17th World Conference on Earthquake Engineering, 17WCEE
Sendai, Japan - September 13th to 18th 2020

NEW INFINITE ELEMENTS FOR SIMULATION OF SATURATED UNBOUNDED MEDIA

K. Edip(1), V. Sheshov (2), J. Bojadjieva(3), T. Kitanovski (4) J. Chaneva(5) and D.Ivanvoski (6)

(1) *Assc.Prof. Phd, Insitute of Earthquake Engineering and Engineering Seismology, Skopje, N.Macedonia, kemal@iziis.ukim.edu.mk*
(2) *Prof. Phd, Insitute of Earthquake Engineering and Engineering Seismology, Skopje, N.Macedonia, vlatko@iziis.ukim.edu.mk*
(3)*Assist..Prof. Phd, Insitute of Earthquake Engineering and Engineering Seismology, Skopje, N.Macedonia, jule@iziis.ukim.edu.mk*
(4)*Assist.,MSc, Insitute of Earthquake Engineering and Engineering Seismology, Skopje, N.Macedonia, tonik@iziis.ukim.edu.mk*
(5)*Assist.,MSc, Insitute of Earthquake Engineering and Engineering Seismology, Skopje, N.Macedonia, dance@iziis.ukim.edu.mk*
(6)*Assist.,Insitute of Earthquake Engineering and Engineering Seismology, Skopje, N.Macedonia, ivanovski@iziis.ukim.edu.mk*

Abstract

The boundary conditions in propagation of elastic waves in saturated porous media are an important topic in earthquake geotechnical engineering. In numerical simulation of wave propagation, the finite elements are used to model the near field, whereas the infinite elements are used to represent the behaviour of the far field. In simulation of saturated soil media, the wave propagation problem is of great importance due to the fact that the boundary conditions limit the soil medium size and thus involve uncertainties due to reflection of wave from the boundaries. Proposed and developed in this paper are new infinite elements for simulation of saturated soil unbounded media.

The first attempts to numerically treat infinite domains involved applying the finite element method directly by simply truncating the outer region. Although this method worked well for static cases, in dynamic analysis, the results diverged enormously due to the reflected waves on the artificially introduced boundaries Formulation and various implementation aspects of the proposed infinite elements are illustrated.

The accuracy and efficiency of the proposed approach is considered by comparing the obtained results with analytical and other numerical results. For better explanation, a couple of examples were analyzed such as one dimensional wave propagation problems arising from the Heaviside step function, two dimensional propagation in which all types of waves are clearly observed. Finally, a soil layer extending to infinity is simulated by using the proposed infinite elements and results compared accordingly.

The new infinite element was developed using the User Programmable Features of the ANSYS software, which enable creating new elements within the ANSYS core algorithm. The main advantage of the proposed infinite elements is that they can be used directly in finite element concept with minor modifications such as the Jacobian matrix and added absorbing properties. The infinite element development is based on mapping functions and viscous layers for propagating waves of both solid and water phases. Coupled finite and infinite elements simulate the wave propagation in saturated media and seismic response of a soil medium which is a common problem of geotechnical earthquake engineering. In simulation of propagating waves both displacement and pore water pressures are considered. The infinite elements for single phase medium have been developed by Shesov et.al [1] which presents the foundation for this paper.

The newly proposed saturated infinite elements efficiently model the far field of soil model in which the wave reflections are suppressed considerably. The performed analysis using the infinite elements shows very promising results and provide a good tool for simulation of boundary conditions.

Keywords: Infinite elements, numerical analysis, wave propagation

1. Introduction

Seismic soil-structure interaction analysis of massive engineering structures such as dams, power plants, high rise buildings is a very complex issue, which has gained an increasing importance for the last decades. The interaction effects con-siderably influence the seismic performance of these structures. Lessons from re-cent strong earthquakes (Christchurch 2010-2011, Great Tohoku 2011, Sichuan 2008, 2013…) undoubtedly show that soil-foundation-structure interaction and lo-cal site effects can significantly increase total damage if they are neglected or not treated appropriately.The present study is focused on numerical simulation of underlying saturated soil media which are unbounded and extend to infinity. In many engineering applications, the numerical treatment of unbounded domains is of a considerable interest especially when fully saturated conditions are the point of interest.

The first attempts to numerically treat infinite domains involved applying the finite element method directly by simply truncating the outer region. Although this method worked well for static cases, in dynamic analysis, the results diverged enormously due to the reflected waves on the artificially introduced boundaries. Artificial boundaries simulating energy radiation towards infinity were proposed by many researchers. In the work of Kausel [2], a layered half space was considered by introducing viscous stress boundaries. Liao [3] developed a system for non-reflecting boundary conditions. Manolis and Beskos [4] used a boundary element method in time domain enabling usage of nonlinear material models in the finite elements domain.

An infinite element attempts to simulate the behaviour of the unbounded domain. The development of infinite elements dates back to a more recent period. The concept is very similar to that of finite elements including the concept of infinity to the element domain. The use of infinite elements together with the well known finite elements is a promising choice for the investigation of such unbounded domains. An infinite element is an element that represents the behaviour of unbounded domains. One of the first publications introducing mapping infinite elements were those of Zienkiewicz and Bettess [4].

There are mainly two types of infinite elements. The first type uses the decay function together with a shape function, which approaches zero at infinity. In the case of the second one, the geometry is mapped from a finite to an infinite domain. The mapping infinite elements have an advantage in the sense that application of standard Gauss [5] integration formulas is possible. Bettess [6] showed that mapped infinite elements work very well for static analysis of elastic media. The application of infinite elements in wave propagation requires more attention to be paid to outwardly propagating waves. The application of infinite elements is extensive and can be used in many fields of engineering. In the work of Sheshov et al. [1], infinite elements are upgraded by absorbing properties at each node and validation is done accordingly. In this work, the infinite element is further developed such that the pore water pressure is included and the mapping functions are arranged accordingly.

The basic idea of the newly developed infinite element is to consider the assumption that the displacement field approaches zero at infinity, absorbing the outward propagating waves. Application of this type of an infinite element in soil-structure interaction problems is preferable due to the formulation, which is similar to that for finite elements. Thus, the exterior domain is partitioned into a finite number of infinite elements, which are directly connected with the finite element mesh of the interior domain.

2. Soil modeling and infinite elements in saturated media

In simulating boundaries in saturated soil elements, numerous approaches have been proposed in the literature to efficiently treat unbounded spatial domains. In the current contribution, the simulation of wave propagation into infinity is realized in the time domain. The near field is discretized with finite elements, whereas the spatial discretisation of the far field is accomplished using the infinite elements. This makes certain the representation of the far-field stiffness in implementing rigid boundaries surrounding the near field. The numerical results have been calibrated by comparison with respective analytical reference solutions. However, in dynamical applications, some additional considerations must be taken into account. In fact, when body waves approach the interface between the FE and the IE domains, they tend to reflect back to the near field due to the fact that quasi-static infinite elements cannot capture the dynamic wave pattern. To overcome this, the waves are absorbed by adding absorbing properties to the nodes of the infinite elements. The idea of adding viscous damping layer is seen in the work of Lysmer and Kuhlemeyer [7], in which damping forces are introduced to get rid of artificial wave reflections. For more information different applications are given in the works by Haeggblad et al.[8], Edip et al.[9] etc.

In saturated porous media the behaviour of transmitting waves basically depend on the frequency of the excitation, the hydraulic permeability, and the mechanical properties of the constituent materials [10] , [Mesgouez(2009)]. As given in the work of Heider [11] in general, three apparent modes of bulk waves can be observed in biphasic solid–fluid aggregates:

1. The compressional waves which are fast with a motion of the solid and fluid constituents. Compressibility of the constituents govern the propagation of this type of waves.

2. The compressional waves which are slow with motion of solid and fluid. This highly damped type of waves cannot propagate in the domain under low-frequency excitations.

3. The transverse shear waves which are transmitted only in the solid skeleton and are mainly governed by the shear stiffness of the solid phase.

In this work the main focus is given to the geotechnical problems, where commonly low-frequency excitations are present. The permeabilities are low and entail very low relative motions between the solid matrix and the pore fluid. Thus, it is accepted that the pore fluid is almost trapped in the solid matrix at far boundaries. Here, only fully saturated poroelastic media with intrinsically incompressible solid and fluid constituents in the low-frequency regime are considered giving rise to only fast compressional and transverse shear waves.

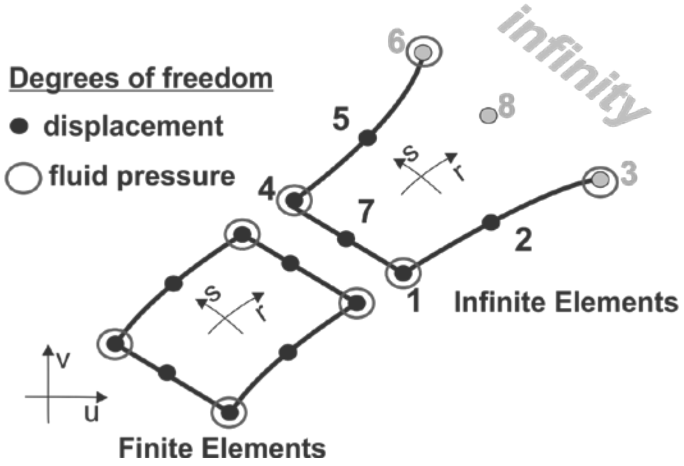

Fig. 1 Coupling of finite and infinite elements

The element displacement in u and v direction is interpolated with the usual shape functions Nu^1, Nu^2, Nu^4, Nu^5 and Nu^7. On the same element the fluid pressure (water or air pressure) is interpolated with the shape functions Np^1 and Np^4.

$$\begin{aligned} u &= [N_u^{\ 1} \quad N_u^{\ 2} \quad 0 \quad N_u^{\ 4} \quad N_u^{\ 5} \quad 0 \quad N_u^{\ 7} \quad 0]\overline{\mathbf{u}} \\ v &= [N_u^{\ 1} \quad N_u^{\ 2} \quad 0 \quad N_u^{\ 4} \quad N_u^{\ 5} \quad 0 \quad N_u^{\ 7} \quad 0]\overline{\mathbf{v}} \end{aligned} \tag{1}$$

$$
\begin{aligned}
p_w &= [N_p^{\,1} \quad 0 \quad 0 \quad N_p^{\,4} \quad 0 \quad 0 \quad 0 \quad 0]\overline{\mathbf{p}}_w \\
p_a &= [N_p^{\,1} \quad 0 \quad 0 \quad N_p^{\,4} \quad 0 \quad 0 \quad 0 \quad 0]\overline{\mathbf{p}}_a
\end{aligned}
$$

In expression (1), $\overline{\mathbf{u}}$ $\overline{\mathbf{v}}$ $\overline{\mathbf{p}}_w$ and $\overline{\mathbf{p}}_a$ vectors of nodal point displacements, water and air pressures in global coordinates. The equation (1) implies that displacements and fluid pressures are set to zero at infinity. The shape functions for displacement and fluid pressures are given in expression (2) as:

$$
\begin{aligned}
N_u^{\,1} &= -(r-1)(-1+s)(s+1+r)/4 \\
N_u^{\,2} &= (r-1)(1+r)(-1+s)/2 \\
N_u^{\,4} &= -(r-1)(1+s)(s-1-r)/4 \\
N_u^{\,5} &= -(r-1)(1+r)(1+s)/2 \\
N_u^{\,7} &= (-1+s)(1+s)(r-1)/2 \\
N_p^{\,1} &= (s-1)(r-1)/4 \\
N_p^{\,4} &= -(s+1)(r-1)/4
\end{aligned} \tag{2}
$$

Based on the isoparametric concept, the infinite element in global coordinate is interpolated onto an element in local coordinate system using the expressions (3) and (4). In the formulation of the infinite element, only the positive r direction extends to infinity. Following Fig. 1 the mapping functions for coordinate interpolation considering displacement degrees of freedom are defined as follows:

$$
\begin{aligned}
r &= [M_u^{\,1} \quad M_u^{\,2} \quad 0 \quad M_u^{\,4} \quad M_u^{\,5} \quad 0 \quad M_u^{\,7} \quad 0]\overline{\mathbf{r}} \\
s &= [M_u^{\,1} \quad M_u^{\,2} \quad 0 \quad M_u^{\,4} \quad M_u^{\,5} \quad 0 \quad M_u^{\,7} \quad 0]\overline{\mathbf{s}}
\end{aligned} \tag{3}
$$

The mapping functions for coordinate interpolation considering fluid pressures as degrees of freedom are defined as follows:

$$
\begin{aligned}
r &= [M_p^{\,1} \quad 0 \quad 0 \quad M_p^{\,4} \quad 0 \quad 0 \quad 0 \quad 0]\overline{\mathbf{r}} \\
s &= [M_p^{\,1} \quad 0 \quad 0 \quad M_p^{\,4} \quad 0 \quad 0 \quad 0 \quad 0]\overline{\mathbf{s}}
\end{aligned} \tag{4}
$$

where

$$
\begin{aligned}
M_u^{\,1} &= -\frac{(1-s)rs}{1-r} \\
M_u^{\,2} &= -\frac{(1-s)(1+r)}{2(1-r)} \\
M_u^{\,4} &= -\frac{(1+s)rs}{1-r} \\
M_u^{\,5} &= -\frac{(1+s)(1+r)}{2(1-r)} \\
M_u^{\,7} &= -\frac{2r(1+s)(1-s)}{(1-r)} \\
M_p^{\,1} &= \frac{1-s}{1-r} \\
M_p^{\,4} &= \frac{1+s}{1-r}
\end{aligned}
\tag{5}
$$

In expression (3) and (4), $\overline{\mathbf{r}}$ and $\overline{\mathbf{s}}$ are vectors of nodal point displacements in local coordinates where it is to be mentioned that, on the side of infinity (r=1), no mappings have been assigned to the nodes as it is taken that displacement and fluid pressure in infinity tends to zero. The number and location of the nodes connecting finite and infinite elements must coincide to guarantee the continuity condition between the elements. The main advantage of the proposed infinite elements is that the number of nodes for displacement on the infinite element allow coupling with finite elements with eight nodes which are used for displacement sensitive problems. The difference of fluid pressure and displacement node numbers is in full agreement with the Babushka Brezzi conditions as given in the work of Pastor et al.[12]. Construction of element matrices is done by using the usual procedures as described above for the infinite elements. The developed infinite element has the advantage concerning the fact that is due to the correct assessment of the boundary conditions. In case of finite elements with truncation approach, the fluid pressure flux through the boundary is free, while in the case of F.E. plus I.E., the fluid pressure flux at the interface between the two element types is governed by the approximated flow at the boundary. The pressure field decays in space even more rapidly than the displacement field. Unfortunately, pore pressures are very sensitive to the boundary conditions introduced when the truncation approach is used and to the polynomial approximation when infinite elements are used. Hence, 1/r decay function is used in the newly programmed infinite elements. The matrix corresponding to the mapped infinite element is very similar to the one used for the standard finite elements.

Construction of element matrices is done by using the usual procedures as described in Bathe [13] . The new coordinate interpolation functions are taken into consideration in the Jacobian matrix as described in Bettess [14]. For the absorbing layer of the infinite element, the Lysmer-Kuhlmeyer approach [7] is used. In all cases, a plane strain two dimensional case is studied. For impact of plane waves on element sides, normal and tangential stresses are derived as follows:

$$
\begin{bmatrix} \sigma^n \\ \tau \end{bmatrix} = \begin{bmatrix} a\rho c^p & 0 \\ 0 & b\rho c^s \end{bmatrix} \begin{bmatrix} \dot{\mathbf{u}}^{\mathbf{n}} \\ \dot{\mathbf{u}}^{\mathbf{t}} \end{bmatrix} \tag{6}
$$

where c^p and c^s indicate the wave velocities for the P wave (compressional) and S wave (shear) respectively. The term ρ stands for density of soil medium. In order to take into account the directions of the incident waves coefficients a and b are used as multipliers. Transformation from local to global coordinates is done by software ANSYS [15] such that there is no need of defining transformation matri-ces. Time derivatives are approximated by the Newmark's method. The program-ming part of the infinite element has been performed using the Programmable Features of the ANSYS software.

3. Verification of infinite elements in satúrated soil media

In order to verify the infinite elements in saturated soil media a fully saturated soil domain is shown in figure 2 below.

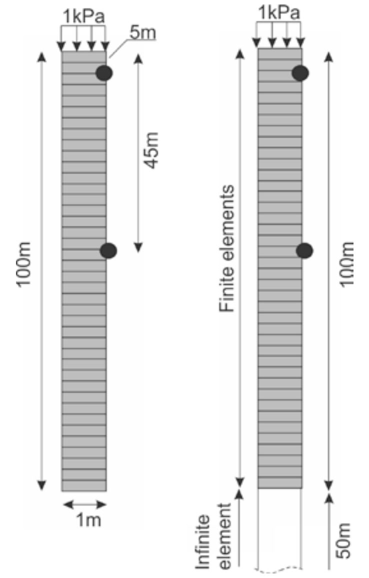

Fig. 2 Fully saturated soil domain discretized by finite and infinite elements

As can be seen in the Fig.2 the vertical soil column is modelled in two ways considering the discretization without and with infinite elements. The soil properties are given in the table below.

Table 1 – Geometric and Mechanical Properties of Bridge Systems

Young’s modulus of elasticity	λ=0.8333kPa
Poisson’s ratio	μ = 0.25kPa
Solid grain density	ρ_s = 0.31 kg/m^3
Bulk modulus of solid grains	$K_s = \infty$
Bulk modulus of water	K_f = 40 kPa
Water density	ρ_f = 0.2977kg/m^3
Initial porosity	n = 0.33
Intrinsic permeability	k = 4.883 x 10^{-3} m^4/(N.s)

In order to show the applicability of infinite elements in saturated soil model, the infinite elements have been placed at the bottom and two points of interest at depths of 5m and 50m have been selected for comparison of results. At the top of the soil layer, a fixed pressure of 1kPa is applied as a transient load.

In Fig. 3 and Fig.4 the time histories of displacements and water pressure at the depths of 5m and 45m for the transient wave propagation problem are presented. It can be observed that there is a good comparison between the analytical solution and the numerical one obtained by using coupled finite-infinite elements.

However, when the fixed boundary is used at the bottom of the finite elements the accuracy of the numerical results become significantly worse because spurious reflections take place at the artificial boundary and the reflected waves propagate back to the near field system. The displacement is considerably underestimated in the case using the artificially fixed boundary. This fact indicates that if the artificially fixed boundaries are used in the analysis the near field of the system should be made large enough to avoid reflections on the artificially truncated boundary within the duration of analysis. Otherwise the numerical results will be affected by the reflected wave. Although a small numerical oscilation in the pore fluid pressure exists in the case of using the absorbing boundaries it decreases quickly as time goes on. Thus it is concluded that the use of the proposed absorbing boundary is an effective and efficient way of modelling the far field of the system for the transient wave problem. As can be seen from the figures in the cases where the infinite elements are used the results show good correlation with the analytical results for both vertical displacement and water pressures. This verifies the correctness of the proposed infinite absorbing infinite elements.

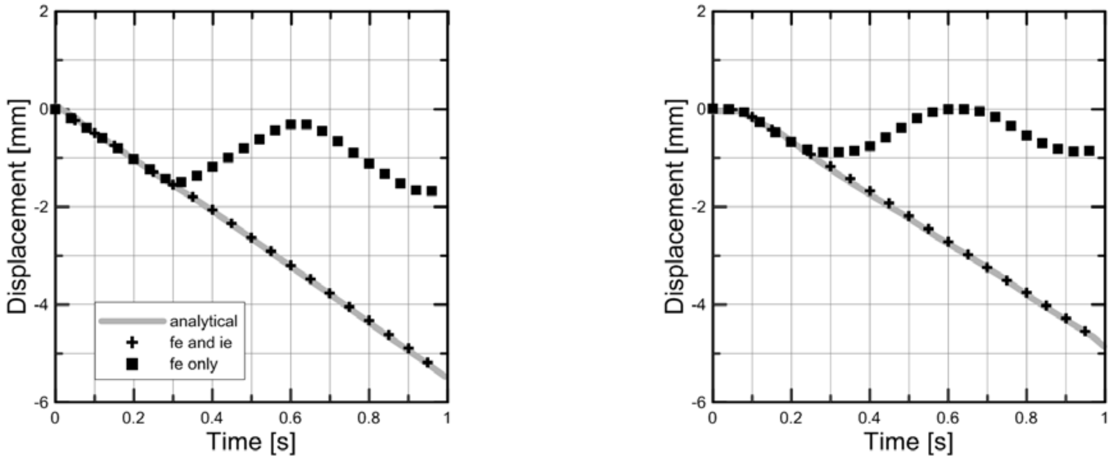

Fig. 3 Time histories of displacement at 5m and 50m depth

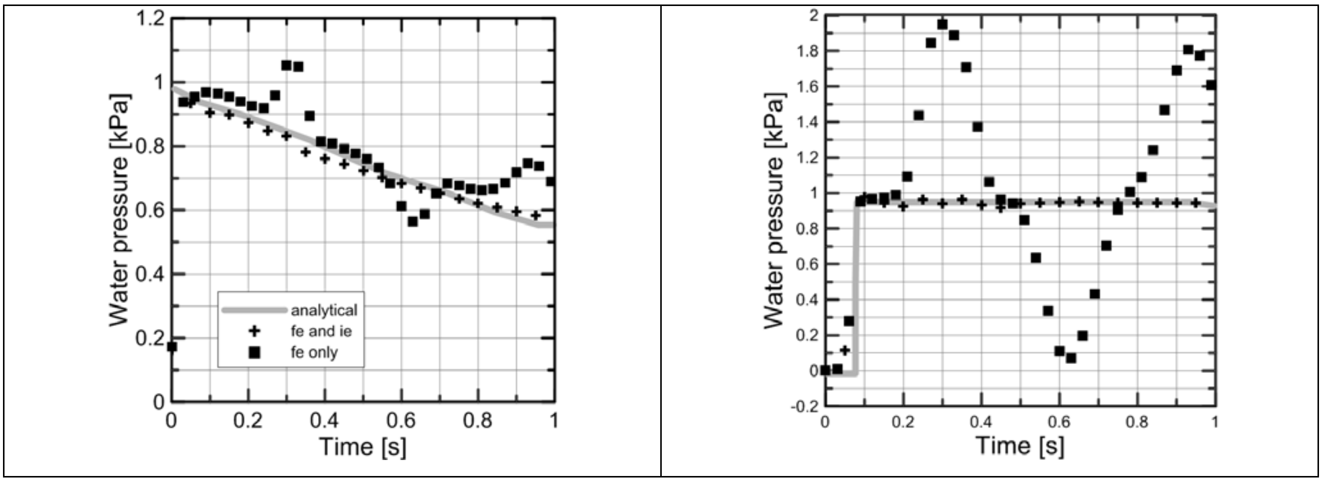

Fig. 4 Time histories of pore water pressure at 5m and 50m depth

4. Conclusions

In this work, a coupled computational method of finite and infinite elements is presented. For the numerical simulation of soil extending to infinity problems, the local region of interest has been modeled by finite elements, which enable simulation of more complex geometries. On the other hand, the surrounding field of the domain has been considered by using infinite elements. By using the coupled finite-infinite elements approach, the number of elements and nodes has been reduced without affecting the accuracy of the results in the near field. The infinite elements with added absorbing characteristics which have been proposed in this study, provide a very general and easy to implement frame of infinite elements. Furthermore the mapping functions and number of nodes proposed in this study increase the accuracy since eight nodded finite elements can be coupled with boundaries. Although the limitation of the proposed approach is that the infinite elements do not represent the real solution at the far field, the finite elements in the coupled approach give an acceptable accuracy for soil extending to infinity problems.

References

1. Sesov, V., et al., *Development of New Infinite Element for Numerical Simulation of Wave Propagation in Soil Media*, in *Experimental Research in Earthquake Engineering*. 2015, Springer. p. 423-436.
2. Kausel Bolt, E.A.M., *Forced vibrations of circular foundations on layered media*. 1974, Massachusetts Institute of Technology.
3. Liao, Z.-P., *Extrapolation non-reflecting boundary conditions.* Wave Motion, 1996. **24**(2): p. 117-138.
4. Zienkiewicz, O. and P. Bettess, *Infinite elements in the study of fluid-structure interaction ppoelems*, in *Computing Methods in Applied Sciences*. 1976, Springer. p. 133-172.
5. Zienkiewicz, O.C., C. Emson, and P. Bettess, *A novel boundary infinite element.* INTERNATIONAL JOURNAL FOR NUMERICAL METHODS IN ENGINEERING, 1983. **19**(3): p. 393-404.
6. Bettess, P., *Infinite elements.* INTERNATIONAL JOURNAL FOR NUMERICAL METHODS IN ENGINEERING, 1977. **11**(1): p. 53-64.
7. Lysmer, J. and R.L. Kuhlmeyer, *Finite Dynamic Model for Infinite Media.* J.Engng Mech. Div., ASCE, 1969. **95,**: p. 859-877.
8. Häggblad, B. and G. Nordgren, *Modelling nonlinear soil-structure interaction using interface elements, elastic-plastic soil elements and absorbing infinite elements.* Computers & Structures, 1987. **26**(1–2): p. 307-324.
9. Edip, K., *Development of three phase model with finite and infinite elements for dynamic analysis of soil media*. 2013, Ss. Cyril and Methodius: Institute of Earthquake Engineering and Engineering Seismology.
10. Mesgouez, A. and G. Lefeuve-Mesgouez, *Study of transient poroviscoelastic soil motions by semi-analytical and numerical approaches.* Soil Dynamics and Earthquake Engineering, 2009. **29**(2): p. 245-248.
11. Heider, Y., *Saturated porous media dynamics with application to earthquake engineering*. 2012.
12. Li, X., X. Han, and M. Pastor, *An iterative stabilized fractional step algorithm for finite element analysis in saturated soil dynamics.* Computer Methods in Applied Mechanics and Engineering, 2003. **192**(35–36): p. 3845-3859.
13. K.J.Bathe, *Finite Element Procedures in Engineering Analysis*. 1982, Englewood Cliffs. NJ, : Prentice-Hall.
14. Bettess, P., *Infinite Elements*. 1992, New Castle,England: Penshaw Press.
15. ANSYS. *Fem Software*. 2006.