



CURVE FITTING APPROACH TO OBTAIN FRAGILITY CURVE FROM BUILDING DAMAGE MATRIX BASED ON SEISMIC GROUND MOTION PARAMETERS

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Abstract

This study proposes a method for damage probability matrix to be presented by fragility curves, which transforms the damage ratio of existing building or actual earthquake damage statistics to the fragility curves. Under the earthquake response of structures follow log-normal distribution hypothesis, we firstly transform the damage probability of the buildings in different intensity levels into exceed damage probability as a transition, combine with the corresponding relationship between intensity and ground motion parameters. Then we calculate the mean values and standard deviation values of fragility functions in different damage limit states by the maximum likelihood estimation method. Finally, we get structures fragility curves corresponding to different ground motion parameters. This paper takes Sichuan Province as an example of structure damage matrix, gives the seismic fragility curve parameters of various types of structure. It is verified that this method has high reliability and accuracy, which can provide the basis for building seismic risk assessment and estimation of earthquake insurance.

Keywords: seismic ground motion parameters, intensity, damage ratio matrix, fragility curve fitting, maximum likelihood estimation method



1. Introduction

Earthquake is one of the most serious disasters in China. Especially with the development of economy and urbanization process, the potential risk of earthquake disaster is gathering constantly. Vulnerability analysis is one of the three main parts of hazard analysis (seismic hazard analysis, vulnerability analysis of structure, loss assessment of disaster), the key target is to establish the relationship between the probability that the structure appears or goes beyond the state of failure and the intensity of the earthquake. So, it has a significant effect for earthquake damage, loss prediction as well as earthquake disaster mitigation to develop the vulnerability analysis of structure.

China has been accumulated many empirical seismic damage matrices based on seismic intensity in the long-term survey of earthquake and seismic hazard prediction. It can be expressed by seismic intensity-earthquake damage matrix which consists of proportion or probability of the damage degree of building structures. However, seismic intensity is mainly based on four kinds of macroscopic phenomena, such as human feeling, the response of artifact, the destruction of building structure and ground surface [1,2]. Therefore, intensity is not a physical quantity with specific meaning, it is directly related to seismic macroscopic destruction phenomenon, and is just a bridge used to roughly evaluate the intensity of ground motion. It also means that with the increase of seismic fortification level of structure, the measurement scale of intensity itself will change in different periods, so the damage matrix based on intensity has a huge uncertainty [3]. But the evaluation of building fragility is commonly based on intensity in most previous earthquake damage prediction earthquake damage matrix expresses the probability of structure damage under different intensities [4]. In order to accurately predict the seismic performance of building structures, develop structural seismic design better, complete the reinforcement and maintenance of existing buildings, accelerate the implementation of earthquake insurance in the nationwide, the vulnerability matrix based on seismic intensity is difficult to meet the actual needs, especially for civil structure, masonry structure and other simple housing, it is very hard to obtain the structure fragility curves through simulation analysis. how to improve the existing damage matrix, and transform the damage matrix based on seismic intensity for that based on a more reasonable ground motion parameter is imperative [5-7].

In this paper, the dual-parameterization approach of maximum likelihood estimation is used to analyze earthquake damage data. We chose the PGA as the parameters of ground motion intensity combine with the corresponding relationship between intensity and ground motion parameters, transform the probability matrix in different damage state based on intensity into that based on PGA. In this way we can obtain the continuous fragility curves based on ground motion parameters.

2. Method

The seismic fragility of building structures refers to the conditional probability of various damage states under different earthquake actions. It quantitatively describes the seismic capacity of structures from the probability. Expression as:

$$F(IM = x) = P(IM \geq IM_c / IM = x) = P(IM_c \leq x) \quad (1)$$

IM is the seismic intensity index, IM_c is a ground motion parameter corresponding to the structure reach the failure state. $F(IM)$ is fragility function.

The vulnerability of building structure is mainly expressed by seismic fragility curve and damage probability matrix. The fragility curve can be obtained by empirical method [8], analytic method [4] and hybrid method [9]. In the process of vulnerability research, different scholars propose many mathematical models for the selection of fragility function. For example, Logistic



regression function [10], Wilson distribution function [11], lognormal distribution function [12-13] and so on. Among them, lognormal distribution is the most commonly used.

There are many fitting methods for fragility curve, such as polynomial fitting, least square fitting, maximum likelihood estimation fitting [12,14-17] and so on. In this paper, the maximum likelihood estimation method is used to calculate and draw the fragility curve by MATLAB program, and the specific process of fragility curve based on ground motion parameters is obtained, as shown in Figure 1:

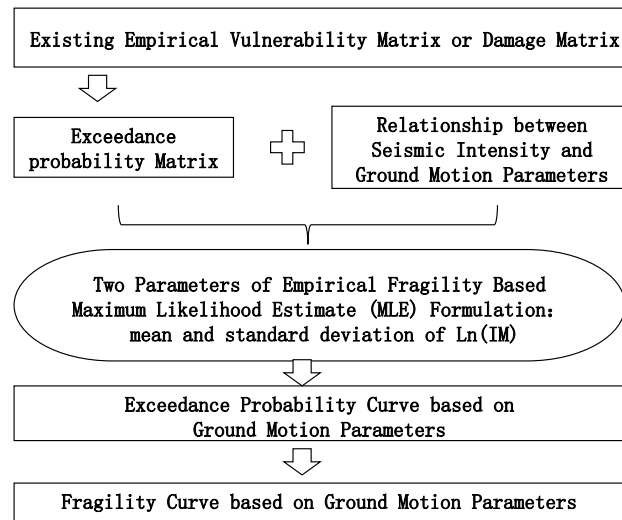


Figure 1 A method for calculating the fragility curve of earthquake damage matrix

2.1 Classification of different failure levels of structures

The failure levels of structures can be divided into five categories, as shown in Figure 2: DS_k ($k=0,1,\dots,4$) are a discrete qualitative description of the overall damage state in structure, describe as None, Slight, Moderate, Extensive and Collapse. LS_s ($s=1,2,3,4$) are the threshold among five kinds of damage states, respectively represent the threshold of slight, moderate, extensive and complete. The X label is ground motion index, Y label is the exceedance probabilities for different failure levels for fragility function.

The criteria for the determination of five damage states are as follows:

None: the load-bearing structure of the building is intact, and the individual non load-bearing components are slightly damaged which need not be repaired can be continued to use.

Slight: there are visible cracks in some load-bearing components, and the non load-bearing components have obvious cracks, which can be continued without repair or minor repair.

Moderate: most of the load-bearing components appear minor cracks, some of the components have obvious cracks, and some non load-bearing components are seriously damaged and need general repair.

Extensive: most of the load-bearing components are damaged seriously or partially collapsed, which need to be overhauled and some individual buildings are difficult to be repaired.

Collapse: most of the load-bearing components are severely damaged, the structure is collapsing or destroyed, and there is no possibility to restore it.

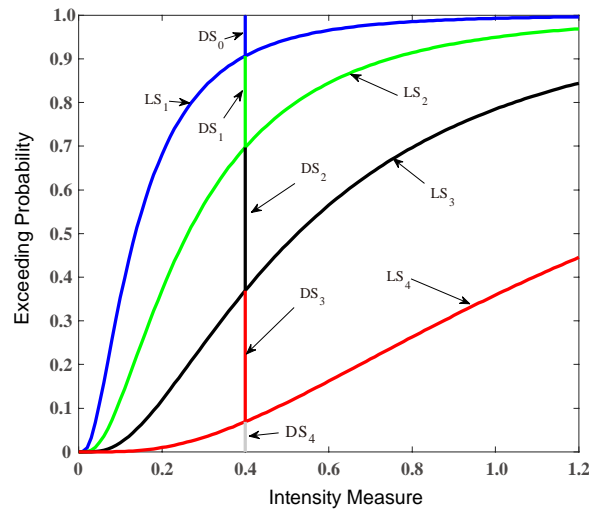


Figure 2 Schematic diagram of fragility curve in different damage states

2.2 Relationship between intensity and ground motion parameters

Seismic intensity describe the ground vibration and intensity of its influence caused by earthquake. Herein the limitation of early monitoring conditions, detailed strong ground motion records are difficult to obtain, but we have accumulated abundant macro seismic intensity data by intensity assessment. Hu Yuxian et al proposed a method for estimating the regional ground motion parameters where is lack of the observation data of strong ground motion [18]. Tian Qiwen et al. carried out a series of studies on estimating the attenuation law of ground motion parameters based on intensity data [19], in particular, the Chinese seismic intensity table gives the corresponding relationship between intensity and peak ground acceleration (PGA) and peak ground velocity (PGV). In this paper, the corresponding values between the intensity and the ground motion parameters given by GB/T 17742-2008 " The China seismic intensity table", from the table, we can get the value of the PGA and PGV that corresponding to each intensity, as shown in Table 1 .

Table 1 The correspondence between seismic intensity and peak ground acceleration and peak ground velocity[2]

Intensity	VI	VII	VIII	IX	X
PGA(m/s ²)	0.63 (0.45~0.89)	1.25 (0.90~1.77)	2.50 (1.78~3.53)	5.00 (3.54~7.07)	10.00 (7.08~14.14)
PGV(m/s)	0.06 (0.05~0.09)	0.13 (0.10~0.18)	0.25 (0.19~0.35)	0.50 (0.36~0.71)	1.00 (0.72~1.41)

2.3 Dual-parameterization obtain by Maximum Likelihood Estimation

In the process of vulnerability research, use the lognormal distribution mathematical model as the fragility probability model has been proved to be a mature and ideal model. In this context, Lognormal Cumulative Distribution Function is used as fragility function. The logarithmic cumulative probability distribution function (CDF) is defined as:

$$F(IM = x) = P(C|IM = x) = \Phi \left(\frac{\ln \left(\frac{x}{\theta} \right)}{\beta} \right) \quad (2)$$



$P(C|IM = x)$ is the probability that the structure reaches a certain limit when the ground motion intensity reaches $IM = x$. $\Phi()$ is the standard cumulative normal distribution function, θ is the median value of fragility function, β is the standard deviation of $\ln IM$.

The density function of the likelihood function at $IM = x_i$ can be obtained through the fragility function defined by formula (2) for the structure that exceeds the different damage states:

$$L = \phi \left(\frac{\ln \left(\frac{x_i}{\theta} \right)}{\beta} \right) \quad (3)$$

Accordingly, the density function of the likelihood function at $IM = x_i$ for the structure that not exceeds the different damage states can be obtained by formula (4):

$$L = 1 - \phi \left(\frac{\ln \left(\frac{x_i}{\theta} \right)}{\beta} \right) \quad (4)$$

Because of the different damage states have different median θ and logarithmic standard deviations β , the likelihood function is defined as:

$$L = \prod_{i=1}^N \phi \left(\frac{\ln \left(\frac{x_i}{\theta} \right)}{\beta} \right)^{t_i} \times \left(1 - \phi \left(\frac{\ln \left(\frac{x_i}{\theta} \right)}{\beta} \right) \right)^{1-t_i} \quad (5)$$

N is the total number of statistical structures, x_i represents the PGA corresponding to the i -th structure. t_i is Bernoulli event, if it reaches the damage state $t_i = 1$ or $t_i = 0$, θ , β is dual-parameterization: the median and logarithmic standard deviations of fragility curve, respectively.

The median θ and logarithmic standard deviations β of the two parameters are calculated by formula (6):

$$\frac{\partial \ln L}{\partial \theta} = \frac{\partial \ln L}{\partial \beta} = 0 \quad (6)$$

3. Structural fragility analysis method

3.1 Collect and statistic earthquake damage data

As a large prototype experimental field, the real earthquake damage experience of engineering structure is an important pillar for earthquake engineering development and human's understanding about how to resist earthquake disasters is also largely from this. Sichuan province has complex geological structure, frequent earthquake activity, various types of building structure, and many earthquakes with magnitude over 7 occurred in history, especially, the 2008 Wenchuan 8 earthquake and the 2013 Lushan earthquake caused



serious economic losses and casualties. Sun Baitao [20] used different classification and refinement method to study the regional north-south seismic belt of various types of building construction and the distribution characteristics of seismic capacity, and Sichuan Province as an example, which can be divided into I regions: the western region, II region, III region: North Central Region: south central region. Based on the investigation of 23964 buildings in 204 zones of 6 cities and the historical earthquake damage experience of Wenchuan earthquake, the seismic fragility matrix of different types of structures in different regions is given by the intensity of the earthquake.

3.2 Fit fragility curve of the civil structure as an example

Firstly, through the calculation of the fragility matrix of different types of structure which beyond different damage state, get different types of exceedance probability of fragility matrix based on intensity, and then fit the structure probability fragility curve, use civil structures in the III District of Sun Baitao etc. [22] statistics are given as an example. The fragility matrix of the civil structure in the III zone is shown in Table 2. The damage ratio of the civil structure in the III zone under different intensities is visually expressed by means of scatter plots and histograms respectively refer to Figure 3.

Table 2 Damage probability matrix of civil structure in III area (%)

Intensity	None DS_0	Slight DS_1	Moderate DS_2	Extensive DS_3	Collapse DS_4
VI	57	29	12	2	0
VII	23	31	25	17	4
VIII	8	10	33	30	19
IX	1	3	19	27	50
X	0	0	3	10	87

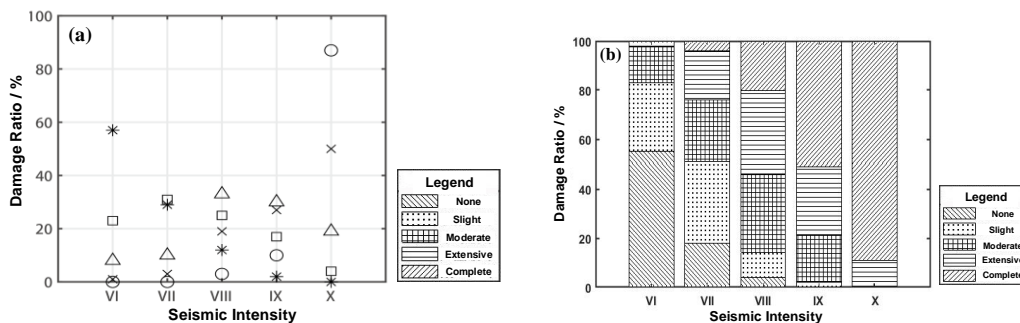


Figure 3 Scatter diagram and histogram of damage probability matrix of civil structure in III zone

Through the fragility matrix based on intensity of civil structure in table 2, we can obtain the exceedance probability matrix, see table 3:

Table 3 Exceedance Probability Matrix of civil structure III area (%)

Intensity	Slight LS_1	Moderate LS_2	Extensive LS_3	Collapse LS_4
VI	43	14	2	0
VII	77	46	21	4
VIII	92	82	49	19
IX	99	96	77	50
X	100	100	97	87

Through the maximum likelihood estimation method, combined with table 1, using the MATLAB write code for formula (5) and (6), we can obtain the civil structure probability, and calculate two parameters the median θ



and logarithmic standard deviation β of the lognormal distribution function. As shown in table 4:

Table 4 Median and standard deviation of the exceedance probability of the civil structure in different limit states in III area

Parameters	Slight LS_1	Moderate LS_2	Extensive LS_3	Collapse LS_4
θ	0.073	0.137	0.263	0.482
β	0.839	0.706	0.779	0.711

Plot the exceedance probability fragility curve based on the ground motion parameter PGA in the III zone by θ and β :

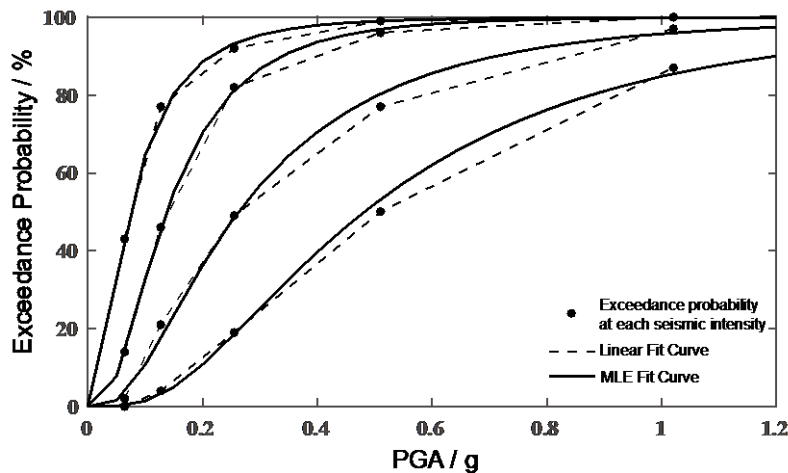


Figure 4 Exceedance probability of civil structure fragility in III zone

The discrete points shown in Figure 4 are obtained by computing the actual earthquake damage investigation, and have a good match with the fitting fragility curve in Figure 4, we can draw a conclusion that the theory of maximum likelihood estimation analysis is reasonable and accurate. Thus, by fitting the fragility curve, we can obtain the exceedance probabilities of different damage states corresponding to different PGA (units: $1g=9.8m/s^2$), as shown in table 5:

Table 5 Exceedance damage probability matrix of civil structure based on PGA

PGA/g	LS_1	LS_2	LS_3	LS_4
0.01	0.89	0.01	0.00	0.00
0.10	64.66	32.78	10.72	1.35
0.20	88.55	70.40	36.28	10.81
0.30	95.41	86.66	56.74	25.26
0.40	97.88	93.55	70.51	39.67
0.50	98.91	96.67	79.55	52.06
0.60	99.40	98.18	85.54	62.10
0.70	99.65	98.96	89.58	70.01
0.80	99.79	99.38	92.36	76.19
0.90	99.86	99.62	94.30	81.01
1.00	99.91	99.76	95.69	84.76

In order to get the probability of various damage states, P_{DS_k} represent the probability under the condition of DS_k . P_{LS_k} represent the probability exceeding the limit of failure. The damage probability under different damage states



is obtained by the following formula:

$$P_{DS0}(x_i) = 1 - P_{LS1}(x_i) \quad (7)$$

$$P_{DSk}(x_i) = P_{LSk}(x_i) - P_{LSk+1}(x_i) = \Phi \left(\frac{\ln \left(\frac{x_i}{\theta_{LSk}} \right)}{\beta_{LSk}} \right) - \Phi \left(\frac{\ln \left(\frac{x_i}{\theta_{LSk+1}} \right)}{\beta_{LSk+1}} \right) \quad (k = 1, 2, 3) \quad (8)$$

$$P_{DS4}(x_i) = P_{LS4}(x_i) \quad (9)$$

According to the results of exceedance probability fragility curves of civil structures, the fragility matrix under different failure states can be obtained by formula (7), (8) and (9), combine with Table 5, as shown in Table 6:

Table 6 Damage probability matrix of civil structure based on PGA

PGA/g	DS_0	DS_1	DS_2	DS_3	DS_4
0.01	99.11	0.88	0.01	0.00	0.00
0.10	35.34	31.88	22.06	9.37	1.35
0.20	11.45	18.14	34.13	25.46	10.81
0.30	4.59	8.75	29.92	31.48	25.26
0.40	2.12	4.33	23.04	30.85	39.67
0.50	1.09	2.25	17.11	27.49	52.06
0.60	0.60	1.22	12.64	23.44	62.10
0.70	0.35	0.69	9.38	19.57	70.01
0.80	0.21	0.41	7.02	16.16	76.19
0.90	0.14	0.25	5.31	13.30	81.01
1.00	0.09	0.15	4.06	10.93	84.76

By the formula (7), (8), (9) plot the fragility curves based on PGA, as shown in Figure 5, then we can visually see the change trend of damage probability with the increase of PGA under various damage states, By this we can accurately assess the damage state of the structure.

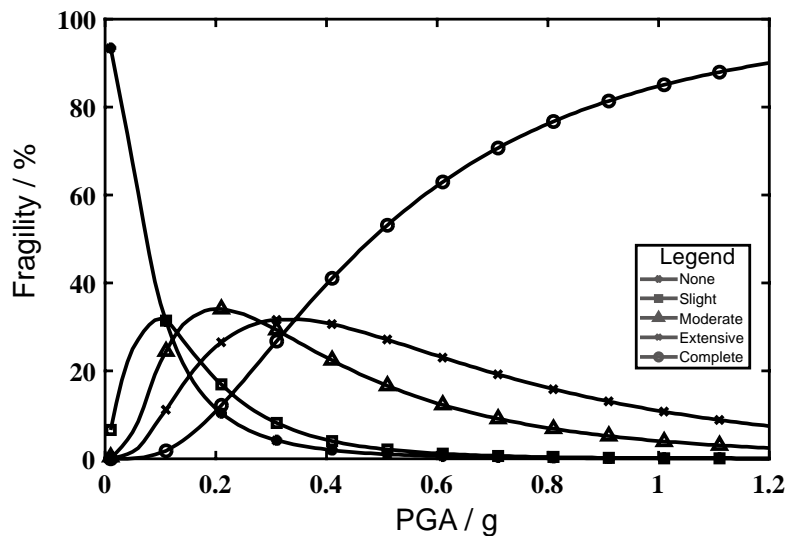


Figure 5 Fragility curve of civil structure



3.3 Fragility curve of various building structures

It is consistent with the calculation of fragility curve of civil structure in III area. By using the curve method of maximum likelihood, and consider the correspondence between intensity and peak acceleration of ground motion, this paper will provide a two parameters θ and β for the fragility function of different structures under different damage states, then plot the fragility curves. As show in Table 7:

Table 7 The two parameters based on PGA in different damage states of different structures: Median θ (unit: m/s) and lognormal standard deviation β

Zone & Structure types		LS_1		LS_2		LS_3		LS_4	
		θ	β	θ	β	θ	β	θ	β
West Region (I)	Mud-Wood Structures	0.070	0.696	0.126	0.675	0.239	0.732	0.467	0.694
	Brick-Wood Structures	0.083	0.703	0.171	0.739	0.346	0.678	0.784	0.720
	Through Type Timber Frame Structures	0.102	0.669	0.212	0.726	0.519	0.848	1.730	0.926
	Stone Structures	0.092	0.689	0.153	0.547	0.238	0.613	0.491	0.845
North-Central Region (II)	Mud-Wood Structures	0.073	0.839	0.137	0.706	0.263	0.779	0.482	0.711
	Brick-Wood Structures	0.090	0.732	0.185	0.737	0.365	0.687	0.808	0.707
	Through Type Timber Frame Structures	0.109	0.666	0.224	0.696	0.558	0.854	1.750	0.851
South-Central and East Region (III)	Mud-Wood Structures	0.073	0.839	0.137	0.706	0.263	0.779	0.482	0.711
	Brick-Wood Structures	0.091	0.723	0.190	0.742	0.371	0.688	0.825	0.694
	Through Type Timber Frame Structures	0.105	0.673	0.213	0.701	0.542	0.868	1.696	0.853
High-Rise Buildings (High-Code)		0.298	0.599	0.737	0.624	1.365	0.476	1.158	0.077
High-Rise Buildings (Moderate-Code)		0.277	0.531	0.634	0.595	1.276	0.514	1.134	0.075
Reinforced Concrete Structures (High-Code)		0.267	0.785	0.540	0.548	0.841	0.506	1.629	0.558
Reinforced Concrete Structures (Moderate-Code)		0.228	0.738	0.481	0.584	0.696	0.466	1.527	0.607
Fortified Masonry Structures (High-Code)		0.176	0.861	0.357	0.734	0.602	0.600	1.624	0.740
Fortified Masonry Structures (Moderate-Code)		0.139	0.845	0.292	0.709	0.510	0.608	1.372	0.828
Unfortified Masonry Structures		0.119	0.770	0.223	0.623	0.396	0.660	1.031	0.786

Based on the fragility curve, the fragility curves of different types of structures in three districts of Sichuan province are plotted, as shown in figure 6:

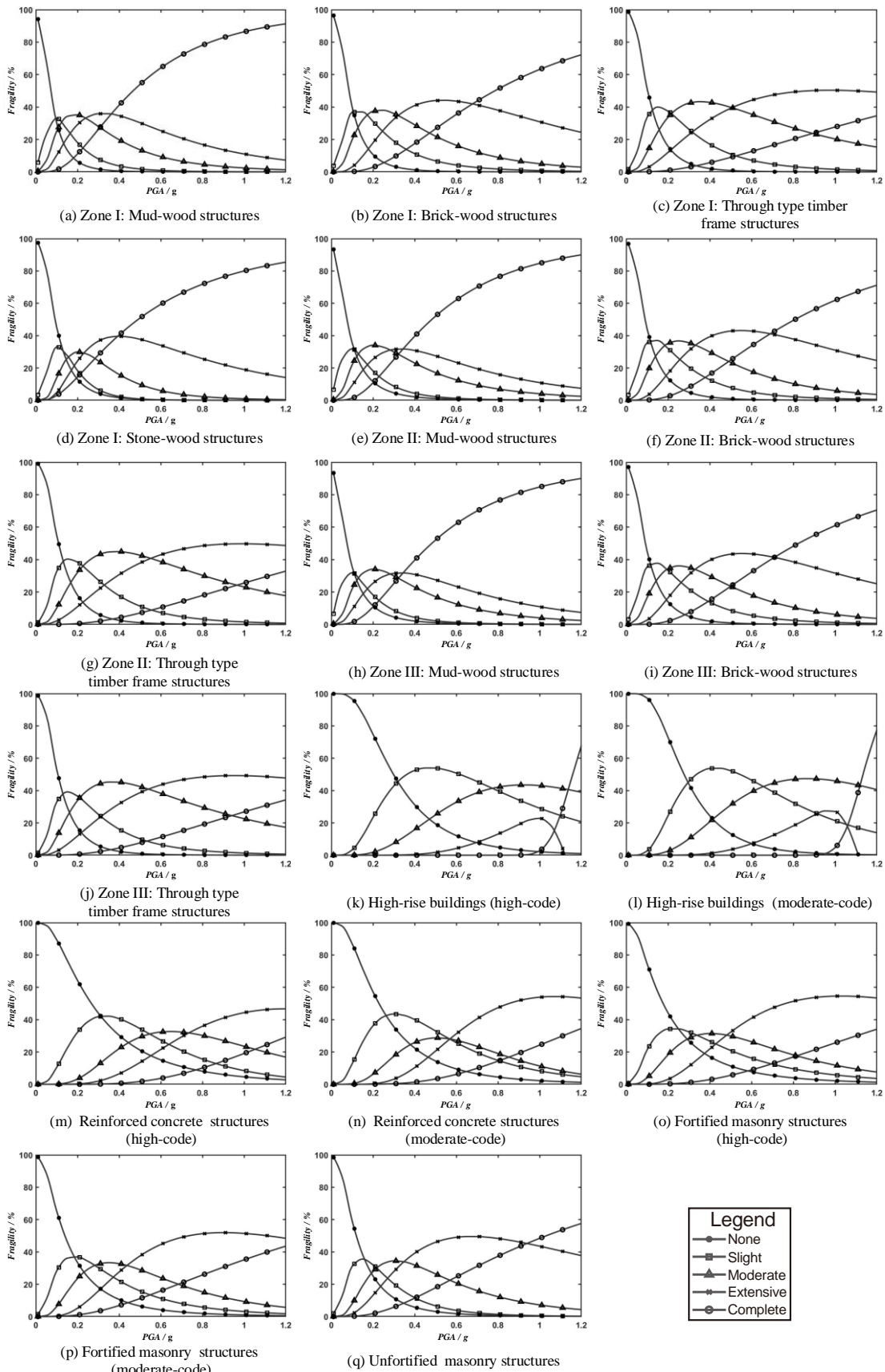


Figure 6 Fragility curves of various types of structure for Sichuan province



4. Conclusion

(1) The curve fitting approach to obtain fragility curve of building fragility matrix presented in this paper, based on the correspondence between intensity and ground motion parameters, adopted the maximum likelihood estimation method, the fragility matrix or the actual damage statistics of the existing building is transformed into a two-parameter fragility curve, then the loss assessment of building structure can be refined. This method makes up for the deficiency of fragility curve based on intensity, provide a simple way to get fragility curve, especially for the structural types which are difficult to simulate, for example civil building, brick building and so on.

(2) Take Sichuan province building damage matrix as an example, through the relationship between intensity and ground motion parameters based on Chinese earthquake intensity scale, we can get the seismic fragility parameters of various building structure types and provide basic data for earthquake damage assessment of buildings in Sichuan Province rely on ground motion parameters. It should be noted that in the actual engineering applications, it is very important to select the appropriate intensity-ground motion parameter for different regional characteristics.

(3) Verified the reliability and applicability of the fragility matrix curve method proposed in this paper, further research work in the application of this method to the existing vulnerability matrix or curve calibration and improvement, to better serve the rapid earthquake hazard assessment, risk assessment and estimation of insurance. The method can be used to calibrate and perfect the existing fragility matrix or curve in the further research work, and has a better apply at the rapid assessment, risk assessment and insurance estimation of earthquake disaster.

(4) The seismic intensity is evaluated based on the macroscopic damage, which is not an accurate scale. In addition, there are many kinds of seismic intensity scales all over the world, and which are different from each other. With the development of earthquake observation technique, it's a bigger trend to utilize ground motion parameters to evaluate the seismic intensity. However, it is worth to consider the uncertainty relationship between seismic intensity and ground motion parameters when use the curve fitting approach mentioned in this paper to obtain fragility curve from building damage matrix.

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