



## NON-LINEAR DECONVOLUTION FOR IDENTIFICATION OF ENGINEERING BEDROCK SEISMIC MOTIONS

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### Abstract

In earthquake engineering, only ground surface records are usually available, given that accelerometers are commonly installed at the ground level. If the motion of the lower soil layers is of interest, it must be estimated after the recorded surface acceleration time series by performing depropagation analyses; certainly, this operation requires a deep knowledge of the soil stratigraphic profile. Such underground accelerograms are relevant in earthquake-resistant design of structures with underground parts: road and railway tunnels, by-passes, underpasses for pedestrians, metro and railway stations, foundations, building basements, mining infrastructure, industrial facilities, pipes (utilities for transporting water or other liquids such as liquefied natural gas, oil, etc.), culverts, sewers, rainwater storage tanks, and sewage treatment stations etc. On the other hand, this operation is also necessary in predicting motions in irregular terrain, in earthquake-triggered landslides, and in some cases of soil-structure interaction, among other common earthquake engineering problems. These considerations highlight the relevance of the analyzed problem.

If the soil behaves linearly, the aforementioned estimation of the lower layers motion is merely an equivalent linear deconvolution; however, in actual situations, especially for severe earthquakes, the soil behavior is non-linear, and, thus, more sophisticated procedures are required. This paper presents an algorithm to estimate, after superficial records, the motion of the ground lower layers. Given that the S (shear) waves are, by far, the most destructive ones, only the propagation of such waves is considered. The aforementioned soil non-linear behavior is represented by a modified Masing model, in which rigidity and damping depend on the shear deformation. The algorithm is 1-D; the soil domain to be analyzed is discretized in layers. The ensuing equations of motion are solved in discrete time by the Newmark method. Given that this problem is numerically ill-conditioned due to the singularity of the mass matrix, a nonlinear Bayesian Kalman Filter-type estimation method is used to obtain the solution. Estimation overlapping windows are used to facilitate the convergence of the backward analysis. Application examples are presented; the accuracy of the proposed model is compared with the one of the DEEPSOIL software.

*Keywords: Seismic records; engineering bedrock; non-linear deconvolution.*



## 1. Introduction

In constructions with underground parts, the seismic excitation is not only the ground surface motion, but also those of their lower layers (covering practically all the underground part depth). In these situations, since the available historical accelerograms usually have been recorded on the surface, it is necessary to estimate the motion in the lower layers. On the other hand, this operation is also necessary in predicting motions in irregular terrain and in some cases of soil-structure interaction.

If the soil behavior is assumed to be linear and elastic, the underground motion can be estimated from the equivalent soil parameters (obtained by linearizing its constitutive equation) by performing a linear deconvolution of the signal measured on the soil surface. However, for severe seismic actions, the soil behavior is usually nonlinear, thus requiring more complex algorithms to solve the corresponding nonlinear deconvolution. This work proposes an algorithm to identify, from surface-recorded seismic accelerograms, the motion of the lower soil layers, even reaching the bedrock (engineering bedrock); this layer is taken as the model base. Only S (shear) waves are considered, as they are, in most situations, more damaging than the P (pressure) waves, which generate vertical movements. The aforementioned non-linear soil behavior is represented by a modified Masing model, in which the soil shear modulus ( $G$ ) and its damping (hysteretic and viscous) depend on the shear strain. This constitutive law is implemented in a discrete model in which the soil is divided into layers. There is a node in each border between adjacent layers; it has only a degree of freedom (Figure 1) corresponding to the horizontal displacement in the direction of study. The differential equations of motion are integrated in discrete time using the Newmark method. Since this problem is ill-conditioned due to the near-singularity of the mass matrix [1], a Kalman filter is used to estimate the solution.

In order to calibrate the proposed model, its results are compared with those provided with the DEEPSOIL program [2-5]. It is capable of carrying out nonlinear convolutions to describe the transmission of upstream waves, but in the downstream direction it can only simulate linear transmission.

## 2. State-of-the-art on propagation and inverse analysis

### 2.1. 1-D propagation model of shear waves

In earthquake engineering, propagation analysis is used to study the variation of the seismic waves when are transmitted from the bedrock through the soil stratigraphic profile. This methodology is based on assuming that the seismic shear waves propagate only in the vertical direction; after this assumption, it is enough to perform one-dimensional analysis. This hypothesis is, in general, suitable for moderate slope sites [6].

If linear soil behavior is assumed (its stiffness and damping do not vary during the earthquake), linear one-dimensional propagation calculations can be performed in the frequency domain [7] by applying the Fourier transformation to the input seismic signal; in that way, its effect on the rest of the stratigraphic profile is expressed by the corresponding transfer functions [6]. However, the ground stiffness and damping vary with its transverse deformation (shear strain); although this variation indicates a non-linear behavior, it can be represented by an equivalent linear calculation consisting of an iterative analysis of seismic wave propagation in the frequency domain. In each iteration, the stiffness and damping corresponding to the effective shear deformation during the earthquake (65% of the maximum achieved deformation) are taken. When the calculated shear effective deformation coincides with the assumed one, within the considered tolerance, the iterative process is terminated [7]. This hypothesis of invariance of the soil mechanical properties during the earthquake is only valid for rigid soil and low intensity earthquakes [8]; for other situations, the following paragraph describes strategies that take this variation into account.

To take into account the soil non-linear behavior, the propagation must be analyzed in the time domain [3]; the soil is discretized as lumped masses connected through spring and dashpot elements representing their rigidity and damping, respectively. The differential matrix equation of motion is integrated (in discrete time) along the input duration; the response of the soil layers to the excitation transmitted by the bedrock is obtained at each instant. The soil can be modeled as rigid or flexible; if rigid / flexible the wave propagation will not / will continue under the bedrock. The hysteretic soil behavior is usually described by the extended Masing rules



[2]. In summary, the resolution in the time domain allows to model the wave propagation with rigor, and this method of analysis provides greater accuracy than the linear equivalent [8].

## 2.2. Nonlinear Bayesian estimation method utilizing UKF (Unscented Kalman Filter)

The back-analysis or refers to the estimation of the excitation (input) of a model (system) so that the result (response, output) approximates as much as possible at a known value (observed, measured). This type of analysis is widely used in engineering; as an example, the Asaoka method [9] provides the oedometric soil properties from the measurements of filler seats on consolidable soils. In the research presented in this article, the model describes the behavior of the stratigraphic profile; the output is the soil surface response and the input is the seismic excitation transmitted by the base (engineering bedrock).

The back-analysis can be deterministic or probabilistic; in the second case, the input is a random variable (or random process, if the system is dynamic). Deterministic back-analysis is commonly used when the system can be easily reversed, thus providing the input in terms of the output; however, in general, the soil behavior is complex and their models cannot be easily inverted, thus probabilistic inverse analyses are usually preferred. In these analyses it is possible to apply the Chapman-Kolmogorov and Bayes theorems to calculate the input probability density function after that of the output; the result of the back-analysis is the input value that maximizes the output occurrence probability.

In the context of probabilistic inverse (back) analysis, the Kalman filters [10] constitute a solution strategy based on imposing that the variance of the estimation error at each moment be minimal. This methodology is used for dynamic (time-dependent) problems. Given a system  $S$  with state  $\mathbf{x}$ , the function that characterizes its temporal evolution is called  $\mathbf{f}(\mathbf{x})$ . The observation function  $\mathbf{h}(\mathbf{x})$  provides the observable results ( $\mathbf{y}$ ) in terms of  $\mathbf{x}$ . This function usually corresponds to the composition of the  $S$  system with an observability function, whose task is to screen the results obtained to provide only those that are observed. The Kalman filter starts from the system expressed by its state ( $\mathbf{f}$ ) and observation ( $\mathbf{h}$ ) equations in a given instant  $k$  as:

$$\mathbf{x}_{k+1} = \mathbf{f}(\mathbf{x}_k) + \mathbf{w}_k \quad \mathbf{y}_k = \mathbf{h}(\mathbf{x}_k) + \mathbf{v}_k \quad (1)$$

In equation (1),  $\mathbf{y}_k$  are the observations, and  $\mathbf{w}_k$  and  $\mathbf{v}_k$  are the modeling (observation) and measurement errors (noise), respectively. In this study,  $\mathbf{x}$  contains the displacements, velocities and accelerations of the degrees of freedom (Fig. 1),  $\mathbf{f}$  is the identity function,  $\mathbf{y}$  is the surface acceleration, and  $\mathbf{h}$  is the restricted identity function.

The minimum variance Kalman filter iterative algorithm is described next:

1. The state for an instant  $k - 1$  is characterized as a normal (Gaussian) random variable with average  $\overline{\mathbf{x}_{k-1|k-1}}$  and covariance matrix  $\mathbf{P}_{k-1|k-1}^x$ .
2. The state average and covariance matrix are actualized for instant  $k$  being propagated with the state equation in (1); the new values are called  $\overline{\mathbf{x}_{k|k-1}}$  and  $\mathbf{P}_{k|k-1}^x$ .
3. The observation average and covariance matrix  $\overline{\mathbf{y}_{k|k-1}}$  and  $\mathbf{P}_{k|k-1}^y$  are determined with the observation equation in (1).
4. The actualized parameters (state and observation) are corrected with the Kalman gain matrix, which provides the values that minimize the variance of the error between the model result  $\mathbf{y}_k$  and the measurements (deterministic)  $\mathbf{Y}_k$ :

$$\overline{\mathbf{x}_{k|k}} = \overline{\mathbf{x}_{k|k-1}} + \underbrace{\mathbf{P}_{k|k-1}^{xy} \left( \mathbf{P}_{k|k-1}^y \right)^{-1}}_{\text{Kalman gain matrix}} \left( \mathbf{Y}_k - \overline{\mathbf{y}_{k|k-1}} \right) \quad (2)$$

$$\mathbf{P}_{k|k}^x = \mathbf{P}_{k|k-1}^x - \mathbf{P}_{k|k-1}^{xy} \left( \mathbf{P}_{k|k-1}^y \right)^{-1} \left( \mathbf{P}_{k|k-1}^{xy} \right)^T \quad (3)$$

This process is repeated until the measurements end. To start the algorithm, some information on the initial values of the parameters to be estimated ( $\overline{\mathbf{x}_{0|0}}$  and  $\mathbf{P}_{0|0}^x$ ) is required. Since this information is not



available, it is assumed that the parameters are represented by a random variable with an arbitrary mean and a covariance matrix with very high values, so that their propagation (by the equations of state and observation (1)) provide a wide range of estimates of the observations (within which the obtained measures are included).

During the Kalman filtering process, it is required to propagate the random variables (that represent the meaningful parameters) so that their value that minimizes the variance of the estimation error (observations vs. actual measurements) can be obtained. There are different types of Kalman filters depending on how this propagation is carried out; they are discussed in the following paragraph.

The original Kalman filter formulation referred exclusively to linear systems, in which the propagation of a random variable is a simple mathematical operation; subsequently, it was tried to apply the filtering algorithm to nonlinear systems, in which this propagation is a more complex task. Initially, it was studied how to linearize the function around the point of estimation by means of Taylor series expansion of the state and observation functions; these methods were named as extended Kalman filters [11]. A limitation of such methods is that the linearization of the system functions provides a poor approximation when they are markedly nonlinear; this circumstance makes back-analysis difficult, since the estimated parameters does not represent accurately the analyzed. A solution is the sigma points Kalman filter; it is based on the transformation of sigma points [12], which allows the precise propagation of random variables through nonlinear functions. The idea behind the sigma point transformation is that it is easier to approximate a random variable in order to propagate it through a nonlinear function, than to linearize it and then propagate the entire random variable (unless the function is only weakly nonlinear). To approximate a random variable, the sigma point transform maps such variable into a set of values chosen in a deterministic way, called sigma points. Weighting coefficients are employed; such coefficients provide the first and second-order moments of the transformed random variable.

Let be  $\mathbf{x}$  a random vector, with average  $\bar{\mathbf{x}} = \boldsymbol{\mu} \in \mathbb{R}^n$  and covariance matrix  $\text{Var}(\mathbf{x}) = \boldsymbol{\Sigma} \in \mathbb{R}^{n \times n}$ . A set of  $m$  sigma points  $\mathbf{X} = \mathbf{X}_1, \dots, \mathbf{X}_m$  with weighting coefficients  $\omega_1, \dots, \omega_m$  are to be selected as:

$$\sum_{i=1}^m \omega_i = 1 \quad \sum_{i=1}^m \omega_i \mathbf{X}_i = \boldsymbol{\mu} \quad \sum_{i=1}^m \omega_i (\mathbf{X}_i - \boldsymbol{\mu})(\mathbf{X}_i - \boldsymbol{\mu})^T = \boldsymbol{\Sigma} \quad (4)$$

There are several ways to solve this system of equations (in terms of sigma points and weights). In the early stages of development of the sigma point transform, it was proposed to select a set of  $m = 2n + 1$  symmetric sigma points [12]. Subsequently, other researchers developed new solutions that allowed reducing the number of sigma points, reaching a minimum of  $m = n + 1$  [13]; such solution is:

$$\mathbf{X} = \mathbf{M} + \sqrt{m} \boldsymbol{\Sigma} \mathbf{U} \quad \omega_i = \frac{1}{m}, \quad i = 1, \dots, m \quad (5)$$

In the calculation of the square root of the matrix  $m \boldsymbol{\Sigma}$ , it should be kept in mind that their eigenvalues are positive, as the covariance matrices are positive definite. In equation (5),  $\mathbf{M} = [\boldsymbol{\mu} \dots \boldsymbol{\mu}] \in \mathbb{R}^{n \times m}$  and  $\mathbf{U} \in \mathbb{R}^{n \times m}$  fulfil:

$$\mathbf{U} \mathbf{1}_m = \mathbf{0} \quad \mathbf{U} \mathbf{U}^T = \mathbf{I} \quad (6)$$

In equation (6),  $\mathbf{1}_m = [1 \dots 1]^T \in \mathbb{R}^{m \times 1}$  and matrix  $\mathbf{U}$  is recursively obtained:

$$\mathbf{U}_j = \begin{bmatrix} \mathbf{U}_{j-1} & \mathbf{0} \\ \boldsymbol{\alpha} & \boldsymbol{\beta} \end{bmatrix} \in \mathbb{R}^{j \times j} \quad \boldsymbol{\alpha} = [a \dots a] \in \mathbb{R}^{1 \times (j-1)} \quad (7)$$

$$a = \frac{1}{\sqrt{j(j-1)}} \in \mathbb{R} \quad \boldsymbol{\beta} = -j a \in \mathbb{R}^{1 \times 1}$$

In these expressions  $j = 2, \dots, m$  and  $\mathbf{U}_j = \begin{bmatrix} 1 & & \\ & \sqrt{2} & \\ & & \sqrt{2} \end{bmatrix}$ .



Thus, to propagate the random variable with the nonlinear function  $\mathbf{g}$  in equation (8), the sigma points are propagated as deterministic variables, and the result is weighted with the coefficients selected to estimate the mean and the covariance of the output variable. In this way, the path of the first statistical moments of the random variable can be traced through any function  $\mathbf{g}$ :

$$\mathbf{y} = \mathbf{g}(\mathbf{x}) \quad \bar{\mathbf{x}} = \boldsymbol{\mu}_x \quad \text{Var}(\mathbf{x}) = \boldsymbol{\Sigma}_x \quad \bar{\mathbf{y}} = \boldsymbol{\mu}_y \quad \text{Var}(\mathbf{y}) = \boldsymbol{\Sigma}_y \quad (8)$$

$$\mathbf{X} = \mathbf{M}_x + \sqrt{m \boldsymbol{\Sigma}_x} \mathbf{U} \quad \mathbf{Y} = \mathbf{g}(\mathbf{X}) \rightarrow \boldsymbol{\mu}_y \approx \sum_{i=1}^m \omega_i \mathbf{Y}_i \quad \boldsymbol{\Sigma}_y \approx \sum_{i=1}^m \omega_i (\mathbf{Y}_i - \boldsymbol{\mu}_y)(\mathbf{Y}_i - \boldsymbol{\mu}_y)^T \quad (9)$$

Since the sigma point transform provides only the first two statistical moments of the random variable, it is necessary to assume their statistical distribution to fully define it. In solution of inverse problems with sigma points Kalman filters, it is commonly assumed that the random variables are Gaussian; this hypothesis has repeatedly proven reasonable [14].

### 3. Proposed deconvolution algorithm

In this research, probabilistic inverse analyses are used to deconvolve a given seismic signal (accelerogram) recorded at the soil surface; the output is another signal at any arbitrary depth, even reaching the bedrock. Such wave depropagation analyses are unidimensional and use Kalman filters of sigma points; given that the soil behavior is nonlinear, the deconvolution will be also nonlinear [2,5]. The proposed algorithm is implemented in the Python programming language.

The soil is discretized in a multiple degrees-of-freedom systems (MDOF); the nodes correspond to the borders between the different layers. Each node has one DOF of transversal displacement. Fig. 1 represents a sketch of this discretization.

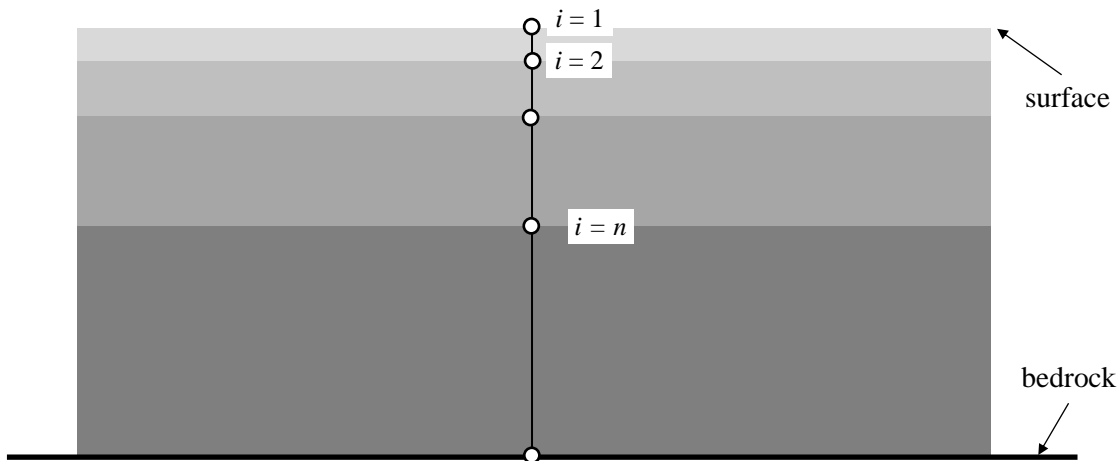


Fig. 1. Soil stratigraphic profile and discretization for unidimensional wave propagation analyses

The discrete model in Fig. 1 incorporates the mechanical properties in the corresponding mass, damping and stiffness matrices. The model base (bedrock) behavior can be either elastic or rigid; for the latter, the wave transmission model in [15] is used.

Two different nonlinear constitutive models are employed: MKZ [3] and GQ/H [2]. The first model considers the typical hyperbolic stiffness degradation curve with respect to shear strain that soils exhibit for cyclic loading; the second model uses a quadratic-hyperbolic approach that allows modeling both shear stiffness and strength of soils for one-dimensional propagation calculations, thus providing more precise results. Both methods use the extended Masing rules, and provide hysteretic damping based on them; however, on irregular cyclic loading, the damping at large strains computed by these rules should be limited in order to obtain realistic values [16]. On the other hand, small-strain damping is modelled as viscous by using a Rayleigh formulation [17] or a frequency-independent damping [16].



The equation of motion is numerically integrated with the Newmark-beta method; by using parameters  $\gamma = 1/2$  and  $\beta = 1/4$ , the algorithm is unconditionally stable.

As a summary, an application written in Python has been developed for obtaining the seismic input at any depth (typically, at the base) of a site profile after an acceleration time series at the surface. This application automates the calculation of the unscented transform, both in its symmetric form [12] and its optimal formulation [13], to use the sigma points Kalman filter for the inverse problem solution. The aforementioned Python code solves the state and observation equations (1), computes the Kalman gain matrix (2) and thus estimates the seismic excitation at the base of the model; the state equation is the identity function  $\mathbf{f}(\mathbf{x}) = \mathbf{x}$  [18]. The model ( $\mathbf{w}_k$ ) and measurement ( $\mathbf{v}_k$ ) noises can be specified by the user.

An analysis procedure based on moving time windows [19] is used to facilitate the convergence of the calculations. This procedure estimates the seismic input at the base of the model for an interval of the seismic excitation defined by the user. Once the results for this interval are obtained, the same problem is solved for another interval of the same duration that starts at a certain time from the previous one. This time delay is called sliding time and can be lesser than or equal to the time window length.

#### 4. Application example

An inverse analysis using the proposed deconvolution algorithm is performed for a sample soil site to highlight the capability of the method to identify the ground motion at the bedrock. The obtained results are compared to those of the well-known one-dimensional shear wave propagation software DEEPSOIL.

The input acceleration is the NS component of the Izmir record of the Kocaeli earthquake (August 17<sup>th</sup> 1999,  $M_w = 7.6$ ) [20]. The base (bedrock) is elastic, the time step is  $\Delta t = 0.005$  s, and the MKZ constitutive model [3] is chosen, along with the frequency-independent damping formulation for small-strain damping. The layering of the sample site is described in Table 1.

Table 1. Soil properties for the application example

Layer	Thickness (m)	$\gamma$ (kN/m <sup>3</sup> )	$v_s$ (m/s)	$D_{\min}$ (%)	$\gamma_{\text{ref}}$	Reference Pressure (kPa)	$\beta$	$s$	P1	P2	P3
1	1.5	22	250	1	$5 \times 10^{-4}$	0.101	0.384	1	1	1	15
2	6	22	500	1	$5 \times 10^{-4}$	0.101	0.384	1	1	1	15
3	8	22	400	1	$5 \times 10^{-4}$	0.101	0.384	1	1	1	15
4	25	22	800	1	$5 \times 10^{-4}$	0.101	0.384	1	1	1	15

In Table 1,  $\gamma$  is the soil unit weight,  $v_s$  represents the shear-wave velocity at each layer,  $D_{\min}$  stands for the small-strain damping ratio,  $\gamma_{\text{ref}}$  and “Reference Pressure” are characteristic values of the soil behavior, coefficients  $\beta$  and  $s$  characterize the hyperbolic equation of the MKZ model by determining the amount of shear stiffness degradation for each shear strain, and P1, P2 and P3 are coefficients that moderate the ever-increasing damping ratio for large strains computed by means of the Masing rules.

The Kocaeli accelerogram is assumed to correspond to the bedrock (Fig. 1), despite that this acceleration time series was actually recorded at the soil surface. Since only the acceleration is available, velocity and displacement are obtained by proper numerical integration. This input ground motion is propagated upwards using both DEEPSOIL and the proposed model; then, the obtained surface accelerogram is deconvolved using the proposed nonlinear algorithm (DEEPSOIL is not considered since it allows for equivalent linear deconvolution only). The accuracy of the proposed procedure is verified by comparing the obtained ground motion at the base with the input Kocaeli accelerogram, and also the motion at each soil layer computed with DEEPSOIL with the proposed algorithm. The mean squared error (MSE) is used to compute the deviation between both approaches; Table 2 displays, for the upward propagation, the MSE between DEEPSOIL and the developed model.



Table 2. Error between DEEPSOIL and the proposed propagation model

DOF	MSE (%)		
	Displacement	Velocity	Acceleration
1 (surface)	0.05	0.22	0.91
2	0.04	0.21	0.83
3	0.13	0.19	0.78
4	0.03	0.14	1.23

Table 2 shows that the proposed propagation model is equivalent to DEEPSOIL for all practical purposes. Fig. 2 presents more detailed comparisons; displacement, velocity and acceleration at surface are plotted (DOF 1, Fig. 1); normalized hysteresis loops obtained with the developed model are displayed.

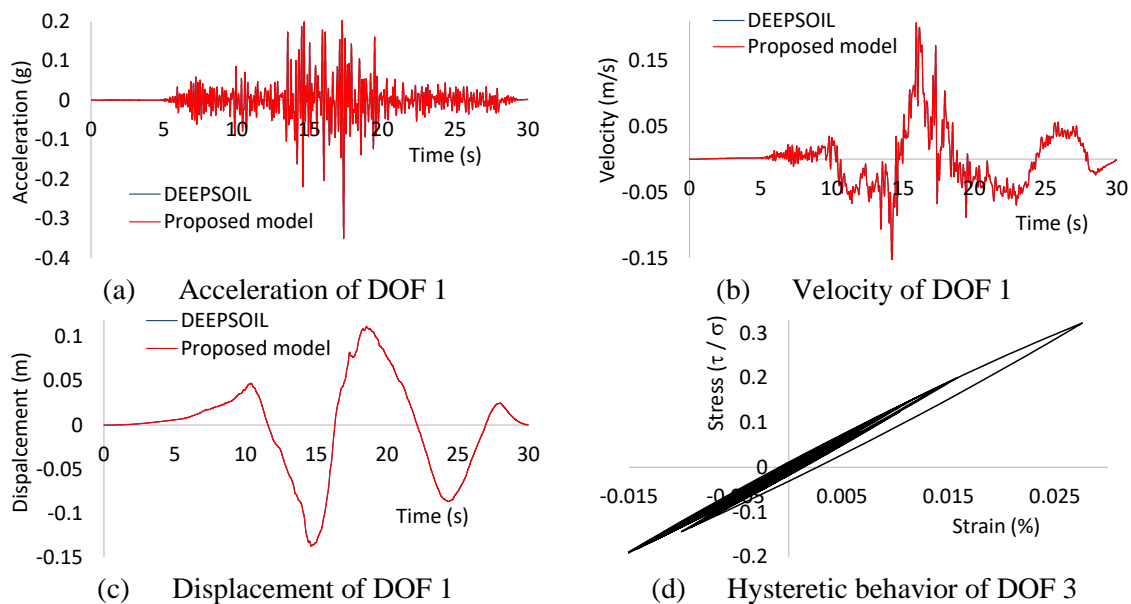


Fig. 2. Comparison of results of propagation results obtained with DEEPSOIL and the proposed model

Fig. 2.a, Fig. 2.b and Fig. 2.c show that the response of DEEPSOIL and the proposed propagation model are practically identical, and Fig. 2.d highlights the nonlinear soil behavior.

Table 3 compares the results from DEEPSOIL and the nonlinear deconvolution algorithm.

Table 3. Error between DEEPSOIL and the proposed nonlinear deconvolution model

DOF	MSE (%)		
	Displacement	Velocity	Acceleration
1 (surface)	0.0	0.06	0.44
2	0.01	0.06	0.38
3	0.08	0.06	0.32
4	0.03	0.16	1.59
Bedrock	0.05	0.46	9.96

Table 3 shows that the comparison between the deconvolved signal and the results produced with DEEPSOIL for DOFs 1 to 4 is satisfactory. Additionally, for the bedrock level, the fit between the input accelerogram and the one computed by nonlinear deconvolution of the ground motion at surface after its propagation is also adequate. Fig. 3 presents a more detailed comparison.

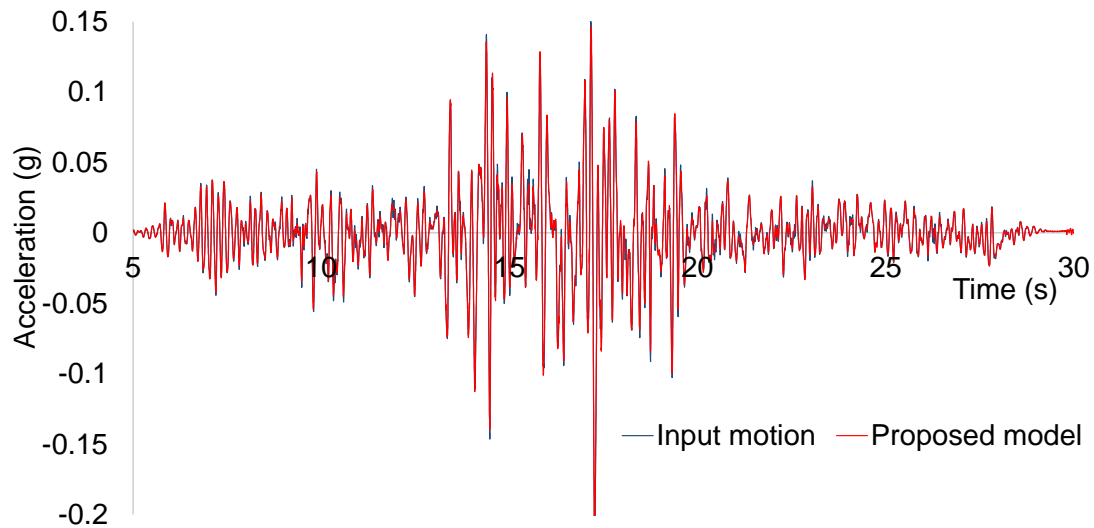


Fig. 3. Comparison between the Kocaeli record and the bedrock results obtained with the proposed nonlinear deconvolution model

Fig. 3 shows that the proposed deconvolution algorithm produces a signal at the base that coincides, for all practical purposes, with the input accelerogram. Hence, the proposed algorithm is accurate and allows for nonlinear deconvolution of a given surface seismic accelerogram.

## 5. Conclusions

In this research, a novel methodology for nonlinear deconvolution of ground motions is presented. This method is based on inverse analysis of surface accelerograms using sigma points Kalman Filters.

This methodology is applied to an application example on a multi-layered soil reaching the bedrock. First, a certain ground motion is propagated nonlinearly upwards through the soil profile, and then the developed nonlinear deconvolution tool is used to recreate the input accelerogram at the base. It is shown that both signals agree closely; this can be understood as the proposed algorithm is sufficiently reliable and accurate.

Future research will account for pore water pressures generated during earthquake excitation. Additionally, the proposed methodology can be extended to identify relevant soil properties.

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