



## ESTIMATION OF STRAIN-DEPENDENT SHEAR MODULI OF SOILS FROM SITE RESPONSE ANALYSIS OF VERTICAL ARRAY

Z. Huang<sup>(1)</sup> & R.P. Orense<sup>(2)</sup>

<sup>(1)</sup> Graduate Geotechnical Engineer, Cook Costello Ltd., New Zealand, [zhua466@aucklanduni.ac.nz](mailto:zhua466@aucklanduni.ac.nz)

<sup>(2)</sup> Associate Professor, University of Auckland, New Zealand, [r.orense@auckland.ac.nz](mailto:r.orense@auckland.ac.nz)

### **Abstract**

Traditionally, laboratory experiments and field geophysical measurements are conducted to determine the dynamic deformation properties of soils. These properties may, however, not well-represent the actual nonlinear behavior of soils since laboratory experiments, as well as in-situ tests, cannot apply the shear stress that is experienced by the soils during earthquakes to soil specimens or the field layers. To make matter worse, soil disturbance during sampling, transport and testing processes often leads to inconsistency among the test results. In order to overcome the shortcomings of the laboratory and in-situ tests, dozens of researchers have developed various techniques to investigate the soils' nonlinear behavior and extract their dynamic parameters, including shear wave velocities, shear moduli and damping ratios, by analyzing recordings measured at vertical array sites. While this approach is promising, due to the fact that the target soil remains undisturbed and is subjected to actual stress and strain fields, most of the studies conducted to date, however, have focused on investigating the Fourier spectra or cross-correlations of time histories to calculate the shear wave velocities or other dynamic properties.

In this paper, a methodology to detect the average shear wave velocity of the 1D soil layer between a pair of vertical array sensors has been analogically conceptualized and satisfactorily confirmed by analyzing and comparing the spatial distributions of shear stresses and shear strains simulated by concurrently executing forward and backward analyses with the recorded earthquake time series. The methodology is validated with idealized models constrained by synthesized motions, as well as with measured field measurements at two vertical array sites (Treasure Island and La Cienega geotech sites, both in California, U.S.A.). It is noted that the shear wave velocity profile numerically extracted by implementing the developed methodology is consistent with those from downhole shooting test but differs from those by suspension PS logging measurements. Finally, it is noted that vertical array recordings carry valuable information associated with strain-dependent shear moduli and these properties can be extracted by performing the methodology developed. These numerically acquired  $G/G_{max}$  values have been compared to those from laboratory tests and empirical data and reasonably validated by conducting site response analyses with these properties acting as input parameters.

*Keywords:* site response analysis; vertical array; shear wave velocity; strain-dependent shear modulus



## 1. Introduction

A good understanding of sites' responses to earthquake shaking in seismically active countries is very important. Thanks to well-developed theory in seismology and geophysics and rapidly-advancing computational technology, dozens of site response analysis methods have been established and put into practice since the 1970s. In essence, these methods have utilized mathematical tools to answer different forms of wave equations in time and/or frequency domains with proper geometric and dynamics properties, which are assigned into the digitized soil models. Thus, finding the right solution to wave equations and extracting representative dynamic properties of soils are of interest to geotechnical earthquake engineers. In theory, the answer to the wave equation should be unique. However, it has been observed that the analysis outcomes from different site response analysis methods differ from each other to a certain degree. For example, the simulation results for Lotung experiment site, Taiwan, from the equivalent linear analysis code EERI and the finite element code SWANDYNEII have presented remarkable variations in forms of displacement time series and spectral responses [1].

Traditionally, laboratory experiments and field geophysical measurements have been commissioned to determine the dynamic deformation properties of soils. These properties, however, may not well-represent the actual nonlinear behavior of soils since the laboratory schemes and conventional in-situ tests cannot apply the shear stresses that are experienced by the in-situ soils during earthquakes to the soil specimens or to the field layers. To make matter worse, soil disturbance during the sampling, transport and testing processes often leads to inconsistent results. In order to overcome these shortcomings, dozens of researchers [2-4] have developed various techniques to investigate the soils' nonlinear behavior and to extract their dynamic parameters, including shear wave velocities ( $V_s$ ), shear modulus ( $G$ ) and damping ratio by analyzing recordings measured at vertical array sites. These studies are promising as, during the measurements with vertical arrays, not only is the target soil undisturbed but the stress and strain fields experienced by the layer are authentic. Most of the studies, however, have been focused on investigating the Fourier spectra [5] or cross-correlations of time histories [6] to calculate the shear wave velocities or other dynamic properties.

This paper proposes a method to estimate the strain-dependent shear moduli of soils from site response analysis of vertical array. To achieve this, a scheme to detect  $V_s$  was developed by implementing forward and backward dynamic analyses with synthesized motions and models. The proposed scheme was used to determine the  $V_s$  profiles at two array sites and the extracted normalized shear modulus ( $G/G_{\max}$ ) parameters were compared with laboratory and empirical data, as well as with the data obtained by another numerical simulation scheme.

## 2. Site response analysis methods

Basically, there are two categories of site response analysis methods: time domain analyses, and frequency domain analyses.

Frequency domain methods are simple and concise to be implemented in site response analysis. In the implementation process, a closed form solution to the wave propagation equation, by which the shear waves propagate through a layered continuous medium, can be presented. Due to its simplicity and the exact solution, frequency analysis methods have been and still will be the mainstream approaches in ground motion analysis. Current efforts when dealing with seismic input motions and estimating the ground motions on the free surface or at depth consist of mainly three steps. (1) break down the input motion from time domain into their frequency components by employing Fourier Transformation; (2) transfer (or amplify) the frequency amplitude spectra of the input motion to those at a depth of interest; and (3) reconstruct the output motion in time domain by summing individual harmonic sinusoidal waves with the help of the Inverse Fourier Transformation technique. It is evident that one of the key elements in frequency-based seismic simulation is the method to transfer or modify the input motions' spectra into what is expected. Most of the frequency analysis methods establish their transfer function during analysis based on the wave multiple reflection theory with an initial set of linear soil properties to calculate the soil's stress and strain



characteristics whereby a new set of linear soil properties can be obtained. Afterwards, by employing an iterative technique, the newly-obtained linear properties are assigned to conduct the next round analysis and compute new properties, and so forth.

In the time domain approach, there are three major procedures employed to get the solutions to the wave equation. First is the characteristic method, which some researchers [7, 8] have named as ray-tracing approach as it deals with both refracted and reflected shear waves transmitted along with the wave rays, by which the solutions to the wave equation may be obtained through some mathematical transformations of the wave equation and the strain-stress relations. In cases where the homogeneous media's dynamic response is analyzed, the solutions using this procedure may be considered as quasi-analytical as it fully follows the wave propagation path without any numerical approximation. A second way is through the finite difference method (hereafter called FD), which is one of the earliest numerical modelling techniques employed in site response analysis. This procedure works out the answers to the equation by approximating the first or second order partial differential derivatives on both sides of the wave equation using the central, forward, and/or backward approximations. Finally, directly integrating the equation of motion is one of the mainstream time-domain techniques used in site response analysis. Typically, a 1D soil column is treated as an MDOF lumped-mass system or a distributed mass continuum with known boundary conditions. A system of coupled equations, which are put together from each equation of motion at nodes or mass units, is solved by implementing numerical time-stepping integration schemes. DEEPSOIL [9], OpenSees [10] and PLAXIS 2D [11] are three of the most popular codes which have adopted the direct integration strategies for site response analysis and the latter two codes discretize the soil model with the help of Finite Element Method (FEM).

### 3. Determination of dynamic deformation characteristics of soils

Determination of dynamic deformation properties of soils is indispensable in a successful site response analysis, as it is abundantly clear that an accurate prediction of ground response to seismic loading relies on the representativeness of those dynamic parameters assigned to the numerical model. Not only have more sophisticated apparatuses been designed and manufactured, the stock of more advanced numerical approaches to investigate those properties has also increased significantly to accommodate the requirements for high-quality input parameters.

#### 3.1 Laboratory and field tests

Traditionally, laboratory tests and in-situ measurements are the main methods to understand dynamic behavior of soils under cyclic loading. Laboratory testing techniques, such as shear strength testing (cyclic simple shear tests and cyclic triaxial tests) and velocity measuring (bender element and resonant column tests), are commonly used, although shaking table and centrifuge tests have offered alternative approaches to evaluating soil behavior. Similarly, field measurements deployed to characterize the dynamic soil properties include strength testing (e.g. by SPT, CPT or DPT) and  $V_s$  measurements (e.g. by conventional geophysical seismic prospecting, vertical wave velocity measurements or surface wave analysis). These methods were and will continue to be an essential part of the acquisition of those dynamic parameters.

#### 3.2 Use of vertical arrays

Identifying dynamic deformation properties of soils by inversely analyzing ground motion recordings, especially those recorded at stations equipped with vertical arrays, has been increasing in popularity over the past decades, as those recordings present considerable opportunities to provide essential insights in understanding soil behavior in seismic environments. Identification of dynamic deformation properties of soils can also be grouped into two classes: frequency domain and time domain.

In terms of the frequency domain, transfer function (TF) is used to deduce some critical parameters, including predominant site frequency and average shear wave velocities. Employing phase spectrum identification technique, Chen & Hsu [5] identified the dominant frequencies of TFs between a series of subsurface recordings and the free surface recording and, from the known geometric feature (i.e. thickness)



of soil, they derived the  $V_s$  for soil layers at Hualien (Taiwan) site. Similarly, Harichane et al. [12] combined soil amplification function (i.e. TF) and least squares minimization technique to identify soil profile characteristics, and they claimed that the approach could determine soil's thickness, damping ratio, shear wave velocity and unit weight from two free field records.

Back analysis in time domain offers a vantage point to derive stress and strain histories of soils subjected to seismic shaking as these two features of soil can be directly used to compute shear modulus and damping ratio. Information extracted by inversely analyzing the measured time series presents a valuable insight into the soil behavior during earthquakes. Zeghal & Elgamal [13] successfully studied the loss in soil stiffness and revealed the liquefaction mechanisms at the Wildlife Site (USA) by analyzing earthquake motions recorded in-situ. Using an inverse analysis framework, Tsai & Hashash [14] simulated downhole array time series in vertical array sites while inferring strain-compatible shear moduli and damping ratios which reasonably matched laboratory and empirical data.

## 4. Proposed method

After exhaustive review of the different methods of investigating dynamic soil properties and comparing laboratory test procedures to field measurements of earthquake motions at vertical array sites, a method is proposed with the following assumptions [15]: Firstly, the soil medium between two vertical array sensors could be seen as a perfect specimen of laboratory soil column but without any disturbance. Secondly, the wave propagation paths in a vertical array are similar to those experienced by a laboratory soil column with the driver and receiver instrumented at its two ends, i.e. the arrangement of a pair of sensors at a vertical array site (one at the surface and another at depth) is the same as that of a fixed-free resonant column device. Also, the shear wave velocity can be determined by analyzing a pair of array recordings on surface and at depth. Most importantly, the reason that resonant column test is capable of detecting the strain-dependent shear modulus is that it can induce different levels of shaking, and thereby capture the pertinently varied resonant frequencies, leading to the revelation of the above shear modulus. Thus, the recordings from vertical arrays would carry valuable information associated with small-strain to high-strain level soil properties provided the event of concern changes intensity during the whole shaking period, which is the case in real earthquakes.

### 4.1 Methodology

In line with the above assumptions, the methodology to determine shear wave velocities and thereby the shear modulus degradation characteristics of soil at a vertical array site is conceptually outlined below.

- 1) Establish a 1D soil model with predefined soil thickness and density;
- 2) Estimate the possible range of the average  $V_s$  over the soil model (i.e. the strata between the surface and the bottom boundaries);
- 3) Assign the lowest  $V_s$  to the model as the initial value and correspondingly calculate the shear modulus;
- 4) Impose the observed time series on the bottom boundary of the above model - the bottom depth must be the same depth at which the observed time series was originally recorded;
- 5) Perform an upward (i.e. forward) site response analysis and derive shear stress and strain time histories within the whole model;
- 6) Apply the observed ground surface time series to the upper boundary of the model and execute a deconvolution analysis (i.e. downward analysis);
- 7) Similarly, derive shear stress and strain for the entire model;
- 8) Compare the two sets of stress-strain time series simulated from forward and downward analyses and compute their correlation coefficients at time instants of interest;



- 9) Assign a new  $V_s$  within the prescribed velocity range and repeat Steps 4 to 8, until all the velocity values in the range have been tested.
- 10) Plot the curves of correlation coefficients vs the assigned shear wave velocities for each time instant;
- 11) If the input motions are weak, the induced strains are in small-strain level; then the largest correlation coefficient values in all curves aforesaid should correspond to approximately the same  $V_s$ . Designate this  $V_s$  as the average  $V_s$  for the soil between the pair of measured motions used in the analyses.
- 12) In cases of large-strain shaking, the  $V_s$  values corresponding to the largest correlation coefficients at some time instants might vary significantly. This observation can be seen as an indication that soils behave nonlinearly as a result of shear modulus degradation. Specify the  $V_s$  values and their corresponding shear moduli, determine the maximum strains at the relevant time instants, and establish the relationship of shear modulus and strain.

#### 4.2 Adopted response analysis methods and verification

In this research, three different seismic response analysis methods (including forward and deconvolution analyses procedures) were investigated: (1) Finite difference (FD) method in time domain; (2) Characteristic line (CL) method in time domain; and (3) Multi-reflection (MR) method in frequency domain. Due to space limitations, details of the relevant equations and calculation processes, as well as computer algorithms developed, are not presented herein; instead, readers are referred to the work by Huang [15]. Note that in the same research, a deconvolution scheme utilizing backward FD algorithms was established and validated. Such deconvolution method in the time domain may supplement and broaden the inverse analysis applications of measured surface recordings and would be helpful in dispelling the myth among some practitioners that deconvolution could only be achieved in frequency domain.

The three analysis schemes mentioned above are originally designed to solve the linear elastic wave equation rather than a nonlinear wave equation. It is rational and appropriate to first verify these methods using relatively ideal motions as input; otherwise, the outcomes from simulation might be complicated to comprehend in terms of the usage of measured motions at depths. For the verification, the three-layer model profile used is shown in Fig. 1(a). The original motion employed was the S-N component of the time history measured on ground surface during the Alamo Earthquake of 5 September 2008 at Treasure Island Geotech Array site [16]; its displacement and amplitude spectrum are presented in Figs. 1(b) and 1(c). Deconvolution is then performed to obtain the base motion which will be used as input in the forward analysis. The reason for performing deconvolution with measured surface motion first and then applying the inversely-modelled base motion to the forward analysis afterwards is because recorded surface motions usually have already been filtered by the subsurface soil media and therefore do not need to take into consideration the effect of viscosity and other damping factors.

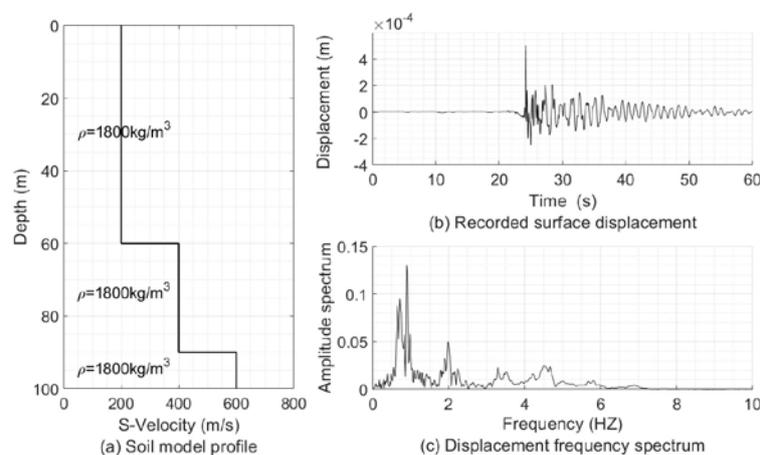


Fig. 1 – Three-layer model and input surface displacement and frequency spectrum used in verification



By performing self- and cross-verifications, Huang [15] showed that, irrespective of the solutions to the wave equation obtained in time domain or frequency domain, site responses to a cyclic excitation calculated with the three analysis methods mentioned above are nearly identical, provided the soils of interest behave linearly, i.e. the solutions to the same wave equation are unique. Fig. 2 shows the comparison of the deconvolved base and simulated surface motions with respect to displacement, Fourier spectrum, and spectral acceleration (SA) from the self-verification using FD, CL, and MR methods. Note that the abnormality in SAs from FD method is due to the extremely small time steps adopted. The solution to this is discussed by Huang (15).

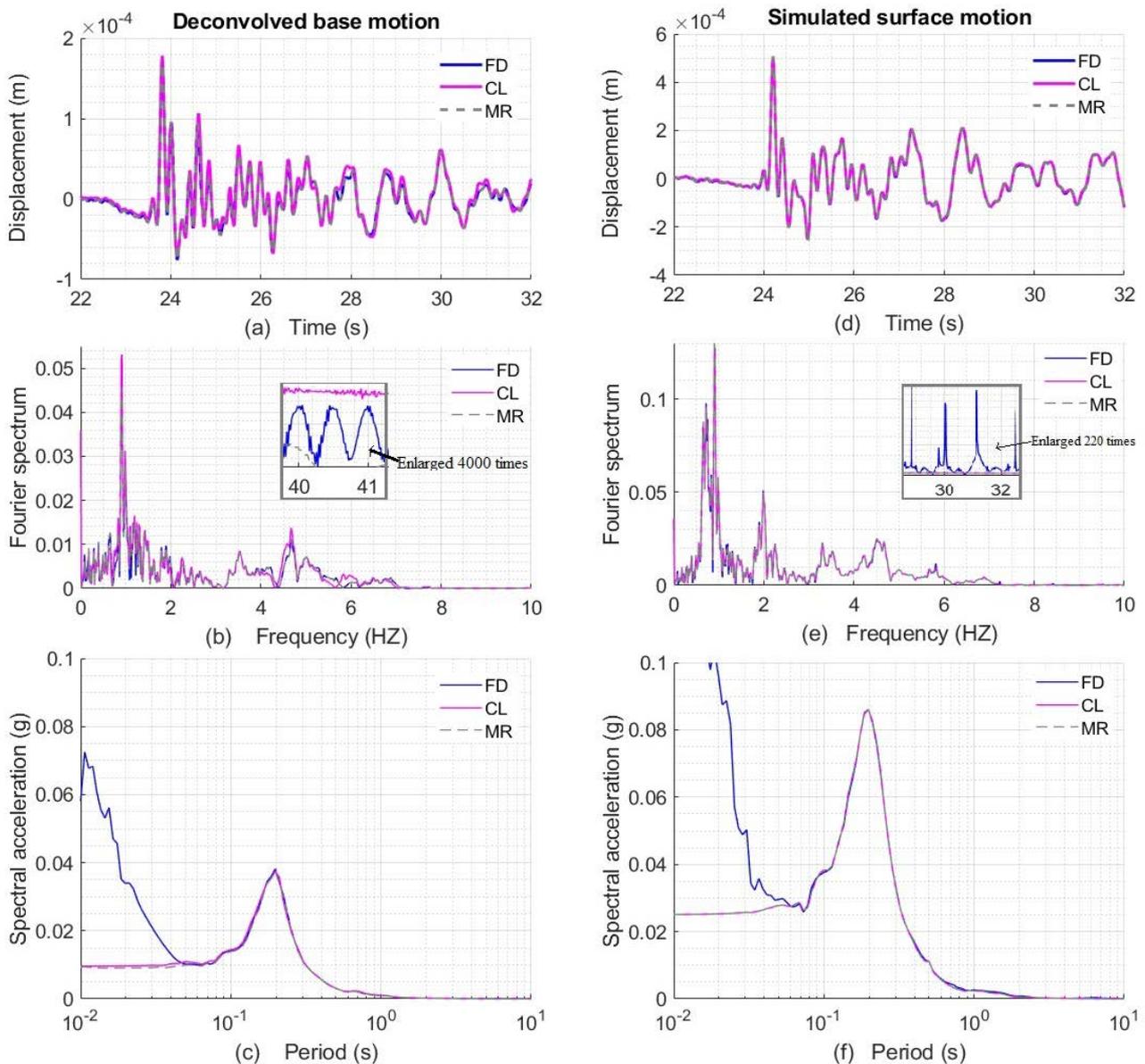


Fig. 2 - Comparisons of displacement time histories, Fourier spectra, and spectral accelerations of motions obtained using FD, CL and MR methods



## 5. Application to vertical array sites

### 5.1 Vertical arrays employed

Several vertical arrays have been extensively studied and considerable site response analysis techniques and soil dynamic property extraction methodologies have been developed and established based on these studies. In this paper, the data from the Geotech Arrays at Treasure Island and La Cienega sites were considered because these two sites have been well documented and therefore could be treated as sources of high-quality benchmarks when validating and comparing results from simulated methods developed here and by others.

The Treasure Island (TI) Geotech Array was installed by the California Strong Motion Instrument Program and National Science Foundation in 1992. Since then, this array has documented more than twenty small earthquakes with local peak ground accelerations (PGAs) ranging from 0.005g to 0.064g [16]. As the site has been equipped with six underground and one surface triaxial accelerometers covering a composite soil ranging from surficial hydraulic fill underlain by natural sand, Bay mud, and gravelly sand and sedimentary sandstone to a depth of 122 m (see Fig. 3a), these small event recordings, along with a huge quantity previous research, have made the site an ideal subject to implement the study of establishing  $V_s$  profiles using the techniques developed in this paper. In this study, recordings from ten small earthquakes, with magnitudes ranging from 3.0 $M_L$  to 4.4 $M_w$  and PGAs varying from 0.004g to 0.064g, were selected.

On the other hand, La Cienega (LC) site is one of the typical deep soil sites in the Los Angeles Basin. The site primarily consists of marine deposits which are overlain by approximately 30 m of fluvial deposits, formed mainly from sands, silts and clays (see Fig. 3b). The  $V_s$  profile of the site has two versions, one obtained from surface-to-borehole logging (i.e. downhole shooting) and another from PS suspension logging. It can be said that the former profile is more suitable to be referenced when investigating dynamic responses of the site under seismic loading as both waves induced by an earthquake underground and a downhole shooting exciter on the ground surface propagate through the same distance covered by a pair of sensors, though the two types of propagation are in opposite directions. Whereas in suspension logging, a pair of transmitter and receiver are fixed at the ends of a cylindrical rod which moves along the borehole. The installation arrangement of the transducers indicates that the velocities measured by this technique may present local elastic characteristics better than the average elastic features at the site. The input motions used in characterizing strain-compatible shear moduli at this site and verifying these moduli are five pairs of recordings measured on the ground surface (Sensor 1) and at a depth of 100.6 m (Sensor 7) whose PGAs range from 0.013g to 0.22g [16], covering small, moderate to strong shakings.

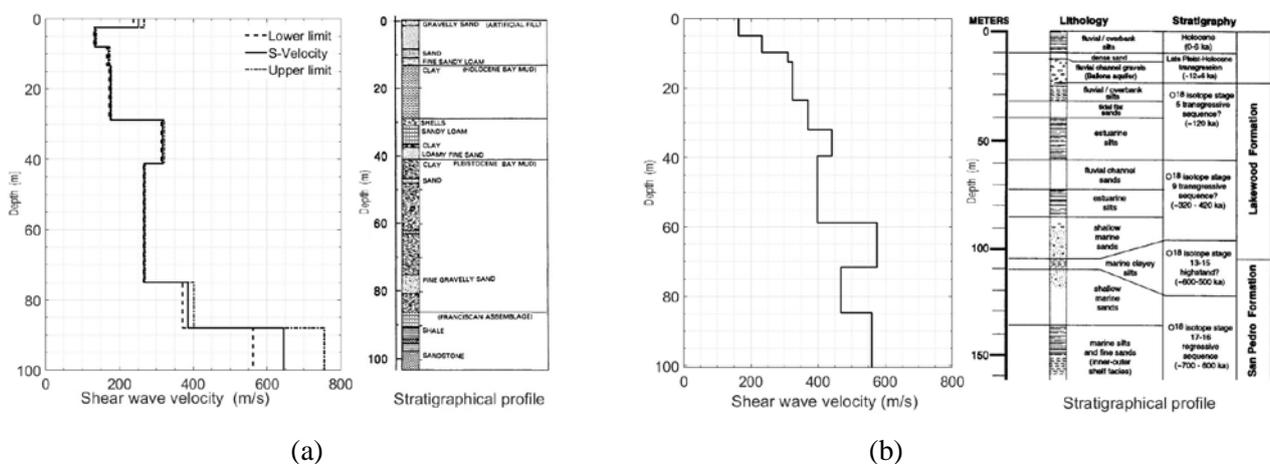


Fig. 3 –  $V_s$  profile and stratigraphy at the: (a) Treasure Island Geotech Array [17]; and (b) La Cienega Geotech Array site [18].



## 5.2 Validation and comparison of analysis results

To check the practicability and accuracy of the proposed site response analysis tools, they were applied to numerically model the seismic responses recorded at these sites. Two pairs of earthquake recordings at TI site and LC site were employed as input and target motions for the validation. In this paper, the results for MR method are presented since it is the most effective method among the three tools developed and is simple to implement. As this frequency domain method is derived from the linear elastic wave equation, its simulation results would be reasonably reliable if the soil of interest is under relatively small cyclic shaking (i.e. the soil behaves in a linear elastic way). The PGAs of the input motions are in a range of 0.014g to 0.034g, which can be considered as small shakings.

Fig. 4 shows the simulated displacements and corresponding spectral accelerations (5% damping) on the ground surface and at depth 44.2 m, along with their measured counterparts at TI site. It may be observed that, overall, there is close agreement between computed and recorded motions (whether time histories or response spectra). In terms of displacements, the simulated motions during the initial period of main shaking (i.e. 22 – 27 sec in Figs. 4(a) and (b)) both at the surface and at depth are in good accord with those measured in-situ; however, there are apparent differences during the period between 28 – 32 sec. In terms of acceleration response, the forward and inverse simulations have generally produced the response shape of the corresponding measured motions during the whole period of interest, although they underestimated by approximately 10% the peak value of the response to the measured records (Figs. (c) and (d)). Overall, the observation from the simulation indicate that the numerical model employed in the analysis represents the site soil's response relatively well, though further improvement of the soil model may be needed.

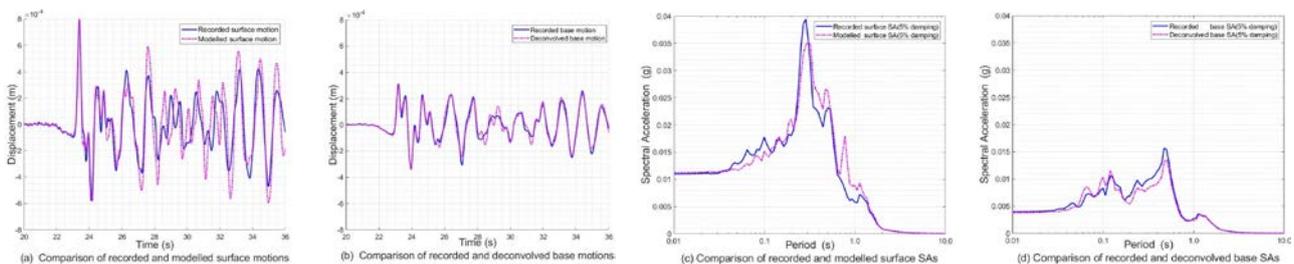


Fig. 4 – Comparison between recorded and modelled motions, and spectral acceleration at free surface and at depth for TI site

Results obtained from LC site analysis reveal that the main shaking (Figs. 5(a) and (b)) is well simulated while there are some evident variations in the surface displacements after that period. The modelled surface spectral acceleration, however, is in perfect agreement with the measured motion in spite of a slight rise in the simulated response at a period of 0.06 sec (see Fig. 5(c)), which may be traced back to the original recorded motion at depth as there is a similar increase in its response value at the same time. Meanwhile, the response spectra of both measured and inversely modelled motions match each other reasonably well even though the main peak of the deconvolved motion reveals a noticeable hike in its main peak (Fig. 5(d)). In general, the forward and backward analysis results from MR method are satisfactory.

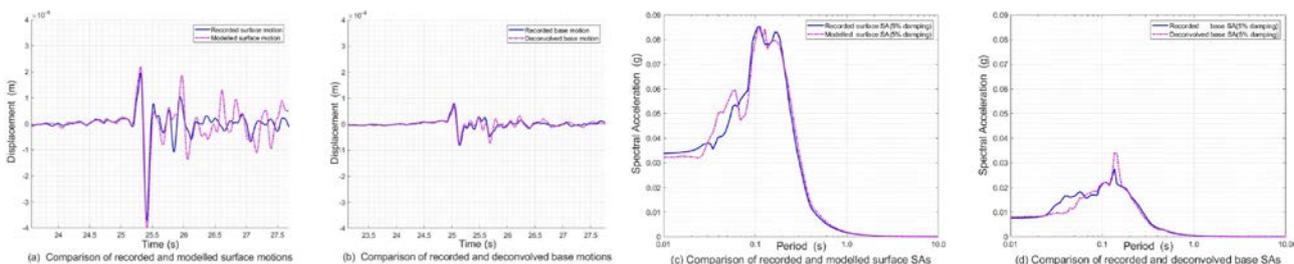


Fig. 5 – Comparison between recorded and modelled motions, and spectral accelerations at free surface and at depth for LC site



It is worthy to mention that the accuracy of the MR method was also investigated by comparing the results with those obtained using two widely-used dynamic analysis programs, i.e. DEEPSOIL and FLAC8.0. During the forward site response analysis, DEEPSOIL time domain linear (which solves the wave equation in an integration manner along with a lumped mass soil model) and frequency domain linear schemes, along with FLAC's dynamic analysis module (which adopts time domain finite difference approximation techniques with the same mass system as that for DEEPSOIL) were utilized. While not presented here due to space constraints, results show that MR method is as competent and reliable as DEEPSOIL in characterizing soil behavior under low levels of shaking. Moreover, it is suggested that when using time domain approaches with lumped mass system in dynamic analyses, they need to be accompanied by an analysis performed with frequency domain methods.

### 5.3 Determination of shear modulus and degradation curve

Based on literature review, there are mainly four approaches proposed to investigate the soils' dynamic properties: (1) using cross-correlation technique to examine the time intervals between the incident and reflected waves thereby shear wave velocities [6, 19] by decomposing the pair of recorded surface and downhole time histories; (2) calculating the ratios of amplitude spectra and/or phase spectra of the same pair of time histories as in the first approach and determining the first modal frequency (i.e. the lowest resonant frequency) of the soil deposits between the two recording depths, then sequentially computing the average  $V_s$  of the soil of interest (similar to a joint application of the transfer function practiced in frequency domain analysis and the resonant column testing in laboratory experiments) [5, 20]; (3) applying a system identification scheme accompanied by the methods of optimization or least-squares to acquire shear wave velocities ( $V_s$ ) or shear modulus ( $G_{max}$ ) [21]; (4) utilizing the recorded surface displacement and base acceleration time histories to obtain the distribution of shear strain and stress [13] between these two recordings and consequently forming a new soil model to iteratively compute the next pair of shear strain and stress until a desirable match is achieved [4, 14].

Based on the above literature review, the following approach has been adopted, together with the methodology outlined in Section 4.1: (1) perform forward and inverse analyses for a pair of recordings concurrently; (2) investigate soil behavior in the form of stress and strain distribution or in its resonant state; and (3) determine  $V_s$  and  $G_{max}$ . Note that according to Chen & Hsu [5], the average  $V_s$  varies in accordance with intensities of earthquake events, hence it might also change during a single event if the shaking intensities vary from low level to a certain high level.

#### 5.3.1 Results for Treasure Island Geotech site

Using the proposed approach, comparisons of the mean surface-to-depth shear wave velocities (i.e.  $V_s$  between the surface sensor and the corresponding sensors at depths) calculated using the proposed method (denoted as 'Thesis SV') and those from in-situ geophysical measurements (lower, mean and upper limits from Gibbs et al. [17] based on downhole shooting tests and data from CESMD [16] derived from PS logging method) are demonstrated in Fig. 6. The differences in profiles derived from the simulation results of shallow and deep pairs of sensors may be due to the relative homogeneity of soil between the pair of sensors. As the height of the 1D soil column used in the analysis increases, the soil as a whole may seem to be more homogeneous compared to the soil in a shorter column, in which the variations in each layer's geometry or physical properties lead to greater heterogeneity. As a result, the simulations with the shallower pair recordings are more sensitive and more unstable than those with broader pair recordings. Good agreement may be noticed between the simulated profile (Thesis SV) and the Gibbs SV Profile (mean SV) as demonstrated in Fig. 6(a). Moreover, in general, the above two profiles are in reasonably good agreement with the CESMD SV Profile, as shown in Fig. 6(b). However, the discrepancies among them are noticeable when comparing the three  $V_s$  profiles. For each pair of sensors (except for the shallowest), the surface-to-depth  $V_s$  of CESMD SV profiles are apparently smaller at mid-depth (and larger at greater depth) than the SVs of the other two profiles, which are quite consistent with each other, despite the fact that both CESMD and Gibbs SV profiles were acquired through geophysical in-situ measurements, whereas the 'Thesis SV' profile was obtained numerically.

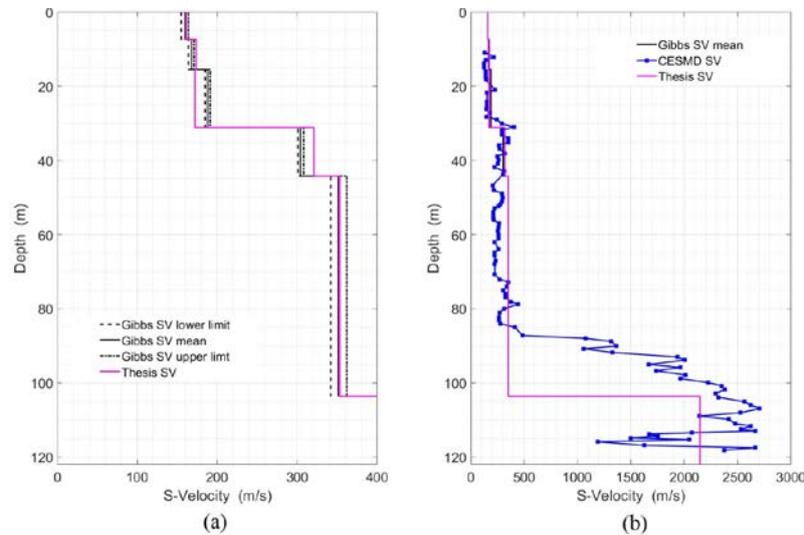


Fig. 6 – Comparisons of  $V_s$  profiles for TI site: (a) SV profile using proposed method vs Gibbs SV profiles [17]; (b) SV profile using proposed method, the mean Gibbs SV profile and CESMD SV profile [16].

According to the above observations, the following conclusions may be drawn: (1) the  $V_s$  profile of a vertical array site can be obtained by analyzing and comparing the stress distributions from forward and downward simulated site responses with pairs of earthquake recordings, one a surface recording and another a recording at depth; (2) the modelled  $V_s$  from shallow pairs of recordings are more sensitive and unstable than those from pairs of sensors that cover a relatively large thickness of soil; and (3) the numerically acquired  $V_s$  profile at TI site is in good accord with that from downhole shooting tests but differs with that from the suspension PS logging method.

### 5.3.2 Results for La Cienega Geotech site

The soil model used in the analysis of La Cienega Geotech site is 100.6 m thick and spatially discretized with a grid size of 0.1 m. After consulting the known  $V_s$  profiles, a range between 250 – 450 m/s with incremental step of 2m/s has been used in the analysis. The time windows in which the spatial distributions of shear stresses and/or strains were calculated and compared differ according to the individual pair of recordings but, in general, they begin slightly ahead of the first arrival of the shear waves in the base acceleration time series and end after 2 – 5 vibration cycles in the surface acceleration time series.

The soil nonlinearity at LC site was studied by applying the scheme outlined in the previous sections. The  $G/G_{\max}$  data derived from this scheme were compared with the commonly accepted nonlinear properties of sands proposed by Seed & Idriss [22], the laboratory testing results commissioned for this site [23] and the numerically modelled data using SelfSim [4]. Again, details of the analyses are provided elsewhere [15]. The  $G/G_{\max}$  vs strain data sets obtained are plotted in Fig. 7 (the green dots are the calculated results). It can be observed from Fig. 7(a) that the  $G/G_{\max}$  data calculated using the proposed method (based on CL approach) are consistent with the upper and mean limits of degradation curves of Seed & Idriss [22]; the slight difference may be due to the fact that the site consists of not only cohesionless sands, but also clayey silts and clay as well. While the computed results are more or less similar to the laboratory-derived data, their difference with those from SelfSim are huge; at this stage, the reason behind the marked difference is unknown as not much is known about the detailed simulation procedures adopted in SelfSim.

Note that the non-linear properties of the upper 100.6 m soil at the LC site obtained using the above procedure were verified by conducting forward site response analysis for the site employing the  $V_s$  profiles from the proposed (CL) method, the laboratory data and the upper and mean degradation curves of Seed & Idriss [22]. Because the three analysis methods developed here are not suitable for non-linear analyses, DEEPSOIL was employed for the validation using its Equivalent Linear and Frequency Linear functions.



The results indicate that the dynamic deformation properties determined by the proposed approach better represent the soil layers at the site in much better way than those obtained by other means [15].

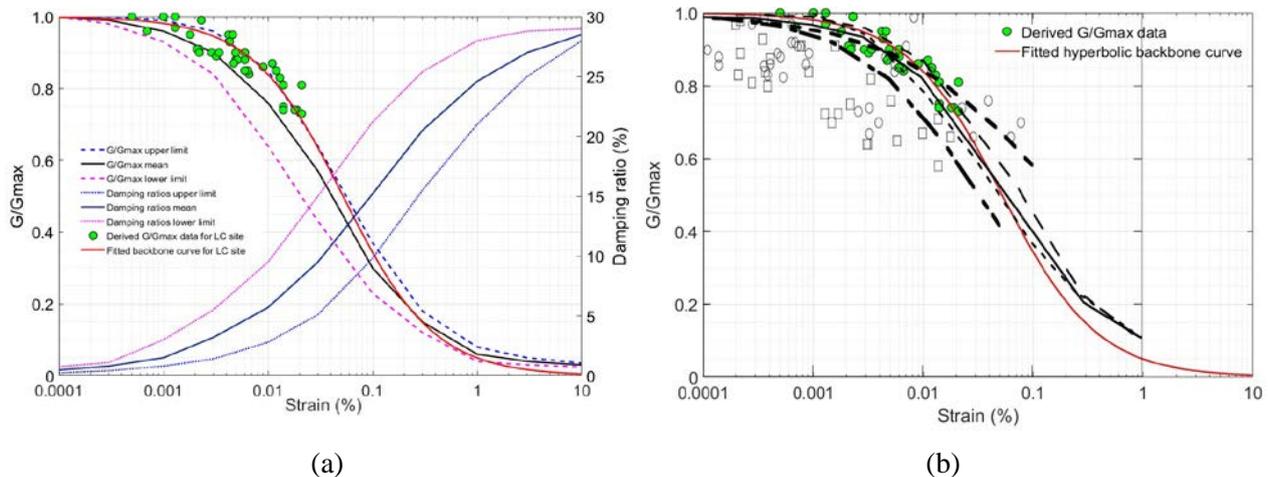


Fig. 7 - Comparison of  $G/G_{\max}$  derived for LC site numerically with the: (a) empirical curves of sand by Seed & Idriss [22]; (b) experimental data (black solid line and two thin dashed lines) [23] and simulated data from SelfSim (open square and circle data) [4].

## 5. Conclusions

In this study, site response analysis methods, both in time domain and frequency domain, were examined in order to investigate the dynamic deformation properties of soils with the help of vertical array data. A methodology to characterize the average  $V_s$  of soils bounded by a pair of seismic recordings has been conceived and then validated by investigating and comparing the spatial distributions of shear stresses and strains computed from simultaneously-modelled forward and backward analyses. This scheme made good use of the readily available vertical array data in some areas in the world and presented an innovative way to study soils' nonlinear behavior in a linear way.

The site response analysis methods developed in the paper were consistent with each other and verified to be credible to be used in practice in terms of small to moderate seismic events. More importantly, the newly-developed scheme to determine the  $G/G_{\max}$  of soils under high strain levels provided an alternative means to investigate the soils' nonlinear characteristics and, hopefully, might give some insights in understanding the mechanisms behind such nonlinear behavior.

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