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VALIDATION OF THE STRESS DENSITY MODEL FOR LIQUEFACTION ANALYSIS

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Abstract

As performance-based design becomes increasingly prevalent in geotechnical earthquake engineering, well-validated constitutive models are becoming more important in practice. In the case of time-history analyses involving liquefaction, these constitutive models must consider the nonlinear behaviour of sands, which significantly depends on the combined effects of relative density and initial confining stress of the soil. Constitutive models that take advantage of nonlinear dynamic effective stress framework incorporate soil fabric and stress history; however, given the complexity of soil behaviour, they require comprehensive verification and validation. The stress density model is a constitutive model for effective stress analyses involving liquefaction. It has been recently implemented in OpenSees, which is a finite element platform widely used in geotechnical research to perform nonlinear dynamic analyses. The stress density model characterises the soil behaviour using the state concept (initial void ratio and normal stress state of soil), a modified hyperbolic stress-strain relation and an adopted stress-dilatancy relation that controls dilation or contraction. Several previously conducted well-documented centrifuge tests provide the high-quality data required for validation. These centrifuge models were spun to 55g in a 9 m radius centrifuge, and dense instrumentation recorded the response of the soil and three different types of structures during several ground motions. Centrifuge tests provide the opportunity for controlled experiments, with extensive instrumentation, while also replicating the in-situ stresses existing in the prototype so that the stress-dependent behaviour of sand can be considered for liquefaction analyses. Evaluation of the stress density model's performance in single-element simulations of cyclic simple shear tests lays the groundwork for subsequent validation of free-field site response and soil-structure interaction analyses using OpenSees. These single element tests aid in the calibration of the loose and dense Nevada sands and dense Monterey sand that comprise the soil profiles in the centrifuge tests. The validation procedure to assess the capabilities and limitations of the model includes comparisons between the numerical model and the centrifuge test in terms of accelerations, settlements, and pore water pressures at critical locations for 1D free-field site response. The ability of the model to capture the amplified motion at the soil surface, settlement, as well as the accumulated pore water pressure in the loose layer is discussed along with potential limitations. Further validation of the stress density model against other reliable centrifuge tests can provide further insight into the model's characteristics and benchmark its applicability for geotechnical engineering purposes.

Keywords: Numerical modelling; Liquefaction; Centrifuge test; Validation

1. Introduction

Liquefaction has caused severe damage in widespread areas during numerous earthquakes (e.g., 1964 Alaska, 1964 Nigata, 1989 Loma Prieta, 1995 Kobe, and the 2010–2011 Canterbury Earthquake Sequence) [1-3]. Numerical simulation of liquefaction and evaluating the efficacy of feasible mitigation actions has long been a focus of interest for researchers and engineers. The growing trend towards performance-based design in geotechnical earthquake engineering has encouraged engineers and researchers to use more advanced numerical tools. Consequently, nonlinear dynamic analyses are becoming prevalent in addition to existing common simplified methods. Therefore, it is critical to investigate the ability of constitutive models to capture the complex behaviour of soil in response to irregular seismic loading, which yields complex pore water pressure generation/dissipation patterns. Numerous validation studies (e.g., [4-7]) in geotechnical earthquake engineering have investigated the capabilities of dynamic liquefaction constitutive models. Validation of a model requires solid evidence and repeatable data. The significant uncertainties in material behaviour, input motions and recorded responses complicate the use of documented field case histories to validate constitutive models. This is partly because large earthquakes occur unpredictably and less frequently, so instrumentation of structures and soil layers becomes expensive and haphazard. Therefore, existing documented case histories may not provide sufficient reliable and repeatable data at desired locations.

Due to the advantages of laboratory measurements and meticulous instrumentation within the soil, centrifuge tests can be beneficial, particularly because they are well-controlled with fewer uncertainties compared to field case histories.

Many researchers have used centrifuge tests to validate the performance of constitutive models (e.g., predicting accelerations, settlements, and pore pressures) in various conditions, including level/sloped ground, single- or multi-layered soil profiles, and with or without structures present in the model. Ramirez et al. [8] compared the PDMY02 [9, 10] and SANISAND [11] liquefaction constitutive models in the finite element software OpenSees and SANISAND in the finite difference software FLAC with a free-field centrifuge experiment. They concluded that the adopted models still need modification to capture free-field volumetric response. Armstrong et al. [12] investigated the capability of the UBCSAND constitutive model to predict the behaviour of soil and pile bridge abutments. The model generally predicted deformation patterns for two cases with or without piles. Kamai and Boulanger [13] investigated the capability of the PM4Sand [14, 15] constitutive model in FLAC to predict the dynamic response of a slope modelled in a centrifuge test. They mainly focused on exploring void ratio redistribution and shear strain localisation effects, which affect the extent of lateral spreading.

As the stress density model [16-18] has been recently implemented in OpenSees [19], the current study initially focuses on the validation of the model in free-field site response analysis to evaluate the model's efficacy in simpler conditions. The stress density model is an elastic-plastic constitutive model tailored for liquefaction analysis, with main features including a vanishing elastic region, hypoplasticity (i.e., the model's flow rule accounts for the rotation of principal stresses), a modified hyperbolic stress-strain configuration and an energy-based stress-dilatancy relationship [20]. The current research explores the strengths and limitations of the current OpenSees implementation of the stress density model in single element simulation and in free-field site response simulations. Furthermore, the PM4Sand model in FLAC [21], which has been more thoroughly validated by previous studies, provides a reference for comparison. Consequently, the results obtained from the two numerical simulations (i.e., PM4Sand model in FLAC and stress density model in OpenSees) are compared against the centrifuge test in terms of accelerations, settlements, and pore water pressures at critical locations for 1D free-field site response. Free-field modelling provides insight into the site effects and aids in predicting the motions at the ground surface for obtaining design response spectra for linear single/multi-degree of freedom systems. This free-field validation is an important step for future validation of more complex 2D analyses involving soil-structure interaction (SSI). Once the model shows reasonable results, it can be used for assessing the liquefaction consequences and mitigation programs.

2. Methodology

2.1 Centrifuge experiment

This study uses data from the centrifuge test performed by Hayden et al. [22], which investigated soil-structure interaction (SSI) and structure-soil-structure interaction (SSSI) using three different types of structures in isolated and adjacent configurations. To match the prototype's stress conditions, Hayden et al. [22] spun the centrifuge to 55g, and all units in this study are presented in prototype scale. The soil model comprises three layers (from the surface to the base of the container): 1.7 m of dense Monterey sand, 4.6 m of loose Nevada sand and 19.3 m of dense Nevada sand. The instrumentation included around 140 sensors to record displacements, pore pressures and accelerations at important locations. Table 1 summarises the soil properties associated with the centrifuge test. The water table was around 0.2 m below the ground surface.
Four input motions were applied consecutively to the base of the model container. Free-field and near-field responses, as well as measurements relating to SSI and SSSI behaviour were obtained, with further details provided in Hayden et al. [22]. Given the complexities of SSI and SSSI, the initial OpenSees implementation of the stress density model presented in this paper focuses on the validation of free-field response, which can provide the foundation for further validation studies considering SSI.

Table 1 – Centrifuge layers and properties

Soil layer	Height (m)	D_R	G_s	$e_{\min}$	$e_{\max}$
Monterey sand **[23]**	1.7	85%	2.64	0.54	0.843
Loose Nevada sand **[23, 24]**	4.6	40%	2.66	0.51	0.89
Dense Nevada sand **[23, 24]**	19.3	90%	2.66	0.51	0.89

2.2 Stress density model

Cubrinovski [16] introduced a constitutive model for sandy soils based on a stress-dependent density parameter, and since then its application has mainly been limited to the finite element software DIANA-J (e.g., Cubrinovski et al. [25]). Cubrinovski and Ishihara [17, 18] subsequently proposed the stress density model, which integrates the combined effects of the initial void ratio and confining stress on sand behaviour using the state index (I_s) introduced by Ishihara [1] and Verdugo [26] within the critical state concept. The aim was to provide a model for liquefaction analysis; however, their elastoplastic model can account for drained cases as well. In addition to elastic parameters, the model contains three types of parameters: 1) state index parameters, 2) stress-strain curve parameters, and 3) stress-dilatancy parameters. The state index (I_s) directly quantifies the relative initial e-p (void ratio versus mean effective normal stress) condition associated with the quasi-steady state (QSS) and upper-reference (UR) lines. The upper reference line can also be regarded as the isotropic consolidation line for the loosest state of the sand. A modified hyperbolic relation characterises the stress-strain curve for the stress density model. In terms of the stress-dilatancy relation, they used the approach proposed by Roscoe et al. [27]. There are four dilatancy parameters: the small strain and cyclic dilatancy coefficients (μ_0, μ_{cyc}), the critical state stress ratio (M) and dilatancy parameter (S_c). Given the relationship of dilatancy parameters with the shear strain and dependence of shear strain development on the state index, the effect of state index is implicitly incorporated into the stress-dilatancy configuration of the constitutive model. The model assumes a continuous yielding or vanishing elastic region, which means that the model does not contain a boundary that separates the purely elastic region and the elastic-plastic region [18, 20]. The model uses an associated flow rule proposed by Gutierrez et al. [28]. The plastic potential formulation accounts for the rotation of principal stress directions.

The stress density model [16-18, 29] has recently been added to OpenSees 3.0.3, a widely used computational platform. Although several previous field case histories suggest that the model reasonably predicts the soil behaviour under liquefaction, further validation against centrifuge tests is necessary to better assess the model's limitations and capabilities. Single element simulation is required for calibration of the model and estimation of dilatancy parameters that control the rate of pore water pressure generation pattern. An advantage of the model is that parameters are applicable for any initial condition of relative density and normal stress for a given sand. While the model is still under testing, this paper presents the first results obtained from the current implementation of the stress density model in OpenSees. Further verification and validation investigations are ongoing.

2.3 Model parameters

Due to the lack of consistent monotonic drained and undrained tests on Nevada and Monterey sands for estimating the stress-strain parameters and elastic parameters, default values that are obtained from rigorous experimental tests on Toyoura sand [17, 18, 30] are used in the present study. It is worthwhile to note that both Nevada and Toyoura sands are sub-angular quartz sand with similar median particle sizes, $D_{50} = 0.19\ mm$ for Toyoura and $0.14 \le D_{50} \le 0.17\ mm$ for Nevada sand [17, 18, 31]. The coefficients of uniformity are reported as 1.67 and 1.7 for Nevada and Toyoura, respectively. Although there are other soil characteristics that can affect the behaviour of these two soil types, given the similarities, the adopted parameters for this study can likely be used with caution as a reasonable approximation. Table 2 presents the calibrated model parameters. The Monterey sand uses the default values for Toyoura sand. The parameters for the Nevada sand reference lines are based on the simplified probabilistic concepts suggested by Bradley and Cubrinovski [29] with additional adjustment. The element permeability values for post-gravity analysis are $1.35\times10^{-4}\,m^2$ for the Monterey, $1.66\times10^{-5}\,m^2$ for the dense Nevada and $5.73\times10^{-6}\,m^2$ for the loose Nevada sand. The input parameters for the PM4Sand model are obtained from Balachandra et al. [32].

Table 2 – Calibrated model parameters

Relation	Parameter	Symbol	Nevada sand		Monterey sand	
Elastic parameters	Shear constant	A	250		250	
	Poisson's ratio	ν	0.2		0.2	
	Exponent	n	0.6		0.6	
Reference lines	UR-line (void ratio and normal stress in kPa)	e_U, p_U	0.782	<400	0.895	<400
	QSS-line (void ratios and normal stress in kPa)	e_U, p_U	0.769	1	0.873	1
		e_Q, p_Q	0.769	10	0.873	10
		e_Q, p_Q	0.752	30	0.873	30
		e_Q, p_Q	0.741	50	0.87	50
		e_Q, p_Q	0.721	100	0.86	100
		e_Q, p_Q	0.699	200	0.85	200
		e_Q, p_Q	0.672	400	0.833	400
Stress-strain parameters	Peak stress ratio coefficients	a_1, b_1	0.592	0.021	0.58	0.023
	Max. shear modulus coefficients	a_2, b_2	291	55	230	65
	Min. shear modulus coefficients	a_3, b_3	98	13	79	16
	Degradation constant	f	4		4	
Dilatancy parameters	Dilatancy coefficient (small strains)	μ_0	0.20		0.22	

Dilatancy coefficient (cyclic)	μ_{cyc}	-0.02 (Dense) 0.01 (Loose)	0
Critical state stress ratio	M	0.607	0.607
Dilatancy strain	S_c	0.0057	0.0055

2.4 Calibration method

One integral component of calibration is to simulate the experimental liquefaction triggering curve for a given soil. Through single element (e.g., cyclic simple shear) tests, a model may show behaviour that is not similar to soil behaviour in practice. In this regard, assessing the stress density model's single-element behaviour lays the groundwork for further application of the model. Similar to other models, for the calibration of the stress density model, input parameters that can be derived from cyclic or monotonic laboratory tests are introduced to a cyclic direct simple shear test simulation program. Calibration in this study was performed by holding the small strain and cyclic dilatancy coefficients (μ_0, μ_{cyc}) constant and adjusting the dilatancy strain parameter (S_c) in an iterative process to achieve the best fit to the experimental liquefaction triggering curve collated from cyclic simple shear experiments performed by Kutter et al. [23], Kammerer et al. [33] and Arulmoli et al. [7]. The small strain dilatancy coefficient is assigned values between 0.15 and 0.25 [18] and primarily affects the stress-strain behaviour in the first few steps. The cyclic dilatancy coefficient is assigned values between -0.1 and 0.1 and influences pore water pressure accumulation from cycle to cycle. Although these two parameters are not the main calibration parameters, they play a key role in replicating experimental strain-stress-path results. Ideally, sufficient experimental data on the given sands can minimise the need for iterative procedures of calibration, but a lack of ample experimental data on the Nevada sand necessitated an iterative process for the calibration of stress density model input parameters in this paper. Fixing the μ_0 and μ_{cyc} can initiate the iterative process to calibrate S_c. Higher S_c values lead to a more dilative behaviour and vice versa. In this study, single element tests were performed to simulate cyclic simple shear conditions with a range of uniform cyclic stress ratios to reach liquefaction (3% single-amplitude shear strain) in each case. Finally, the CSR values required to trigger liquefaction in 15 cycles were introduced into the single-element simulation driver to calibrate the dilatancy parameter for the loose and dense Nevada sands. Fig. 1 shows the results of single-element simulations for Nevada sand with various relative densities against cyclic simple shear experiments [7, 33, 34].

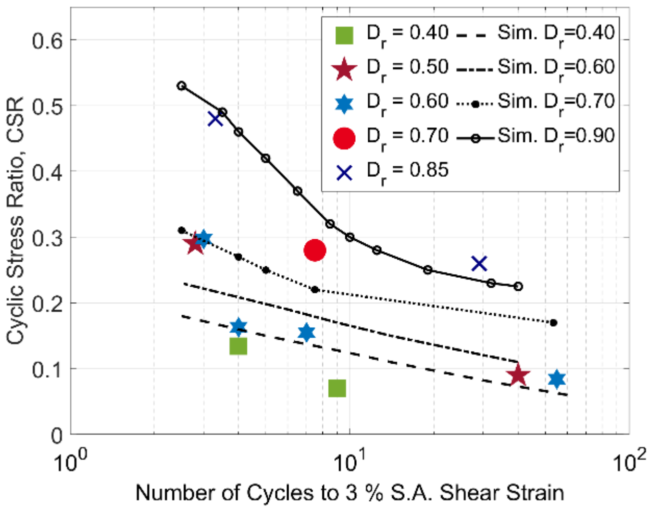

Fig. 1 – Cyclic simple shear results of Nevada sand from numerical simulations (lines) and experimental tests (symbols)

2.5 Simulation

One dimensional site response analysis was conducted to study the free-field response using OpenSees, which uses the finite element approach. The model used $0.1 \times 0.1\ m$ mesh (i.e., 1 element in the horizontal direction and 256 elements in the vertical direction) to allow accurate wave propagation at reasonably high frequencies. Given this discretization, the model recorders were within 0.05 m of the sensor locations from the centrifuge. A soil column was modelled in OpenSees using the SSPquadUP element [35], which is a four-node element with three degrees of freedom at each node accounting for the horizontal and vertical displacements and pore water pressure. The input motions applied to the 1D numerical model are the recorded motions at the base of the centrifuge container. Given the metal base of the centrifuge container, a rigid base is a reasonable representation and the two nodes at the base were fixed in both directions. Nodes at the same elevation were tied together to act as a periodic boundary suitable for 1D response. Rayleigh damping of 2% was applied to the model. The FLAC model used for the PM4Sand comparison generally followed a similar approach, although there are some differences, particularly given FLAC is a finite difference code. More details can be found in Balachandra et al. [32]. This paper presents the soil response to two of the motions applied in the centrifuge test, a Port Island (PRI) motion [25], associated with the 1995 Kobe earthquake in Japan, scaled to PGA values of 0.045 and 0.230 g, referred to as the Small and Moderate PRI events, respectively.

3. Results

This section compares the stress density model and PM4Sand simulated time series for acceleration, pore pressure and displacement to records from the centrifuge test. Fig. 2 shows that the computed acceleration time histories for both input motions roughly agree with the centrifuge test in terms of the amplitudes throughout the time series. However, the stress density model poorly captures the response after liquefaction occurs. Both models show larger-than-observed acceleration spikes, which is likely due to the cyclic mobility characteristics of the models. The Small PRI results suggest that the current OpenSees implementation of the stress density model may encounter issues when dealing with the small strain damping, which is currently being investigated further.

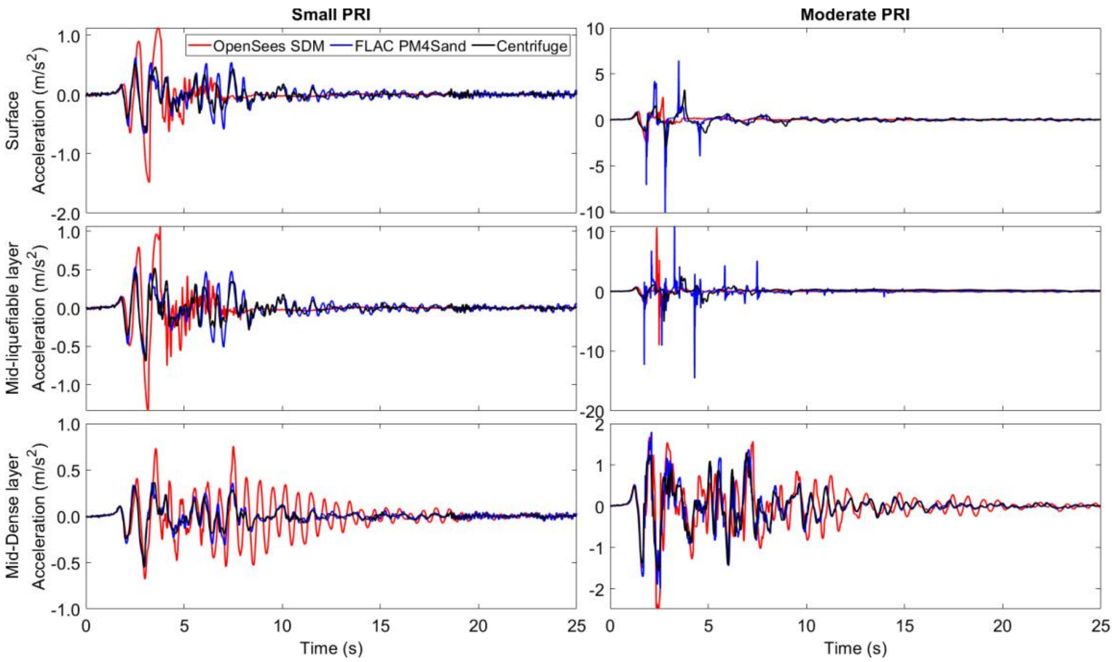

Fig. 2 – Acceleration time series for the computed and measured responses

Fig. 3 shows reasonable agreement between the computed and measured pore pressures at three depths for the Moderate PRI motion. In the dense layer and top of the liquefiable layer, the stress density model captures the top pore water pressure threshold after the liquefaction better than the PM4Sand. Both computed and measured responses do not imply liquefaction triggering in the dense layer in either motion. In the Small PRI motion, the current OpenSees implementation of the stress density model significantly overestimates the excess pore pressures in the middle of the liquefiable layer and predicts the triggering of liquefaction, which is inconsistent with the observations. Even at the surface, the stress density implies an excess pore pressure ratio approaching one, which is not realistic, since the small motion is not capable of inducing liquefaction at any depth. The discrepancies between the initial computed and measured pore pressure in the Small PRI motion are likely due to the higher than expected water table in the centrifuge test for this particular event. The difference is smaller for the moderate motion as the water table subsided towards its expected level.

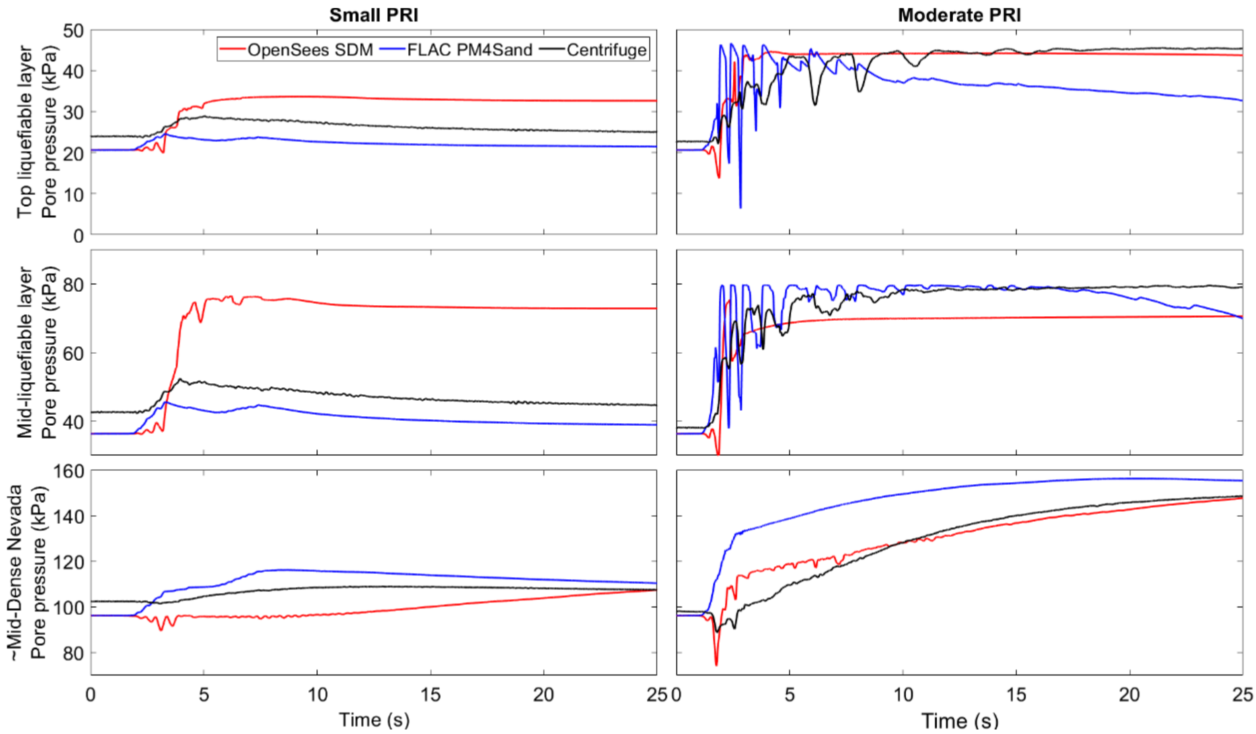

Fig. 3 – Pore pressure time series for the computed and measured responses

The PM4Sand model and the centrifuge test acceleration spectra at the ground surface are in good agreement for the Small PRI motion, as shown in Fig. 4. However, the stress density model overestimates the response at all periods. In the case of the moderate motion, both constitutive models are fairly inconsistent with the centrifuge response. In the low-period range, the PM4Sand model predicts stronger than measured response, and the stress density model is in a reasonable agreement with the centrifuge. For the longer-period components, they both underpredict the response compared to the centrifuge.

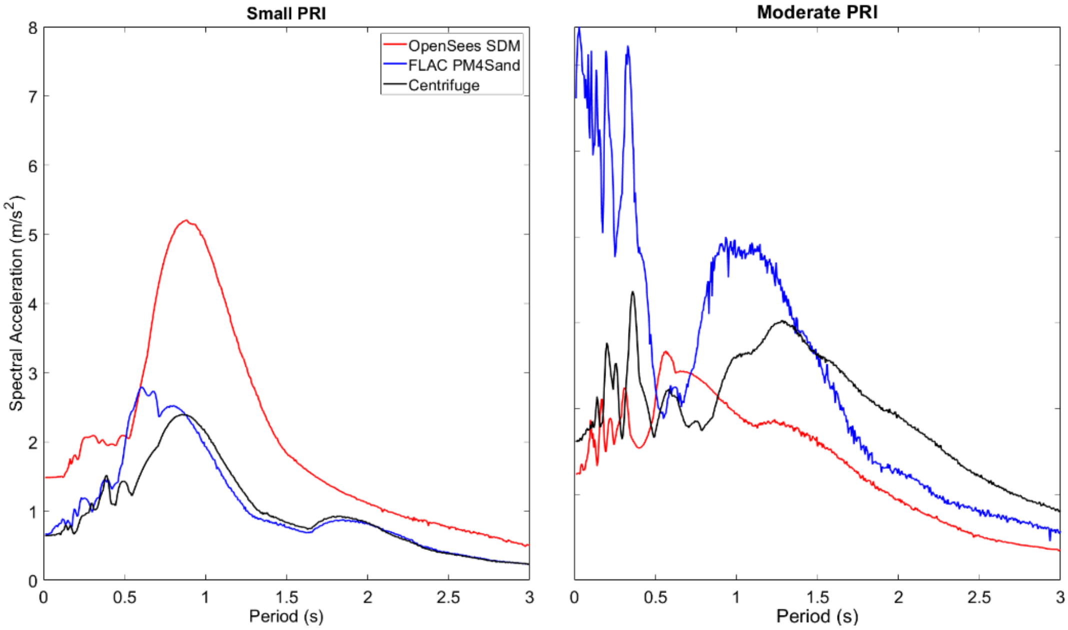

Fig. 4 – Spectral acceleration (5% damping) at the ground surface for the computed and measured outputs

Both models significantly underpredict free-field settlements. This is mainly due to the underestimation of the volumetric strains by the constitutive models, as reported by other researchers [5, 36, 37]. Although the PM4Sand model predicts some settlement, it underestimates the centrifuge recordings by a factor of around 10 after 25 seconds of applying the moderate motion. Given that the liquefaction does not occur, settlements in the centrifuge due to the small motion are minor. Even so, the PM4Sand model still predicts settlement at the end of the analysis 40% smaller than measured in the centrifuge, and the stress density model predicts almost negligible settlement.

4. Conclusion

This study evaluates the performance of the new OpenSees implementation of the stress density model in the prediction of free-field site response involving liquefaction. Accelerations, pore pressures and displacement at important locations are compared with the centrifuge results as well as with numerical simulations carried out using a different constitutive model (PM4Sand) in a finite difference platform (FLAC) to better assess the capabilities and limitations of the stress density model. The stress density model provides realistic pore pressure results for the Moderate PRI motion, but not the Small PRI motion. The acceleration response obtained using this model illustrates some limitations of the model in the current OpenSees implementation. However, comparing the small with moderate motion results suggests that the stress density model results improve in terms of acceleration time histories, pore pressures and spectral responses when subjected to larger strains (i.e., in the moderate motion). Based on these findings, the current version of the stress density model as implemented in OpenSees 3.0.3 would benefit from further verification and validation efforts, which are being undertaken to enhance its performance. Moreover, uncertainties in the estimation of input parameters affect the numerical analysis. Therefore, the stress density model results presented in this paper are preliminary and will be investigated further through a more comprehensive examination of other experimental and numerical cases.

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