STOCHASTIC IDA FOR EFFICIENT HAZARD-CONSISTENT COLLAPSE RISK EVALUATION – PROOF OF CONCEPT

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Abstract

Quantifying collapse risk in seismically active regions is one of the key elements to inform decision making for building design and establishment of public policies to promote seismic safety and resilience. To estimate collapse risk, earthquake engineers typically use incremental dynamic analysis (IDA) or multiple-stripes analysis (MSA). The recently proposed hazard-consistent IDA (HC-IDA) brought IDA on par with MSA by enabling hazard-consistent collapse risk estimation while retaining analysis simplicity of using a single set of ground motions. However, both of these approaches are still numerically expensive limiting their use for complex structures. In this paper, we present 'stochastic IDA', an approach for conducting hazard-consistent IDA with a drastically reduced numerical cost. Unlike in IDA, the proposed approach does not require searching for the lowest spectral acceleration values, typically $Sa(T_1)$, at which collapses occur. Rather, scales at which to analyze the ground motions are selected randomly while the bounds on how to select the scales are obtained from a rough initial estimate of the non-hazard-consistent collapse fragility. The method is tested using an archetype 20-story reinforced concrete moment frame representative of modern construction in California. The results suggest that stochastic IDA yields excellent results compared to hazard-consistent IDA while requiring only about 25% of the computational cost. In fact, very good results can be obtained by running only a single analysis for each ground motion while the accuracy of the procedure is not too sensitive to the choice of the initial estimate of collapse fragility. Opportunities for potential applications of stochastic IDA, including its application for efficient regional risk estimation, are discussed.

Keywords: IDA; collapse risk; hazard-consistency; regression; numerical efficiency

1. Introduction

In order to protect the lives and safety of building occupants during large earthquakes it is required that the structural collapse risk is acceptably low. To obtain an estimate of structural collapse risk, specifically the mean annual frequency of collapse ($\lambda_{collapse}$), earthquake engineers typically use incremental dynamic analysis (IDA) or multiple-stripe analysis (MSA) approaches. While IDA is practically simpler in that it uses only a single set of ground motions in the analysis, the resulting collapse estimate is not consistent with seismic hazard estimate for any specific site. In contrast, multiple sets of hazard-consistent ground motions are used in MSA allowing for site-specific collapse estimates at the cost of additional ground motion selection considerations. The recently proposed hazard-consistent IDA (HC-IDA, [1]) brought IDA on par with MSA by enabling hazard-consistent collapse risk estimation while retaining analysis simplicity of using a single set of ground motions. However, all of these approaches are still numerically expensive limiting their use, especially for complex structures. In this paper, we present a numerically efficient approach for conducting HC-IDA. This approach, which we term 'stochastic IDA', leverages censored regression for predicting collapse capacities resulting with a drastically reduced analysis cost.

The details on HC-IDA method are available in [1] but a brief summary is provided here for reader's convenience. The approach first requires performing an IDA for the structure using an appropriate generic set of ground motions. For each ground motion, IDA yields the lowest spectral acceleration Sa at the fundamental period T_1, $Sa(T_1)$, at which that ground motion causes collapse of the structure. This data is used in the second step where a collapse response surface, i.e. an estimate of $Sa(T_1)$@collapse, is obtained using regression on selected intensity measures ***IM***. Specifically, study [1] recommends to use linear regression, in log space, to predict $Sa(T_1)$@collapse with Sa_{ratio} and 5 to 75% significant duration ($D_{a,5\text{-}75\%}$) used as predictive features. To yield an estimate of the collapse fragility, the collapse response surface is combined with hazard-consistent conditional distribution of **IM**, using the following equation:

$$P(C|Sa(T_1)) = \int P(C|\boldsymbol{IM}, Sa(T_1))\mathrm{f}(\boldsymbol{IM}|Sa(T_1))\boldsymbol{dim}, \tag{1}$$

where $Sa(T_1)$ is the conditioning intensity, $\mathrm{f}(\boldsymbol{IM}|Sa(T_1))$ is the conditional distribution of intensity measures ***IM*** given $Sa(T_1)$, and $P(C|\boldsymbol{IM}, Sa(T_1))$ is the estimate of probability of collapse given ***IM*** and $Sa(T_1)$. Since $Sa(T_1)$@collapse is estimated via linear regression, it follows that $P(C|\boldsymbol{IM}, Sa(T_1))$ can be computed using:

$$P\big(C\big|\boldsymbol{IM}, Sa(T_1)\big) = \Phi\left(\frac{Sa(T_1) - \mu(\boldsymbol{IM})}{\sigma}\right), \tag{2}$$

where Φ represents the standard normal CDF while $\mu(\boldsymbol{IM})$ and σ are the outputs of the regression model for $Sa(T_1)$@collapse. A flowchart outlining the HC-IDA approach is given in Fig. 1.

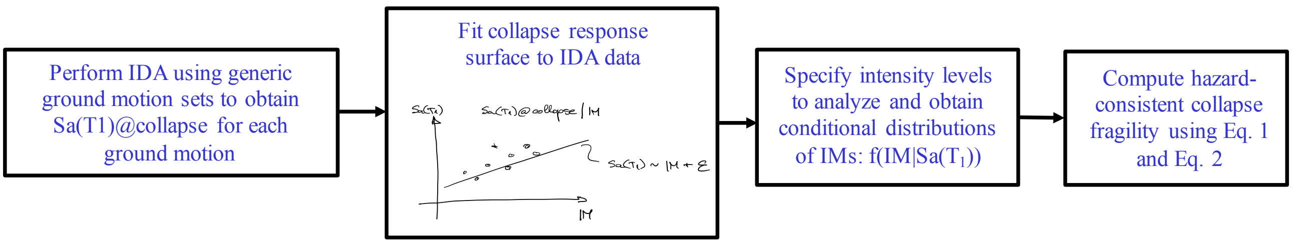

Fig. 1 – Flowchart of hazard-consistent IDA as introduced in [1]

2. Stochastic IDA approach

2.1 Basic idea

The key idea behind the stochastic IDA approach is to use censored regression to estimate the collapse response surface. Unlike in HC-IDA, where one uses IDA to search for the lowest $Sa(T_1)$ at which collapse occurs (indicated as $Sa(T_1)$@collapse) by examining an exhaustive set of scales for each ground motion, in the stochastic IDA approach the scales at which to run each ground motion are selected randomly. In this way, each data point contains the information whether the analyzed ground motion induced collapse at that intensity or if no collapse was observed. If the ground motion did cause collapse, then the actual value of $Sa(T_1)$@collapse is either at that intensity or at a lower one. Conversely, if no collapse is observed, then $Sa(T_1)$@collapse for that ground motion is at a higher intensity. In other words, such data is censored in terms of observing $Sa(T_1)$@collapse and censored regression is a useful statistical tool for making inferences in these situations. The following paragraphs provide a brief overview of the censored regression model.

The dependent variable in the censored regression model can be either left-censored, right-censored or both, and the lower and upper limits of the variable can be any number:

$$y_i^* = x_i'\beta + \varepsilon_i \tag{3}$$

$$y_i = \begin{cases} a & if\ y_i^* \leq a \\ y_i^* & if\ a < y_i^* < b \\ b & if\ y_i^* \geq b \end{cases} \tag{4}$$

where a is the lower limit and b is the upper limit of the dependent variable. Note that in our application the lower and upper limit can change depending on the observation. Censored regression models are typically estimated by the maximum likelihood method. Assuming that the ε term follows a normal distribution with mean 0 and variance σ^2, the log-likelihood function is [2]:

$$\log L = \sum_{i=1}^{N} \left[I_i^a \log \Phi\left(\frac{a - x_i'\beta}{\sigma}\right) + I_i^b \log \Phi\left(\frac{x_i'\beta - b}{\sigma}\right) + (1 - I_i^a - I_i^b)\left(\log \phi\left(\frac{y_i - x_i'\beta}{\sigma}\right) - \log \sigma\right)\right], \tag{5}$$

where the ϕ and Φ represent the PDF and the CDF of the standard normal distribution, and I_i^a and I_i^b are indicator functions with

$$I_i^a = \begin{cases} 1 & if\ y_i = a \\ 0 & if\ y_i > a \end{cases} \tag{6}$$

$$I_i^b = \begin{cases} 1 & if\ y_i = b \\ 0 & if\ y_i < b \end{cases} \tag{7}$$

The log likelihood of the censored regression model can be maximized using the readily available optimization algorithms. This implementation of the censored regression model is available in censReg package [2] for statistical environment R.

Once the model for collapse response surface is obtained, the computation of the hazard-consistent collapse fragility using stochastic IDA approach essentially follows the same steps as in HC-IDA framework. The only difference is that the collapse response surface is obtained using censored regression (see Fig. 2). Additionally, the analyst needs to decide how to sample scales at which to run ground motions as well as how many scales to use. Although the scales can be selected randomly, the quality of approximation of the collapse response surface depends on how well the selected scales bound the actual values of $Sa(T_1)$@collapse as obtained by IDA. One approach to achieve this is to use an initial estimate of the collapse fragility (such as could be quickly obtained from SPO2IDA approach [3]) and randomly sample the scales between specified bounds. For instance, if one desires to use five scales per ground motion, then one option would be to sample the scales from a uniform distribution on an interval bounded with $Sa(T_1)$ values corresponding to 5% and 95% probabilities of collapse as obtained from the initial estimate of collapse fragility. The application of this approach is demonstrated in the following section, and it will be seen that a good estimate of the hazard-consistent collapse fragility can be obtained even if a poor estimate is used for the initial collapse fragility for sampling of scales.

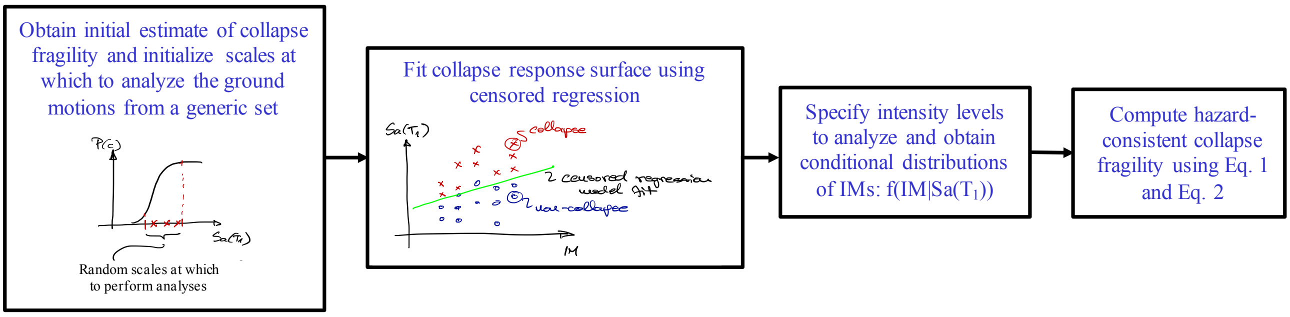

Fig. 2 – Flowchart of the stochastic IDA approach for hazard-consistent collapse risk estimates

2.2 Application of stochastic IDA for a 20-story tall building model

2.2.1 Case study building

The tall building used in this study is an archetype model of a 20-story reinforced concrete special moment frame representative of modern office buildings in California. The 20-story building was designed as a part of a previous benchmark study [4], according to the governing provisions of the 2003 International Building Code (IBC), American Society of Civil Engineers (ASCE)7-02, and American Concrete Institute (ACI) 318-02. The structural system is a perimeter frame with three bays at 6 m width (archetype ID1020). Whereas the height of the bottom story is 4.6 m, the upper stories have a height of 4 m. The frame was designed for a design base shear coefficient of 0.044g. Initial member sizes (beam depths, column dimensions) were determined by drift limits and column–beam compatibility considerations. Beam strengths were controlled by force demands, particularly lateral forces. Column strengths were determined by strong-column weak-beam ratios except in the bottom story, where flexural considerations controlled and in the upper stories where minimum reinforcement ratios controlled. Column concrete was increased to 41.4 MPa in the lower stories to meet joint shear requirements. Beam stirrups were controlled by shear capacity design. The column stirrups were controlled by both the shear capacity design and the confinement requirements. Strength and stiffness are stepped over the height as would be done in common design practice.

The frame is idealized as a 2D analysis model using OpenSees [5], in which the first three modal periods are 2.63, 0.85, and 0.45 s, respectively. The nonlinearities are captured in concentrated plasticity models in panel zones and plastic hinges at the ends of columns and beams. Lumped plastic hinges are modeled using the phenomenological Ibarra–Medina–Krawinkler model [6], which has been previously calibrated to capture the deterioration of concrete members out to large deformations. Rayleigh damping of 5% critical is assigned to periods T_1 and $0.2T_1$, in which T_1 is the period of the fundamental mode. For additional details regarding design and modeling assumptions, see [4].

2.2.2 Ground motion sets and test site

The ground motion set used to conduct this study is a generic set of 88 recorded ground motions that was assembled by [1] as part of a study investigating the effects of ground motion duration on seismic collapse risk. The ground motion set is generic in the sense that it does not represent seismic hazard at any specific site. 44 of these ground motions were taken from the FEMA P695 far-field set, which contains relatively short duration ground motions (with $D_{a,5\text{-}75\%} < 25$s), recorded from shallow crustal earthquakes. The remaining 44 ground motions were selected from long duration ground motions (with $D_{a,5\text{-}75\%} > 25$s) recorded from both large magnitude interface earthquakes like the 2011 Tohoku (Japan), 2010 Maule (Chile), and 1985 Michoacan (Mexico) earthquakes, and large magnitude crustal earthquakes like the 2008 Wenchaun (China) and 2002 Denali (USA) earthquakes. Each of the 44 long duration ground motions was selected to have a similar response spectrum to one of the short duration ground motions. Detailed information about the selected records is available in [1].

These ground motions are used to perform HC-IDA as well as the proposed stochastic IDA to obtain collapse risk estimates at a selected site. The site used in this study, codenamed WNGC, is a site in Los Angeles basin for which seismic hazard data was simulated as part of the Southern California Earthquake Center (SCEC) CyberShake project. Fine details on how the CyberShake calculations are performed are presented in [7]. For the purposes of this paper, suffice it to say that CyberShake project performs probabilistic seismic hazard assessment (PSHA) by completely relying on numerical simulations of earthquakes for all the ruptures sampled from the assumed earthquake rupture forecast. In other words, for each considered rupture, which has an associated annual rate of occurrence λ_{rup}, (obtained from UCERF2 in CyberShake), a resulting seismogram is generated by solving the wave propagation equation through the linear medium (a 3D representation of sedimentary basins and other near-surface structures). As a result, around 500,000 seismograms are simulated at each CyberShake site including WNGC used here. The reason for using this site is that it was previously extensively studied by the authors [8], and benchmark results in terms of mean annual frequencies of collapse ($\lambda_{collapse}$) are available for the case study tall building at this site. Additionally, availability of simulated seismograms representing the full hazard at this site allows for straightforward computation of PSHA results, for instance the conditional distributions of intensity measures as required in Eq. (1).

2.2.3 Results

Stochastic IDA is performed for the 20-story tall building model using the FEMA short and long duration sets and collapse responses are computed for the WNGC site. The results are contrasted with HC-IDA performed for the same site and the benchmark collapse risk obtained in a previous study [8]. Shown in Fig. 3a is the exact collapse fragility as obtained by IDA using the FEMA sets, while the Fig. 3b shows the collapse response surface estimated using HC-IDA approach. The green points on Fig 3b correspond to collapse capacities (i.e. the lowest $Sa(T_1)$ causing collapse for each of the ground motions) and the green plane is the collapse response surface fitted to the data using linear regression. Predictive features used in regression are $Sa_{average}$ and $D_{a,5\text{-}75\%}$ as proposed in [1] and the regression is performed in log-space. Contrasted to this is the result of stochastic IDA approach for estimating the collapse surface as shown in Fig. 3c. The magenta points indicate scales at which the used ground motions caused collapse. Conversely, the blue points indicate the scales at which collapses were not observed. In this example case five scales were used per ground motion. The collapse response surface indicated in red is fitted to the data using censored regression, specifically linear censored regression (in log-space, using $Sa_{average}$ and $D_{a,5\text{-}75\%}$ as predictors) as described in a previous section. It can be seen from the plot that the collapse response surface is trying to separate the collapse-inducing data from data not inducing collapse.

Once the collapse response surfaces are obtained, the hazard-consistent collapse fragilities for the case-study site can be obtained using the flowcharts in Fig. 1 and 2. Shown in Fig. 4a is the comparison of collapse response surfaces obtained using HC-IDA (green) and stochastic IDA with five scales per ground motion (red). A close match between surfaces can be seen indicating that stochastic IDA does approximate the HC-IDA. This similarity between collapse response surfaces also results in a close match in collapse fragilities. Specifically, the HC-IDA collapse fragility and the collapse fragility from stochastic IDA with 5 scales are practically identical, as seen in Fig. 4b. Given for reference in Fig. 4c,d are the collapse fragilities obtained using stochastic IDA with three scales and one scale per ground motions. It can be seen that the median collapse capacities in all cases are in very close agreement while there are some slight discrepancies in the dispersions. Since utilizing HC-IDA in this case required performing on average 12 nonlinear response history analyses for each ground motion, or 1056 analyses in total, the results suggest that stochastic IDA can yield excellent approximation with a total of 3*88=264 analyses, or 25% of the numerical cost. In fact, a very good approximation can be obtained with only a single analysis per ground motion or about 8% of the numerical cost of performing HC-IDA.

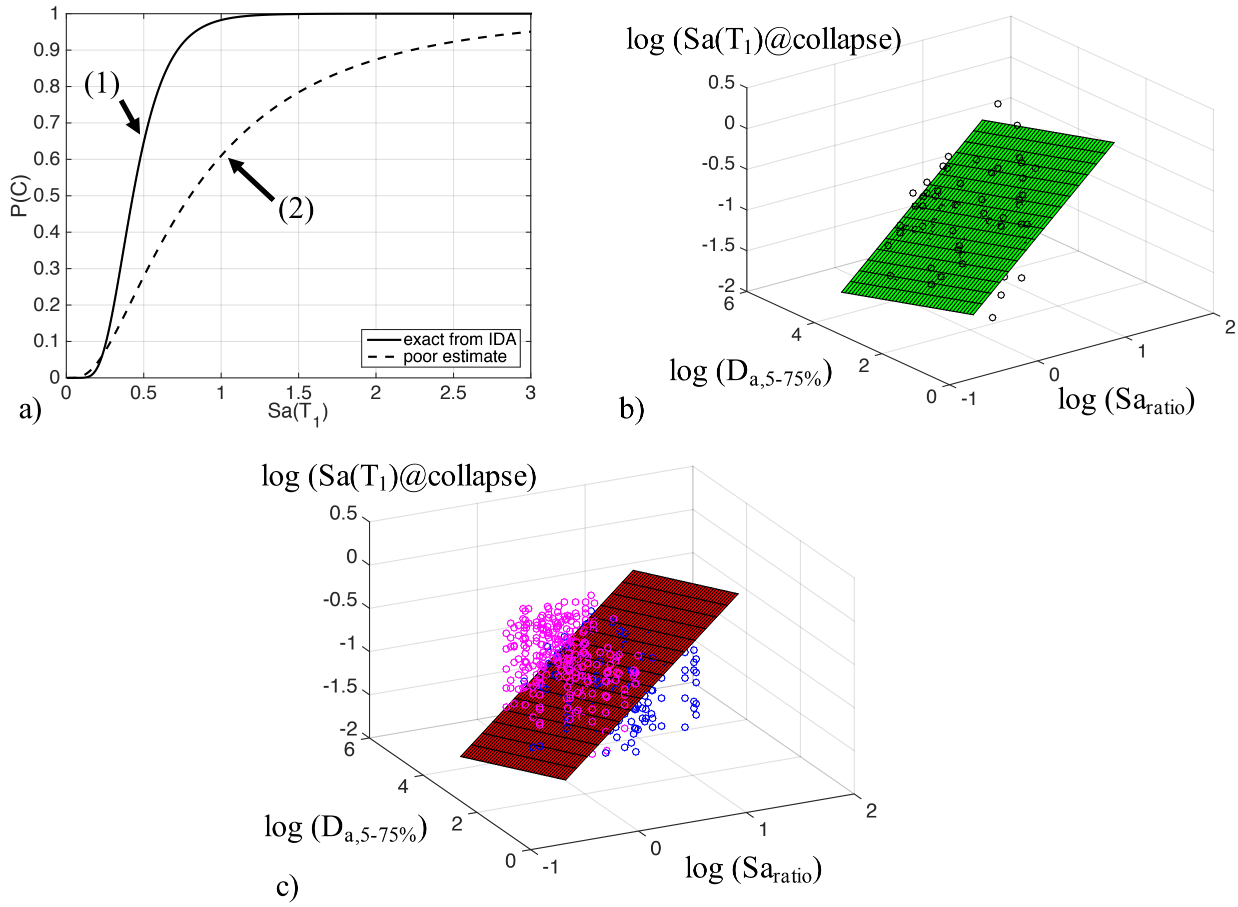

Fig. 3 – Examples of stochastic IDA estimates for the 20-story building: (a) exact collapse fragility obtained using IDA (not hazard-consistent); (b) collapse surface obtained using HC-IDA; (c) realizations of scales and collapse surface obtained using stochastic IDA

These results used the exact collapse fragility from IDA as an initial guess for sampling of scales in stochastic IDA which in a way defeats the purpose of performing stochastic IDA in a practical application as this information is not known a priori. Additionally, since the scales at which to perform the analysis are sampled at random, the quality of approximation does depend on how lucky one is with the sampling. To further investigate the issue of stability of collapse risk estimate depending on the number of scales used as well as the sensitivity of results to the used initial guess for collapse fragility used to sample the scales, we perform Monte Carlo (MC) analysis with stochastic IDA. Specifically, we sample a hundred realizations for scales used in stochastic IDA considering sampling of five, three and one scales per ground motion. Additionally, we consider the following two cases for the initial estimate of collapse fragility: (1) we use the exact collapse fragility as obtained with IDA with median collapse capacity of 0.43g and dispersion 0.4, and (2) we use a purposefully bad estimate of collapse fragility with median of 0.8g and dispersion of 0.8 (collapse fragilities are compared in Fig. 3a). The use of exact collapse fragility (1) is intended to mimic the best possible performance we can expect from stochastic IDA, while the use of collapse fragility (2) is to examine the influence of a poorly informed initial estimate for the collapse response.

Given in Fig. 5 are the results of MC analyses with stochastic IDA to obtain mean annual frequencies of collapse ($\lambda_{collapse}$) at the WNGC site. Shown with a full black horizontal line is the benchmark value of $\lambda_{collapse}$ as obtained by direct analysis of all seismograms at the site [8], while the dashed black line shows the estimate as obtained from HC-IDA. A very close match between the benchmark result and the HC-IDA estimate can be seen. The box plots indicate the MC results using stochastic IDA with five, three, and one scale per ground motion. The red lines indicate the medians, the boundaries of box plots indicate 25th and 75th percentiles, while the whiskers indicate the smallest and the largest values. A couple of observations can be made. First, the median values from stochastic IDA are in all cases very close to the HC-IDA estimate indicating an efficient approximation regardless of the number of scales used. Second, increasing the number of scales improves the quality of approximation by decreasing the dispersion of the estimates. In case when the exact collapse fragility is used for sampling of scales (Fig. 5a) there are diminishing returns beyond using three scales per ground motion. When using a poor initial estimate of collapse fragility (Fig. 5b), the method performs less well but is fairly stable and accurate with three to five scales per ground motion. Note that in this case the median collapse capacity for the initial collapse fragility is two times larger than the exact value. We expect that in practical applications, where one might obtain an initial estimate of collapse fragility using for instance SPO2IDA approach [3], the results would be between these two extremes. We leave such considerations for future study.

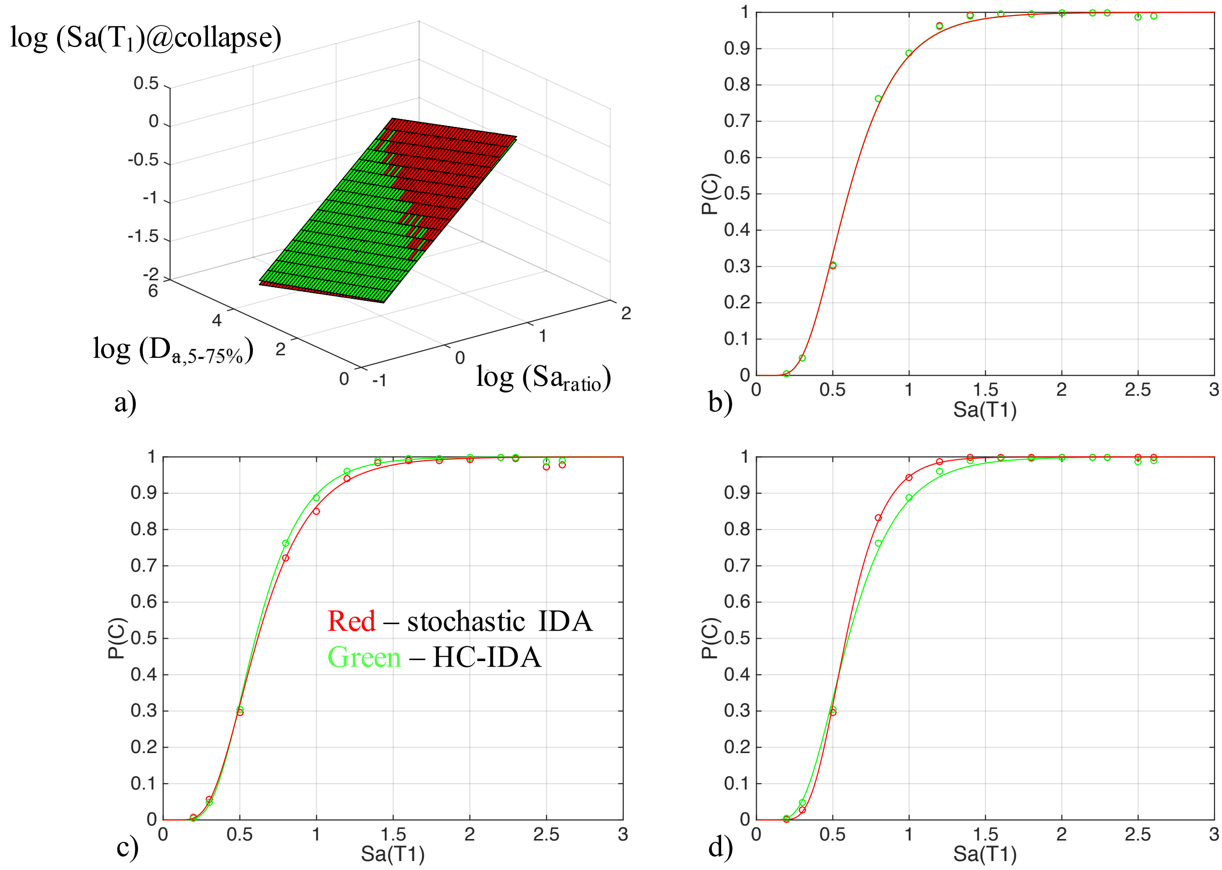

Fig. 4 – Comparison of stochastic IDA (red) and HC-IDA (green) estimates: (a) collapse response surfaces where five scales per ground motion were used in stochastic IDA; collapse fragilities obtained using (b) five, (c) three, and (d) one scale per ground motion in stochastic IDA. Exact collapse fragility from IDA (Fig. 3a) was used as the initial estimate of collapse fragility for selection of scales in stochastic IDA

3. Conclusions

This paper presented the stochastic IDA, a numerically efficient approach for obtaining hazard-consistent collapse risk estimates. The method builds on the hazard-consistent IDA approach by leveraging censored regression to mimic the collapse process allowing for a drastically reduced numerical cost. The method is tested by computing collapse risk of an archetype 20-story tall building located at a site in Los Angeles basin.

The results suggest that stochastic IDA effectively approximates hazard-consistent IDA and that excellent results can be obtained with 10% to 20% of the numerical cost.

This proof of concept for stochastic IDA opened up a number of areas for further exploration. For instance, methods for appropriately selecting the scales in stochastic IDA should be investigated including the considerations of approximations of the initially used collapse fragilities. Further, the sensitivity of the results to the number of scales used should be comprehensively studied including additional building models. More generally, issues such as ground motion selection for performing IDA as well as the utility of different intensity measures in regression should be investigated. This will be particularly important for general utilization of stochastic IDA as well as HC-IDA in different geologic settings and applications such as regional collapse risk investigations. Nevertheless, stochastic IDA is a promising tool for efficient hazard-consistent collapse risk estimation especially for complex structures that are numerically expensive to analyze.

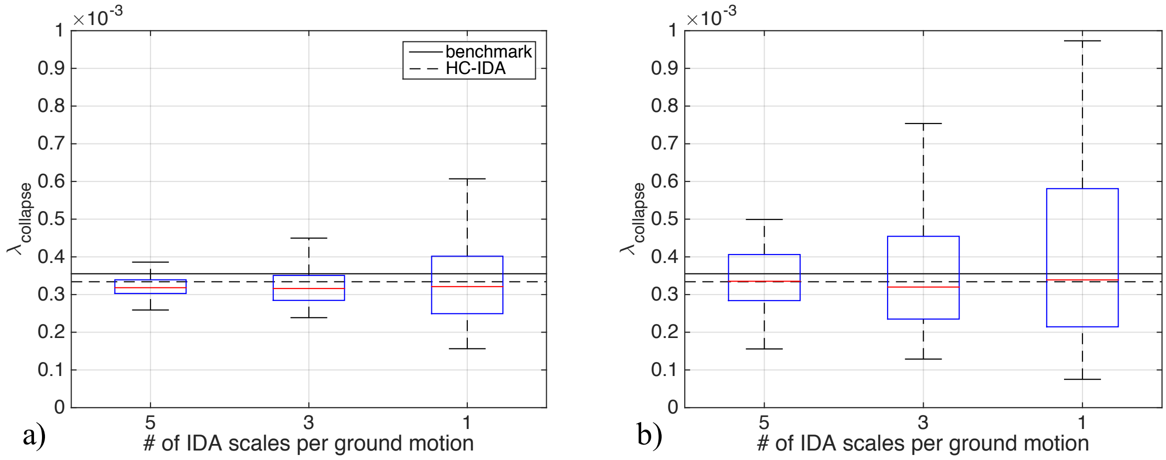

Fig. 5 – Monte Carlo estimates of mean annual frequency of collapse ($\lambda_{collapse}$) at the WNGC site using 100 realizations of stochastic IDA approach with different number of scales per ground motion: (a) exact collapse fragility from IDA (median = 0.43g, dispersion = 0.4) used as initial estimate for stochastic IDA; (b) collapse fragility with median = 0.8g and dispersion = 0.8 used as initial estimate (initial estimates for collapse fragilities indicated in Fig. 3a)

4. Acknowledgements

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