



## CALCULATION OF CRITICAL BUCKLING LOAD OF PILE UNDER LIQUEFIED SOIL CONDITIONS

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### **Abstract**

Buckling instability has been cited as a possible mechanism of pile failure in liquefiable soils recently, and this failure mechanism is currently missing in all design codes. Therefore, it is necessary to accurately calculate the critical buckling load of pile to avoid the buckling instability of pile in liquefiable soils. However, the existing calculation methods for the critical buckling load of pile in liquefied soils do not consider the influence of geometric imperfections and nonlinear behavior of pile. In this paper, an efficient approach using the Beam on Nonlinear Winkler Foundation (BNWF) model, is proposed to calculate the critical buckling load of pile in liquefied soils. Both geometric imperfections and nonlinear behavior of pile have been considering in this method, and the results of finite element method and centrifuge tests were used to verify the validity of the method.

*Keywords: Liquefaction, Pile foundation, Buckling instability, Critical Buckling Load, BNWF model*



## 1. Introduction

Extensive damage to pile foundations in liquefiable soils has been observed in many strong earthquakes, such as the collapse of the pile-supported Showa Bridge during the 1964 Niigata earthquake [1, 2]. In order to reduce liquefaction damage to pile foundations, many attempts have been made to explore the seismic response of pile foundations in liquefiable soils, and various design guidelines have been formulated, such as JRA (2017), Eurocode 8: part 5 (1998), and USA code [3-6].

Recently, buckling instability has been postulated as a possible mechanism of pile failure in liquefiable soils [7-10]. When the soil around the pile loses much of its stiffness and strength owing to liquefaction, the pile will become an unsupported long slender column and could buckle under the high axial load from the superstructure. The buckling failure mechanism has already been confirmed by centrifuge tests, case histories, and analytical works [3, 8, 11, 12]. However, consideration of this failure mechanism is currently missing in all design codes [2]. Therefore, to improve the design code of the pile, it is essential to accurately calculate the critical buckling load of pile, which could avoid the buckling instability of pile in liquefiable soils.

So far, only a few researchers have devoted efforts to studying the calculation of critical buckling load of pile in liquefied soils. Shanker et al. [13] proposed an analytic method to predict the critical buckling load of pile based on an extension of the Mindlin solution, and the pile and the surrounding soil are assumed to be elastic. Nadeem et al. [14] investigated the buckling response of straight and initially bent-end-bearing piles without considering soil liquefaction through the finite element method. Knappett and Madabhushi [15] studied the buckling behavior of pile groups using the Riks post-buckling analysis method, which is implemented in ABAQUS, but did not significantly consider the influence of geometric imperfections of pile. Moreover, some studies, such as Bhattacharya [16], Haldar et al. [17], and Haldar and Babu [18], treat the pile in liquefied zone as laterally unsupported slender columns and use the Euler's buckling formula to calculate the critical buckling load of pile, without considering the influence of the residual shear strength of liquefied soil and the nonlinear behavior of pile. To the best of the authors' knowledge, no studies have been reported yet on the calculation of critical buckling load of pile in liquefied soils consider the influence of geometric imperfections and pile nonlinear behavior simultaneously. However, these factors have a significant influence on the critical buckling load of pile [13-15]. Therefore, it is necessary to propose a more accurate calculation method of the critical buckling load of pile in liquefied soils.

In this study, a method for calculating the critical buckling load of pile in liquefied soil is proposed based on the BNWF model. This method takes into consideration the material and geometric nonlinearities of pile, soil liquefaction, and geometric imperfections of pile. The method is then verified by the results of finite element method and centrifuge tests.

## 2. Calculation of critical buckling load using BNWF method

A method for calculating the critical buckling load of pile in liquefied soil is proposed in this section. This method is based on the BNWF model (Fig. 1a). All the numerical simulations were performed using the Open System for Earthquake Engineering Simulation, OpenSees (<http://opensees.berkeley.edu> [19]).

### 2.1 $p$ - $y$ modeling of liquefied soil

As observed by Bhattacharya et al. [9] and Knappett and Madabhushi [15], buckling failure of the end-bearing pile normally occurs when the soil is fully liquefied. Therefore, when predicting the critical buckling load of pile in liquefiable soils, only the soil that has fully liquefied needs to be considered as pile buckling in partially liquefied soil would require a higher axial load.

In this BNWF model, the interaction between the soil and the pile was modeled using discrete nonlinear springs represented by  $p$ - $y$  curves for lateral loading (Figs. 1a and 1b), where  $p$  refers to the lateral soil



pressure per unit length of the pile, and  $y$  refers to the corresponding relative soil-pile horizontal displacement [20]. The  $p$ - $y$  curves (Fig. 1b) proposed by Dash et al. [21] is adopted in the present study. This  $p$ - $y$  curves retain the essential features of liquefied soil as observed in both element and physical model tests, and can be easily constructed from a typical field bore log data [21]. The  $p$ - $y$  curve is given by Eq. (1).

$$p = \omega \frac{p_1}{y_1} y + A(1-\omega) \left[ \frac{p_u + p_1}{2} + \frac{p_u - p_1}{2} \tanh \frac{2\pi}{3(y_u - y_1)} \left( y - \frac{y_u + y_1}{2} \right) \right] \quad (1)$$

where  $p_u$  is ultimate lateral resistance,  $y_u$  is ultimate lateral displacement,  $p_1$  is initial lateral resistance,  $y_1$  is initial lateral displacement,  $A=0$  for  $y=0$ ,  $A=1$  for  $y \neq 0$ , and  $\omega$  is a weight function can be calculated from Eq. (2).

$$\omega = \frac{1}{2} \left\{ 1 - \tanh \left[ \frac{6\pi}{y_u} \left( y - \frac{4y_1 + y_u}{6} \right) \right] \right\} \quad (2)$$

The specific methods for calculating  $p_u$ ,  $y_u$ ,  $p_1$ , and  $y_1$ , from the field bore log data can be referred to Dash et al. [21].

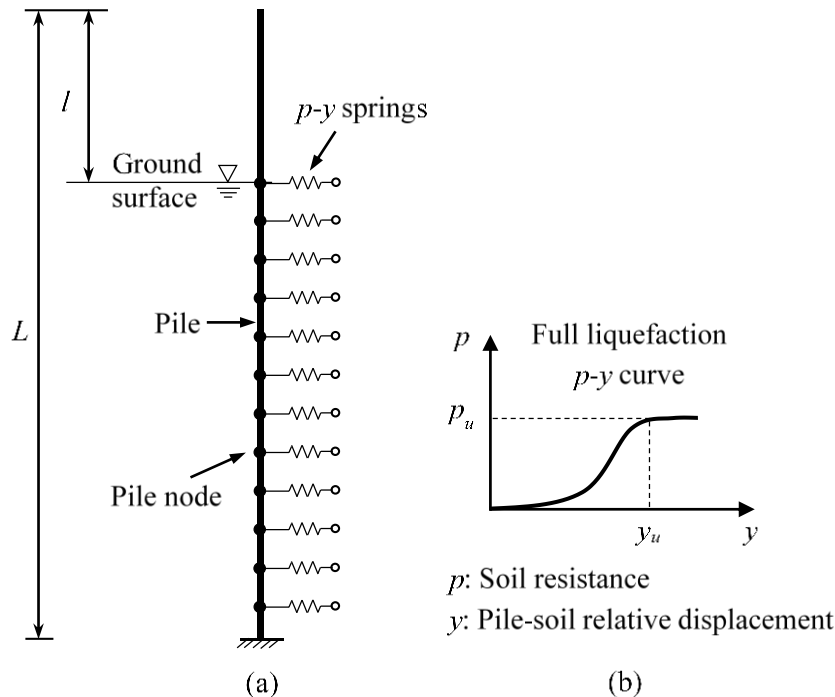


Fig. 1 – Numerical model: (a) Schematic of BNWF model; and (b) full liquefaction  $p$ - $y$  curves (after Dash et al. [21])

## 2.2 Nonlinear modeling of pile

The pile was modeled as displacement-based nonlinear beam-column elements [19]. The pile nodes are created with three dimensions and six degrees of freedom (three translational and three rotational). A moment-curvature relationship can be defined for an inelastic pile section in simulating its nonlinear material response [22]. A ‘‘Corotational transformation’’ was adopted to considering geometric nonlinearity of pile [22, 23], which has been confirmed can be used effectively in detecting the buckling load as reported by Denavit and Hajjar [24].

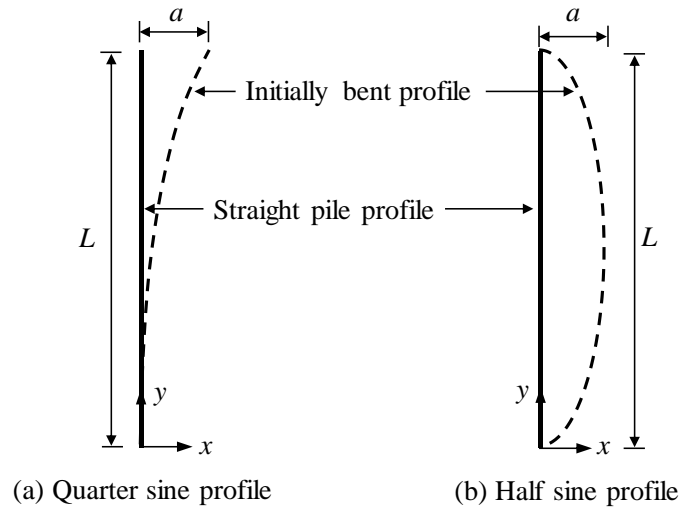


Fig. 2 – Two typical initially bent pile profiles: (a) Quarter sine profile; and (b) Half sine profile (after Nadeem et al. [14])

### 2.3 Geometric imperfections of pile

Real piles are not perfectly straight and usually have some geometric imperfections that may affect their buckling behavior [14]. Therefore, it is necessary to consider the geometric imperfections when calculating the critical buckling load of piles. As reported by Nadeem et al. [14], there are two typical initially bent profiles could be used considering the geometric imperfections, i.e., quarter sine profile, as represented by Eq. (3) (Fig. 2a), and half sine profile, as in Eq. (4) (Fig. 2b). The initial imperfection amplitude ( $a$ ) was defined to describe the imperfection magnitude, as shown in Fig. 2. In the numerical modeling, these two initially bent profiles could be described by defining the node coordinates of pile, and the x-coordinate can be calculated from Eqs. (3) and (4) as:

$$x = a \sin \frac{\pi y}{2L} \quad (3)$$

$$x = a \sin \frac{\pi y}{L} \quad (4)$$

where  $a$  is the initial imperfection amplitude,  $L$  is the length of pile (Fig. 2).

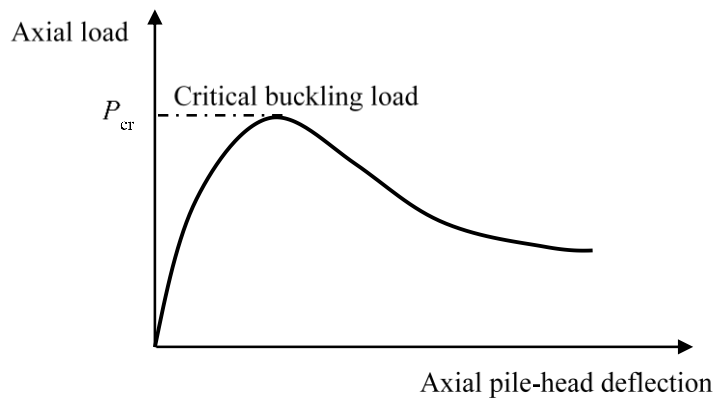


Fig. 3 – Typical load-deflection curve for buckling analysis (after Nadeem et al. [14])



## 2.4 Prediction of critical buckling load

A BNWF model considering the material and geometric nonlinearities of pile, soil liquefaction, and geometric imperfections of pile can be established using the above method. Based on this model, nonlinear buckling analysis was performed by applying a displacement-controlled axial compression on the pile head [25]. Then, a complete axial-load versus axial pile-head deflection curve (i.e., the well-known load-deflection curve [14, 15]) can be obtained, and the typical load-deflection curve is shown in Fig. 3. The peak axial load in the load-deflection curve could be regarded as the critical buckling load ( $P_{cr}$ ) of the pile (Fig. 3) [14, 15].

## 3. Validation of the proposed prediction method

The proposed prediction method is subsequently assessed for its accuracy and reliability via the consideration of two case studies. Firstly, buckling analysis of initially bent columns, comparing the results with the finite element simulation results reported in Nadeem et al. [14]. Secondly, buckling analysis of piles in liquefied soils, comparing the present predicting results with the centrifuge test results given by Bhattacharya [8].

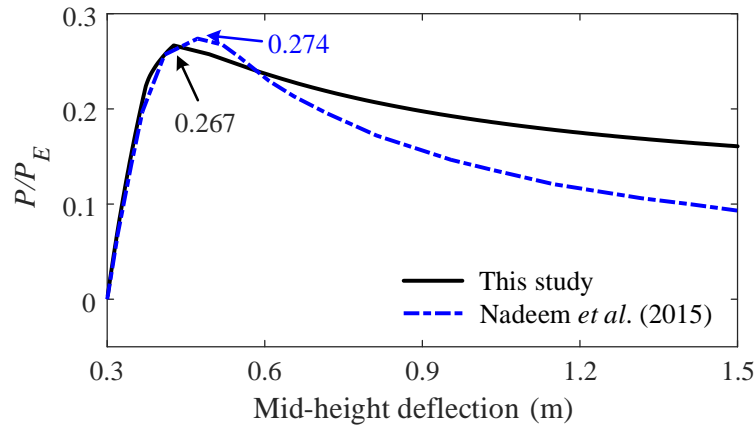


Fig. 4 – Comparison of the load-deflection curve from this study with finite element analysis of Nadeem et al. [14]

### 3.1 Compare with the finite element model

In the first validation analysis, the buckling analysis of initially bent columns which is 2 m in diameter and 40 m in length was carried out. An imperfection in the form of a half sine profile bent (Fig. 2b) along the centroidal axis has been incorporated in the column, the initial imperfection amplitude ( $a$ ) is taken as 0.3 m. The column was simulated as a linear-elastic perfectly plastic material, and the material elasticity modulus  $E=160$  Gpa and the yield strength  $\sigma_y=200$  MPa. The load-deflection curve obtained by the method of this study and Nadeem et al. [14] is shown in Fig. 4. The  $P_E$  in Fig. 4 is the Euler buckling load of the column could be calculated using Eq. (5).

$$P_E = \pi^2 \frac{EI}{L_{eff}^2} \quad (5)$$

where  $EI$  is the flexural rigidity of column, and  $L_{eff}$  is the effective length of the column. Then, the critical buckling load predicted by this study is  $0.267P_E$ . It is almost the same as the finite element simulation result in Nadeem et al. [14] which is  $0.274P_E$  (Fig. 4).



Table 1 – Pile properties (Bhattacharya [8])

Parameters	Pile 8	Pile 10
Test ID	SB-04	SB-06
Axial load, $P$ (kN)	2180	1838
Pile length (m)	11.13	11.13
Pile outside diameter (mm)	465	465
Pile inside diameter (mm)	425	425
Young's modulus, $E$ (MPa)	$7.0 \times 10^4$	$7.0 \times 10^4$

### 3.2 Compare with the centrifuge tests

Bhattacharya [8] performed dynamic centrifuge tests to investigate pile buckling instability in liquefiable soils. It was found that Pile 8 in Test SB-04 and Pile 10 in Test SB-06 failed in buckling mode during shaking. The results of these two damaged piles were used to verify the proposed method. All values described below are presented in the prototype scale.

Both Pile 8 and Pile 10 are pipe pile, and the piles were fixed at the base of the soil box. The pile top was applied with a superstructure load of 2180 kN and 1838 kN for Pile 8 and Pile 10, respectively. A specially designed frame was used to restrain the head mass against inertial action, in other words, only the effect of the axial load was considered in these two piles. The pile properties are given in Table 1. The soil model in the centrifuge tests consisted of one layer of saturated Fraction E silica sand. The relative density of sand were 43% and 40% for Test SB-04 and Test SB-06, respectively. The bulk unit weight of the fully saturated soil was approximately  $18.68 \text{ kN/m}^3$ .

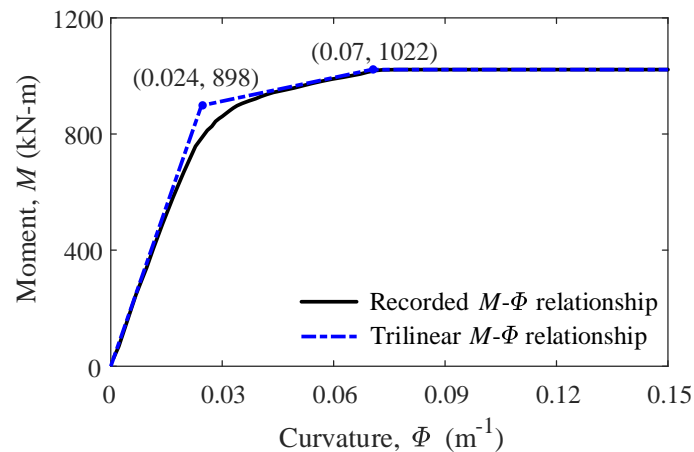


Fig. 5 – Nonlinear moment-curvature relationship of the pile (after Bhattacharya [8])

For the second validation analysis, two BNWF models were established using the above method to calculate the critical buckling load of Pile 8 and Pile 10. A trilinear relationship was adopted to characterize the recorded moment-curvature relationships for the pile, as shown in Fig. 5. The pile bottom was fully fixed to simulate the boundary condition of the end-bearing pile. Moreover, as the pile is under the combined action of 1 g gravity acceleration and 50 g centrifugal acceleration during the test, and these two accelerations are perpendicular to each other. The pile may be associated with geometric imperfections in the type of quarter sine profile (Fig. 2a). The initial imperfection amplitude  $a$  is taken as 0.5%-, 1%-, and 2%-



times the length of pile ( $L$ ), owing to there is no accurate test data on the initial imperfection amplitude. The calculated critical buckling loads based on the proposed method are shown in Table 2. If the axial load of the pile is higher than the critical buckling load, the pile will fail in buckling instability. Therefore, according to the method proposed in this paper, Pile 8 and Pile 10 will fail in buckling instability, which is consistent with the centrifuge test results (Table 2).

Table 2 – Compare with the centrifuge test results by Bhattacharya [8]

Pile No.	Axial load, $P$ (Experimental)	Critical buckling load, $P_{cr}$ (Computed)			Predicted by the proposed method	Observed in the tests
		$a/l=0.5\%$	$a/l=1.0\%$	$a/l=2.0\%$		
Pile 8	2180 kN	1446 kN	1109 kN	949 kN	Failure ( $P > P_{cr}$ )	Failure
Pile 10	1838 kN	1417 kN	1086 kN	930 kN	Failure ( $P > P_{cr}$ )	Failure

Since the calculation results of the proposed method are in good agreement with the finite element results and the centrifuge test results, it can be concluded that the proposed method can be reliably used to calculate the critical buckling load of pile in liquefied soils.

#### 4. Conclusions

This study provides better insight into the critical buckling load of pile in liquefied soils. A calculation method of the critical buckling load of pile considering the material and geometric nonlinearities of pile, soil liquefaction, and geometric imperfections of pile is proposed based on the BNWF model. The validity of this method is confirmed by the results of centrifuge test and finite element method.

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