LIQUEFACTION CONSTITUTIVE MODEL VALIDATION USING PORE PRESSURE RECORDS FROM THE CANTERBURY EARTHQUAKE SEQUENCE

T. Khansari [(1)], C. Hayden [(2)], L. Wotherspoon [(3)], S. van Ballegooy[(4)]

[(1)] *PhD Candidate, Dept. of Civil & Environmental Engineering, University of Auckland, tkha758@aucklanduni.ac.nz*
[(2)] *Lecturer, Dept. of Civil & Environmental Engineering, University of Auckland, connor.hayden@auckland.ac.nz*
[(3)] *Associate Professor, Dept. of Civil & Environmental Engineering, University of Auckland, l.wotherspoon@auckland.ac.nz*
[(4)] *Technical Director, Geotechnical, Tonkin & Taylor Ltd, Auckland, New Zealand, SVanBallegooy@tonkin.co.nz*

Abstract

Liquefaction-induced strength loss in a soil deposit can cause settlement or tilt of the foundations of any overlying structures. Liquefaction-induced ground movements can also devastate infrastructure. Therefore, it is important to investigate the potential liquefaction susceptibility of a site to reliably assess potential ground movements. Well-validated liquefaction constitutive models are increasingly important as non-linear time history analyses become relatively more common in industry for key projects. PM4Sand is a plasticity model for liquefaction in sandy deposits, and previous validation efforts of PM4Sand have generally focused on centrifuge tests. Pore pressure transducers installed at several free-field sites during the Canterbury Earthquake Sequence (CES) 2010-11 in Christchurch, New Zealand, where several earthquakes caused liquefaction in some parts of the city, provide a relatively unique dataset to validate PM4Sand. This study presents 1-D effective stress site response analyses performed in the finite difference software FLAC to examine the capability of PM4Sand to capture the generation of excess pore pressures during these earthquakes. Extensive cone penetration test (CPT) investigations previously carried out in Christchurch were used to develop the soil profile and the parameters associated with PM4Sand. Soil properties necessary to calibrate PM4Sand, including relative density, shear wave velocity and cyclic resistance ratio (CRR), were estimated based on CPT correlations. As relative density is the main parameter affecting the liquefaction potential of sands, it was used to subdivide the soil profile into different representative layers. A Canterbury-specific CPT-shear wave velocity correlation was used to estimate the shear wave velocity for each layer. Given the significant depth to bedrock in Christchurch, the Riccarton Gravels formation, which is the uppermost firm layer, was used as the base of the numerical model. Deconvolved ground motions from a nearby recording station served as the input motions for the numerical model. The time histories of excess pore pressure from this free-field FLAC model using PM4Sand compare well to records during several real-world earthquakes in the CES.

Keywords: Liquefaction; PM4Sand; Numerical Modelling; Site Response

1. Introduction

Liquefaction can damage structures and infrastructure as emphasized by the 2010-11 Canterbury earthquake sequence (CES) in Christchurch, New Zealand. Liquefaction-induced ground movements can cause settlement or tilt of the foundations of overlying structures [1] as well as devastating other infrastructure such as bridges, pipelines and dams [2]. The rapid seismic loading of contractive sands leads to the transfer of normal stress from the sand matrix to the pore water, resulting in strength reduction, large strains and potential flow failure. Therefore, it is crucial to investigate the liquefaction susceptibility of a site to assess potential impacts. Current practice often employs empirical liquefaction triggering procedures, such as those by Boulanger and Idriss [3] and Youd et al. [4], along with correlations for volumetric strain-induced settlements. However, these empirical methods cannot account for system level response such as discontinuity and interaction between liquefiable layers [5]. Hence, more advanced projects in industry may require dynamic numerical models, and validation is essential to demonstrate whether liquefaction constitutive models can capture soil behavior.

Geotechnical centrifuge experiments have been used to study seismic behavior of liquefiable soil in both the absence and presence of structures [6-8]. Centrifuge tests have a number of desirable features including, being a controlled experiment with well characterized soils in relatively simple layering and can include numerous instruments to record accelerations, excess pore pressures, and settlements. Therefore, centrifuge data can be ideal for validation of numerical models. Although case histories from real-world earthquakes are generally more challenging to use in validation efforts due to increased complexity (e.g., in soil stratigraphy and properties) and uncertainty (e.g., in the input motion, lack of recorded response), they are more similar to real-world scenarios that practicing engineers ultimately are concerned with. Therefore, real-world cases histories play an important role in validation of liquefaction constitutive models.

Researchers have performed analysis involving liquefaction using numerical modelling to evaluate the capabilities and limitations of various liquefaction constitutive models in capturing the soil behavior. Zeghal et al. [9] carried out dynamic time history analysis in the finite element software OpenSees [10] and compared the results with two sets of centrifuge tests of sloping saturated sand deposits. The numerical model simulated the dilative acceleration spikes in the shallow depths on the downslope direction of the deposit, although their amplitude and the time of occurrence did not fully match the experiments. Karimi and Dashti [11] assessed the capability of the pressure dependent multiple-yield-surface (PDMY02) constitutive model [12-14] using OpenSees to capture free-field response and soil-structure interaction simulated in previous centrifuge tests. They found that the excess pore pressures and accelerations within the soil in the free field and beneath the foundation computed with the constitutive model showed reasonable agreement with the results recorded by centrifuge tests. Hashash et al. [15] compared results from the site response software DEEPSOIL [16] and OpenSees with selected soil constitutive models to centrifuge results, which showed reasonable agreement in terms of acceleration as well as strain and lateral displacement profiles.

Several studies have used centrifuge tests to validate the Plasticity Model for Sand (PM4Sand) [17], which is a stress-ratio controlled and critical state compatible plasticity model for capturing the nonlinear response of sand. PM4Sand is a modified version of the plasticity model initially developed by Dafalias and Manzari [18], which was improved to capture the stress-strain response of sandy soil during earthquakes. Kamai and Boulanger [19] used PM4Sand, with calibration based on laboratory tests, to simulate lateral spreading and found reasonable agreement with deformations measured in a centrifuge test. Dynamic site response and surface settlement obtained by centrifuge model tests of piled bridge abutments in embankments on liquefiable soil agreed fairly well with the PM4Sand predictions [20]. Balachandra [21] and Luque [22] found that PM4Sand generally captured soil-structure interaction behavior for structures founded on liquefiable soil.

Although centrifuge tests can provide a wealth of data to validate numerical models against, there have been several efforts to validate PM4Sand against real-world case histories. Markham et al. [23] found that the surface acceleration response spectra for liquefiable sites predicted using the finite difference software FLAC [24] with PM4Sand compared reasonably well to the recorded spectra. Their study used deconvolved motions from Christchurch as input motions. Site response analysis carried out by PM4Sand matched better in terms of the surface recorded data and stress-strength behavior than did similar analyses performed in DEEPSOIL. Luque and Bray [25] performed soil-structure interaction analyses using FLAC and PM4Sand for two buildings that were affected by liquefaction-induced ground movement in Christchurch, New Zealand. The final liquefaction-induced settlements beneath these structures agreed well with the settlements from the simulation in their studies.

Given the increasing importance of numerical modelling in performance-based design, more validation efforts are warranted, so that the models can be used more confidently in practice. Although previous studies simulating real-world case histories, such as those performed by Markham et al. [23] and Luque and Bray [25], focused on acceleration time histories or the settlement of structures, respectively, they did not have any pore water pressure readings available. The present study uses real-world excess pore pressure data collected from pore pressure transducers installed in Christchurch, New Zealand during the Canterbury Earthquake Sequence to validate PM4Sand in 1D, nonlinear effective stress site response analyses. Given the rarity of pore pressure time recordings in liquefiable layers during earthquakes, this presents a unique opportunity to evaluate the numerical model capabilities against real-world excess pore pressure time histories.

2. Methodology

2.1 Site description

Christchurch is located in the Canterbury Plains, which were deposited by rivers originating in the Southern Alps and flowing eastward into the Pacific where they form a sedimentary basin [26]. The surficial deposits are dominated by the Springston and Christchurch formations, which overlie the Riccarton Gravels formation. The Christchurch Formation consists of estuarine, lagoon, dunes, and swamp deposits of sand, silt, clay and peat, while the Springston Formation is comprised of alluvial gravels, sands and silts. The site investigated in the current study is located in Burwood to the north-east of the Christchurch Central Business District, not far from Avon River and Horseshoe Lake. The site showed moderate to severe liquefaction manifestation during the CES [2]. The pore pressure transducer installed at this site by Tonkin and Taylor Limited at a depth of 6 m recorded pore pressure data from March 2011 to February 2012.

In the absence of laboratory tests, in-situ tests, such as the cone penetration test (CPT), can provide estimates of soil parameters required for modelling. The numerous CPT investigations performed during or after the CES provide a thorough means to evaluate the soil stratigraphy and estimate relevant properties. The CPT data were collected from the New Zealand Geotechnical Database (NZGD) [27]. This comprehensive database was used to characterize the most representative soil profile.

The significant depth to the bedrock in Christchurch (greater than 1000 m) makes reaching it impractical for many geotechnical investigations. The uppermost firm layer is the Riccarton Gravel Formation, which has a significantly higher shear wave velocity (V_s) than the overlying layers, making it a suitable location for input motions in seismic site response analyses in Christchurch [28]. The depth to the Riccarton Gravel was estimated to be 30 m deep based on the well records included in the map layer provided by the NZGD. This estimate of 30 m is consistent with the depth that the Riccarton Gravel encountered in adjacent strong motion stations presented by Wotherspoon et al. [28].

The considerable depth to the bedrock results in a lack of representative outcrop motions in Christchurch, making use of deconvolved motions preferred. Markham et al. [23] deconvolved motions for two strong motion stations that were not susceptible to liquefaction and had comparatively shallow Riccarton Gravel deposits, so were expected to have more linear response than other sites: the Riccarton High School (RHSC) and Christchurch Aero Club (CACS). The present study used the fault-normal deconvolved motions at the RHSC recording station and adjusted them using the distance scale factors presented by Markham [29]. This study examines two earthquakes: the moment magnitude (M_w) 6.0 13 June 2011 event and the M_w=5.8 23 December 2011 event. The pore pressure transducer readings for these two events at Burwood are provided in Fig. 1 (variable sample interval of 5 seconds or greater). It is notable that the two spikes in both events are due to the occurrence of two earthquakes on both days.

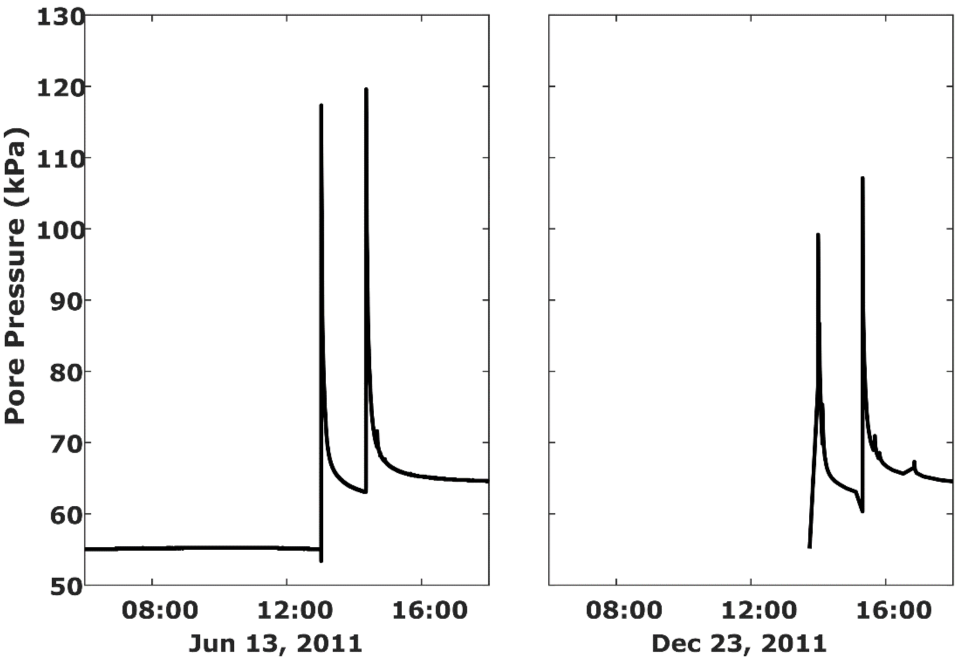

Fig. 1 – Pore pressure transducer recordings at Burwood during 13 June 2011 and 23 Dec 2011 events

2.2 Constitutive model calibration

The PM4Sand model uses three primary parameters, which need to be calibrated for the desired site, and numerous secondary parameters, which can remain as the default values unless additional soil data is available. The relative density (D_r) was the first primary parameter estimated [30]. A range of extensive experimental tests undertaken on different soils has led to the development of several empirical correlations between D_r and CPT penetration resistance by multiple researchers [3, 31-34]. The D_r used in this study was a weighted average of the estimates from these correlations, with the Baldi [31] correlation assigned twice the weight compared to the other correlations, as shown in Fig. 2a. This higher weighting was based on laboratory tests performed by Taylor [35], which showed a close agreement between the D_r based on the correlation proposed by Baldi [31] and that of Christchurch sands. Given the significance of D_r on liquefaction behavior, the idealized stratigraphy for use in FLAC was developed based on D_r. The idealized D_r in each layer is the mean of weighted average D_r at different depths within a given soil layer. Fig. 2a shows the CPT-based D_r estimated by the correlations, as well as the weighted average D_r, and the idealized layering and assigned D_r at the Burwood site versus depth.

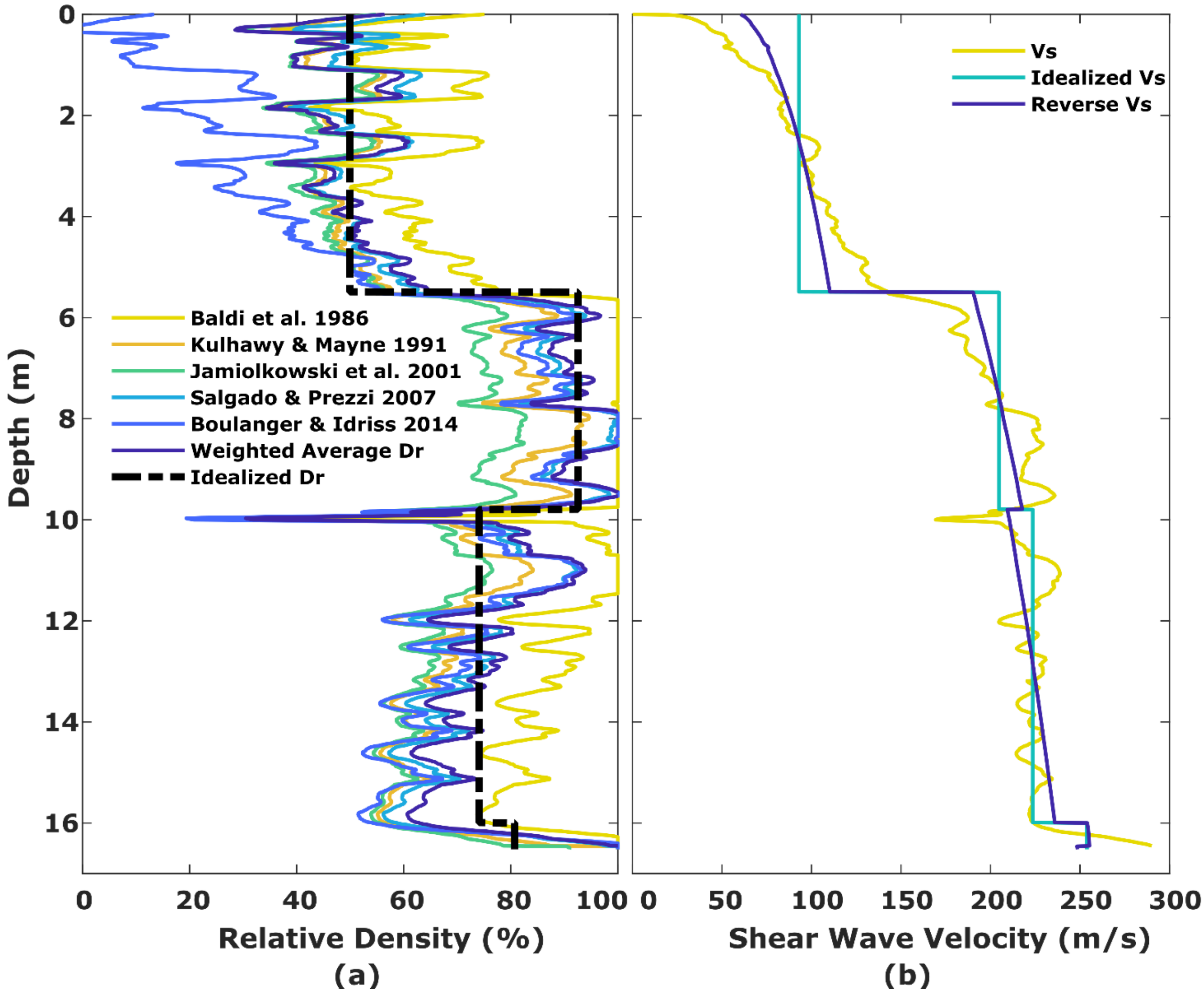

Fig. 2 – CPT-based D_r and shear wave velocity with depth at Burwood: (a) CPT-based D_r correlations by Baldi [31], Kulhawy and Mayne [32], Salgado et al. [33], Boulanger and Idriss [3], and Jamiolkowski et al. [34], the average D_r, and idealized D_r (b) The shear wave velocity profile using the CPT-V_s correlation by McGann et al. [36]

The PM4Sand primary parameter G_0 controls the elastic (or small strain) shear modulus, G, based on the soil stress conditions. The Canterbury-specific CPT-V_s correlation proposed by McGann et al. [36] was used to estimate the V_s profile as shown in Fig. 2b. The G_0 parameter was estimated based on the mean V_s of the given layer in the soil profile for each layer. The stress-dependent reversed V_s implied by PM4Sand is shown in Fig. 2b.

The third PM4Sand primary parameter hp_0 controls the contractiveness of the model. This parameter was calibrated based on the cyclic resistance ratio (CRR), using the PM4Sand cyclic direct shear stress test driver by Boulanger and Ziotopoulou [30]. As lab measured cyclic resistance ratios were not available in this study, the CRR was estimated based on CPT data using the correlation developed by Boulanger and Idriss [37].

Table 1 summarizes the parameters estimated for each layer. Borehole 1717, carried out at the site, indicated that the fines content is less than 5% up to 9 m depth, with no indication of significant fines content down to 30 m. Hence, the soil was considered as clean sand. The unit weight values and permeabilities presented in Table 1 are based on the correlations proposed by Robertson and Cabal [38]. When applicable, secondary parameters were adjusted to Christchurch-specific values estimated based on other studies. Taylor [35] performed laboratory tests on Christchurch soil and found a critical state friction angle of 35° and e_{min}=0.56 and e_{max}=0.95, which were adopted in the current study. Fitting parameters R, Q for cohesionless soils were set to be 0.98 and 7.90 [29], respectively. In addition, the hydrostatic pore pressure head recorded by the pore pressure transducers before the events occurring at the Burwood site indicated that the water table was 0.4 m below ground level.

Table 1 – Soil profile parameters

Layer	**Depth (m)**	**D_r (%)**	**Unit weight (kN/m^3)**	**Permeability (10^{-5} m/s)**	**G_0**	**Hp_0**
1	0 – 5.5	50	16.4	2.8	392	0.53
2	5.5 – 9.8	93	20.0	2.1	1164	10.0
3	9.8 – 16	74	19.6	5.2	1076	0.90
4	16 – 30	81	20.0	3.4	1249	0.95

2.3 Site response analysis

A 30-meter-tall soil column was modelled using the previously calibrated parameters. The soil profile was subdivided into zones with 0.1 m heights, limiting the numerical distortion of the high frequency waves travelling to the surface. Static soil stresses were initialized using a gravity analysis with an elastic material model to vary appropriately with depth. Boulanger and Ziotopoulou [30] and the FLAC manual [24] recommend the application of a small amount of Rayleigh damping (e.g., 0.5%) to limit low amplitude oscillations for most problems. Based on this value and the shear wave velocity profile of the soil, a minimum damping of 0.0037 at a centre frequency of of 3.5 Hz were introduced to the model.

The base of a soil profile can be modelled as rigid bedrock or as elastic bedrock [39]. The rigid base is an acceptable approximation when there is a high impedance ratio between the bedrock and the soil [24]. However, as discussed earlier, the impedance ratio is fairly high between the Riccarton Gravels ($V_s \approx 400$ m/s) and the loose overlying sediments it is not high enough to behave as rigid bedrock. Therefore, a compliant (elastic, assuming a unit weight of 22 kN/m^3) base was used in this study, which was achieved with a quiet boundary to allow energy radiation through the model base. The sides of the soil column were modeled using a periodic boundary condition.

3. Results and Discussion

As the pore pressure transducer at the Burwood site was installed at a depth of 6 m, this is where the predicted and measured pore pressure time histories can be compared. However, it is useful to first consider the general observations of pore pressure profiles and acceleration time histories for any relevant trends. Fig. 3a shows the profiles of excess pore pressure ratio (r_u) at different times of interest during the 13 June 2011 event. As expected, the uppermost layer with the lowest D_r liquefied, with r_u exceeding 90% throughout much of this layer. Note that the top 0.4 m of the soil is unsaturated, so r_u is plotted as zero in this region. The acceleration time histories shown at different depths in Fig. 3b demonstrate only minor amplification from the base up to around 6 m, just below the liquefiable layer, where the pore pressure transducer was installed. However, the peak ground acceleration (PGA) at the surface is around five times greater than the PGA at the interface with the bedrock, due to the presence of significant spikes as a result of cyclic mobility. This is consistent with the observation that liquefaction occurred from around 6 m below ground level and the surface. Furthermore, the surface acceleration time histories illustrated in Fig. 3b and Fig. 4b show PM4Sand produces dilative acceleration spikes at the surface similar to those observed by other researchers [6, 8].

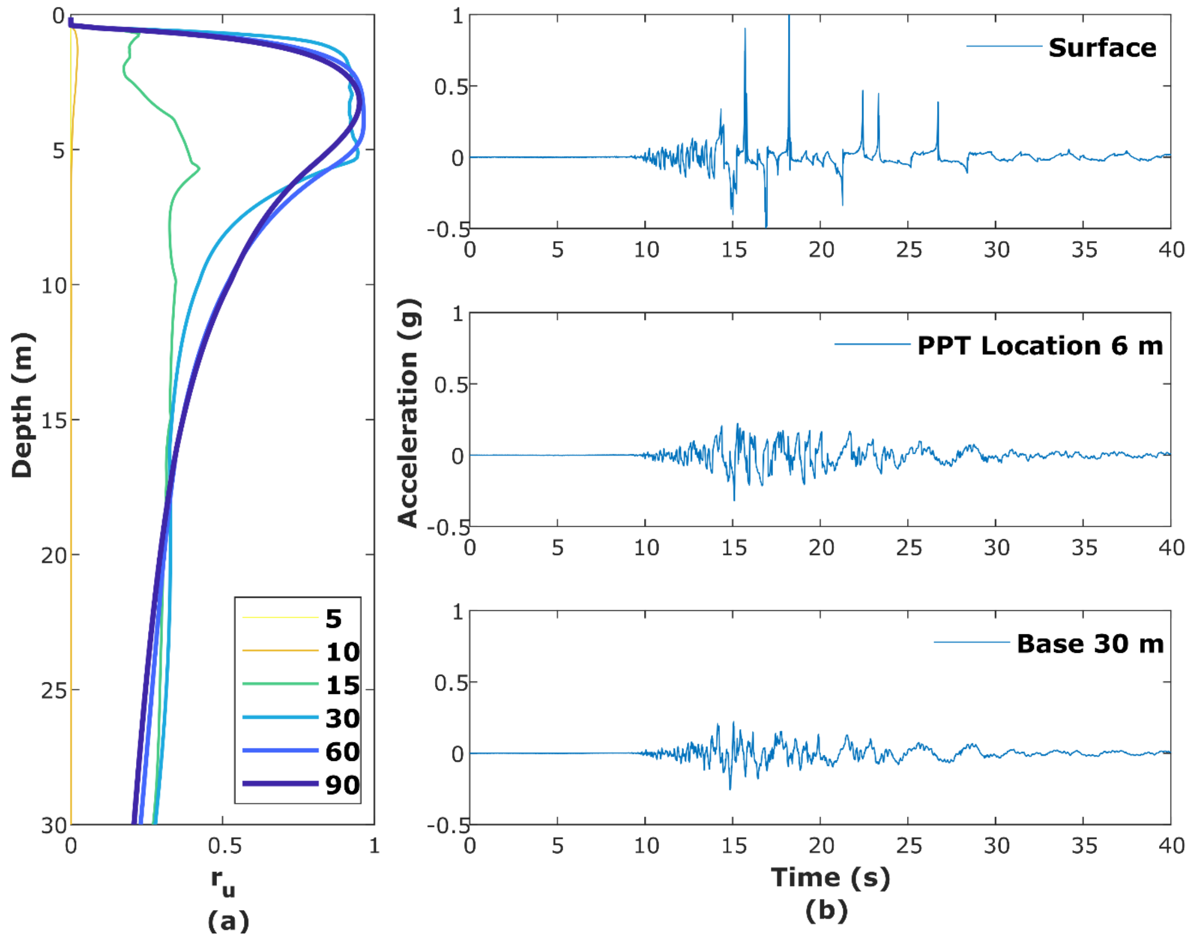

Fig. 3 – 13 June 2011 earthquake results: (a) Excess pore pressure ratio of the soil column at different times, (b) Acceleration time histories at different depths

The model predictions from the 23 December 2011 event, as shown in Fig. 4, are similar to those from the 13 June 2011 event, shown in Fig. 3. Given the similarities in PGA between these two events, it is unsurprising that similar levels of excess pore water pressure were predicted, and the overall trends in Fig. 4 are similar to Fig. 3. Moreover, the surface acceleration time histories shown in Fig. 3b and Fig. 4b demonstrate the notable changes after liquefaction occurs based on the r_u profiles and resulting in soil softening.

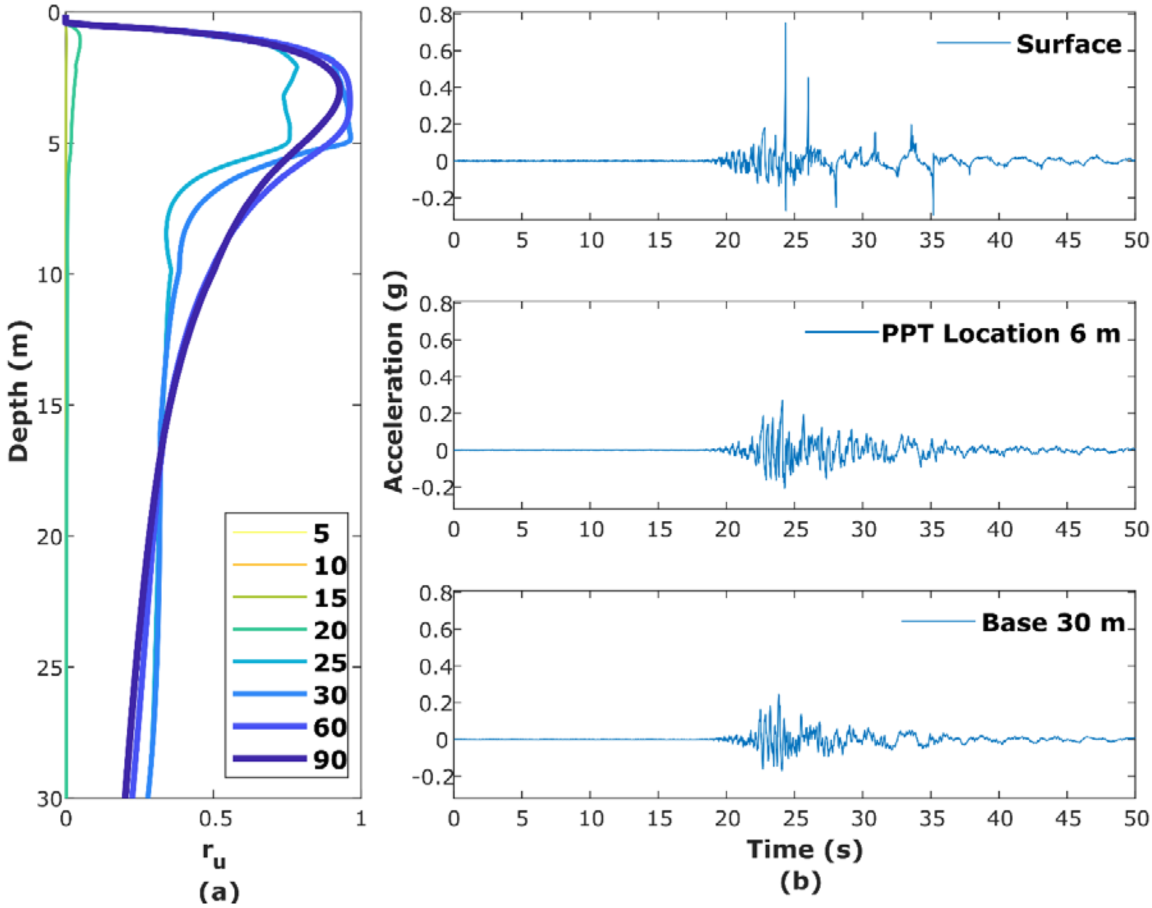

Fig. 4 – 23 December 2011 earthquake results: (a) Excess pore pressure ratio of the soil column at different times, (b) Acceleration time histories at different depths

The changes in pore pressure measured by the pore pressure transducer installed at the Burwood site provide a unique opportunity to validate the ability of PM4Sand to predict excess pore pressures generated during real-world seismic events. Fig. 5 compares the simulated pore pressure time histories from PM4Sand to the recordings of pore pressure transducer for the M_w=6.0 13 June 2011 event, which occurred around 2 pm, and the M_w=5.8 23 December 2011 event, which occurred around 2 pm. The general trend of generation and dissipation of the pore pressure during both events are well simulated by the constitutive model. However, the simulated pore pressure for the 13 June 2011 event is somewhat lower than the recording. This is likely partially due to excess pore pressures not fully dissipating from an event that occurred around two hours previously, as can be observed in Fig. 1. However, the comparison between the recorded and simulated data for the event at 23 December 2011 illustrates that the generated pore pressures are quite similar. The predicted free-field settlements were found to be quite small and are not discussed further as there are not quantitative records to compare the predictions to and previous researchers (e.g., Macedo et al. [40]) have found that PM4Sand significantly underestimates volumetric strains.

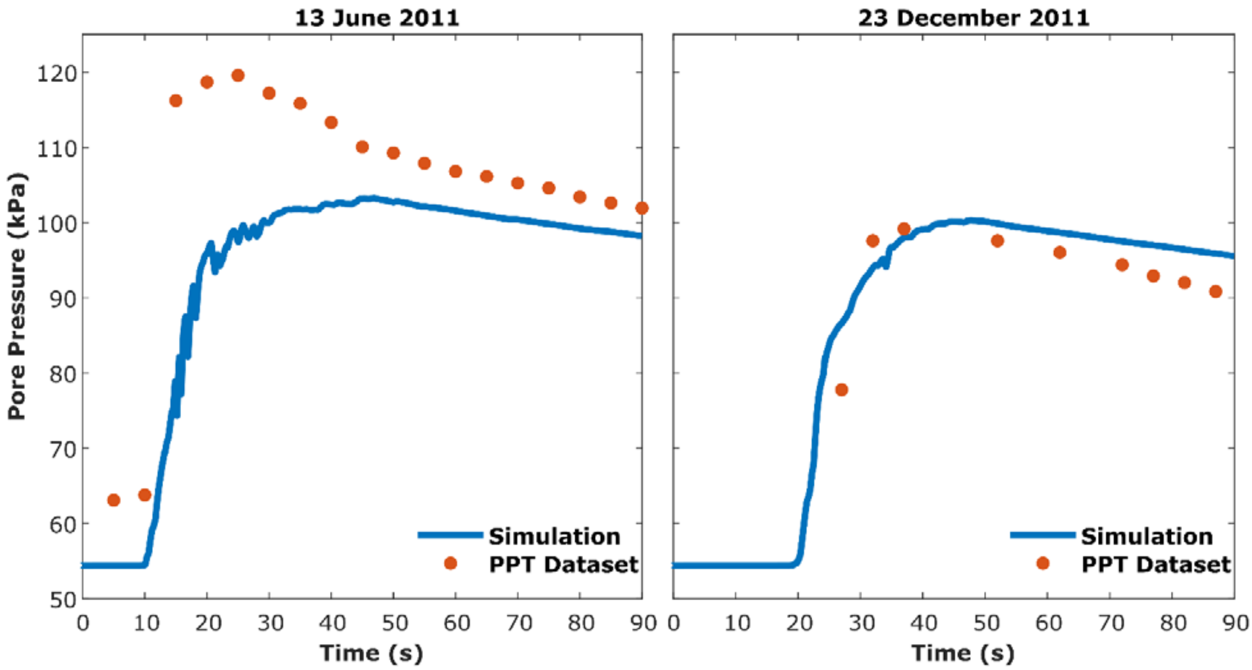

Fig. 5 – PM4Sand simulated results versus pore pressure transducer recordings

4. Conclusions

The inherent limitations of simplified liquefaction assessment methods and the increasing use of performance-based design necessitate the use of advanced numerical models in assessing liquefaction consequences for complex projects. However, these models should be thoroughly validated against physical data to ensure reliability prior to being used in practice. Centrifuge data as well as real-world case histories can play a key role in validation, but a typical limitation for most real-world case histories is a lack of recorded data, particularly pore pressure data. Therefore, the pore pressure transducer installed at the Burwood site in Christchurch, New Zealand provides a relatively unique dataset to validate the PM4Sand constitutive model against the real-world excess pore pressure time histories from several earthquakes. Extensive CPT investigations performed at Christchurch were used to estimate a representative soil profile at the location of the pore pressure transducer. Examination of the results from the free-field effective stress analysis shows that similar levels of liquefaction occurred in the top layer during both events, with roughly similar PGA values. This study examined several aspects of PM4Sand. The overall trend of amplification of surface acceleration due to the liquefiable layer is reasonably achieved by PM4Sand. The onset of acceleration spikes at the surface corresponds to the times at which significant excess pore pressures develop. PM4Sand performed well in predicting the generation of excess pore pressure compared to recorded pore pressures, which further supports the suitability of PM4Sand. The free-field volumetric strains and resulting settlement were quite small, which is consistent with the observations from other numerical studies. The results from this site will be incorporated with future results from several additional sites with pore pressure transducers to provide more comprehensive validation results.

Acknowledgements

The first author of this paper is supported by a University of Auckland Doctoral Scholarship. We would like to thank Tonkin & Taylor Ltd for providing the pore pressure dataset and Cong Ding for some preliminary modelling efforts.

References

[1] Cubrinovski M, Bray JD, Taylor M, Giorgini S, Bradley B, Wotherspoon L, and Zupan J (2011): Soil liquefaction effects in the central business district during the February 2011 Christchurch earthquake. *Seismological Research Letters*, **82** (6), 893-904.

[2] Cubrinovski M, Bradley B, Wotherspoon L, Green R, Bray J, Wood C, Pender M, Allen J, Bradshaw A, and Rix G (2011): Geotechnical aspects of the 22 February 2011 Christchurch earthquake. *Bulletin of the New Zealand Society for Earthquake Engineering*, **44** (4), 205-226.

[3] Idriss IM, Boulanger RW (2008): *Soil liquefaction during earthquakes*. Earthquake Engineering Research Institute.

[4] Youd T, Idriss I, Andrus R, Arango I, Castro G, T. Christian J, Dobry R, Finn L, F. Harder Jr L, P. Koester J, S. C. Liao S, and F. Marcuson W (2001): *Liquefaction Resistance of Soils: Summary Report from the 1996 NCEER and 1998 NCEER/NSF Workshops on Evaluation of Liquefaction Resistance of Soils*. Vol. 127.

[5] Cubrinovski M, Rhodes A, Ntritsos N, and Van Ballegooy S (2019): System response of liquefiable deposits. *Soil Dynamics and Earthquake Engineering*, **124**, 212-229.

[6] Dashti S, Bray JD, Pestana JM, Riemer M, and Wilson D (2010): Centrifuge testing to evaluate and mitigate liquefaction-induced building settlement mechanisms. *Journal of geotechnical and geoenvironmental engineering*, **136** (7), 918-929.

[7] Dashti S, Bray JD, Pestana JM, Riemer M, and Wilson D (2010): Mechanisms of seismically induced settlement of buildings with shallow foundations on liquefiable soil. *Journal of geotechnical and geoenvironmental engineering*, **136** (1), 151-164.

[8] Hayden CP, Zupan JD, Bray JD, Allmond JD, and Kutter BL (2014): Centrifuge tests of adjacent mat-supported buildings affected by liquefaction. *Journal of Geotechnical and Geoenvironmental Engineering*, **141** (3), 04014118.

[9] Zeghal M, Goswami N, Manzari M, and Kutter B (2017): Performance of a Soil Liquefaction Model. in *Poromechanics VI*, 370-377.

[10] Mazzoni S MF, Scott M, Fenves G. (2006): Open System for earthquake engineering simulation user command-language. Berkeley, CA: NEES.

[11] Karimi Z, Dashti S (2015): Numerical simulation of earthquake induced soil liquefaction: validation against centrifuge experimental results. in *IFCEE 2015*, 11-20.

[12] Elgamal A, Yang Z, and Parra E (2002): Computational modeling of cyclic mobility and post-liquefaction site response. *Soil Dynamics and Earthquake Engineering*, **22** (4), 259-271.

[13] Yang Z, Elgamal A, and Parra E (2003): Computational model for cyclic mobility and associated shear deformation. *Journal of Geotechnical and Geoenvironmental Engineering*, **129** (12), 1119-1127.

[14] Yang Z, Lu J, and Elgamal A (2008): OpenSees soil models and solid-fluid fully coupled elements user's manual.

[15] Hashash YMA, Dashti S, Romero MI, Ghayoomi M, and Musgrove M (2015): Evaluation of 1-D seismic site response modeling of sand using centrifuge experiments. *Soil Dynamics and Earthquake Engineering*, **78**, 19-31.

[16] Hashash YMA GD, Musgrove M, Park D, Phillips C, Tsai C-C. (2011): DEEPSOIL V5.0, manual and tutorial. Urbana, IL: Board of Trustees of University of Illinois at Urbana-Champaign.

[17] Boulanger R, Ziotopoulou K (2012): PM4Sand (version 2): A sand plasticity model for earthquake engineering applications. Report No. *UCD/CGM-12/01*, Center for Geotechnical Modeling, Department of Civil and Environmental Engineering, University of California, Davis, CA. (Revision 1 posted July 2012 – Boulanger_Ziotopoulou_Sand_Model_CGM-12-01_2012_rev1), March, 100.

[18] Dafalias YF, Manzari MT (2004): Simple plasticity sand model accounting for fabric change effects. *Journal of Engineering mechanics*, **130** (6), 622-634.

[19] Kamai R, Boulanger RW (2013): Simulations of a centrifuge test with lateral spreading and void redistribution effects. *Journal of Geotechnical and Geoenvironmental Engineering*, **139** (8), 1250-1261.

[20] Armstrong R, Boulanger R (2015): Numerical simulations of liquefaction effects on piled bridge abutments. *6th International Conference on Earthquake Geotechnical Engineering*, Christchurch, New Zealand.

[21] Balachandra A (2019): *Validating SSI Response of Buildings on Liquefiable Deposits using Centrifuge Testing and Numerical Modelling*. Master's Thesis. The University of Auckland.

[22] Nuques RXL (2017): *Numerical Analyses of Liquefaction-Induced Building Settlement*. Ph.D. Thesis. University of California, Berkeley.

[23] Markham CS, Bray JD, Macedo J, and Luque R (2016): Evaluating nonlinear effective stress site response analyses using records from the Canterbury earthquake sequence. *Soil Dynamics and Earthquake Engineering*, **82**, 84-98.

[24] Itasca Consulting Group I. (2016): Fast Lagrangian Analysis of Continua, Ver. 8.0. Minneapolis: Itasca.

[25] Luque R, Bray JD (2017): Dynamic analyses of two buildings founded on liquefiable soils during the Canterbury earthquake sequence. *Journal of Geotechnical and Geoenvironmental Engineering*, **143** (9), 04017067.

[26] Brown LJ, Weeber JH, and Weeber J (1992): *Geology of the Christchurch Urban Area:-Scale 1: 25,000*. Institute of Geological & Nuclear Sciences.

[27] CanterburyGeotechnicalDatabase (2015): Ground Motion. in *Map Layer CGD5170* - 30 June 2015, retrieved April 2019 from https://canterburygeotechnicaldatabase.projectorbit.com/.

[28] Wotherspoon L, Orense R, Bradley B, Cox B, Wood C, and Green R (2013): Geotechnical characterization of Christchurch strong motion stations.

[29] Markham CS (2015): *Response of liquefiable sites in the central business district of Christchurch, New Zealand*. Ph.D. Thesis. University of California, Berkeley.

[30] Boulanger R, Ziotopoulou K (2017): PM4Sand (version 3.1): A sand plasticity model for earthquake engineering applications. Report No. *UCD/CGM-17/01*, Center for Geotechnical Modeling, Department of Civil and Environmental Engineering, University of California, Davis, CA, March, 112.

[31] Baldi G (1986): Interpretation of CPT's and CPTU's. 2nd Part: Drained penetration of sands. *Proc. 4th International Geotechnical Seminar on Field Instrumentation and In Situ Measurements*, Singapore: School of Civil and Structure Engineering, Nanyang Technology Institute.

[32] Kulhawy FH, Mayne PW (1991): Relative density, SPT, and CPT interrelationships. *Calibration Chamber Testing*, 197-211.

[33] Salgado R, Boulanger R, and Mitchell J (1997): Lateral stress effects on CPT liquefaction resistance correlations. *Journal of geotechnical and geoenvironmental engineering*, **123** (8), 726-735.

[34] Jamiolkowski M, Lo Presti DCF, and Manassero M (2001): Evaluation of relative density and shear strength of sands from cone penetration test and flat dilatometer test. *soil behavior and soft ground construction (GSP 119)*, 201-238.

[35] Taylor ML (2015): *The geotechnical characterisation of Christchurch sands for advanced soil modelling*. Ph.D. Thesis. The University of Canterbury.

[36] McGann CR, Bradley BA, Cubrinovski M, Taylor ML, and Wotherspoon LM (2014): Development and evaluation of CPT-Vs correlation for Canterbury, New Zealand soils of the shallow Christchurch and Springston formations.

[37] Boulanger RW, Idriss I (2015): CPT-based liquefaction triggering procedure. *Journal of Geotechnical and Geoenvironmental Engineering*, **142** (2), 04015065.

[38] Robertson PK, and, and Cabal KL (2015): Guide to Cone Penetration Testing for Geotechnical Engineering. *Gregg Drilling & Testing, Inc.*

[39] Kramer SL (1996): *Geotechnical earthquake engineering*. Prentice-Hall Civil Engineering and Engineering Mechanics Series. Upper Saddle River, N.J.: Prentice Hall c1996.

[40] Macedo J, Bray JD (2018): Key trends in liquefaction-induced building settlement. *Journal of Geotechnical and Geoenvironmental Engineering*, **144** (11), 04018076.

