A NEW MODEL FOR SOIL-STRUCTURE INTERACTION OF HORIZONTALLY VIBRATING STRUCTURES

Zs. B. Pap[(1)], L. P. Kollár [(2)]
[(1)] *PhD student, Budapest University of Technology and Economics, pap.zsuzsa@epito.bme.hu*
[(2)] *Professor, Budapest University of Technology and Economics, lkollar@eik.bme.hu*

Abstract

In case of seismic design of structures the deformability and damping of the soil layers should be considered, which can be performed in several ways. A simple spring-dashpot system (with constant characteristics) can be applicable in those cases, when the soil is infinite, but also for these cases neglecting the frequency dependency may cause significant errors. To approximate the dynamic impedance function of a soil layer more complex models can be also applied, for example a layered cone model, echo constants to take into account the effect of the refracted waves or more complex lumped parameter models for the different excitations.

2D (e.g. strip foundations) problems are considered. The depth of the soil above the rock is finite, while the horizontal dimensions can be infinite or finite. The rock under the soil is excited by earthquakes and the horizontal response of the structure is investigated. To consider the effect of a finite layer a new simple model based on a physical approach is given for the horizontal excitation of strip foundations. A physical representation and analytical solution is given for an infinite bar on elastic foundation connected parallelly to a mass-spring system. The advantage of using this model is that it contains only very few parameters and results in a 1D problem instead of a 2D one. Numerical verification is presented, and the parameter range is determined, where the application of the new model is recommended, since applying a simple spring-dashpot model results in significant errors.

Keywords: soil-structure interaction, simplified model, identification, finite soil layer

1. Introduction

In earthquake resistant design it's important to consider the effect of soil-structure interaction (SSI). It can be modelled in several ways. The most accurate method is the so-called direct approach (Fig. 1 a), where the structure and a segment of the soil are modeled together. In this case the nonlinear properties of the soil can also be included in the computation, but the computation time and effort can be enormous. In linear cases the soil can be replaced by the system's impedance function, which can be represented as frequency dependent spring and dashpot elements (Fig. 1 b). However, the practical use of springs and dashpots with frequency dependent characteristics is also difficult; therefore, for earthquake analysis different simplified models are used. The simplest model is a spring-dashpot system with constant characteristics (Fig. 1 c), this can be applicable in those cases, when the soil is infinite [1], but also in these cases neglecting the frequency dependency may cause significant errors [2,3]. When the vertical dimension of the soil is finite the error can be substantially higher [4,5].

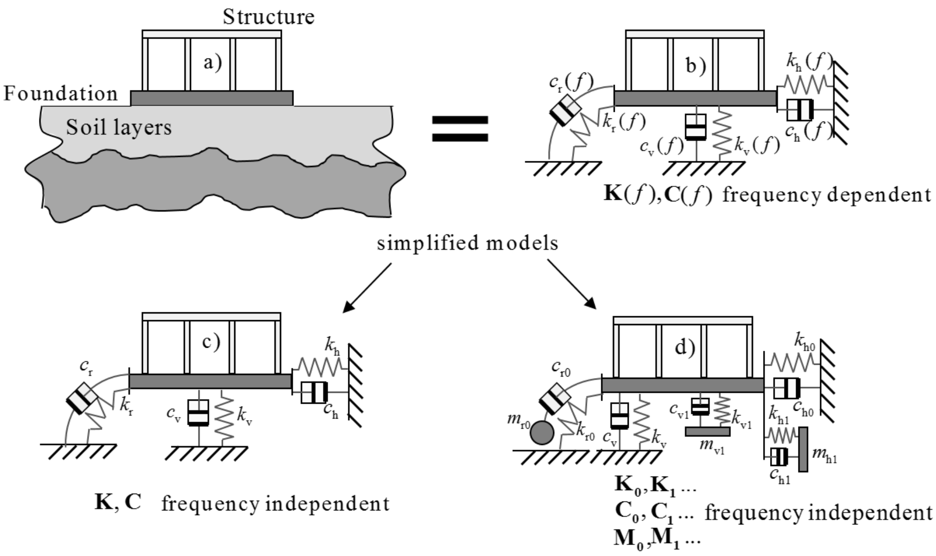

Fig. 1 – The modelling levels of soil effect: a) direct approach, b) impedance function, c) one, frequency independent spring and dashpot element for all directions, d) more complex lumped models

To approximate the dynamic impedance function of a soil layer more complex models were also applied (Fig. 1 d). Meek and Wolf used a layered cone model and developed echo constants to take into account the effect of the refracted waves [6]. Wolf also developed more complex lumped parameter models for the different excitations [6–11]. Saitoh also constructed a more complex lumped model with frequency independent parameters and suggested a new element type [12]. The more complex model is used, the better accuracy can be reached [3]. As a possible representation of the soil, an axially constrained bar is given in [9], however it is not given, how it is properly applicable to model the soil.

2. Problem statement and approach

A rigid object resting on the surface of the ground is investigated. The depth of the soil above the rock is finite and may vary with the horizontal coordinates, while the horizontal dimension is infinite. The rock under the soil is excited by earthquakes and the horizontal response of the structure is investigated.

Our aim is to give a simplified model (Fig. 2), which is based on the real physical behavior of a soil layer, its response is able to produce the phenomenon of cut-off frequency and radiation damping, and simple formulas can be used to calculate the model parameters.

The Rayleigh-Ritz method will be used to reach an approximate solution of the 2D problem (strip foundation on the top of a horizontally infinite vertically finite regular soil layer), and to derive a simplified (1D) model. The latter one will be analyzed by directly solving its differential equation (DE). Simple closed-formed expressions are given to calculate the model parameters in case of the 2D problem, for regular soil layers. To analyze the dynamic effects and validate the model both the direct method (time-history analysis), and the harmonic analysis are applied.

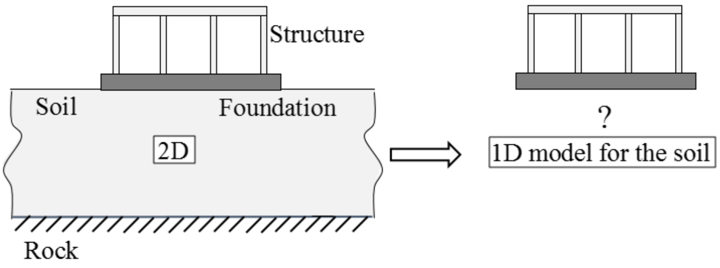

Fig. 2 – Example of 2D problem

3. Horizontal excitation of rigid structures on a finite soil layer

3.1 Model of regular soil layer with strip foundation

To obtain a simplified model first the case, when there is no object on the soil is considered. Then it is extended, and an object with finite size is also taken into account.

According to the Rayleigh-Ritz method the displacement field (for zero Poisson's ratio, ν=0) can be assumed in the following form (Fig. 3):

$$u_{2D}(x,z,t) = u(x,t)\sin\left(\frac{\pi z}{2h}\right), \qquad v_{2D}(x,z,t) = 0, \tag{1}$$

where $u(x,t)$ is the displacement function in the x direction, and h is the thickness of the soil layer.

According to this, after straightforward mathematical manipulations we can obtain the total mechanical energy:

$$\Pi(u(x,t)) = \frac{Gh}{2}\int_x \left(\frac{du}{dx}\right)^2 dx + \frac{G\pi^2}{16h}\int_x u^2 dx - \frac{\rho h}{4}\int_x \left(\frac{du}{dt}\right)^2 dx, \tag{2}$$

where G is the shear modulus ($G=\rho v_s^2$, vs is the shear wave velocity), ρ is the density of the soil layer. The differential equation of the problem can be derived mathematically as the Euler-Lagrange equation of the stationary condition (Π=stationary):

$$-Ghu'' + \frac{G\pi^2}{8h}u + \frac{\rho h}{2}\ddot{u} = 0. \tag{3}$$

The DE of an axially constrained bar is $-\overline{EA}u'' + \kappa u + \mu\ddot{u} = 0$, where $\overline{EA}$ is the normal stiffness of the bar, κ is the stiffness of the elastic foundation and μ is the mass per unit length of the bar (Fig. 3). It may be observed that this equation is equivalent to the DE of the approximation of the 2D problem provided that:

$$\overline{EA} = Gh\,;\ \kappa = \frac{G\pi^2}{8h}\,;\ \mu = \frac{\rho h}{2} \tag{4}$$

Now we investigate the problem when there is an object on the top of the layer with a total mass of $2m$. When the size $2b$ is finite, Rayleigh's method must be reconsidered. We assume that the displacement field is uniform with respect to x, under the object:

$$u(x,z,t) = u(b,z,t) \text{ if } 0 \le x \le b. \tag{5}$$

Introducing Eq. (5) into the expression of the potential energy (Eq.(2)), and determining the Euler-Lagrange equation, we obtain an axially constrained bar, however there is a replacement spring and mass at the end:

$$k_0 = \frac{G\pi^2 b}{8h}, \tag{6}$$

$$m_0 = \frac{\rho h}{2} b, \tag{7}$$

both are proportional to the size b.

When the foundation size is small compared to the thickness of the soil layer (approximately b/h<5), the additional mass-spring system can be neglected.

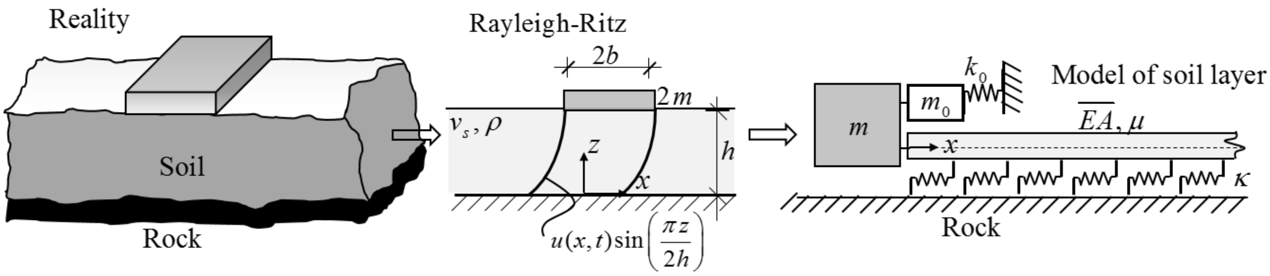

Fig. 3 – Simplified model of a soil layer with an object

3.2 Solution of the model for harmonic excitation

In case of an axially constrained bar, the response for harmonic excitation depends on the stiffness of the foundation, or for a given foundation on the frequency of the end displacement. The frequency which separates the two behavior is the so-called cut-off frequency (ω_c) [13]. When the frequency of excitation is above the cut-off frequency, $\omega > \omega_c$, the bar behaves similarly as a bar without foundation. When $\omega < \omega_c$ the behavior changes considerably. For this case, even for a harmonic excitation, which is applied infinitely long, only a finite length of the bar will be affected. The phase angle and energy dissipation is also different in the two cases. For $\omega > \omega_c$ the phase angle is $\phi = 90^\circ$, and there is energy dissipation, while for $\omega < \omega_c$, $\phi = 0^\circ$, and there is no energy dissipation. Note that at $\omega = \omega_c$ the response is singular. The analytical solution of the model can be given for harmonic force (Fig. 4 a) and base excitation (Fig. 4 b). Here only the steady-state solution is presented for ξ=0, the whole solution (including the transient solution) and its verification is given in [14].

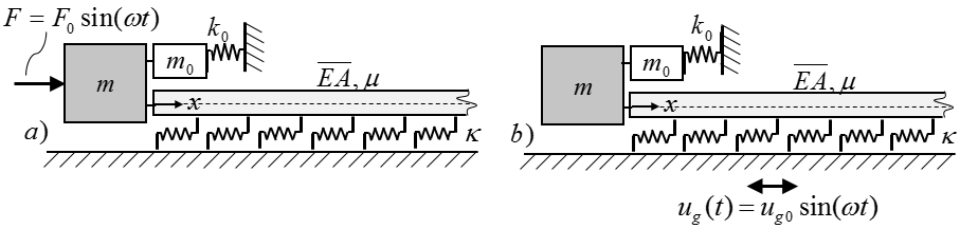

Fig. 4 – Analyzed cases

For the first problem (Fig. 4 a) the steady-state solution is the following:

$$u = \bar{D}_K e^{i(kx-\omega t)}, \tag{8}$$

where $k = \omega\sqrt{\frac{\mu}{\overline{EA}}\left(1-\frac{\kappa}{\mu\omega^2}\right)}$ is the wave number, and $\bar{D}_K$ is:

$$\bar{D}_K = \frac{F_0 / \omega^2 (m+m_0)}{\frac{\omega_{02}^2}{\omega^2}\sqrt{1-\frac{\omega^2}{\omega_c^2}} - 1 + \frac{\omega_{03}^2}{\omega^2}}, \tag{9}$$

where $\omega_c = \sqrt{\kappa/\mu}$ is the cut-off frequency, $\omega_{02} = \sqrt{k_s/(m+m_0)}$ and $\omega_{03} = \sqrt{k_0/(m+m_0)}$.

The natural frequency is as follows:

$$\omega_n = \sqrt{-\frac{\omega_{02}^4}{2\omega_c^2} + \omega_{03}^2 + \sqrt{\left(\frac{\omega_{02}^4}{2\omega_c^2} - \omega_{03}^2\right)^2 - \left(\omega_{03}^4 - \omega_{02}^4\right)}}. \tag{10}$$

The last expression (Eq.(10)) was verified by a 2D finite element solution (Fig. 5), the maximum difference is 8%. Despite of the major simplifications the frequencies of the model (Eq. (10)) and that of the 2D problem are close to each other.

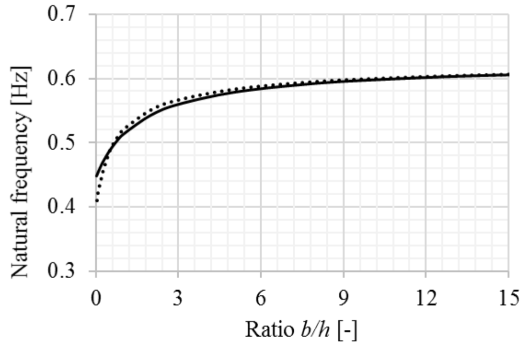

Fig. 5 – Natural frequency of the simplified model (Eq. (10)) and 2D model (ρ=1800 kg/m³, v_s=100 m/s, m=1800 t)

The solution for base excitation (Fig. 4 b) was also derived, the result is:

$$\bar{D}_{Kr} = u_r(x=0,\omega) = \left(\frac{1-\dfrac{\omega_{03}^2}{\omega^2}}{\dfrac{\omega_{02}^2}{\omega^2}\sqrt{1-\dfrac{\omega^2}{\omega_c^2}}+\dfrac{\omega_{03}^2}{\omega^2}-1} + 1 \right) u_{g0} \frac{1}{1-\dfrac{\omega^2}{\omega_c^2}}. \quad (11)$$

Harmonic analysis is performed to compare the 2D and 1D models. The horizontal displacement is calculated for horizontal harmonic force excitation (Fig. 4 a). Fig. 6 shows the numerical solution of the 2D model (soil layer with strip foundation) and the analytical solution of the 1D model. It can be observed that the two solutions are close to each other in the proximity of the first eigenfrequency.

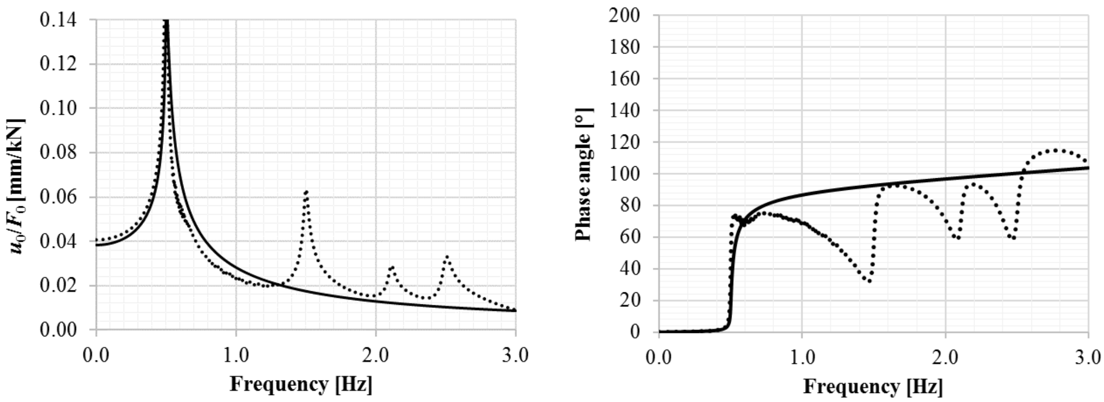

Fig. 6 – Steady-state solution of the different models (ξ=0)
(h=50 m, ρ=1800 kg/m^3, v_s=100 m/s, m=1800 t, 2m=720 t, 2b=20 m)

In reality besides the radiation damping the soil also has damping ($\xi \neq 0$). In this case the analytical solution is not presented here, only numerical solution is performed. The impedance function of a damped, axially constrained infinite bar is given in [15]. In case of an SDOF system the maximum value of the amplification factor (which is maximum of the inverse impedance divided by the static stiffness) is $1/2\xi$, however in infinite systems, where the radiation damping plays an important role, this value is significantly smaller. For an axially constrained infinite bar it is $1/(2\xi)^{0.5}$ [15]. In case of our model, which is an axially constrained infinite bar connected parallelly to a mass-spring system, is between these two values. The comparison of the 2D and 1D models for ξ=5% is given in Fig. 7, here numerical harmonic analysis was performed for both cases.

In this case the functions are even closer to each other than without damping, because the higher modes are not dominant.

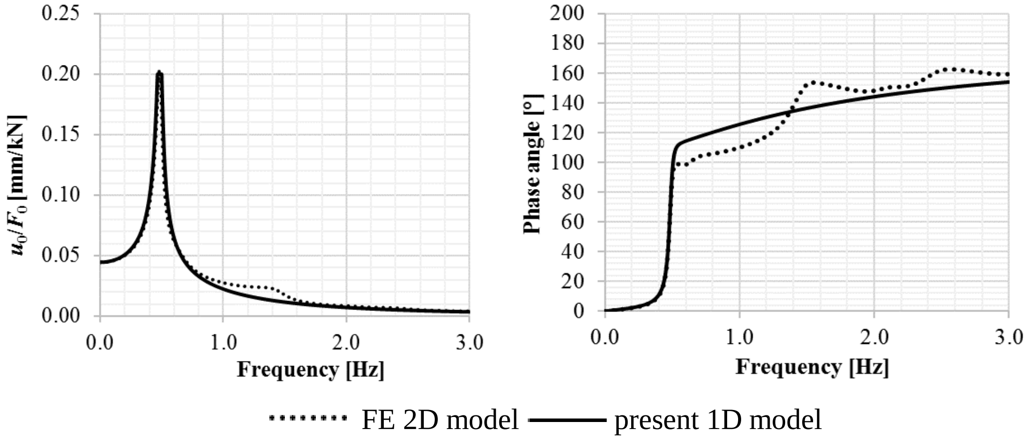

Fig. 7 – Steady-state solution of the different models (ξ=0.05)
(h=50 m, ρ=1800 kg/m^3, v_s=100 m/s, m=1800 t, 2m=720 t, 2b=20 m)

3.3 Numerical example

As a numerical example a time-history analysis is performed for a real earthquake record [16]. First the system has no damping (ξ=0), then it is set to ξ=5%. The 2D model is excited at the bottom of the soil layer.

The horizontal displacements are shown in Fig. 8 when ξ=0, and in Fig. 9 when ξ=5%. In both cases the results of the displacements of the 2D and 1D models are very close to each other.

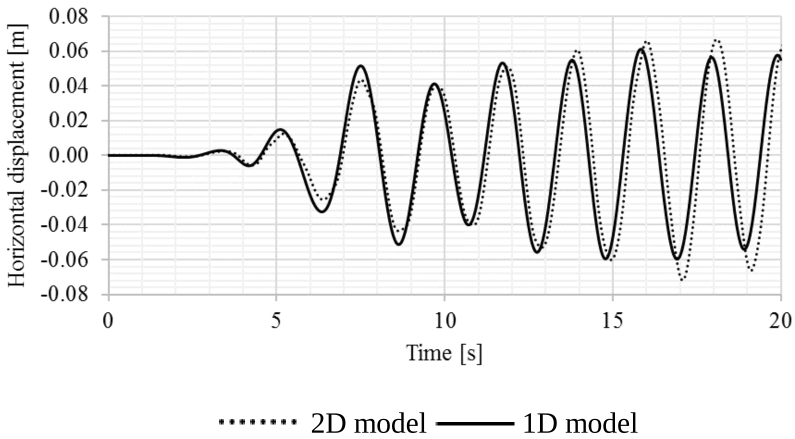

Fig. 8 – Solution of the different models for an earthquake record ([50] record no. 32) (ξ=0)
(h=50 m, ρ=1800 kg/m^3, v_s=100 m/s, m=1800 t, 2m=720 t, 2b=20 m)

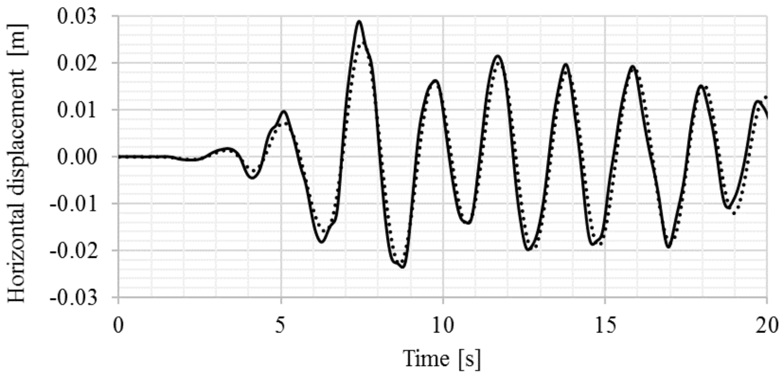

Fig. 9 – Solution of the different models for an earthquake record ([50] record no. 32) (ξ=0.05) (h=50 m, ρ=1800 kg/m^3, v_s=100 m/s, m=1800 t, 2m=720 t, 2b=20 m)

4. Conclusions

A rigid structure resting on a finite depth soil layer was investigated. The size of the object resting on the soil influences the behavior of the layer. When the structure is long in one direction the 2D problem can be reasonably well modelled by an axially constrained bar, where the bar stiffness ($\overline{EA}$), mass per unit length (μ) and the coefficient of elastic foundation (κ) depend on the soil parameters and on the stiffness of the soil layer. In the model a spring (k_0) and a concentrated mass (m_0) must be taken into account, which depend on the size of the foundation. The model parameters can be calculated with simple, closed-formed formulas (Eqs. (4), (6) and (7)), which are based on analytical derivation. The concentrated mass can be interpreted as the soil which directly moves together with the object. The recommended model can be considered as two sub models connected parallelly: a spring-mass system and an axially constrained infinite bar (Fig. 10).

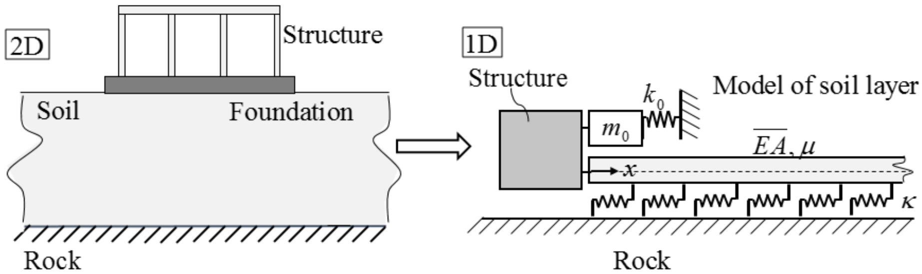

Fig. 10 – Simplified model of the soil

The presented model takes into account two resonant points between the rigid structure and the supporting soil, the first natural frequency and the cut-off frequency. In the 2D solution there are further resonance points, since in the soil layer higher modes can develop. These might be taken into account by the combination (serial or parallel) of our simple model, however this is not the task of this paper.

5. Acknowledgements

The authors gratefully thank the support of the OTKA 115673 project of the Hungarian National Research, Development and Innovation Office. Support of grant BME FIKP-VÍZ by EMMI is kindly acknowledged.

6. References

[1] Wolf J.P. and Deeks A.J. (2004) Foundation vibration analysis: a strength-of-materials approach. Elsevier, Oxford.

[2] Celebi E., Firat S. and Cankaya I. (2006) The evaluation of impedance functions in the analysis of foundations vibrations using boundary element method. *Applied Mathematics and Computation*, **173**, 636–67. http://dx.doi.org/10.1016/j.amc.2005.04.006

[3] Andersen L. (2010) Assessment of lumped-parameter models for rigid footings. *Computers and Structures*, Elsevier Ltd. **88**, 1333–47. http://dx.doi.org/10.1016/j.compstruc.2008.10.007

[4] Mylonakis G., Nikolaou S. and Gazetas G. (2006) Footings under seismic loading : Analysis and design issues with emphasis on bridge foundations. *Soil Dynamics and Earthquake Engineering*, **26**, 824–53. http://dx.doi.org/10.1016/j.soildyn.2005.12.005

[5] Dunn P.W. (2010) Comparison o cone model and measured dynamic impedance.

[6] Meek J.W. and Wolf J.P. (1992) Cone Models for Soil Layer on Rigid Rock. *Journal of Geotechnical Engineering*, **118**, 686–703.

[7] Wolf J.P. (1991) Consistent lumped-parameter models for unbounded soil: frequency independent stiffness, damping and mass matrices. **20**, 33–41.

[8] Wolf J.P. (1991) Consistent lumped-parameter models for unbounded soil:physical representation. *Earthquake Engineering and Structural Dynamics*, **20**, 11–32.

[9] Wolf J.P. (1994) Foundation vibration analysis using simple physical models. Prentice Hall, Englewood Cliffs.

[10] Wolf J.P. (1997) Spring-dashpot-mass models for foundation vibrations. *Earthquake Engineering and Structural Dynamics*, **26**, 931–49.

[11] Meek J.W. and Veletsos A.S. (1973) Simple models for foundations in lateral and rockinf motion.

[12] Saitoh M. (2007) Simple Model of Frequency-Dependent Impedance Functions in Soil-Structure Interaction Using Frequency-Independent Elements. *Journal of Engineering Mechanics*, **133**, 1101–14.

[13] Hagedorn P. and DasGupta A. (2007) Vibrations and Waves in Continuous Mechanical Systems. John Wiley & Sons.

[14] Pap Z.B. and Kollár L.P. (2019) Model of Soil-Structure Interaction of Structures Resting on Finite Depth Soil Layers. *Periodica Polytechnica Civil Engineering*, **63**, 1204–16. http://dx.doi.org/https://doi.org/10.3311/PPci.14459

[15] Pap Z.B. and Kollár L.P. (2019) Dynamic Response of Long Rectangular Floors Subjected to Periodic Force Excitation. *Materials*, **12**, 1–16. http://dx.doi.org/10.3390/ma12091417

[16] FEMAP695. (2009) Quantification of Building Seismic Performance Factors. *FEMA P695*,.

