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SHAKING TABLE TEST ON THE SEISMIC RESPONSE OF GROUP PILES IN LIQUEFIABLE SOIL

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Abstract

Group piles are widely used in for their excellent bearing capacity, including the lateral resistance which can withstand seismic loading. However, soil liquefaction during earthquakes degrades the stiffness and the bearing capacity of group piles, especially in the lateral direction because they are mainly attributed to the shallower soil layers, which generally exhibit higher liquefaction potential then deeper layers if the ground is liquefiable. In order to investigate the seismic behavior of group piles during liquefaction, shaking table tests of a 1/25 scale model of 2×2 group piles were conducted in this study. A 2.5 m (L) × 2.5 m (W) × 3.0 m (D) large laminar shear box was utilized as the soil container, and its laminar flexible boundary comprised by layers of movable frames can minimize the boundary effect so as to reproduce the seismic response of the ground. A ground specimen with a thickness of 2 m composed of fine silica sand was prepared by wet sedimentation method. The scale model of the group piles was designed for the jacket foundation of an offshore wind turbine. It comprised four 2-m-in-length and 81-mm-in-diameter aluminum alloy pipes as piles, which has a lower Young's modulus than steel pipes to achieve a more reasonable simulation of the flexural behavior of the pile. The pile heads were connected by a relatively rigid steel frame as a pile cap. 2 Hz sinusoidal waveforms with various acceleration amplitude were used as input motions. Piezometers were installed in the soil to monitor the generation of excess pore water pressure so that the development of liquefaction can be captured. Accelerometers were installed at different depths in the ground specimen, on the movable frames composing the laminar boundary of the shear box and along the pile to observe the response of the ground and the piles. Under the sinusoidal excitation with an amplitude of 50 gal, the upper half of the ground specimen reached initial liquefaction, while in the 75-gal amplitude test almost the entire ground was liquefied. During the tests, the recorded accelerogram in the soil body is close to that on the movable frame at the same elevation prior to liquefaction, verifying the effectiveness of the flexible boundary of the shear box. After liquefaction, the acceleration amplitude of the ground specimen significantly reduced, while the pile as well as the movable frames showed high frequency chatters other than the 2-Hz sinusoidal response, probably due to the loss of lateral confinement provided by the soil resistance. The change of natural frequency of the group-pile-soil system after each excitation implies the influence of post-liquefaction densification of soil as well as its attached mass effect to the pile.

Keywords: shaking table test; laminar shear box; group piles; soil liquefaction

1. Introduction

Soil liquefaction is one of the most devastating geotechnical phenomena during earthquakes. In addition to settlement and lateral spreading of the ground, liquefaction lowers the stiffness and strength of soil, which subsequently degrades the stiffness and bearing capacity of the foundation. Pile foundations, especially group piles, are widely used for their excellent bearing capacity and high stiffness which represents the displacement control performance. The lateral resistance is one of the most important functions of piles in seismic active area such as Japan and Taiwan because it helps to withstand seismic loading. However, the lateral resistance of piles is mainly attributed to the shallower soil layers, which generally exhibit higher liquefaction potential then deeper layers if the ground is liquefiable because they undergo lower confining pressure. Therefore, the seismic behavior of group piles may be significantly influenced by soil liquefaction.

In this study, shaking table tests of a 1/25 scale model of 2×2 group piles embedded in saturated loose sandy ground which was considered liquefiable were conducted to investigate the seismic response of group piles as well as the ground during liquefaction. These are pilot tests of a scale model testing program of a 5-MW offshore wind turbine supported by the jacket foundation [1]. A 2.5 m (L) × 2.5 m (W) × 3.0 m (D) large laminar shear box to serve as a soil container for geotechnical physical model tests [2] was used, as shown in Fig. 1. It has laminar flexible boundary comprised by layers of movable frames that can minimize the boundary effect of a finite-volume specimen to represent the semi-infinite nature of the ground, which enables reasonable simulation of the seismic response of layered ground under the earthquake excitation. The 8 m × 8 m high-performance shaking table in Tainan Laboratory of National Center for Research on Earthquake Engineering (NCREE) was utilized, as shown in Fig. 1. It has a maximum horizontal stroke and velocity of ±1.0 m and ±2.0 m/s, respectively [3], and has a maximum payload of 250 ton and a maximum horizontal acceleration of ±2.5 g at bare table and ±0.75 g at full payload, making it more than sufficient to accommodate the mentioned laminar shear box for seismic simulation tests of large-scale geotechnical physical model. Sinusoidal excitation tests with various acceleration amplitude were adopted, and the excess porewater pressure and acceleration within the soil body as well as the acceleration of the piles and the movable frames of the shear box were instrumented during the tests. Thus, the seismic response of the ground and the piles were observed, and the soil-pile interaction during the excitation and the influence of the liquefaction can be thus investigated.

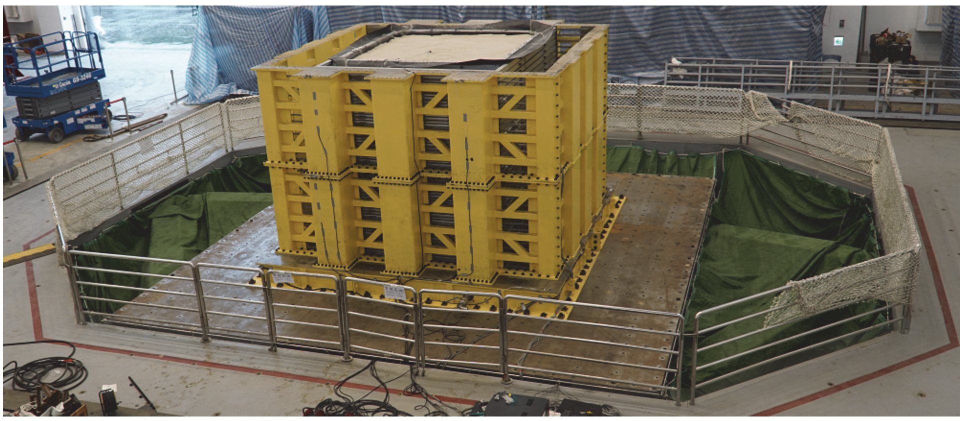


Fig. 1 – Large laminar shear box and high-performance shaking table at NCREE Tainan Laboratory

2. Test Setup

2.1 Sandy ground specimen

The sandy ground specimen was composed of Malaysia silica sand with grain sizes ranged from 0.15 mm to 0.425 mm and can be regarded as uniform fine sand, and had a thickness of 2 m, which was the same as the pile length because the piles were bottom-fixed. Concerning the specimen preparation, the wet sedimentation method along with a special pluviator designed for the mentioned large shear box was utilized, which had been verified that the obtained sandy ground specimen can be satisfactorily homogeneous, almost fully saturated and sufficiently loose to be liquefiable [4]. The initial relative density (D_r) achieved was 10.6% and can be categorized as very loose.

2.2 Scale model of group piles

2.2.1 Scaling considerations

In this study, a 1/25 scale model of group piles was used according to its following scale model testing program of a jacket- foundation-supported offshore wind turbine based on the prototype offshore wind turbine structure proposed in [5], which had a tower height of 80.3 m (including the transition piece), a jacket substructure height of 59.6 m and four piles with a length of 70 m. The scaling factors are listed in Table 1. $\lambda = 25$ was adopted considering the limitation of the test facilities, which gave a model tower height of 3.2 m (including the transition piece) and a model jacket substructure height of 2.4 m. The tower and the jacket substructure of the model were made of steel, same as the prototype, so $\lambda_\rho = \lambda_E = 1$. For a 1-g shaking table test to simulate earthquake excitation, the acceleration of the model is preferred to be representative of the prototype. Therefore, $\lambda_T = 5$ was adopted to have $\lambda_a = 1$, which subsequentially gave $\lambda_f = 1/5$. Due to the critical concern of resonance for wind turbine structures, it is important to keep the ratio of the natural frequency of the wind turbine structure to the loading frequency in the model test same as the prototype. Because the model tower was made of same material as the prototype, $\lambda_k = 25$ considering the lateral stiffness of the tower $k_T = 3E_T I_T / L_T^3$, where E_T, I_T and L_T are the Young's modulus, cross-sectional moment of inertia and the length of the tower. Thus, the mass of the rotor-nacelle-assembly (RNA) of the model was adjusted to give $\lambda_m = 625$ so that the natural frequency of the prototype would be 1/5 of the model with a simplified consideration that the natural frequency of the wind turbine structure $\omega_n = \sqrt{k_T/m_{RNA}}$, where m_{RNA} is the lumped mass of RNA.

Table 1 – Scaling factors for following offshore wind turbine model test

Items	Dimension	Scaling factors (prototype/model)
Length	[L]	$\lambda = 25$
Density	[ML^{-3}]	$\lambda_\rho = 1$
Young's modulus	[ML^{-1}T^{-2}]	$\lambda_E = 1$
Time	[T]	$\lambda_T = \lambda^{0.5} = 5$
Acceleration	[LT^{-2}]	$\lambda_a = \lambda\lambda_T^{-2} = 1$
Frequency	[T^{-1}]	$\lambda_f = \lambda_T^{-1} = 1/5$
Lateral stiffness of tower	[MT^{-2}]	$\lambda_k = 25$
RNA mass	[M]	$\lambda_m = \lambda^2 = 625$
Inertia force acting on RNA	[MLT^{-2}]	$\lambda_F = \lambda_m\lambda_a = 625$

2.2.2 Model group piles

The model group piles consisted of four piles with their heads connected to a rigid frame to have a 2x2 layout at a center-to-center spacing of 0.5 m, as shown in Fig. 2, in which piles were aluminum alloy pipes with an outer diameter of 81 mm and a thickness of 4 mm to simulate the prototype steel pipe piles with an outer diameter of 2 m and a thickness of 40 mm [5]. The outer diameter of model piles was the nearest available size in market to meet $\lambda = 25$, and the thickness was also due to availability, yet the needed flexural behavior was achieved using aluminum alloy pipes. The length of the model piles was 2 m due to the depth limit of the shear box, shorter than 2.8 m based on the prototype pile length of 70 m and $\lambda = 25$, yet it was long enough to behave as a slender pile based on the suggestion in [6]. The lower ends of piles were fixed at the bottom of the shear box, yet the pile can be approximately regarded as floating because of its slenderness.

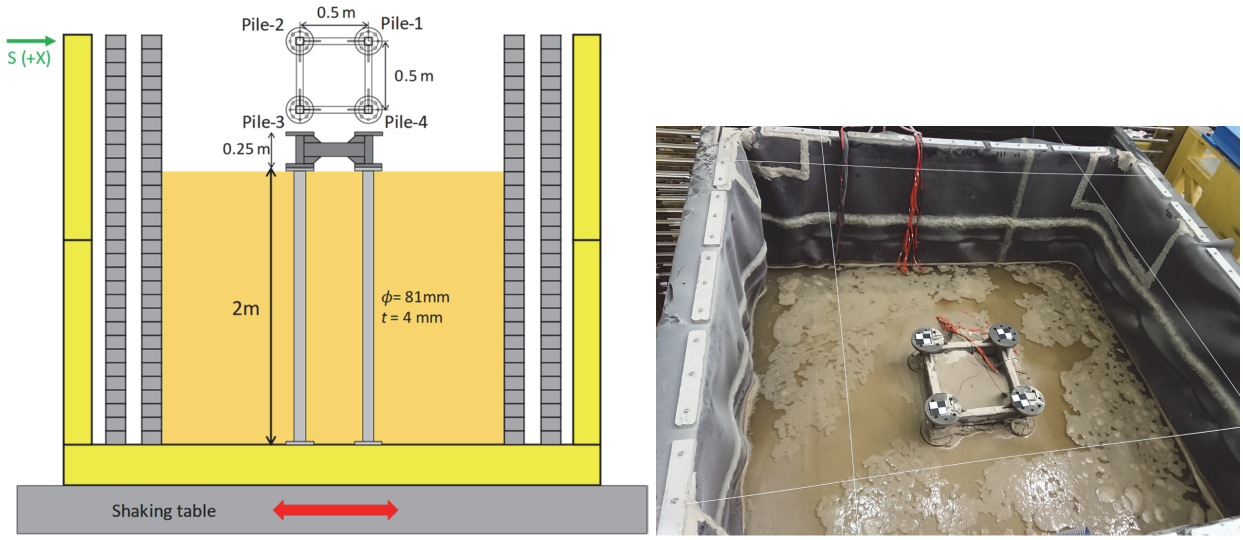


Fig. 2 – 1/25 scale model of group piles

2.3 Input motions and instrumentation layout

The 2 Hz sinusoid was adopted for the input motion in this pilot test for its easier to analyze than earthquake excitations, and it was applied uniaxially in the X-direction as defined in Fig. 2. The acceleration amplitudes of 30 gal, 50 gal and 75 gal were used, and the excitation that reached the target amplitude continued around 10 seconds (20 cycles). Fig. 3 shows the actually achieved 50-gal and 75-gal table motions, and it is noted that right after the ramp-up stage there was one or two cycles of which the amplitude exceeded the target, probably due to the inertial effect from the around 60-ton weight of the ground specimen along with the shear box. The white noise input motion with a spectral content from 0.1 Hz to 30 Hz which had a root-mean-squared (RMS) acceleration of 15 gal was applied before, between and after these sinusoidal excitations (at least 30 minutes afterwards) to identify the natural frequency group-piles-soil interaction system.

Fig. 4 shows the layout of instrumentation for the tests. Piezometers were installed in soil every 0.25 m in depth from the ground surface, that is, at elevations of 1.75 m, 1.50 m, 1.25 m, 1.00 m, 0.75 m, 0.50 m and 0.25 m to monitor the generation of excess pore water pressure during the excitation so that the occurrence and extent of liquefaction can be assessed. Accelerometers were installed in soil at depths of 0.25 m and 1.25 m, respectively, or elevations of 0.75 m and 1.75 m taking the bottom of the shear box as datum. The mentioned sensors embedded in soil were in between the movable frames of the shear box and the group piles so that they can be regarded as located in the free field. Furthermore, accelerometers were installed on the movable frames that compose the laminar boundary of the shear box, on the rigid frame that connected the piles to represent the pile head response, and along Pile-1 at elevations of 0.75 m, 1.25 m and 1.75 m.

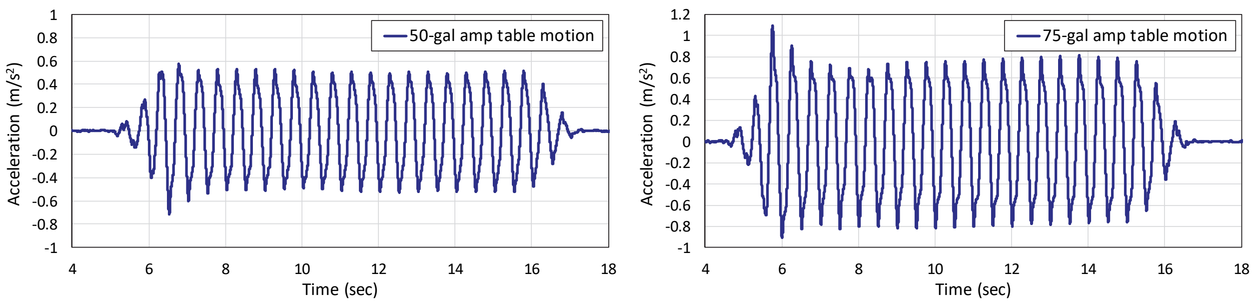


Fig. 3 – Actually achieved table motion in 50-gal and 75-gal amplitude tests

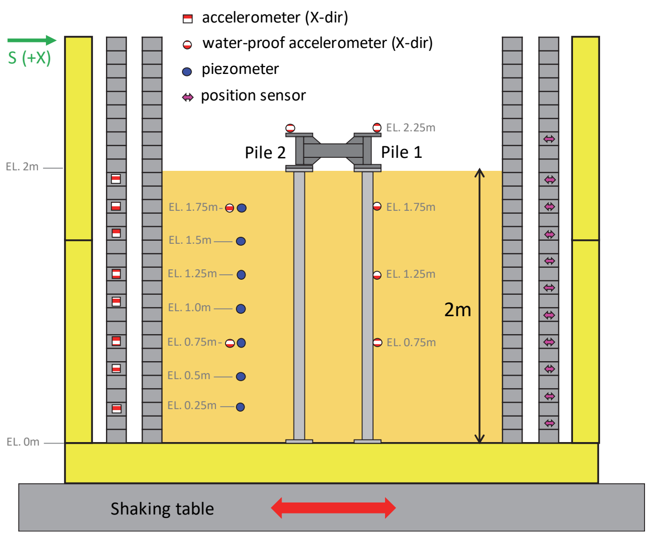


Fig. 4 – Layout of instrumentation

3. Ground response

3.1 Excess pore water pressure

Because the focus of this research is on the seismic response of group piles in liquefiable soil, the excess pore water pressure generated during the shaking table test will be discussed firstly. In the 30-gal amplitude test, the excitation was not strong enough to induce no significant excess pore water pressure at all the depths, so only the results of 50-gal and 75-gal amplitude tests will be presented in this section.

3.1.1 Excess pore water pressure ratio in 50-gal amplitude test

Fig. 5 depicts the excess pore water pressure ratio (r_u) at each elevation in the 50-gal-amplitude test. r_u denotes the ratio of the excess pore water pressure to the effective overburden pressure and helps to determine the occurrence of liquefaction; when $r_u \approx 1$, soil is considered completely liquefied. r_u increased rapidly in the earlier part of the excitation at all elevations, reached its maximum at about 10 sec (13 sec for EL. 1.75 m), and then showed no significant change at the remains of the excitation, that is, around 10-17 sec (13-17 sec for EL. 1.75 m). For EL. 0.75 m and lower, or, depths no smaller than 1.25 m, $r_u \leq 0.7$, that is, the soil was not liquefied; besides, r_u decreased right after excitation, indicating the dissipation of excess pore water pressure. At EL.1.00 m, $r_u \approx 0.9$ around 10-17 sec, yet r_u declined as the excitation ceased, so the soil could be close to initial liquefaction. Regarding EL. 1.50 m and EL. 1.75 m, r_u remained roughly constant after the excitation, indicating that the dissipation of excess pore water pressure still yet to occur, until 25-30 sec. This is typical for liquefied soil but maximum values of r_u were only 0.85-0.9, so probably only initial liquefaction was induced at this stage, and also possibly because of the discrepancy between the specified and actual elevations of the piezometers leading to errors of the effective overburden pressure calculation. It is noted that the shallowest piezometer (at EL. 1.75 m) showed a delay in the onset of r_u reaching its maximum compared to those at other elevations. This delay could be owing to the propagation of pore water pressure, as well as the disturbance of the wave near the ground surface caused by the shaking because the water table was slightly higher than the ground surface during the tests for full saturation. To sum up, the upper half of the ground specimen was initially liquefied after about 5 seconds of shaking in the 50-gal-amplitude excitation.

3.1.2 Excess pore water pressure ratio in 75-gal amplitude test

Concerning the excess pore water pressure generation in the 75-gal-amplitude test, r_u increased sharply at the end of the ramp-up stage of the excitation, probably related to the over-target amplitudes mentioned in Section 2.3. Consequently, r_u reached 0.95-1.0 at every instrumented elevation during the remains of the excitation, indicating liquefaction at nearly all depths of the ground specimen. The excess pore water pressure kept undissipated after the excitation for various durations at different elevations except EL. 0.25 m, and the deeper location the shorter duration. Besides, it is noted that at EL. 1.75 m r_u gradually increased after the excitation to a value of 1.13, which is presumed to be due to the gradual settlement of the piezometer. Briefly speaking, almost the entire ground specimen was liquefied at about 7 sec, which was right after the ramp-up stage of the excitation, and reached complete liquefaction as the excitation continued in the 75-gal-amplitude test.

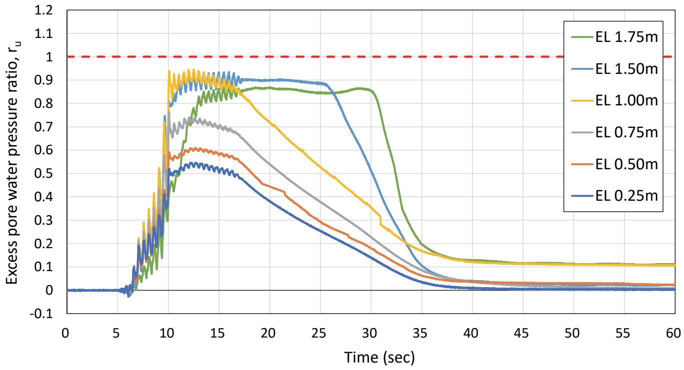


Fig. 5 – Time histories of excess pore water pressure ratio in 50-gal amplitude test

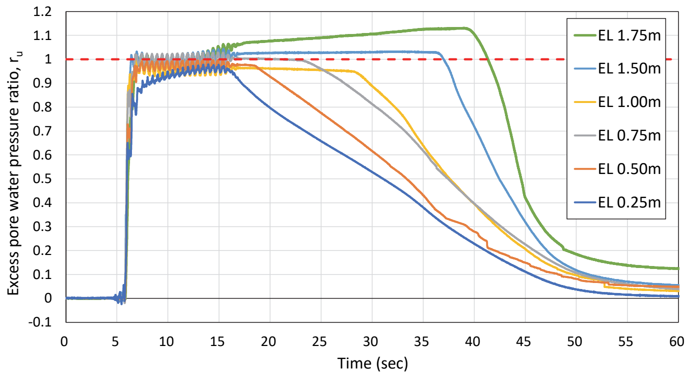


Fig. 6 – Time histories of excess pore water pressure ratio in 75-gal amplitude test

3.2 Ground settlement

The elevation of the ground level was measured at the initial status and after each excitation. Thus, the settlement due to each shaking can be obtained as listed in Table 2. Although 30-gal excitation did not cause liquefaction at all, some settlement was still observed because the soil was very loose at the initial stage. As the excitation got more intense, the settlement was more significant due to liquefaction. The 50-gal excitation, in which about half of the ground specimen liquefied, caused a 23-mm settlement increment and thus D_r ascended to about 17%; while the 75-gal excitation, which induced a nearly thorough liquefaction, caused a considerable settlement increment of 70 mm and increased D_r to nearly 30%.

Table 2 – Settlement and relative density of the ground specimen after each excitation

Excitation case	Initial	30-gal amp.	50-gal amp.	75-gal amp.
Ground level elevation	2.029 m	2.018 m	1.995 m	1.925
Total Settlement	—	11 mm	34 mm	104 mm
Settlement increment	—	11 mm	23 mm	70 mm
Relative density	10.60%	12.64%	16.90%	29.94%

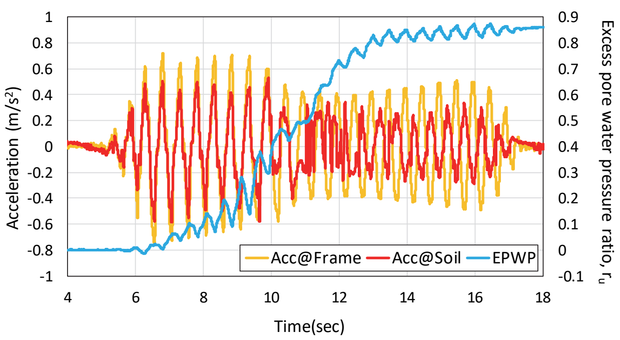
3.3 Ground acceleration

3.3.1 Ground acceleration in 50-gal amplitude test

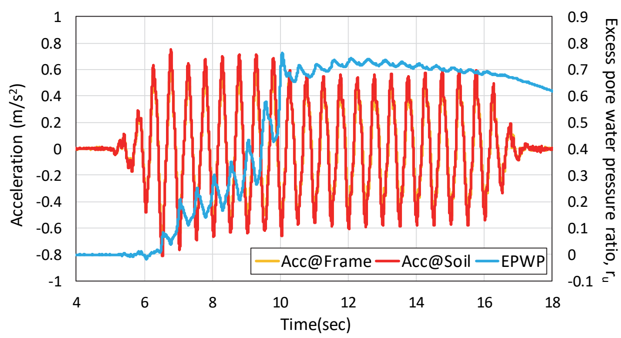
Fig. 7 give the measured acceleration time histories in soil as well as on the movable frame of the shear box at EL. 1.75 m and EL. 0.75 m, and the excess pore water pressure (EPWP) in terms of r_u at the same elevation is also included for comparison. At EL. 1.75 m acceleration in soil is smaller than on the frame in the earlier part of the excitation, yet difference is limited, showing the effectiveness of the flexible boundary of the shear box; while the amplitudes of both declines considerably after 10 sec, when initial liquefaction occurred.After 10 sec the accelerations in soil and on the frame are nearly 180° out of phase, implying the dramatic state change of the liquefied soil. As for EL. 0.75 m, the accelerations in soil and on the frame are almost identical, and the amplitudes of both slightly decreases when r_u reached its maximum, or, after 10 sec, despite the soil was not liquefied at this elevation in this case. Therefore, overlaying liquefied layers may influence the seismic response of deeper layers because the dynamic characteristics of the entire ground system has been changed.

3.3.2 Ground acceleration in 75-gal amplitude test

Fig. 8 depict the acceleration time histories in soil and on the frame at EL. 1.75 m and at EL. 0.75 m. As mentioned, liquefaction occurred at about 7 sec at both elevations, and before that the acceleration in soil shows similar tendency yet is somewhat smaller than that on the frame at EL. 1.75 m, while at EL. 0.75 m they are very similar. However, after liquefaction, the acceleration amplitude dropped remarkably at both elevations, especially in soil, causing its lager difference from that on the frame. The accelerations in soil and on the frame are approximately 180° out of phase at EL. 1.75 m, while at EL. 0.75 m they remain in phase, showing the complicated behavior of the liquefied ground. Additionally, some high-frequency chatters of the frame showed after liquefaction at both elevations, possibly because the lateral confinement from the soil to the frame almost vanished when the ground completely liquefied so that high-frequency response of the frame itself was excited.

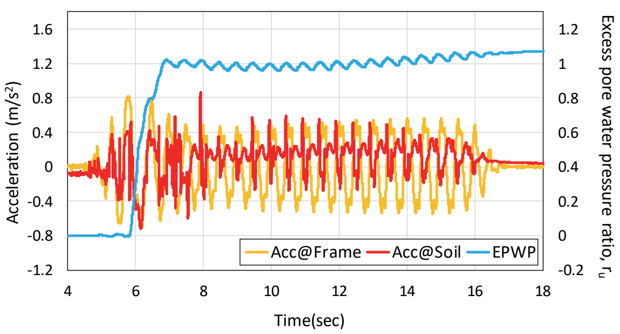


(a) At EL. 1.75 m

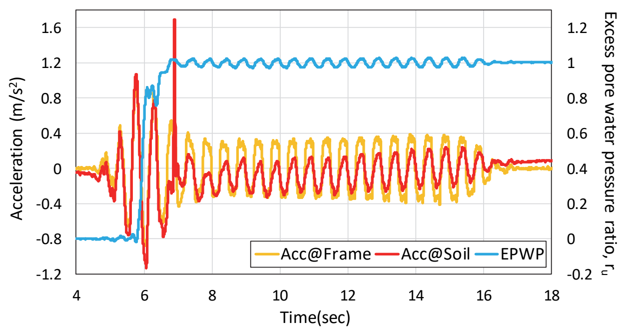


(b) At EL. 0.75 m

Fig. 7 – Time histories of ground acceleration in 50-gal amplitude test



(a) At EL. 1.75 m



(b) At EL. 0.75 m

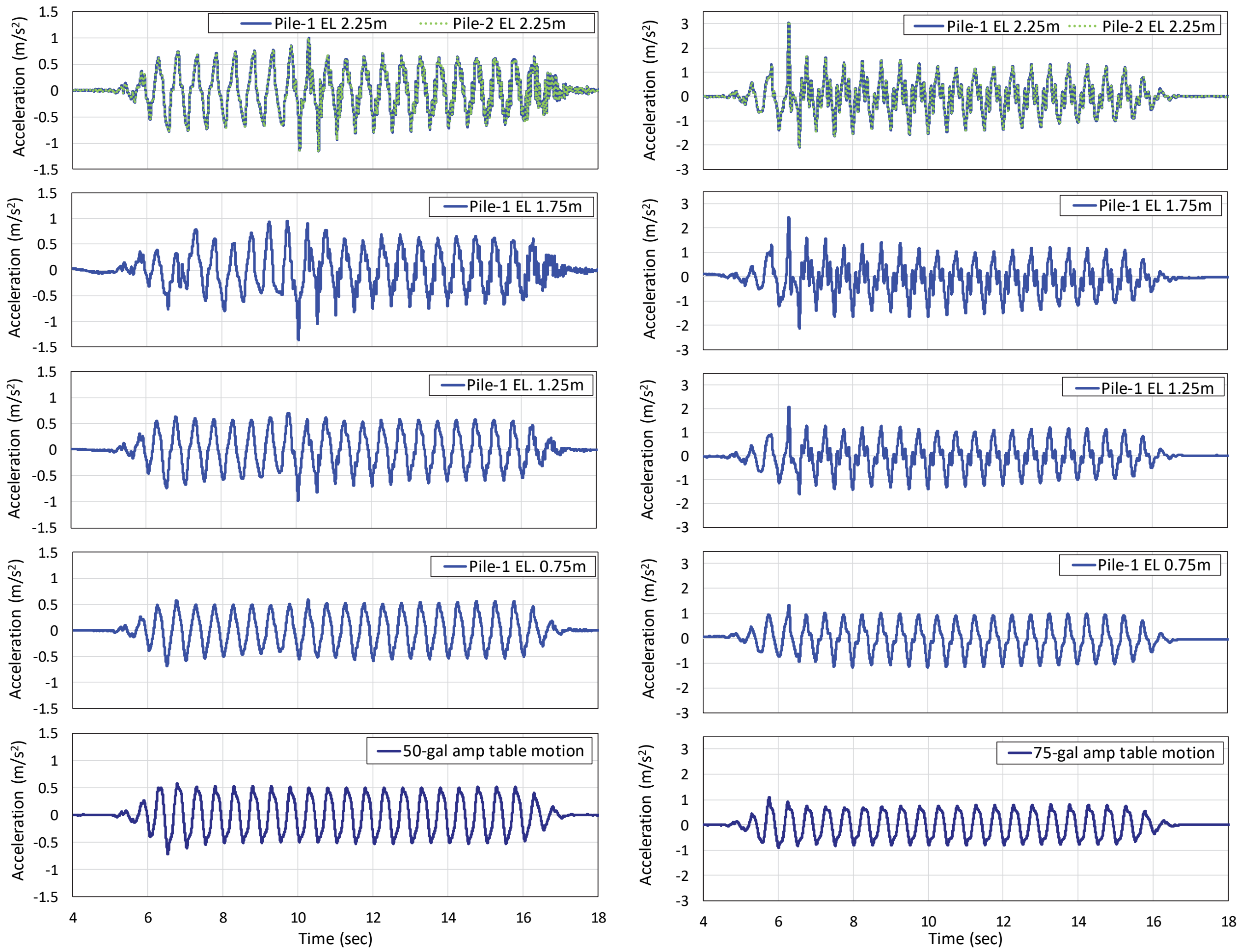
Fig. 8 – Time histories of ground acceleration in 75-gal amplitude test

4. Response of group piles

4.1 Acceleration along the pile

Fig. 9(a) shows the acceleration time histories in 50-gal amplitude test at the table, which is considered same as at the fixed (lower) end of the pile, and along the pile including at EL. 0.75 m, at EL. 1.25 m, at EL. 1.75 m and at the top of the rigid frame that connected the piles, which is denoted as EL. 2.25 m. It is noticed that the acceleration of Pile-1 and Pile-2 at EL 2.25 m are nearly identical due to the constraint of the rigid frame. Before liquefaction (10 sec), the waveforms along Pile-1 are similar to that at the table, especially for the lower part of the pile. At the onset of liquefaction, acceleration impulse is observed at EL 1.25 m and higher part of the pile; at the same positions high-frequency chatters showed up after liquefaction, just like what observed on the frame, and the higher elevation the more obvious. This is possibly because the lateral confinement from the soil was reduced due to liquefaction so that high-frequency pile response was excited. It is however noted that at EL. 0.75 the response is very close to the table motion both in waveform and amplitude, probably because no liquefaction occurred here.

The acceleration time histories along the pile in 75-gal amplitude are given in Fig. 9(b). In this case, almost the whole ground specimen was liquefied at around 7 sec, right after the ramp-up stage of the excitation. Similar to the previous case, response of Pile-1 and Pile-2 at EL 2.25 m are almost identical, waveforms along



(a) In 50-gal amplitude test (b) In 75-gal amplitude test

Fig. 9 – Time histories of acceleration along the pile

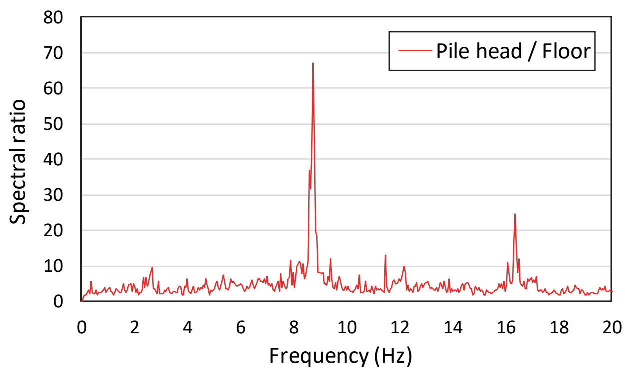
lower part of Pile-1 are similar to the table motion before liquefaction, and pile response with higher frequency content than the excitation is observed. In fact, it seems the pile response composed of the harmonics of 2-Hz sinusoid was excited, probably because the complete liquefaction of nearly entire ground specimen in this case caused total loss of the soil confinement. It is noted that, different from the previous case, at EL. 0.75 the response shows different waveform with some high-frequency content and larger amplitude to some extent compared to the table motion. This could be owing to liquefaction at this elevation in this case.

Nevertheless, the acceleration amplification of the pile is generally not remarkable except the impulse at the onset of liquefaction, possibly because the group piles model is much stiffer than soil, and its natural frequency, which will be discussed later, is much higher than the excitation frequency.

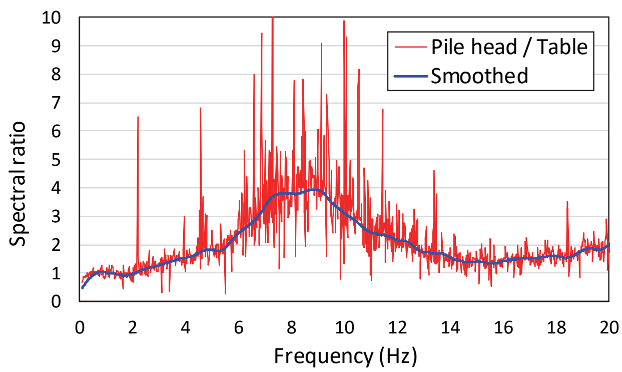
4.2 Natural frequency of group piles

Before this shaking table test, the group piles model was fixed on the strong floor at the NCREE Tainan Laboratory with no soil surrounded to identify its natural frequency based on its response excited by the ambient vibration, which had an RMS of merely 0.0024 gal. Fig. 10(a) shows the transfer function amplitude of the pile head response with respect to the floor motion. It has a very narrow-banded peak indicating a natural frequency (f_N) of 8.75 Hz.

Regarding the group-piles-soil system, the tests using the white noise input motion in its initial state and after each sinusoidal excitation test can be analyzed. As mentioned, all these white noise excitations were made at least 30 minutes after the sinusoidal excitation when the excess pore water pressure can be considered totally dissipated. Fig 10(b) depicts the transfer function amplitude of the pile head response with respect to the table motion in the initial state, which has in contrast a wide-banded peak, implying much larger damping than in the unembedded state. The smoothed curve is used to identify its natural frequency in each test stage, as listed in Table 2. At the initial state, f_N = 8.86 Hz, which is slightly higher than in the unembedded condition, showing the contribution of the stiffness of surrounding soil, yet its influence is minor because soil was very loose then. After 30-gal amplitude excitation, f_N kept almost unchanged because no liquefaction occurred at this stage; while after 50-gal amplitude test, f_N was raised to 9.36 Hz probably because the liquefaction somewhat densified the soil, with D_r increased from 12.64% to 16.90%. However, after 75-gal amplitude excitation, though D_r was increased to 29.94%, f_N was lowered to 8.20 Hz. It is presumed that the rearrangement of soil grains after complete liquefaction leaded to some attached mass effect to the pile, which could be more influential than the densification in this case.



(a) Unembedded, under ambient vibrations



(b) Embedded (initial state), under white noise

Fig. 10 – Transfer function amplitude of the group piles model

Table 3 – Natural frequency and damping ratio of the group-piles-soil system after each excitation

Excitation case	Initial state	30-gal amp.	50-gal amp.	75-gal amp.
Natural frequency (f_N)	8.86 Hz	8.88 Hz	9.36 Hz	8.20 Hz

5. Conclusions

Based on the results of shaking table tests on group piles in liquefiable soil presented in this paper, the following conclusions are drawn:

(1) According to the development excess pore water pressure, the upper half of the ground specimen reached initial liquefaction under the 2-Hz 50-gal amplitude sinusoidal excitation, while almost the entire ground was completely liquefied in the 75-gal amplitude test.

(2) Before the liquefaction of the ground specimen, acceleration in soil is close to that on the movable frame of the shear box at the same elevation, verifying the effectiveness of the flexible boundary of the box. After liquefaction, the ground acceleration amplitude significantly reduced and showed some discrepancy from the frame motion; besides, high frequency responses of the pile and movable frames other than the 2-Hz sinusoid were excited, probably due to the loss of lateral confinement from the soil as it was liquefied.

(3) The rise of natural frequency of the group-pile-soil system after the 50-gal amplitude excitation indicate the influence of post-liquefaction densification, while its decline after the 75-gal amplitude excitation might imply the attached mass effect to the pile from rearrangement of the soil grains.

6. Acknowledgements

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