17th World Conference on Earthquake Engineering, 17WCEE
Sendai, Japan - September 27th to October 2nd, 2021

SIMPLIFIED ANALYTICAL SOLUTION OF BURIED PIPELINE SUBJECTED TO PIPE BURSTING UNDERNEATH

C. H. Chaudhuri[(1)], D. Choudhury[(2)]

[(1)] *Ph.D. Research Scholar, Department of Civil Engineering, Indian Institute of Technology Bombay, Powai, Mumbai 400076, India, e-mail: chaudhuri.ch@iitb.ac.in*
[(2)] *Institute Chair Professor, Department of Civil Engineering, Indian Institute of Technology Bombay, Powai, Mumbai 400076, India, e-mail: dc@civil.iitb.ac.in*

Abstract

An existing old buried pipe can be replaced with a new pipe by static pipe bursting technique without the conventional cut and cover method. The pipe bursting method involves less excavation and hence less disturbance on the ground surface in terms of social activities. However, ground heave is produced during pipe bursting operation, and it may have significant influence on the surrounding underground structure. In this regard, the present study investigated the impact of ground movement resulting from static pipe bursting on adjacent buried pipeline through simplified closed-form analytical solution. For analytical solution, Gaussian error function is used to replicate the ground deformation pattern induced from pipe bursting. Further, the pipe (Euler beam) is modelled as linear elastic isotropic material, and the soil is considered as single-parameter linear elastic foundation (Winkler foundation). The proposed closed-form analytical analysis is validated with the results procured from an earlier full-scale experimental study. From the comparative study, it is perceived that the analytical formulation provides overestimation in results but within the acceptable limit. The present analysis will be helpful in the initial design stage to understand the behavior of buried pipeline subjected to ground deformation induced from pipe bursting underneath. Further, the method involves less time and less complexity as compared to three-dimensional finite-element-based numerical or rigorous experimental study.

Keywords: pipe bursting; closed-form solution; pipe-soil interaction; ground movement; Gaussian function

1. Introduction

The static pipe bursting technique is commonly used for rehabilitation, repairing, or replacing the buried pipeline. The method involves less excavation compared to the conventional cut and cover method. For replacement of the existing pipe by static pipe bursting method, the new pipe is attached at the end of the expander, and the expander is pulled along the existing pipe using a pulling rod. The expander's outer diameter is larger than the diameter of the existing pipe so that the expander can disintegrate the existing pipe. During this operation, three-dimensional ground movement will propagate [1], and it may have drastic influence on surrounding structures [2-4]. The Greenfield ground heave pattern obtained from pipe bursting can be described analytically using Gaussian function. It has been observed that the ground heave is symmetric with respect to the axis of the replaced pipe [4, 5]. Cholewa *et al*. [2] experimentally investigated the influence of pipe bursting on surrounding polyvinyl chloride (PVC) pipe. Later on, Shi *et al.* [4] performed a similar study through simplified numerical analysis. Further, analytical analyses were also performed to predict the ground deformation and corresponding expansive force during pipe bursting operation [6, 7]. The influence of ground deformation resulting from seismic landslide and permanent ground deformation on buried continuous pipeline has also been investigated based on the concept of beam on elastic foundation [8-10]. The present study is the extension of the prior mentioned studies to examine the response of buried pipeline subjected to pipe bursting underneath. In the current study, the pipe is considered as Euler Bernoulli's beam, and surrounding soil is idealized as single parameter isolated linear elastic Winkler foundation. The proposed methodology will be helpful in the preliminary design stage as the method involves fewer input parameters and less time-consuming compared to details experimental study or three-dimensional continuum analysis.

2. Problem definition

From literature, it is understood that the pattern of Greenfield ground deformation obtained from tunneling or static pipe bursting is similar. Both upward ground heave patterns induced from static pipe bursting or downward ground settlement profiles due to tunneling can be well represented by a Gaussian function [4, 11, 12]. Hence, following the existing literature, the present study used the Gaussian error function expression to represent the ground heave pattern induced by static pipe bursting. The Gaussian function to define the upward ground heave can be expressed as

$$y_g = \delta \exp\left(-\frac{x^2}{2i^2}\right) \tag{1}$$

Where y_g is the profile of upward ground displacement induced by static pipe bursting underneath, δ is the peak ground deformation, and i is the width of the settlement trough.

The following expression is used in the present study to estimate the value of 'i' of equation (1) [13].

$$i = 0.175Y_0 + 0.325(Y_0 - Y) \tag{2}$$

Where Y_0 is the distance from the ground surface to the pipe bursting location, and Y is the distance from the ground surface to the adjacent buried pipe's location.

Mair *et al.* [13] proposed the prior mentioned expression of i (equation 2) for the case of tunnel in clay soil. The researchers proposed the expression in terms of the new and adjacent tunnel position based on

several data obtained from field measurements and centrifuge model tests. Later on, Zhang *et al.* [11] also adapted the same expression of i in sandy soil to get the response of an existing tunnel to a new tunnel excavation underneath and successfully validated the results using the expression of i as proposed by Mair *et al.* [13] with the centrifuge results obtained by Marshall *et al.* [14]. So, in the absence of Greenfield soil profile at the adjacent pipe or tunnel location, the approximate value of i can be estimated using equation (2).

Fig. 1 depicts the idealization of the present problem. Where L is the adjacent pipeline's length and x_1 is half of the total pipe length (i.e., $x_1 = L/2$). It is assumed that the adjacent buried pipe is transverse to the pipe bursting direction, and pipe bursting induced upward soil heave is symmetric with respect to the axis of the replaced pipe. The spring stiffness of the Winkler foundation (K) is calculated using the following expression proposed by Shi *et al.* [4].

$$K = K_u^{0.1} K_d^{0.9} \tag{3}$$

Where K_u and K_d are the subgrade modulus for pipe-soil relative movement in upward and downward direction, respectively. ASCE guidelines [15] are used for estimating the value of spring stiffness. Shi *et al.* [4] mentioned the non-linear force-displacement response of pipe-soil interaction for both downward and upward pipe-soil relative movement and stated that the non-linear response can be simplified as linearly elastic perfectly plastic model. Whereas, in the present closed-form analytical study, the soil is idealized as single parameter linear elastic foundation i.e., Winkler foundation. Hence, the present study considered only the linear part of the simplified linearly elastic and perfectly plastic model as shown by Shi *et al.* [4] in their numerical analysis. During pipe bursting process, the ground will move upward and heave will generate. Hence, the relative movement between the overlying adjacent pipelines and surrounding soil is mainly in the downward direction. Therefore, in equation (3), maximum weightage is given for K_d, i.e., for downward pipe-soil relative movement.

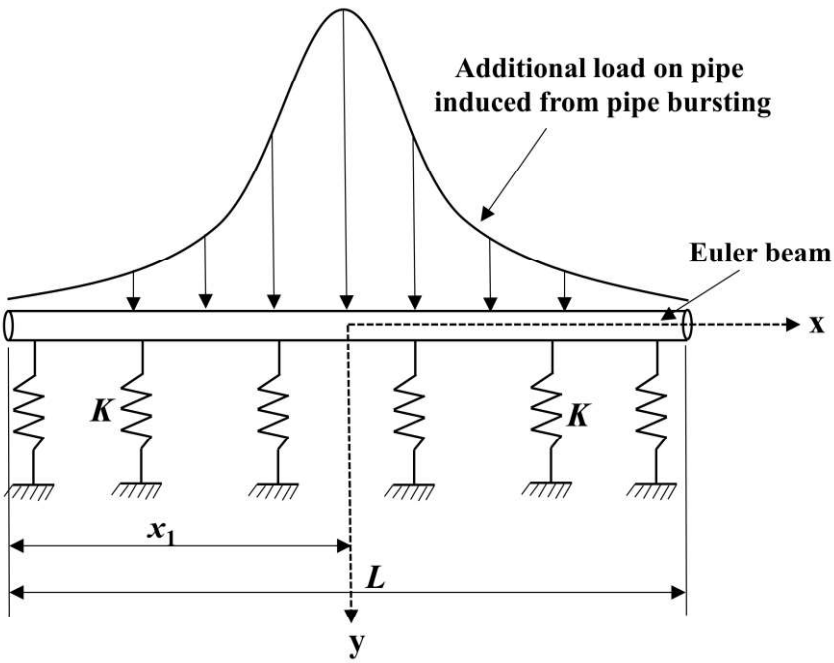

Fig. 1 – Idealization of beam-spring interaction model

3. Governing differential equation

In the present analysis, the buried pipe is idealized as conventional Euler Bernoulli's beam, and the surrounding soil is considered as linear elastic isolated Winkler foundation. Fig. 2 illustrates the free body diagram of an infinitesimally small length of the beam segment. Anticlockwise moments and upward forces are considered as positive sign conventions. From equilibrium of force following expression can be obtained:

$$\frac{dV}{dx} = q(x) \tag{4}$$

Where V is the shear force and $q(x)$ is the pressure acting on the pipe. Further, from the equilibrium of moment following expression is obtained

$$\frac{dM}{dx} = V \tag{5}$$

Where M is the moment. Further, we have

$$M = -EI\frac{d^2y}{dx^2} \tag{6}$$

Where EI is the bending stiffness of the beam. Equations (4), (5), and (6) lead to the following expression

$$EI\frac{d^4y}{dx^4} + q(x) = 0 \tag{7}$$

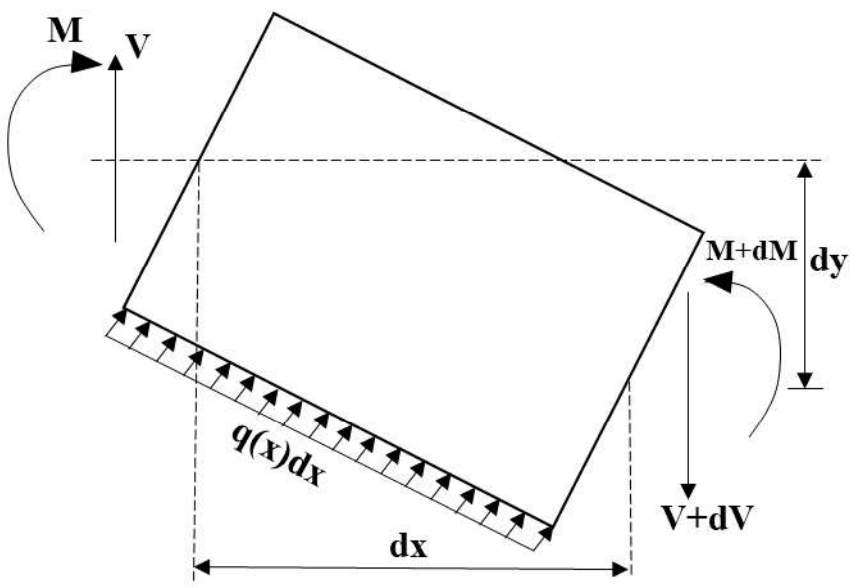

Fig. 2 – Forces acting on an infinitesimally small length of pipe

The pressure $q(x)$ on the pipe will be proportional to the pipe-soil relative displacement. Hence, for single parameter Winkler foundation, the expression of $q(x)$ can be obtained as

$$q(x) = K(y - y_g) \tag{8}$$

Equations (1), (7), and (8) lead to the following governing differential equation of pipe deformation

$$EI\frac{d^4y}{dx^4} + Ky = K\delta\exp\left(-\frac{x^2}{2i^2}\right) \tag{9}$$

4. Closed-form solution

The complementary solution of equation (9) is

$$y_C(x) = e^{\lambda x}(c_1\cos\lambda x + c_2\sin\lambda x) + e^{-\lambda x}(c_3\cos\lambda x + c_4\sin\lambda x) \tag{10}$$

Where $\lambda = (K/(4EI))^{1/4}$, and c_1, c_2, c_3, and c_4 are unknown constant co-efficient.

Fourier series is adapted in the present formulation to get the particular solution of equation (9). The right-hand side term of equation (9) is a function of x, and it can be expressed as

$$f(x) = K\delta\exp\left(-\frac{x^2}{2i^2}\right) \tag{11}$$

Equation (11) is an even function i.e., $f(-x) = f(x)$. So, equation (11) can further be expressed in terms of Fourier cosine series

$$f(x) = C_0 + \sum_{n=1}^{n=\infty} C_n\cos\frac{n\pi}{L}x \tag{12}$$

Where $C_0 = \frac{1}{L}\int_0^L f(x)dx$, and $C_n = \frac{2}{L}\int_0^L f(x)\cos\frac{n\pi}{L}xdx$

Equations (9), (11), and (12) lead to the following expression

$$EI\frac{d^4y}{dx^4} + Ky = C_0 + \sum_{n=1}^{n=\infty} C_n\cos\frac{n\pi}{L}x \tag{13}$$

Now, the particular solution of equation (13) is

$$y_P(x) = \frac{C_0}{K} + \sum_{n=1}^{n=\infty} a_n\cos\frac{n\pi}{L}x \tag{14}$$

Where $a_n = \dfrac{C_n}{K + EI\left(\frac{n\pi}{L}\right)^4}$

The complete solution of equation (9) can be obtained using equations (10) and (14)

$$\begin{aligned} y(x) &= y_C(x) + y_P(x) \\ &= e^{\lambda x}(c_1 \cos\lambda x + c_2 \sin\lambda x) + e^{-\lambda x}(c_3 \cos\lambda x + c_4 \sin\lambda x) + \\ &\frac{C_0}{K} + \sum_{n=1}^{n=\infty} a_n \cos\frac{n\pi}{L}x \end{aligned} \tag{15}$$

To get the values of four unknown co-efficient (c_1, c_2, c_3, and c_4), four boundary conditions are required. Following Fig.1, the available boundary conditions are

$$\frac{d^2y}{dx^2}(x = -x_1) = 0 \tag{16}$$

$$\frac{d^3y}{dx^3}(x = -x_1) = 0 \tag{17}$$

$$\frac{d^2y}{dx^2}(x = x_1) = 0 \tag{18}$$

$$\frac{d^3y}{dx^3}(x = x_1) = 0 \tag{19}$$

Further, equations (16) to (19) lead to the following four equations

$$c_1D_{11} + c_2D_{12} + c_3D_{13} + c_4D_{14} = R_1 \tag{20}$$

$$c_1D_{21} + c_2D_{22} + c_3D_{23} + c_4D_{24} = R_2 \tag{21}$$

$$c_1D_{31} + c_2D_{32} + c_3D_{33} + c_4D_{34} = R_3 \tag{22}$$

$$c_1D_{41} + c_2D_{42} + c_3D_{43} + c_4D_{44} = R_4 \tag{23}$$

Where D_{11}, D_{12}, ….., D_{44}, and R_1, …, R_4 are known constant co-efficient and expression of these coefficients are as follows (equations 24 to 43)

$$D_{11} = 2\lambda^2 \sin(\lambda x_1)\exp(-\lambda x_1) \tag{24}$$

$$D_{12} = 2\lambda^2 \cos(\lambda x_1)\exp(-\lambda x_1) \tag{25}$$

$$D_{13} = -2\lambda^2 \sin(\lambda x_1)\exp(\lambda x_1) \tag{26}$$

$$D_{14} = -2\lambda^2 \cos(\lambda x_1)\exp(\lambda x_1) \tag{27}$$

$$D_{21} = -2\lambda^3 \cos(\lambda x_1)\exp(-\lambda x_1) + 2\lambda^3 \sin(\lambda x_1)\exp(-\lambda x_1) \tag{28}$$

$$D_{22} = 2\lambda^3 \cos(\lambda x_1)\exp(-\lambda x_1) + 2\lambda^3 \sin(\lambda x_1)\exp(-\lambda x_1) \tag{29}$$

$$D_{23} = 2\lambda^3 \cos(\lambda x_1)\exp(\lambda x_1) + 2\lambda^3 \sin(\lambda x_1)\exp(\lambda x_1) \tag{30}$$

$$D_{24} = 2\lambda^3 \cos(\lambda x_1)\exp(\lambda x_1) - 2\lambda^3 \sin(\lambda x_1)\exp(\lambda x_1) \tag{31}$$

$$D_{31} = -2\lambda^2 \sin(\lambda x_1)\exp(\lambda x_1) \tag{32}$$

$$D_{32} = 2\lambda^2 \cos(\lambda x_1)\exp(\lambda x_1) \tag{33}$$

$$D_{33} = 2\lambda^2 \sin(\lambda x_1)\exp(-\lambda x_1) \tag{34}$$

$$D_{34} = -2\lambda^2 \cos(\lambda x_1)\exp(-\lambda x_1) \tag{35}$$

$$D_{41} = -2\lambda^3 \cos(\lambda x_1)\exp(\lambda x_1) - 2\lambda^3 \sin(\lambda x_1)\exp(\lambda x_1) \tag{36}$$

$$D_{42} = 2\lambda^3 \cos(\lambda x_1)\exp(\lambda x_1) - 2\lambda^3 \sin(\lambda x_1)\exp(\lambda x_1) \tag{37}$$

$$D_{43} = 2\lambda^3 \cos(\lambda x_1)\exp(-\lambda x_1) - 2\lambda^3 \sin(\lambda x_1)\exp(-\lambda x_1) \tag{38}$$

$$D_{44} = 2\lambda^3 \cos(\lambda x_1)\exp(-\lambda x_1) + 2\lambda^3 \sin(\lambda x_1)\exp(-\lambda x_1) \tag{39}$$

$$R_1 = \sum_{n=1}^{n=\infty} a_n \left(\frac{n\pi}{L}\right)^2 \cos\frac{n\pi}{L}x_1 \tag{40}$$

$$R_2 = \sum_{n=1}^{n=\infty} a_n \left(\frac{n\pi}{L}\right)^3 \sin\frac{n\pi}{L}x_1 \tag{41}$$

$$R_3 = \sum_{n=1}^{n=\infty} a_n \left(\frac{n\pi}{L}\right)^2 \cos\frac{n\pi}{L} x_1 \tag{42}$$

$$R_4 = -\sum_{n=1}^{n=\infty} a_n \left(\frac{n\pi}{L}\right)^3 \sin\frac{n\pi}{L} x_1 \tag{43}$$

Fig. 3 outlines the proposed methodology of analyzing buried pipeline under pipe bursting induced ground deformation.

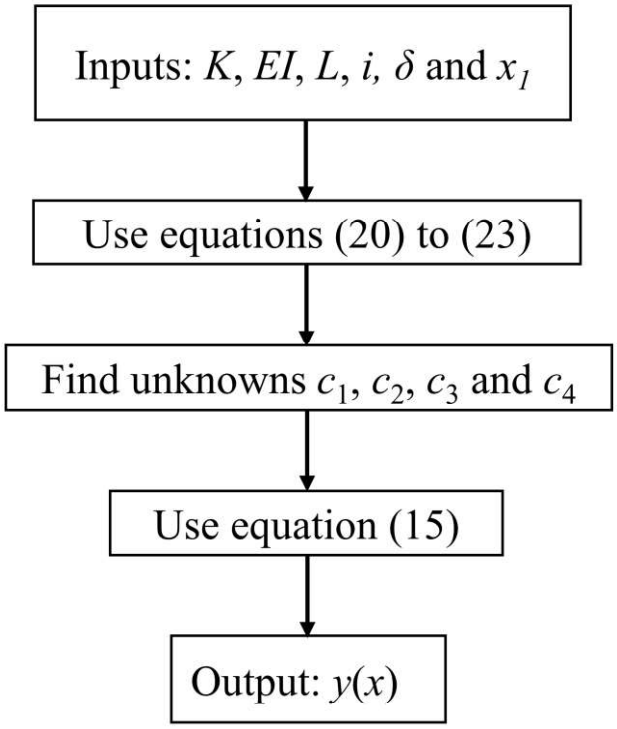

Fig. 3 – Flow chart of the proposed methodology

5. Validation of present analytical formulation

The present analytical formulation is validated with full-scale pipe bursting experimental results performed by Cholewa *et al.* [2]. The experimental test has been performed in a test pit having size of 8 m × 8 m × 3 m. Well graded sand and gravel soil were utilized to perform the test. A concrete pipe was replaced with a high-density polyethylene (HDPE) pipe during pipe bursting operation. The center of the pipe bursting was located at a depth of 1.50 m from the ground surface. The outer diameter and wall thickness of concrete pipe were 0.229 m and 0.038 m, respectively. The outer diameter and wall thickness of HDPE pipe were 0.168 m and 0.011 m, respectively. The maximum diameter of expander was 0.202 m. Another PVC water pipe was placed at 0.45 m above the concrete pipe. The PVC pipe was placed in a transverse to the pipe bursting direction. The outer diameter and wall thickness of PVC pipe were 0.122 m and 0.007 m, respectively. During pipe busting operation, the response of adjacent PVC pipe was recorded. Fig.4 and Fig.5

Table 1 – Details of pipe and soil parameters used in the present study

Pipe data	Material	PVC pipe
	Young's modulus, E (GPa)	3.3
	Poisson's ratio, ν	0.2
Soil data	Unit weight, γ (kN/m^3)	20.9
	Friction angle, φ	40°
	i value (equation 2)	0.46
	δ value at the level of adjacent pipe centerline [4], (mm)	6.7

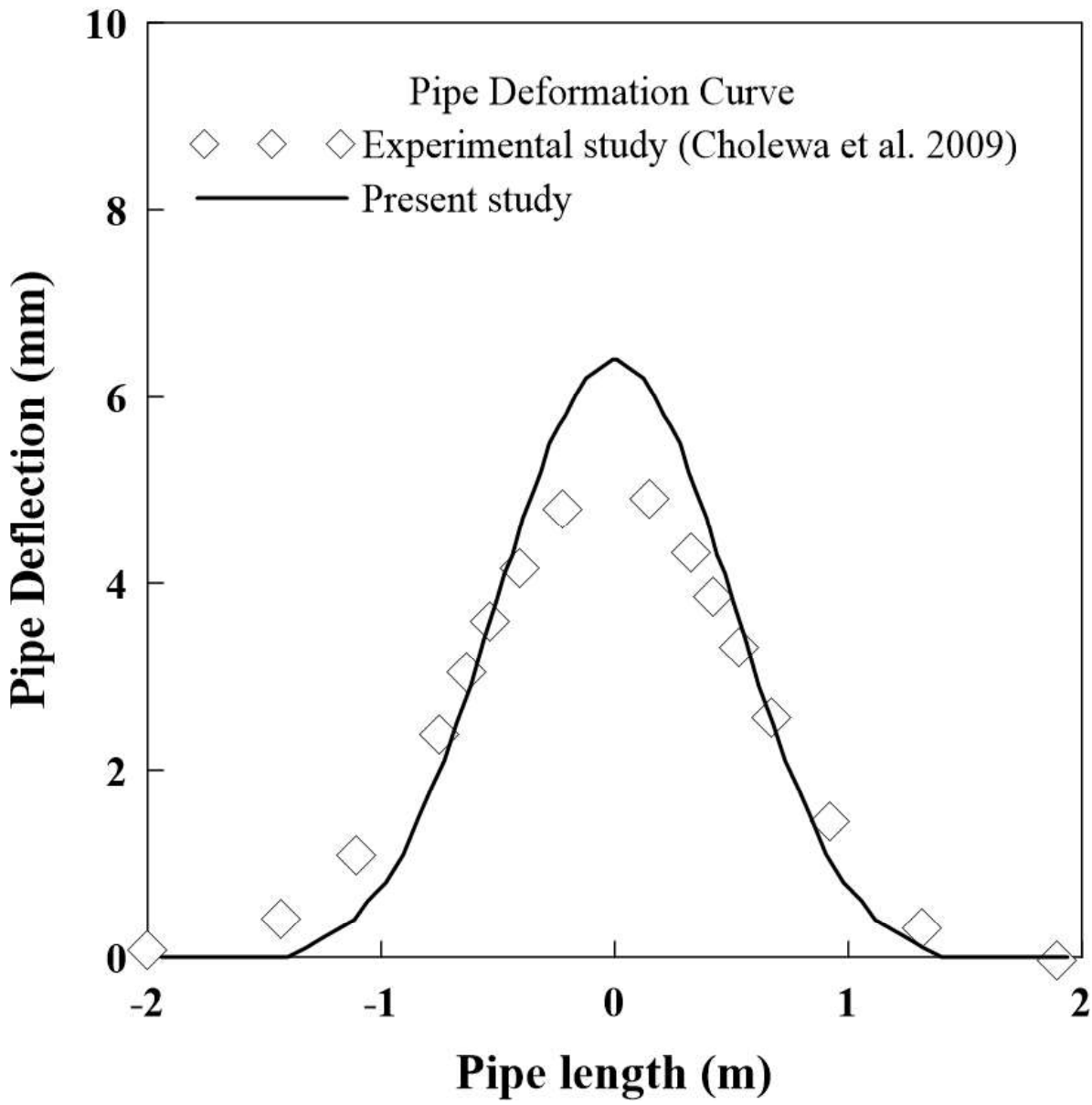

Fig. 4 – Comparison of pipe deformation

depicts a comparison study of pipe deflection and pipe curvature of adjacent PVC pipe, respectively, obtained from the present analytical study and earlier experimental test. The soil and pipe parameters used in the present study are shown in Table 1. From Fig.4 and Fig.5, it is perceived that deviations between the present study and experimental results are within acceptable limits, and the patterns are also similar. However, the present study gives overestimation in results due to simplified assumptions in the current analytical study.

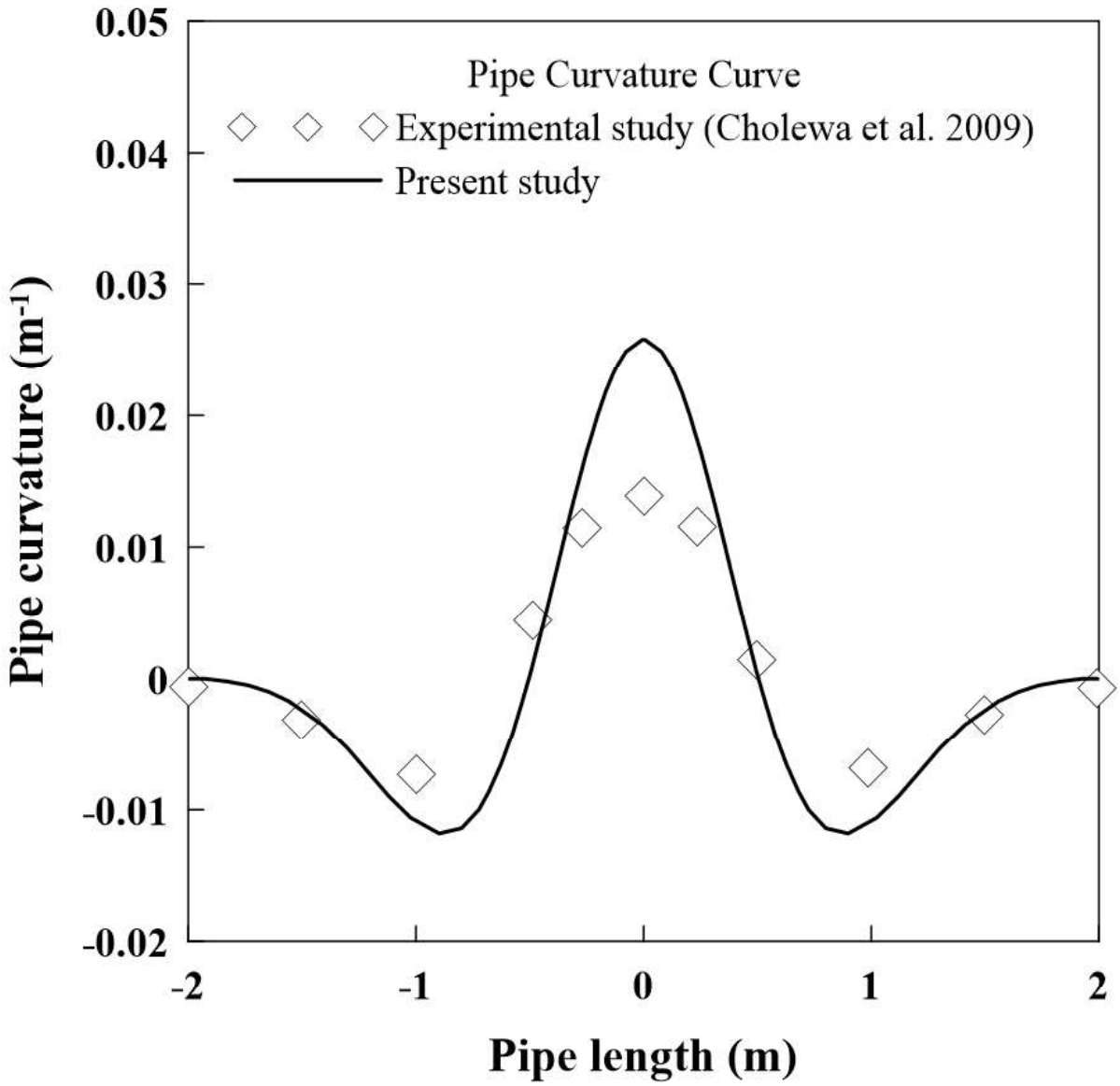

Fig. 5 – Comparison of pipe curvature

6. Summary and conclusions

The present study proposed an analytical formulation to investigate the response of an adjacent buried pipeline subjected to ground deformation resulting from pipe bursting operation. In the proposed method, the pipe is idealized as Euler Bernoulli's beam, and the soil is idealized as single parameter Winkler foundation. The present study is successfully validated with the published experimental results. Although the study does

not incorporate the non-linearity and plasticity of the soil and pipe material, but the present study has the advantage in terms of simplicity, calculation efficiency, and time along with moderate accuracy. Hence, the present study will be useful in the preliminary stage before performing any rigorous numerical and experimental analysis.

7. References

[1] Brachman RWI, McLeod HA, Moore ID, Take WA (2010): Three-dimensional ground displacements from static pipe bursting in stiff clay. *Canadian Geotechnical Journal*, 47, No. 4, 439-450.

[2] Cholewa JA, Brachman RWI, Moore ID (2009): Response of a polyvinyl chloride water pipe when transverse to an underlying pipe replaced by pipe bursting. *Canadian Geotechnical Journal*, 46, No. 11, 1258-1266.

[3] Rahman KM (2013): Three dimensional numerical analysis of static pipe bursting. *Ph.D. thesis*, Dept. of Civil Engineering, Queen's Univ. at Kingston, Ontario, Canada.

[4] Shi J, Wang Y, Ng CW (2013): Buried pipeline responses to ground displacements induced by adjacent static pipe bursting. *Canadian Geotechnical Journal,* 50, No. 5, 481-492.

[5] Lapos B, Brachman RW, Moore ID (2004): Laboratory measurements of pulling force and ground movement during a pipe bursting test. *NASTT*, No-Dig March, 22-24.

[6] Rogers C, Chapman DN, Sagaseta, Vafaeian (1998): Analytical modelling of ground movements associated with trenchless pipelaying operations. *Proceedings of the Institution of Civil Engineers-Geotechnical Engineering,* 131, No. 4, 210-222.

[7] Hou Ngan K, Yi Y, Rostami A, Bayat A (2017): Estimation of Soil Expansion Force in Static Pipe Bursting: Comparison between Numerical and Analytical Solutions. *International Journal of Geomechanics*, 17, No. 3, 04016065.

[8] Chaudhuri CH, Choudhury D (2020a): Effect of Earthquake Induced Transverse Permanent Ground Deformation on Buried Continuous Pipeline Using Winkler Approach. *Geo-Congress 2020: Geotechnical Earthquake Engineering and Special Topics*, ASCE, 274-283.

[9] Chaudhuri CH, Choudhury D (2020b): Buried pipeline subjected to seismic landslide: A simplified analytical solution. *Soil Dynamics and Earthquake Engineering*, 134, 106155.

[10] Chaudhuri CH, Choudhury D (2021): Semi-analytical solution for buried pipeline subjected to horizontal transverse ground deformation. *Journal of Pipeline Systems Engineering and Practice*, ASCE, USA, (accepted, in press with doi: 10.1061/(ASCE)PS.1949-1204.0000541).

[11] Zhang DM, Huang ZK, Li ZL, Zong X, Zhang DM (2019): Analytical solution for the response of an existing tunnel to a new tunnel excavation underneath. *Computers and Geotechnics*, 108, 197-211.

[12] Wang Y, Shi J, Ng CW (2011): Numerical modeling of tunneling effect on buried pipelines. *Canadian Geotechnical Journal*, 48(7), 1125-1137.

[13] Mair RJ, Taylor RN, Bracegirdle A (1993): Subsurface settlement profiles above tunnels in clays. *Geotechnique*, 43, No. 2, 315-320.

[14] Marshall AM, Klar A, Mair RJ (2010): Tunneling beneath buried pipes: view of soil strain and its effect on pipeline behavior. *Journal of Geotechnical and Geoenvironmental Engineering*, 136, No. 12, 1664-1672.

[15] ASCE Committee on Gas and Liquid Fuel Lifelines (1984): Guidelines for the seismic design of oil and gas pipeline systems. *American Society of Civil Engineers* (ASCE), New York.

