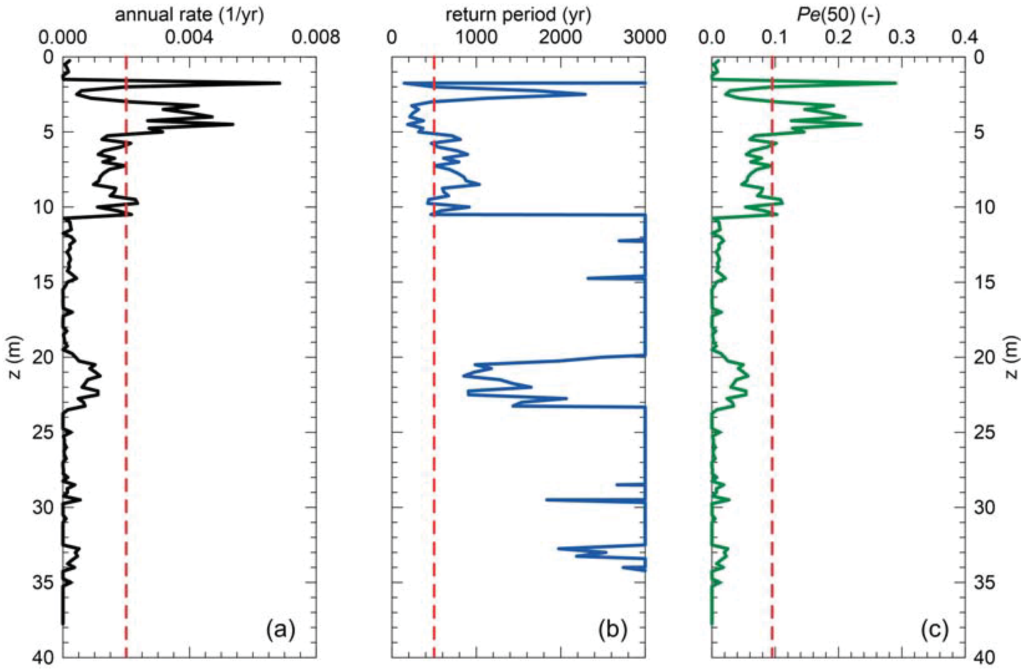

Fig. 5 – Results of Probabilistic Liquefaction Hazard Analysis for our example. a) Annual rate of occurrence of liquefaction; b) Return period of the liquefaction occurrences; c) Probability of liquefaction in the next 50 years.

6. References

[1] Seed BH (1979) Soil liquefaction and cyclic mobility evaluation for level ground during earthquakes. *J. Geotech. Eng. Div.* **105**, 201–255.

[2] Seed BH, Idriss IM (1971) Simplified Procedure for Evaluating Soil Liquefaction Potential. *J. Soil Mech. Found. Div.* **97**, 1249–1273.

[3] Andrus RD, Stokoe KH (2000) Liquefaction resistance of soils from shear-wave velocity. *J. Geotech. Geoenvironmental Eng.* **126**, 1015–1025.

[4] Robertson PK, Wride CE (1998) Evaluating cyclic liquefaction potential using the cone penetration test. *Can. Geotech. J.* **35**, 442–459.

[5] Youd TL, Idriss IM, Andrus RD, Arango I, Castro G, Christian JT, Dobry R, Finn DL, Harder Jr LF, Hynes ME, Ishihara K, Koester JP, Liao SSC, Marcuson III WF, Martin GR, Mitchell JK, Moriwaki Y, Power MS, Robertson PK, Seed RB, Stokoe KH (2001) Liquefaction resistance of soils: Summary report from the 1996 NCEER and 1998 NCEER/NSF workshops on evaluation of liquefaction resistance of soils. *J. Geotech. Geoenvironmental Eng.* **127**, 817–833.

[6] Seed BH, Idriss IM (1982) *Ground motions and soil liquefaction during earthquakes*, , Oakland, California.

[7] Cetin KO, Bilge HT (2012) Performance-based assessment of magnitude (Duration) scaling factors. *J. Geotech. Geoenvironmental Eng.* **138**, 324–334.

[8] Ku C-S, Juang CH, Chang C-W, Ching J (2012) Probabilistic version of the Robertson and Wride method for

liquefaction evaluation: development and application. *Can. Geotech. J.* **49**, 27–44.

[9] Esteva L (1967) Criterios para la construcción de espectros para diseño por sismo. In *3er Simposio panamericano de estructuras* , Caracas, venezuela.

[10] Cornell CA (1968) Engineering seismic risk analysis. *Bull. Seismol. Soc. Am.* **63**, 9–48.

[11] Darendeli MB, Stokoe KH (2001) *Development of a new family of normalized modulus reduction and material damping curves*, , Geotechnical Engineering Report GD01-1, University of Texas at Austin, Austin.

[12] Seed BH, Idriss IM (1970) *Soil moduli and damping factors for dynamic response analysis*, , UCB/EERC-70/10, University of California, Berkeley.

[13] Robertson PK, Cabal KL (2014) *Guide to cone penetration testing for geotechnical engineering*, Gregg Drilling & Testing, Inc., Signal Hill, California.

[14] Ordaz M, Martinelli F, Aguilar A, Arboleda J, Meletti C. and D'Amico V. (2019). R-CRISIS, Program for computing seismic hazard. Instituto de Ingeniería. Universidad Nacional Autónoma de México. Mexico City, Mexico.

