IMPACT OF TIME-DEPENDENT EARTHQUAKE RECURRENCE MODELLING ON PROBABILISTIC SEISMIC HAZARD ANALYSIS

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Abstract

The (re)insurance industry has traditionally used a memoryless, time-independent (i.e., Poissonian) model for representing earthquake recurrence in seismic loss (risk) calculations. However, time-dependent renewal models, which account for the time elapsed since the last event, are more appropriate for modelling the long-term characteristics of cyclical mainshock occurrences in fault-based seismic hazard assessments. This study first reviews the main features and advantages of some of the most used time-dependent models for mainshock recurrence, and provides a critical discussion on their calibration and possible combination. A simple case-study fault is used to quantify the changes in seismic hazard estimates resulting from the use of time-dependent Brownian Passage-Time (BPT) models instead of the conventional Poisson process. The considered fault is the Ohariu Fault in New Zealand, which is one of the major sources of earthquake hazard for the city of Wellington. BPT model parameters are calibrated using the maximum likelihood estimation (MLE) method together with paleoseismic data published in the literature. Results from this study show that the use of a time-dependent BPT model can lead to a significant over- or under-estimation of the seismic hazard compared to the time-independent Poisson model, depending on the ratio between the time elapsed since the last event and the mean recurrence time of the fault. The simple single-fault case study also highlights the potential need for a combination of time-dependent models in actual earthquake risk models, since the single BPT model produces unrealistically low seismic hazard estimates for time periods in the immediate aftermath of an earthquake occurrence.

Keywords: Catastrophe Risk Modelling, Time-Dependent Probabilistic Seismic Hazard Analysis, Earthquake Recurrence, Brownian Passage-Time Model

1. Introduction and Motivation

Probabilistic catastrophe (CAT) risk models are becoming increasingly popular tools for estimating potential loss due to natural hazards. These models incorporate advanced scientific understanding of highly complex physical phenomena related to natural hazards and their effect on the built environment. This paper focuses on earthquake CAT models, which are used for calculating seismic losses within the (re)insurance industry. These models follow a modular structure [1] and involve: (a) conducting numerical simulations of earthquake occurrence and ground-motion intensity prediction (within a hazard module); (b) assessing damage (within an exposure/vulnerability module); and (c) calculating seismic losses (within a loss/financial module). One of the key outputs of an earthquake CAT model is the loss exceedance probability (EP) curve for a building/infrastructure portfolio of interest, within a prescribed period of time.

The hazard module of an earthquake CAT model relies on models and tools from probabilistic seismic hazard analysis (PSHA) ([2], [3]). PSHA is a probabilistic framework ([1], [2], [3]) for quantifying the probability of exceeding various ground-motion levels at a site (or an ensemble of sites) in a given time period. One of the key inputs of PSHA is an earthquake rupture model, which describes source geometry, the magnitude probability distribution and the probability of earthquake occurrences in time. Earthquake recurrence in these models is typically represented as a homogeneous Poisson process, which assumes that the inter-arrival times between two events are independent, identically distributed exponential random variables [3]. This implies that the occurrence of events in a specific observation time window (W) is only dependent on the extent of the time frame of interest and not on the time elapsed since the last event (t_e), i.e., there is no memory of past earthquakes.

However, it is now well recognized in the literature that earthquakes tend to interact with each other in time and space (e.g. [4] and [5]). Hence, time-independent approaches are not able to fully capture actual earthquake occurrence and the associated hazard/risk. In particular, they do not properly model:

(i) the long-term time-dependency of mainshocks on specific fault segments, i.e., the fact that soon after a segment-rupturing earthquake, the probability of having a similar magnitude earthquake might be low. This phenomenon is explained using elastic rebound theory [6]. During the so-called seismic cycle, faults accumulate elastic strain energy and release it when the internal strength of the fault rocks is reached. After an earthquake, the accumulated/stored energy is assumed to be at or near zero;

(ii) the interaction between adjacent faults, i.e., the fact that an earthquake on one fault can result in a tectonic loading change in surrounding faults, which may delay or promote the occurrence of other events on those faults;

(iii) the hazard produced by foreshocks and aftershocks clustered in both space and time. This is not included in the majority of the PSHA studies but that can affect the short-term hazard significantly [7].

The aim of this study is to first review the main features and advantages of some of the most popular long-term time-dependent models (point (i) above) relative to the time-independent homogenous Poisson process, and then discuss their calibration and possible combination. A case-study of the Ohariu Fault in New Zealand is used to quantify the changes in seismic hazard that result from the use of time-dependent Brownian Passage-Time (BPT) models instead of a conventional Poisson model. BPT model parameters are calibrated using the maximum likelihood estimation (MLE) method together with paleoseismic data published in literature.

2. Long-Term Earthquake Recurrence Models

Although suitable for modelling the recurrence of earthquakes on several (independent) sources [8], the homogeneous Poisson process is not as appropriate for fault-based hazard assessments (as described previously). Various 'renewal' (i.e. time-dependent) models have been proposed to capture more accurately the nature of the inter-event time between consecutive mainshocks occurring on the same fault segment ($\tau_i = t_i - t_{i-1}$, where t_i is the occurrence time of the i^{th} mainshock earthquake, see Fig. 1). In a renewal process, the expected time before the next event does not depend on any of the details of the last event except its time of occurrence. Renewal processes incorporate the hypothesis of a characteristic mainshock earthquake, which is meant to represent individual faults or fault segments that tend to produce earthquakes of about the same magnitude ([9], [10]). The statistical distribution of time-dependent models is generally defined by their probability density function (PDF), $f(\tau)$, and the corresponding cumulative density function (CDF), $F(\tau)$.

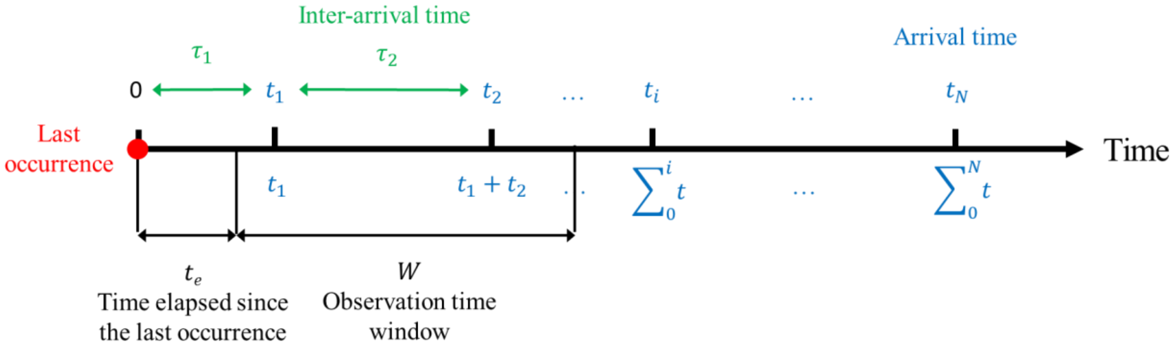


Fig. 1 – Sketch of the inter-arrival time between consecutive events occurring on the same fault segment (τ), the time elapsed since the last event on the segment (t_e), and the time window of observation (W).

The hazard function $h(\tau)$, which is widely used in the analysis of the failure of engineering systems, is a convenient way of comparing alternative models of inter-arrival times. It is defined as the instantaneous rate of occurrence of an event:

$$h(\tau) = \frac{f(\tau)}{1 - F(\tau)} \quad (1)$$

The hazard function multiplied by an infinitesimal increment of time (dt) represent the conditional probability of occurrence of an earthquake in a small increment of time ($\tau; \tau + dt$), given that no event occurred until time τ (i.e., τ is equivalent to t_e in Fig. 1) [8].

The trend of the hazard function can provide useful insights into the physics of the mainshock earthquake generation process under investigation [11]. For instance, a monotonically increasing trend of the function (i.e., a positive derivative of $h(\tau)$ with respect to τ) implies a growing probability of occurrence of a mainshock as time elapses since the last event. A monotonically decreasing trend of the hazard function implies that the probability of mainshock occurrence decreases as time elapses since the last event.

Some of the most popular long-term recurrence models in PSHA (including the homogeneous Poisson process) are now described, including their major features and possible applications. To facilitate comparison and discussion, we also report for each model the corresponding functional form of the PDF, the mean recurrence time between consecutive events (μ) and the coefficient of variation (CoV, i.e. ratio between the standard deviation of the recurrence times and μ).

2.1 Homogeneous Poisson process

An earthquake generation process is called a homogeneous (or stationary) Poisson process if the seismic source (e.g., fault) of interest has the following characteristics: (a) independent increments, i.e., the numbers of events occurring in two disjoint time intervals are independent of each other; (b) stationarity, i.e., the probability distribution of the number of events falling in a time interval only depends on the length of the time interval; and (c) simplicity, i.e., there is never an occasion that two or more events occur simultaneously. It can be shown that a homogeneous Poisson process has the following properties: (i) the number of events occurring in an interval of length W has a Poisson (discrete) distribution; (ii) the time τ between two consecutive events is exponentially distributed as follows:

$$f(\tau) = \lambda\, exp(-\lambda\tau)$$
$$\mu = 1/\lambda \quad (2)$$

where λ is the only parameter to be specified and is the long-term rate of events in a unit interval of time.

Using Eq. (1) and (2), it can be proven that $h(\tau) = \lambda$. This means that the probability of occurrence of earthquakes in a future small increment of time remains constant regardless of the elapsed time since the last event or its magnitude. The homogeneous Poisson process is extensively used to model seismicity in homogeneous seismotectonic areas, which include many different independent faults.

2.2 Weibull distribution

The Weibull distribution has been most used to describe the fatigue failure of rocks. Hagiwara (1974) [12] proposed the use of this distribution based on the assumption that the Earth-crust rupture can be modelled as a fatigue failure. The PDF of the Weibull distribution is:

$$f(\tau) = \frac{b}{a}\left(\frac{\tau}{a}\right)^{b-1} exp(-(\tau/a)^b)$$
$$\mu = a\Gamma\left(1+\frac{1}{b}\right) \quad CoV = \sqrt{\Gamma\left(1+\frac{2}{b}\right)\Big/\Gamma\left(1+\frac{1}{b}\right) - 1} \tag{3}$$

where b is the shape parameter (roughly proportional to $1/CoV$) and a is the scale parameter of the probability distribution. The CoV has a marked effect on the shape of the function. A Weibull distribution with a $CoV = 1$ (i.e., $b = 1$) reduces to an exponential function with constant hazard function and rate equal to $1/a$ (i.e. the process is a random time-independent process). If $CoV > 1$ (i.e., $b < 1$), $f(\tau)$ results in a decreasing hazard function (i.e. short-term clustering), while for $CoV < 1$ (e.g. $b > 1$) the hazard function is increasing.

2.3 Lognormal distribution

The lognormal distribution has been used to model mainshock recurrence intervals since its first appearance in Nishenko and Buland (1987) [13], where it was applied to model the recurrence of mainshocks along specific faults and subductions zones in the circum-Pacific region. It was found that the lognormal distribution fitted historic and geologic recurrence data better than the Gaussian or the Weibull distributions. The Working Group on California Earthquake Probabilities (WGCEP) 1995 [14] and following studies used the lognormal distribution in computing earthquake probabilities in California for time-dependent seismic hazard analysis. The PDF of the lognormal distribution is:

$$f(\tau) = \frac{1}{\tau\sigma^*\sqrt{2\pi}} exp\left(-\frac{(log(\tau)-\mu^*)^2}{2\sigma^{*2}}\right)$$
$$\mu = exp\left(\mu^* + \frac{1}{2}\sigma^{*2}\right) \quad CoV = \sqrt{exp(\sigma^{*2}) - 1} \tag{4}$$

where σ^* is the shape parameter (i.e. the standard deviation of the logarithm of the recurrence times) and μ^* is the scale parameter (i.e. the mean logarithmic recurrence interval).

The hazard function is zero for small values of τ, which leads to a null probability of occurrence of a mainshock event for a short period after an earthquake. Following the peak of $f(\tau)$, the lognormal distribution hazard function decreases asymptotically to zero for very high τ/μ ratio (i.e. much higher than 10).

2.4 Brownian Passage Time model

The Brownian Passage-Time (BPT) model was proposed by Ellsworth et al. 1999 [15] and Matthews et al. 2002 [16] to model mainshock recurrence on a single fault. The BPT model can be considered a "physically-motivated" renewal model for earthquake recurrence, as it represents the link between elastic rebound theory [6] and statistical distributions commonly used to model earthquake recurrence.

Elastic rebound theory was developed by Reid in 1910 [6] and has been extensively used to describe the main characteristics of stress and the strain accumulation processes on a fault (i.e. seismic cycle). Assuming that elastic rebound theory is able to describe the complex mechanisms behind earthquake generation (which is yet to be fully proven [10]), the BPT model builds upon the so-called Brownian relaxation oscillator (BRO), which is a superposition of a constant tectonic loading and a Brownian perturbation.

The BRO model belongs to the family of stochastic renewal processes identified by four parameters: the mean loading (drift) rate, the perturbation rate, the ground state, and the failure state. However, it can be proven that recurrence times due to the BRO are described by an inverse Gaussian distribution, characterized by only two parameters: the mean recurrence time (μ) between events and the aperiodicity (α) of the mean recurrence time, which is equivalent to the coefficient of variation ($\alpha = CoV$). The PDF of the BPT model is:

$$f(\tau) = \sqrt{\frac{\mu}{2\pi\alpha^2\tau^3}} exp\left(-\frac{(\tau-\mu)^2}{2\mu\alpha^2\tau}\right) \quad (5)$$

The mean recurrence time between events (μ) is the scale parameter in the BPT formulation, which rescales the distribution in time. The aperiodicity (α) is the shape parameter (i.e. it modifies the shape of the distribution) and it represents a dimensionless measure of the irregularity in the event sequence. A perfectly periodic sequence has an $\alpha = CoV = 0$, while the BPT tends to a random (i.e., Poissonian) process as the α (CoV) increases.

The hazard function of the BPT distribution is always zero at $\tau = 0$, which implies that after a mainshock the cycle is returned to its starting state and the probability of having another event is zero. The hazard function increases and reaches the peak value; then, it decreases asymptotically to $1/(2\mu\alpha^2)$. The asymptotic decrease implies that when τ is large, the probability of having a new earthquake becomes time-independent, i.e., it is a constant that depends only on the properties of the fault itself.

2.5 Discussion

Fig. 2 compares the mainshock recurrence models described in Section 2 (i.e., Poisson, Weibull with a shape parameter higher than 1, Lognormal and BPT) in terms of $f(\tau)$ and $h(\tau)$ for $\mu = 250yr$ and $CoV = 0.3$ (first row), $CoV = 0.5$ (second row), $CoV = 0.7$ (third row). The independent variable used to produce Fig. 2 was normalized using μ to ease the discussion.

At a null time elapsed since the last event (i.e., $\tau = 0$), the Poisson process produces a non-zero probability of occurrence, while the lognormal distribution, the BPT and the Weibull distributions with shape parameters larger than 1 have zero probability of occurrence of next earthquakes. This is conceptually in agreement with the interpretation of elastic rebound theory and the characteristic earthquake model proposed by Schwartz and Coppersmith (1984) [9] (i.e. when the fault segment ruptures, it experiences a near-complete stress drop). The left panels in Fig. 2 show $f(\tau)$ for the described recurrence models. When $\tau/\mu < 0.6, 0.5, 0.4$ for $CoV = 0.3, 0.5, 0.7$ respectively, the lognormal distribution and the BPT curves are below the Weibull distribution, which has a steep probability increase (i.e. the probability of new events increases rapidly with time). The mode of all time-dependent models shown in Fig. 2 (left plots) is located to the left of the mean of the distributions ($\tau/\mu = 1.0$), which means that most of the earthquakes occur at times less than the average time of occurrence. It is also worth noting that the BPT and lognormal $f(\tau)$ are very close to each other for $\tau/\mu < 2.0$, which is often the case for actual known faults.

One of the major differences between BPT, lognormal distribution and Weibull is the "tail behavior" of the corresponding $h(\tau)$ (right panels in Fig. 2). The BPT $h(\tau)$ tends to a constant value at large values of τ, the Weibull with $CoV < 1$ (shape parameter higher than 1) is characterized by a monotonically increasing $h(\tau)$, while the lognormal distribution $h(\tau)$ decreases asymptotically to zero. The tail behavior is indeed the reason why most PSHA studies in the literature have given less weight (or none at all) to the Weibull or lognormal than to the BPT model. Most of the existing studies suggest that the hazard due to a single fault should tend to a constant positive value rather than to zero (lognormal) or to infinity (Weibull) (e.g. [11], [17]).

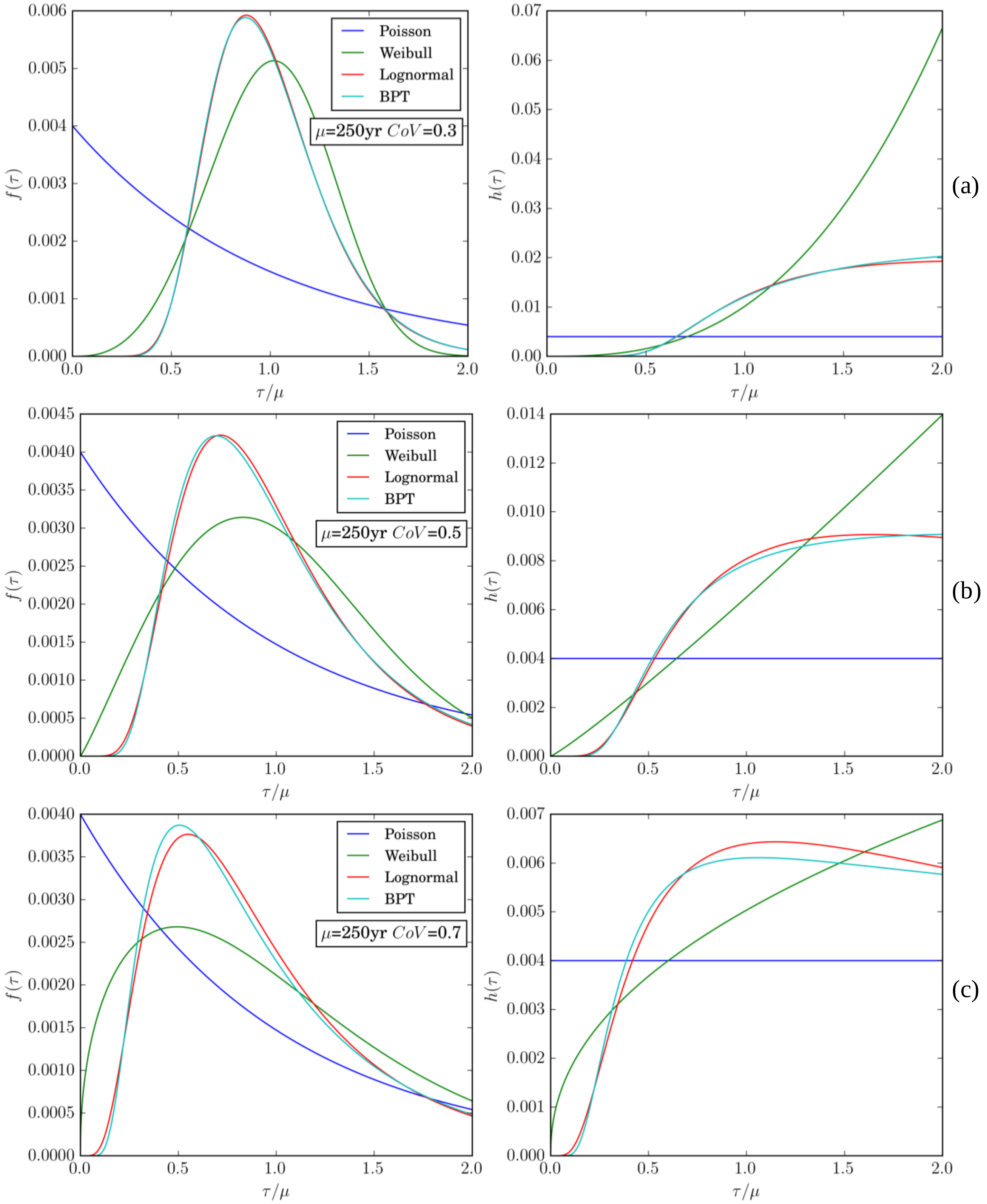


Fig. 2 – Comparison between the Poisson, Weibull (shape parameter higher than 1), Lognormal and BPT models in terms of PDF, $f(\tau)$ (left) and hazard function, $h(\tau)$ (right) for $\mu = 250yr$ and $CoV = 0.3$ (a), $CoV = 0.5$ (b) and $CoV = 0.7$ (c).

3. Model Calibration

When several mainshock recurrence models are considered within a PSHA model, a logic tree procedure is usually used to weight the different models according to expert opinion. Different parameters for each model are considered as additional logic tree branches [10], to incorporate epistemic uncertainties. The non-trivial choice of suitable recurrence models and model calibration are also sources of uncertainty. The calibration procedure depends on available data.

If only a few geological and geomorphology data are available (e.g. long-term average slip rate, average event slip displacement), the "direct method" [14] is usually used to calibrate the mean recurrence time:

$$\mu = \frac{\overline{D}}{V} \tag{6}$$

where $\overline{D}$ is the average earthquake slip displacement on the segment and V is the long-term average slip rate. Often, information on $\overline{D}$ is not available and the "segment total seismic moment rate conservation" [19] is instead used:

$$\mu = \frac{M_0}{G\,VA} \tag{7}$$

where M_0 is the seismic moment (depending on the moment magnitude of the characteristic earthquake), G is the earth shear modulus (usually taken equal to $30GPa$), V is long-term average slip rate, and $A = H \cdot L$ is the rupture area (H is the seismogenic thickness of the brittle crust and L is the length of the segment). By comparing Eq. (6) and (7), it can be seen that the only difference is the assumption that $\overline{D} = M_0/(\mu A)$. In both cases above, the shape parameter (e.g. aperiodicity for BPT or the standard deviation for the lognormal distribution) is often assumed to be equal to a set of commonly used values (e.g. 0.3, 0.5, 0.7 for the aperiodicity of the BPT model). The associated epistemic uncertainties are usually dealt with in a logic tree as different mutually exclusive branches weighted from expert judgement. When information on the range of slip rate and maximum magnitude is available, the variance σ_μ^2 applied to the mean recurrence time μ can be formally calculated with the method proposed by [20] and applied in the Fish Code ([21]).

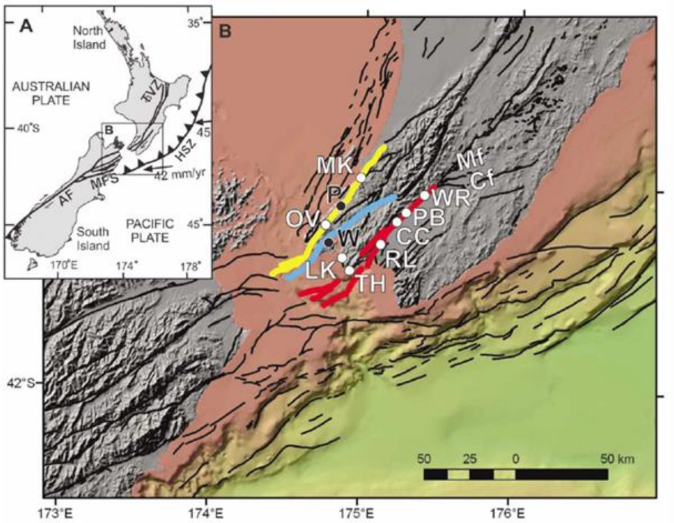
When historical data are available (e.g. past event dates), the adjusted maximum likelihood estimation (MLE) method [15] can be used for model calibration: the parameters are estimated through a given set of observations. Ellsworth et al. (1999) [15] suggests that at least five historical intervals (hence, six past events) should be present for a meaningful result with the MLE method.

Often, a mixture of historical and paleoseismic (trench) data are available. In this case, a Monte Carlo-based MLE procedure is possible [21]. This method primarily consists of multiple realizations of possible recurrence time from the paleoseismic distributions of potential event dates; for each realization, the MLE method is applied, and the resulting parameter values are stored in the form of histograms or heatmaps. The final pair of parameters for the time-dependent model according to this method is the one corresponding to the mode of the 2D frequency distribution (example in Fig. 4).

For "mature" faults where both probabilistic geological/geomorphology data and historical/paleoseismic data are available, several methods exist in the literature to capture the variability and information of both datasets (which are often not consistent). The most popular methods are the Parsons (2008) [22] method, which relies on Monte Carlo simulations, and Bayesian frameworks ([17], [23]), which mix each probability distribution (with different pairs of parameters) with a non-null likelihood of being the "correct" one based on the available data. (Note that Bayesian calibration methods are also useful for combining different time-dependent models, as they do not rely on expert opinion for assigning model weights).

4. Single-Fault Case Study Application

This case study demonstrates an example calibration of a BPT model with the Monte Varlo MLE method proposed by [21], using paleoseismic data from the Ohariu Fault segment in New Zealand [24], which extends through Porirua city and within 5 km of central Wellington city (Fig. 3). Along with the Wellington-Hutt Valley segment of the Wellington Fault and the Wairarapa Fault, the Ohariu Fault represents one of the major sources of hazard and loss in Wellington. The paleoseismic data for the Ohariu Fault from [24] are shown in the table of Fig. 3. The results in terms of seismic hazard were compared with the Poisson model, which was also calibrated with the Montecarlo MLE method.



Event	Time distribution of past ruptures for the Ohariu Fault (years before 2010)
Event I	Uniform (1060, 1110)
Event II	Trapezoidal (3320, 3425, 4470, 4870)
Event III	Trapezoidal (4470, 4870, 5110, 5330)

Fig. 3 – Figure adapted from [24]. Tectonic setting of New Zealand (inset left). Active faults of central New Zealand (main figure): the southern Wairarapa Fault (red), the Ohariu Fault (yellow), and the Wellington-Hutt Valley segment of the Wellington Fault (blue). Timing of past events on the Ohariu Fault (table right).

The Monte Carlo MLE method proposed by [21] consists of: (a) establishing the probability distribution of past events; (b) drawing a number of random realizations of potential recurrence times; (c) applying the MLE method to estimate the mean recurrence time μ and the aperiodicity α (CoV) of the BPT model; (d) discretizing the possible values of μ (between the minimum and the maximum of the realizations) and CoV (assumed between zero and one) into discrete bins; and (e) producing a frequency plot (e.g. heatmap) showing the number of times the estimated μ-CoV pairs fall into a specific bin (Fig. 4). Even though the statistical significance of the results is questionable due to the limited number of past events, the mode ($\mu = 1987yr$ and $CoV = \alpha = 0.23$) is used hereinafter. The Poissonian rate was calculated as $\lambda = 1/1987yr = 5.03 \cdot 10^{-4}$.

The OpenQuake software [25] was used to carry out the single-fault PSHA calculations. The Ohariu Fault was modelled as a "Non-parametric fault" [26], which assumes rupture of the entire fault plane for any event occurrence. According to [18] and published files (https://github.com/nzshm/nshm-2010 last accessed 30th Jan 2020), rupture of the entire Ohariu Fault would produce a magnitude $M_w = 7.2$ characteristic earthquake, which is used herein. The occurrence probabilities for the BPT model were computed assuming $W = 1yr$ and using the calibrated parameters discussed above. 100,000 one-year long stochastic catalogues were generated through Monte Carlo simulation starting from 2010 and assuming different t_e/μ ratios (i.e. $0.5, 0.6, 1.0, 2.0$). The actual t_e/μ ratio of the Ohariu Fault is somewhere between 0.53 and 0.56 (i.e. the last event occurred between 1060 and 1110 years before 2010). Poissonian occurrence probabilities were calculated through the closed form solution given by Eq. (8):

$$\Pr(k) = \frac{\lambda^k \exp(-\lambda)}{k!} \quad (8)$$

where λ is the Poissonian rate and k is the number of events. The calculations used the McVerry et al. 2006 [27] ground-motion model, which is used for the New Zealand seismic hazard map [18]. Several PSHA studies were performed for different BPT occurrence probabilities (i.e. different values of t_e/μ for the stochastic simulations). The resulting hazard curves for Wellington (Longitude: 174.78°, Latitude = -41.29°) are shown in Fig. 5, in terms of peak ground acceleration (PGA) and spectral acceleration at 1s period (S_{A1s})

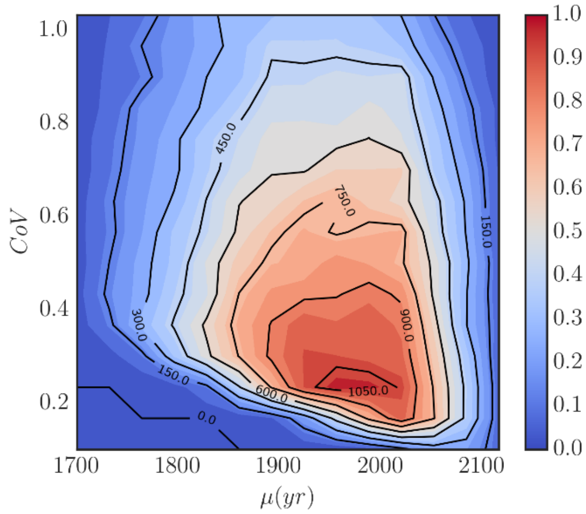


Fig. 4 – Resulting heatmap of the Montecarlo MLE method for the estimation of the BPT model parameters (mean recurrence time μ, aperiodicity α).

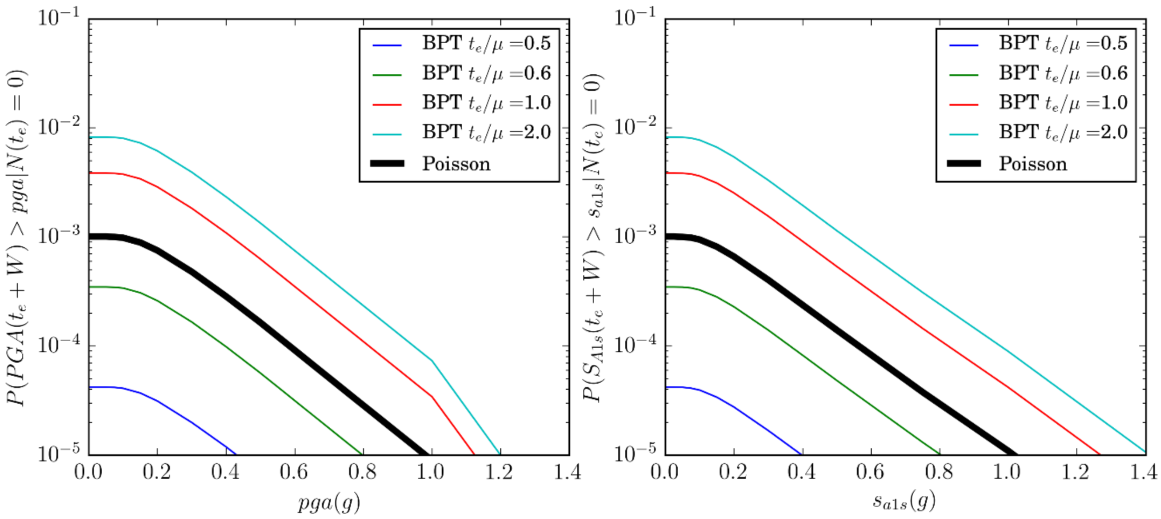


Fig. 5 – Single-fault-based hazard curves for PGA and S_{A1s} at Wellington due to the Ohariu Fault. Time-independent (Poisson) results are shown along with time-dependent (BPT) hazard, assuming $t_e/\mu = 0.5, 0.6, 1.0, 2.0$

Even though this example is based only on paleoseismic data (for simplicity), it offers several relevant and interesting insights about the use of time-dependent mainshock recurrence models in PSHA and CAT model calculations. It is seen in Fig. 5 that the BPT hazard curves for $t_e/\mu = 0.5, 0.6$ are lower than the Poissonian hazard curve; conversely, the BPT hazard curves for $t_e/\mu = 1.0, 2.0$ are higher than the Poissonian one. As mentioned above, the t_e/μ ratio of the Ohariu Fault is between 0.53 and 0.56, which mean that the hazard curve would fall between the one corresponding to $t_e/\mu = 0.5$ and $t_e/\mu = 0.6$. Physically, the ratio between the time elapsed since the last event (t_e) and the mean recurrence time (μ) relates to whether a specific fault is at an early stage of the earthquake cycle or if the stress state on the fault is close to reach a rupture state. If the t_e/μ ratio is low (i.e. an earthquake just occurred), the BPT seismic hazard for the fault is lower than that of the Poisson model. Conversely, if the t_e/μ ratio is high (i.e. long time has passed since the last event such that the probability of having a rupture on the fault is large), the BPT seismic hazard is higher than that of the Poisson model. These results are consistent with the findings of previous published studies (e.g. [17] and [28]).

Assuming the BPT model as a benchmark, the memory-less feature of the Poisson process (time-independent model) can result in a non-negligible under- or over-estimation of the hazard (and therefore related risk) due to a specific fault segment. Use of the BPT model instead of the Poissonian process can help to better describe the evolution of the seismic cycle and the recurrence of mainshock events on the fault. It is worth noting however, that the BPT model can produce low seismic hazard curves when the time elapsed since the last event is short, which may not be valid in real cases. For example, the stress drops due to a mainshock event may not be complete (or nearly complete), the strain accumulation may be nonlinear (i.e. recovery may be rapid soon after the event and then progressively slow down), and the stress build-up on a fault segment may be affected by nearby faults. This highlights the potential need for combining different recurrence models to avoid producing unrealistically low seismic hazard in the period soon after the last event has occurred.

Another limitation of the BPT model (and renewal models in general) is that it refers to a single fault (or fault segment); hence, it cannot take the interaction between adjacent faults into account and it cannot model the aftershock triggering process. In fact, mainshocks occurring on one fault can trigger other events on adjacent faults [29] and aftershocks [7] as a result of the tectonic loading change in surrounding areas. Future efforts will also focus on the interaction between faults and the incorporation of aftershock clustering.

5. Conclusions

This study has reviewed the main features of some of the most popular long-term time-dependent seismic hazard models, and discussed possible approaches for their calibration and combination. A simple case-study fault was presented to quantify the changes in seismic hazard resulting from the use of time-dependent Brownian Passage-Time (BPT) models instead of a memoryless conventional Poisson model. Model parameters were calibrated using the maximum likelihood estimation (MLE) method and paleoseismic data of the Ohariu Fault (New Zealand) published in the literature.

The case study demonstrated that the widely used Poisson process (time-independent seismic hazard model) is not able to fully capture the long-term recurrence of mainshock earthquakes on single faults. It was shown to either under- or over-estimate the hazard relative to that of the BPT model, depending on the ratio between the time elapsed since the last event and the mean recurrence time of the fault. While time-dependent models (e.g., the BPT model) can better capture the evolution of the seismic cycle and the recurrence of mainshock events than Poissonian models, they can produce unrealistically low seismic hazard curves when the time elapsed since the last event is short. This highlights the potential need for combining a number of time-dependent models (e.g. BPT, Weibull, etc.) in seismic hazard calculations. Calibration methods are also important to consider for future applications of time-dependent models in PSHA and CAT models. Different calibration methods can produce different outcomes, and the choice of the preferred method is often driven by the amount of available data on the fault segment of interest. Future efforts will also focus on the interaction between faults and the incorporation of aftershock clustering, which cannot be modelled by the application of any mainshock recurrence model on a single fault segment.

In summary, the careful use of long-term time-dependent models is recommended in fault-based seismic hazard assessments, which will have significant effects on the risk calculations commonly used in earthquake catastrophe modelling practice.

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