RE-APPRAISING THE ROLE OF FOURIER AMPLITUDE SPECTRA (FAS) IN SEISMIC HAZARD AND RISK ANALYSIS

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Abstract

The relation between the response of an elastic single degree of freedom oscillator subject to seismic shaking and the seismological characteristics of the input waveforms represented in terms of the Fourier amplitude spectrum (FAS) has become a topic of growing interest in the engineering seismological community in recent years. As simulations of ground motions assume a greater prominence in engineering design and databases of both strong and weak motion records expand exponentially, the need to reconcile the influence of the controlling seismological properties of the motions with their potential impacts on structural response is becoming ever more important. Prediction of ground motion in the Fourier amplitude domain has several key advantages when compared to that of the response spectrum, namely a closer relationship to the physical seismological properties of the source, path and site that can be inferred from more abundant small magnitude and weak motion data, as well as maintained linearity of site response at all frequencies. Recognising this, new empirical ground motion models have been developed in terms of FAS [1, 2], in addition to an inter-frequency correlation model that can facilitate the definition of conditional spectrum compatible empirical and simulated ground motion records for design [3]. In spite of these advantages, the usage of FAS in seismic hazard and risk analysis remains limited to the scaling of simulated ground motions in the development of median ground motion models using random vibration theory (RVT). One major factor behind this is that translation from FAS to response spectra via RVT requires joint characterization not only of ground motions across a range of frequencies but also of strong motion duration. However, this limitation could potentially be overcome if fragility functions were to be derived directly in terms of FAS. It is for this purpose that a comparison is made in this paper between the efficiency of intensity measures based on the FAS and those based on conventional response spectra for a set of simple building fragility models of the type commonly used in seismic risk analysis. The feasibility of achieving end-to-end loss estimation exclusively in the Fourier amplitude domain is subsequently explored. While its full range of benefits and limitations will require further study, the potential for embedding seismological theory and data more deeply into engineering applications is appealing for the future practice of seismic design and risk analysis.

Keywords: seismic hazard and risk; ground motion models; Fourier amplitude spectrum; fragility

1. Introduction

The characterisation of strength transient strong ground motion that a structure may experience during its expected lifetime is one of the most critical objectives for seismic hazard and risk analysis. For a wide variety of applications, a probabilistic approach to the calculation of hazard and risk from earthquakes is preferred, otherwise referred to as probabilistic seismic hazard and/or risk analysis (PSHA/PSRA). This requires not only a prediction of the expected ground motion for a given scenario, but also an accurate an unbiased estimation of the expected variability. Conventionally, such ground motion models (GMM) take the form of a lognormal distribution that is fit to empirically observed ground motions, which consist of a median ground motion that is an explicit, physically-based formula dependent on magnitude, distance and site properties, and a variance that may be homoscedastic (independent of scenario) or heteroskedastic (dependent upon the scenario).

To represent the strength of the ground shaking, the vast majority of GMMs adopt as an intensity measure (IM) the pseudo peak acceleration of the elastic response a single-degree-of-freedom (SDOF) oscillator with fractional damping, ξ, and natural period, T. From an engineering perspective, the pseudo peak oscillator acceleration, which for a range of periods is referred to as the response spectrum (or SA), is an idealisation of the peak elastic response of a structure, from which post-elastic response can subsequently be characterized by means of a ductility or force reduction factor. As a representation of the ground motion itself, however, the response spectrum is a convolution of both the input ground motion and the structural response. Therefore, SA at a given period, T, is influenced not only by the strength of shaking at a given frequency but also the shape of the underlying Fourier spectrum and the duration of motion.

Historically, GMMs have been constructed by fitting common functional forms to observations of strong ground motions in a particular locality or tectonic environment. This approach has its limitations, however, as strong ground motions represent only a small subset of the total number of motions recorded, and the density of accelerometer networks can be highly variable regionally, with some active regions of the world recording few or no motions, whilst even densely instrumented regions may not capture well the specific distribution of source, path and site properties relevant for ground motion at a site of interest. When applying ground motions in PSHA, particularly taking models from areas of high seismicity and dense instrumentation and applying them to regions of low seismicity and/or sparse or even absent instrumentation, it is common to encounter the need to adjust existing GMMs from their host environment to the target environment [4, 5]. Specifically, adjustments are commonly made to the source stress drop scaling, the frequency dependent path attenuation (Q), and the local site amplification. These adjustments may affect different frequency bands of the spectrum, and differences between the host and target regions may often be inferred not from the response spectra of strong motion but from weak or low intensity motion Fourier spectra of shaking. Consequently, to adjust response spectrum GMMs, the relation between the Fourier spectrum of the observed shaking and the peak response of the elastic SDOF oscillator is critical to our understanding [6].

Why can this be a problem in engineering practice? Although weak motions may be more abundant as a means of constraining local differences in source, path and site amplification, the adjustments inferred by differences in the Fourier spectra do not scale linearly when applied to response spectra. The explanation as to why this is the case can be found in random vibration theory (RVT), from which it can be shown that the response spectrum at short periods is dependent on the shape of the entire frequency range of the Fourier spectrum [6, 7]. When attempting to calibrate local site amplification in terms of response spectra, this leads to scenario-dependence (specifically magnitude and distance) of the resulting amplification functions [7]. Consequently, local or site-specific adjustments to existing ground motion models made on the basis of frequent, lower intensity shaking observations cannot necessarily be assumed to scale to larger ground motions.

Recognising these limitations to the adjustability of GMMs fit entirely to response spectra, recent years have witnessed the introduction of GMMs for ground motions in terms Fourier amplitude spectra (FAS) [1, 2, 8, 9]. Though such models have been derived typically for the purposes of facilitating regional adjustments in ground motion scaling [1, 8, 9], they could also be applied directly within a seismic hazard and risk framework and potentially circumvent the need to constrain response spectrum scaling altogether. There is an intuitive conceptual appeal to adopting such an approach. It separates the representation of the ground motion, which is itself dependents only on the seismological properties of the source, path and site and their respective aleatory and epistemic uncertainties, from that of the SDOF oscillator response. Furthermore, as nonlinear dynamic analysis of structures for both design and risk assessment becomes more established in engineering practice, the need to define input ground motion in a matter consistent with simpler idealisations of the structural response (such as design code spectrum) in order to facilitate closed-form static structural analysis procedures may no longer assume primary importance.

This paper explores some of the possibilities for running seismic hazard and risk assessment in terms of Fourier spectra, and looks at the potential implications of doing so. Using a recently derived Fourier amplitude ground motion model, we illustrate a simple site-specific PSHA application with the aim of producing typical seismic hazard outputs that could be used for defining the input seismic motion for design, identifying controlling scenarios and generating hazard-consistent time-histories for dynamic structural analysis. We then proceed to explore the feasibility of using Fourier amplitudes as a ground motion IM for predicting the level of damage to structures. The results of this exploration can be used to derive fragility functions for use in seismic risk assessment, and their efficiency with respect to current response spectrum practices. Finally, a simple seismic risk calculation is performed to illustrate the consequences of adopting FAS as the ground motion IM. In doing so, we indicate the potential costs of a Fourier-only approach versus the conventional response spectrum approach and outline a possible strategy that may be able to combine the benefits of both.

2. Fourier Spectra in Seismic Hazard Analysis

Before exploring the uses and impacts of adopting FAS as the preferred ground motion metric in seismic risk analysis, it is important to address whether it can be integrated into PSHA. This is, of course, facilitated by recent efforts toward the development of FAS ground motion models based on both empirical strong motion records [e.g. 1, 2, 8], stochastic simulations [e.g. 10] or some combination of the two [9, 10]. Such models can be readily integrated into existing seismic hazard assessment software in order to yield outputs commonly associated with response spectra, e.g. curves of annual rates and/or probabilities of exceedance of ground motion, uniform hazard spectra, disaggregation, etc. An application of seismic hazard analysis entirely in the Fourier domain is illustrated in the following example, for which we considered a site in northern Italy and adopt the seismogenic source model constructed as part of the 2013 European Seismic Hazard Model (ESHM13) [11].

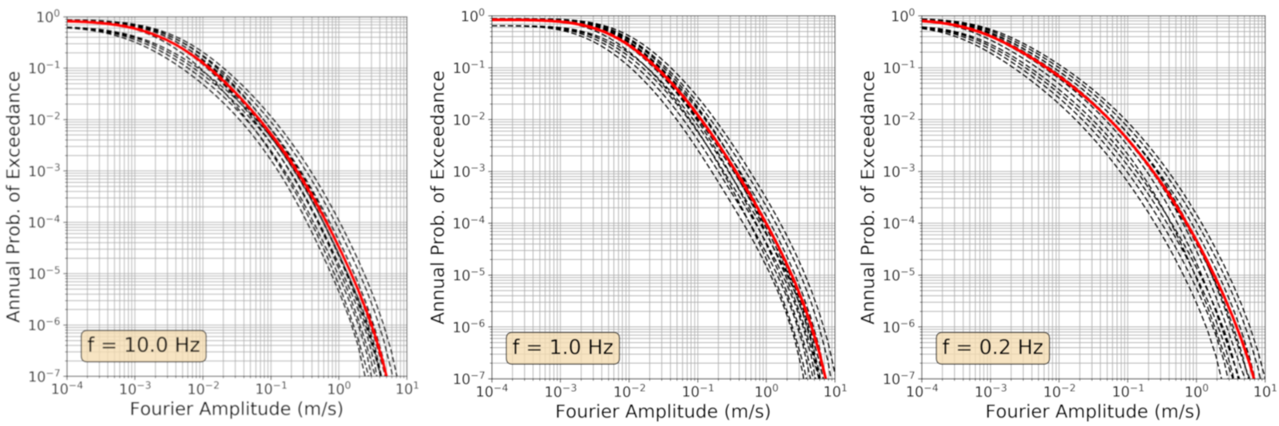

Fig. 1 – Seismic hazard curves describing the annual probability of exceedance of Fourier amplitude at the selected target site (assuming a V_{S30} of 800 m/s) for frequencies of 10 Hz,1 Hz and 0.2 Hz.

The FAS ground motion model adopted in the hazard example and the subsequent risk analyses, is that of Bora et al. (2019) [8], which is calibrated on recordings from shallow crustal earthquakes within the NGA-West 2 database, spanning a magnitude range $3.2 \leq \mathbf{M} \leq 7.9$ and a distance range $0 \leq R_{RUP}$ (km) ≤ 300. A critical factor in the selection of this GMM is the definition of the horizontal ground motion, which is for the individual (or arbitrary) horizontal component, as opposed to the effective amplitude spectrum (EAS) of a pair of horizontal motions [2, 12]. This is for consistency with the derivation of the fragility curves described in the subsequent section, which are fit to the resulting displacements of an inelastic single-degree-of-freedom oscillator subject to only one as-recorded component of ground motion at a time.

A simple logic tree of 12 branches is constructed, with three source model branches taken from the ESHM13 (area source, fault source plus background, and smoothed seismicity) and four ground motion logic tree branches in which the median ground motions are scaled by factors of 0.8, 1.0, 1.2 and 1.4, with weights of 0.2, 0.5, 0.2, and 0.1, respectively. This represents a simple "scaled-backbone" logic tree [e.g. 13] in which the scaling factors are intended to account for epistemic uncertainty in the local stress parameter. Seismic hazard curves for the FAS (in m/s) are shown in Fig. 1 for spectral frequencies 10 Hz, 1Hz and 0.2 Hz. Naturally, all other common PSHA outputs can be obtained from the seismic hazard curves. Corresponding uniform hazard spectra (UHS) for the 475-year and 2475-year return period are also shown in Fig. 2 for both the mean hazard and the 16th to 84th percentiles.

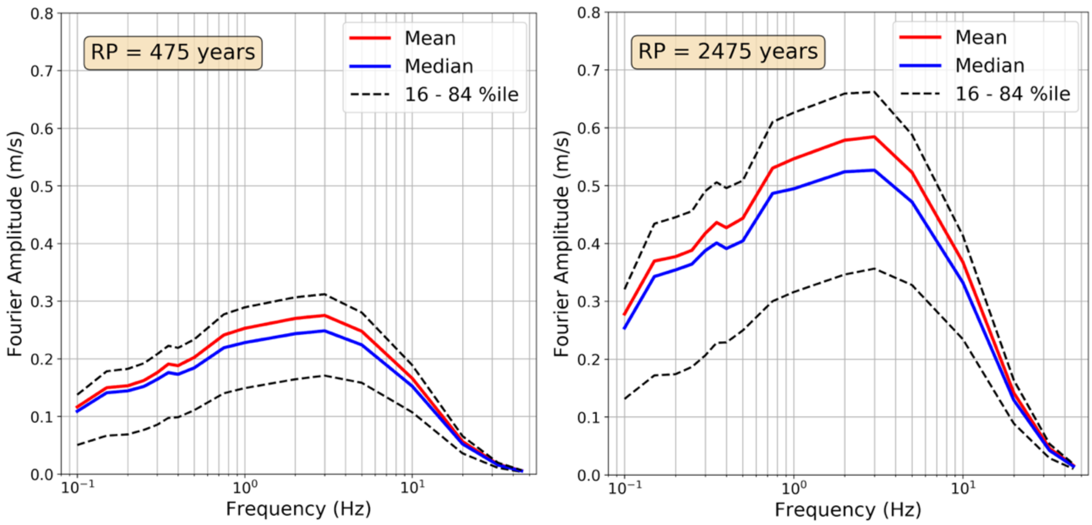

Fig 2. – Uniform hazard Fourier spectra for a return period of 475 years (left) and 2475 years (right)

In addition to the hazard curves and UHS, PSHA in terms of FAS can be extended further into the engineering application context to develop potential target spectra for scaling of ground motion records. Fig. 3 demonstrates the results of a disaggregation of the 1.0 Hz seismic hazard curves for the 2475-year return period. From this we identify a controlling scenario of M_W 6.25 and rupture distance of R_{RUP}, of 12.5 km, which can be seen to require a scaling of approximately 1.283σ in order to match the UHS. Whilst the UHS itself could be adopted as a target Fourier amplitude spectrum for scaling ground motion records, arguments against this practice for target UHS in the response spectra in terms of excessive conservatism apply similarly for FAS. Specifically, generation of time-series from the Fourier-based UHS would inevitably result in unphysical ground motions owing to the greater ratio of high frequency motions with respect to lower frequency motions. Instead, however, the conditional spectrum can be constructed from the disaggregation outputs [14, 15]. For this purpose, we adopt the interfrequency FAS correlation model of Stafford (2017) [3], replacing the between-event, between-site and site-corrected within-event components of ground motion variability with those returned by the original GMM [8]. This particular interfrequency correlation model is selected as it is determined from a Fourier GMM for the arbitrary component of ground motion rather than a resolved horizontal pair [3]. One point of note is that the interfrequency correlation model is fit using unsmoothed FAS, whilst the GMM utilises the smoothed FAS. This inconsistency is not necessarily a critical issue in generating the target spectrum conditioned upon a single target frequency (f^*) but may be of greater relevance if defining as a target spectrum an envelope of multiple conditional spectra anchored to several different frequencies. The relation between the median scenario spectrum (for M_W 6.25 and R_{RUP} = 12.5 km), the UHS and the conditional spectrum (including the mean and standard deviation) can be seen for the case of the 2475-year return period with $f^* = 1$ Hz in Fig 3.

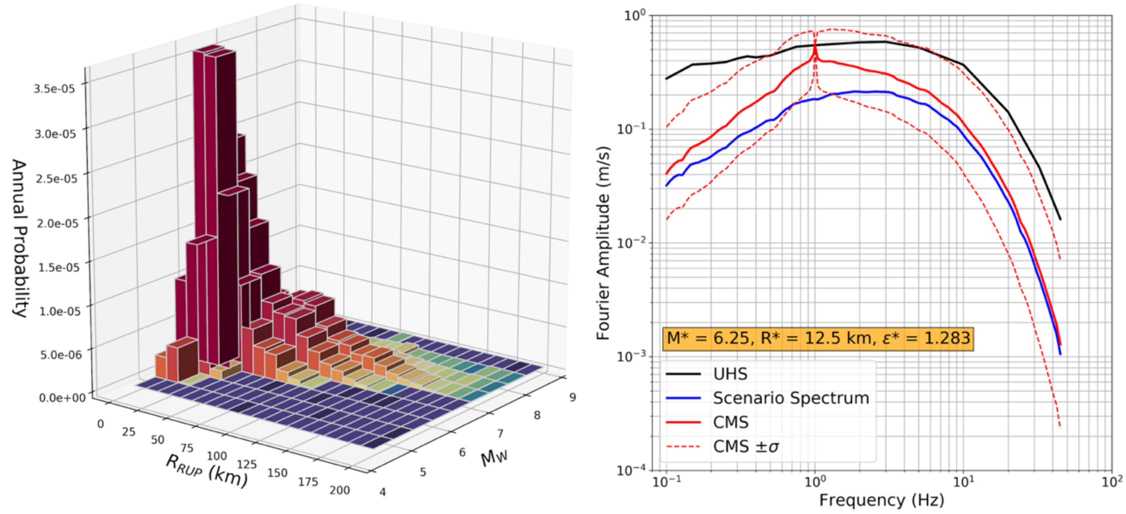

Fig. 3 – Disaggregation for the 1 Hz seismic hazard and the 2475-year return period (left) and the corresponding uniform hazard spectrum, median scenario spectrum (using the Bora et al. (2019) [8] GMM) and conditional spectrum for the mean scenario earthquake

Taken to its natural conclusion for an engineering application, the conditional spectra, presented in Fig. 3, and its related inputs can be used directly for generation of time-series consistent with the controlling scenario. This is achieved in the manner described by Stafford (2017), who generates CS compatible FAS by sampling from a multivariate Gaussian distribution, taking the Inverse Fast Fourier Transform to define a synthetic stationary time-series and then applying an envelope in the manner described by Boore [16] Whilst some discrepancies in the variability of the energy content of time-series (represented by the Arias intensity) generated in this manner exist with respect to observed records at smaller magnitudes, there is good agreement for moderate-to-large magnitudes [3]. This illustrates the potential for this approach to generate larger suites of hazard scenario-compatible time-series for nonlinear dynamic analysis of structures that could *supplement* (rather than replace) records of observed strong motions for cases where availability of such records is limited.

Though not the main objective of this paper, this illustrative example of a realistic PSHA calculation shows that it is possible to describe the probability of exceeding given amplitudes of ground motion, the controlling earthquake scenario, and even site- and context-specific relevant stochastic ground motion time-series without requiring the definition of the response spectrum at any stage in the process. Further possible advantages of this process, such as the scenario independence of the linear site effects in the Fourier domain [7], offer yet further reasons to consider adopting a Fourier-only approach to hazard characterisation for certain applications. For seismic risk analysis, however, consideration must be given to the relation between the Fourier amplitude of the ground motion and the response of the structure(s) of interest before one can fully judge its suitability in wider application.

3. Fragility and Vulnerability Functions using Fourier Amplitude Spectra

The framework used to develop fragility models in the present work is similar to that described in Martins and Silva (2018) [17], and consists of representing each structural class by means of an equivalent single-degree of freedom (SDOF) oscillator and subjecting them to a series of non-linear time-history analyses (NLTHAs) using real earthquake records. These records were collected from several ground motion databases worldwide (e.g. Pacific Earthquake Engineering Research (PEER) NGA-West, Chilean Geological Institute, Colombian Geological Service, Universidad Nacional Autónoma de México – Engineering Instituting, European Strong Motion Database) for active shallow and subduction tectonic environments, resulting in an initial set of 3500 records with a minimum peak ground acceleration (PGA) of 0.05g. From this set, 174 ground motion records were randomly selected in order to ensure a uniform distribution of intensities between 0.05g and 2.0 g. The non-linear behaviour of the SDOF systems is defined by means of capacity curves derived assuming a trilinear elasto-plastic model, in most cases, and a quadrilinear model for the cases of typologies containing infill walls [18, 19]. The expected period of vibration as well as the global drift values at the yield and ultimate capacity of each building class are determined from available literature (e.g. numerical studies, experimental tests, damage observations), and further adjusted based on expert judgment. [16]. Four damage states are considered (slight, moderate, extensive and complete damage), each associated with a displacement threshold estimated from the yield and ultimate displacement capacity of each SDOF system, as shown in Table 1.

Table 1 – Displacement thresholds and loss ratios associated with each damage state.

Damage State	**Threshold Engineering Demand (Displacement)**	**Consequence Model**	
		Mean Loss Ratio	**Coefficient of Variation**
DS1	$0.75 \cdot S_{dy}$	0.05	0.10
DS2	$0.5 \cdot S_{dy} + 0.33 \cdot S_{du}$	0.20	0.15
DS3	$0.25 \cdot S_{dy} + 0.67 \cdot S_{du}$	0.60	0.15
DS4	S_{du}	1.00	0.15

A comprehensive set of 490 structural classes (classified using an updated version of the GEM Building Taxonomy [20]) are analysed herein, covering a wide range of materials (wood, concrete-steel composite, steel, masonry, reinforced concrete) and lateral load-resisting systems (frames, walls, dual), with fundamental periods T of up to 2.8 seconds but higher density of cases for T < 1 s and, in particular, around 0.5 s. While in practice it is common to select a few fundamental periods to be used for risk assessment and associate the periods of the classes to these few selected options (e.g. using T=0.3 s to represent the period of both a structure with T=0.27 s and another with T=0.32 s), the exact values are used herein to facilitate the comparison of fragility functions obtained with different intensity measures.

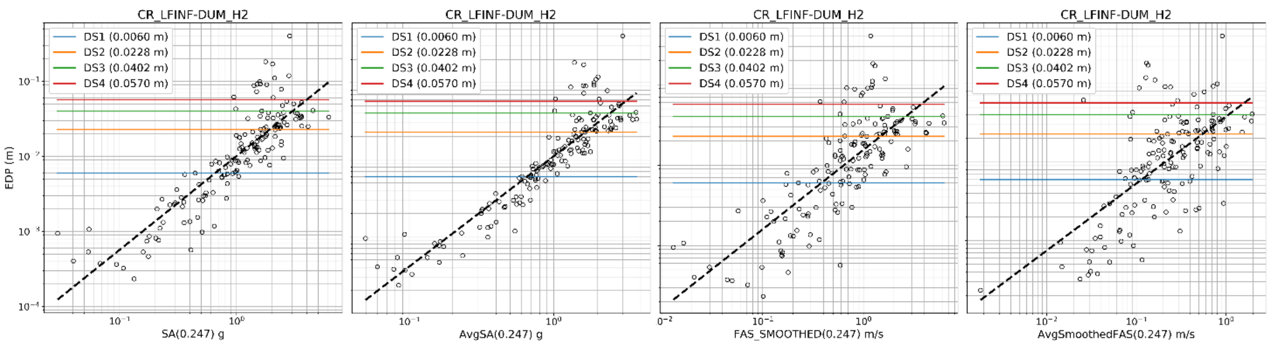

Fig. 4 – Example of linear EDP-IM models in log-log space for structural class CR/LFINF-DUM/H2 for the four IMs considered herein (in order): SA, AvgSA (0.2T – 15 T), FAS, AvgFAS (0.2T – 1.5T).

For each structural class, fragility functions are fit using censored cloud analysis, as implemented in the GEM's Risk Modeller's Toolkit. The censorship imposes the condition that no displacement greater than 1.5 times the threshold for DS4 could be attained without global instability of the system, and thus capping the maximum displacements obtained from the NLTHAs to these values. The procedure consists of first fitting a linear relationship in log-log space between the maximum displacement of the system and each of the IMs considered herein (see Fig. 4), to then calculate the probability of a series of pre-defined values of the IM exceeding the displacement threshold of each damage state assuming a lognormal distribution of displacements with expectation equal to the fitted linear relationship and the standard deviation associated with the fit (and an additional fixed value of 0.3 to account for building-to-building variability). The final fragility curves result from maximum likelihood estimation of the mean (θ) and logarithmic standard deviation (β) that define a log-normal cumulative distribution function (CDF) that fits the resulting probabilities.

In order to be able to investigate the efficiency of FAS to characterize the fragility of structures, fragility curves are derived both in terms of FAS and SA. Moreover, as recent studies [e.g. 21] have shown that considering an average of SA at several periods of interest for any particular structure is more efficient than using a single period, fragility curves are calculated as well in terms of average SA (AvgSA) and average FAS (AvgFAS), defined by $AvgSA(T) = \frac{1}{N}\prod_i^N Sa(T_i)$ and $AvgFAS(T) = \frac{1}{N}\prod_i^N FAS(T_i)$ respectively. In both cases we take an average from 50 periods in the range 0.2T-1.5T (where T is the period of the SDOF system). FAS are smoothed using a Konno and Ohmachi [21] filter with a bandwidth of 40.

The efficiency of the four IMs considered is evaluated here by means of the comparison of their associated Pearson product moment correlation coefficient (R^2) between the IM and the maximum displacements of the SDOF system (the engineering demand parameter, EDP), amongst other possible measures of variability in the correlation. All three parameters of efficiency indicate that for all 490 structural classes FAS and AvgFAS perform worse than both SA and AvgSA. When plotting R^2 against the fundamental elastic period of the SDOF systems it is observed that the efficiency improves with period, as shown in Fig.5. While this observation is true for all four IMs, it appears to be particularly marked for FAS and AvgFAS.

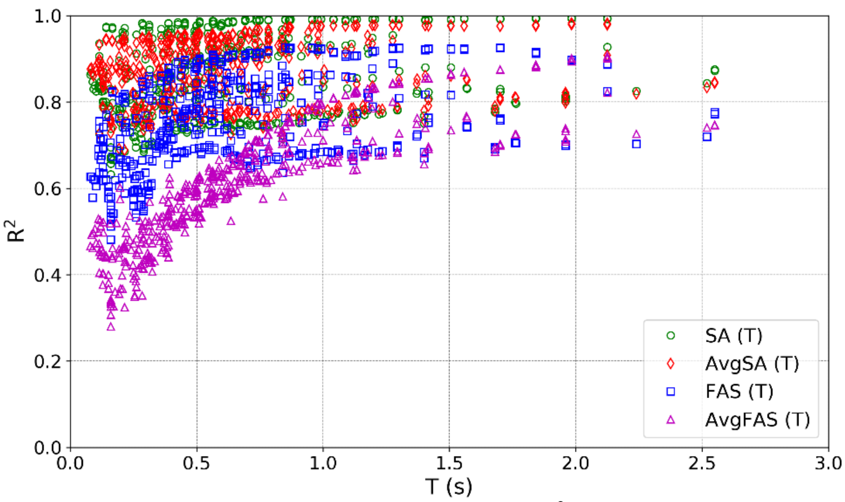

Fig. 5 – Pearson product moment correlation coefficient (R^2) between the IM and the maximum displacements of the SDOFs systems versus period of vibration, for each of the four IMs.

The overall poorer efficiency of FAS with respect to SA is not a surprising outcome in itself, but the improvement in efficiency with period is noteworthy and offers some clue as to the cause. The explanation for this outcome is once again rooted in random vibration theory [6, 7], which shows that response spectral ordinates are proportional to the square root of the zeroth spectral moment (m_0) of the SDOF oscillator with fundamental frequency f_{osc} and damping ξ:

$$m_k(f_{osc}, \xi) = 2\int_0^\infty (2\pi f)^k |Y_{SDOF}(f, f_{osc}, \xi)|^2 df \quad (1)$$

where $|Y_{SDOF}(f, f_{osc}, \xi)|^2$ is the product of the squared FAS of the time series and the square of the SDOF transfer function ($I(f, f_{osc}, \xi)$). The resulting effect is that SA and FAS scale mostly linearly at lower frequencies, before diverging at frequencies approaching the peak of the input FAS. The extent of the divergence and the frequency at which it initiates are dependent on many factors that control the shape of the FAS including the source, path and site effects. They are thus record dependent. This too may account for why AvgFAS is less efficient than FAS, a situation opposite to that of AvgSA in relation to SA, as the averaging over the rapidly decaying high frequency part of the spectrum may exaggerate the divergence in scaling between FAS and SA. Exploration of the sufficiency of the fragilities with respect to the record duration found that whilst SA was largely sufficient, the residuals of the correlation between the logarithms of FAS and the resulting EDPs were persistently negative for longer duration records. This too would suggest that differences in source and path effects may will contribute significantly to the overall variability the scaling between EDP and FAS and therefore deteriorating the efficiency.

By comparing the fragility models for more than 490 structural classes, we do believe that the trends seen in this comparison are relatively robust for a broad range of structure types, albeit with the caveat that only one type of EDP and NLTHA methodology have been explored. It is possible that differences might arise when considering more complex multi-degree of freedom structures, but there is little to suggest that improvements in the efficiency of FAS should be anticipated in these cases. Recognising then that greater inefficiency of FAS in predicting structural response is likely persistent, the obvious questions emerge as to what extent this influences the estimations of loss, whether the costs in terms of increased uncertainty in structural response outweigh the potential benefits of characterising ground motion in terms of FAS rather than SA, and what, if anything, could be done to better reconcile the two approaches.

4. Seismic Risk in terms of Fourier Spectra

Having established the means of running a complete PSHA in its entirety using only Fourier spectra, and then subsequently developing fragility functions using Fourier spectra as the input intensity measure, it is a relatively straightforward task to implement this process within a complete seismic risk calculation. For this purpose, an idealised, but not atypical, seismic risk calculation is undertaken using the same input source model, ground motion model and site configuration as that previously seen in the PSHA example. Although fragility models have been constructed for an extensive set of 490 building types, we focus here on just three classes that are representative of low and mid-rise structures commonly encountered in Europe:

i. **CR/LFINF-DUM/H2:** Two-storey reinforced concrete (RC), moderate ductility moment frame with infills.
ii. **CR/LDUAL-DUL/H4**: Four-storey RC with dual system (frames and walls) and low ductility.
iii. **CR/LDUAL-DUM/H8**: Eight-storey RC with dual system (frames and walls) and moderate ductility.

For the sake of comparison, ground motions are calculated for the fundamental elastic periods of the structures, which are found to be 0.250 s, 0.407 s and 0.772 s (4 Hz, 2.457 Hz and 1.295 Hz) for the CR/LFINF-DUM/H2, CR/LDUAL-DUL/H4 and CR/LDUAL-DUM/H8 typologies, respectively. From the analyses of efficiency, these three structure types were found to have a Pearson product moment correlation coefficient (R^2) between the intensity measure level and engineering demand parameter of 0.75, 0.82 and 0.8, respectively for SA, and 0.60, 0.72 and 0.71 respectively for the smoothed FAS. For the current illustrative purposes, we do not consider the average spectral intensity measures.

To understand how the use of an exclusively Fourier-based seismic hazard and risk calculation compares against alternative conventional response spectrum approaches, we carried out three alternative calculations, one using FAS from start to end, another using FAS to define input but SA as the predictor of structural response, and one using SA from start to end. The two alternative means of obtaining losses through SA are:

SA-RVT: Conversion of the Fourier spectra produced by the ground motion model of Bora *et al.* (2019) into response spectra, using the random vibration theory procedure, adapted from Boore (2003) [16], described by the authors and adopting their definition and empirical model of ground motion duration (D_{RVT}):

$$\ln D_{RVT,ij}(f_{osc}) = g\left(M_i, R_{RUP,ij}, V_{S30,j}, f_{osc}\right) + \delta E_i + \delta S_j + \delta W_{ij} \tag{2}$$

where $\delta E_i(f_{osc})$, $\delta S_j(f_{osc})$ and $\delta W_{ij}(f_{osc})$ are the event-to-event, site-to-site and site-corrected within-event variability, which are normally distributed variates with zero means and standard deviations of $\tau_E(f_{osc})$, $\phi_{S2S}(f_{osc})$ and $\phi_0(f_{osc})$, respectively. A full description of the RVT procedure is omitted for space, but it is noted that in order to determine the peak response of a SDOF oscillator at a given oscillator frequency it is necessary to describe the entire Fourier spectrum for the scenario. For this purpose, Bora *et al.* (2019) suggest using the mean spectrum (rather than the medium) such that:

$$\ln FAS_{ij}(f)^{mean} = \ln FAS_{ij}\left(M_i, R_{RUP,ij}, V_{S30,j}, f\right) + \frac{\tau_E^2(f)+\phi_{S2S}^2(f)+\phi_0^2(f)}{2} \quad (3)$$

and

$$\ln D_{RVT}(f_{osc})^{mean} = \ln D_{RVT}\left(M_i, R_{RUP,ij}, V_{S30,j}, f_{osc}\right) + \frac{\tau_E^2(f_{osc})+\phi_{S2S}^2(f_{osc})+\phi_0^2(f_{osc})}{2} \quad (4)$$

To constrain the variability of the resulting SA, Bora *et al.* (2019) deconstruct the residual differences between the observed spectral acceleration of records in their dataset and the median spectral acceleration derived using the mean FAS and random vibration theory. The resulting variances are found to be in reasonable agreement with those of the other NGA West 2 models, albeit slightly higher for larger magnitudes and higher frequencies. The resulting heteroskedastic standard deviation model for SA is adopted in this implementation.

SA-B14: Seismic risk assessment using a scaled backbone logic tree to the Boore et al. (2014) NGA West 2 ground motion model for response spectra [23]. This ground motion model is chosen as it shares a common dataset with the Bora *et al.* (2019) model and is shown to scale comparably to the resulting RVT-constructed response spectra. The Boore *et al.* (2014) model is chosen in favour of the alternative NGA West models largely because it shares a similarly degree of parameterisation as that of Bora *et al.* (2019), avoiding complex hanging wall and/or basin amplification terms that may skew differences between the two.

The seismic risk calculations are undertaken using the OpenQuake-engine classical risk calculator [24]. The vulnerability functions are derived by combining the fragility functions obtained in Section 3 with the consequence model shown in (Table 1). The mean loss ratios are those adopted in the European SERA project for the 2020 European Seismic Risk Model [25]. The seismic hazard curves for each location (in this case co-located with the asset) are computed using the classical PSHA approach, from which a probability of occurrence for each ground motion intensity measure level in the loss ratio matrix can be determined. The total probability of exceeding each loss ratio level is then calculated by summing the probabilities per loss ratio for each asset and subsequently multiplied by the value of the asset in order to obtain the monetary loss exceedance curve. As we are investigating the relative impact of adopting the different intensity measure types, all structures are located at the same site and we consider a unit value per asset; therefore, the resulting loss curves are in terms of the probability of exceeding a given loss ratio. A comparison of the risk curves, in terms of both the mean and the individual branches of the 12-branch logic tree, can be seen in Fig 6.

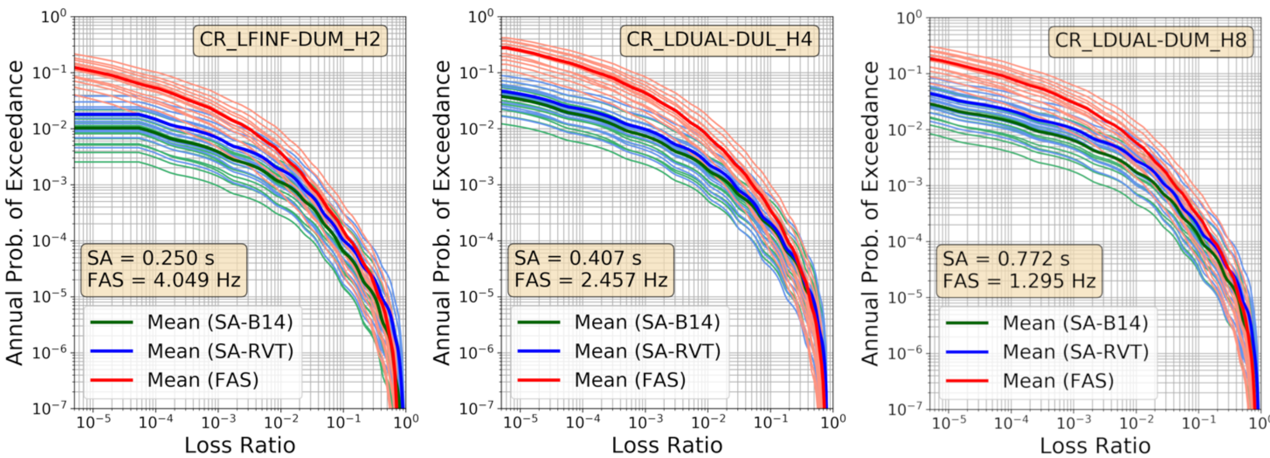

Fig. 6 – Mean loss curves for seismic risk calculations conducted using Fourier hazard and fragilities (FAS – red), Fourier hazard converted to SA using random vibration theory (SA-RVT - blue), and SA defined using only using the GMM of Boore *et al.* (2014) (SA-B14 – green)

The differences between the response spectrum cases (SA-RVT and SA-B14) and between the exclusively FAS case shown in Fig. 6 are quite substantial, with the exclusively FAS-based losses considerably larger for the lower range of the losses and higher probabilities of exceedance. This trend is persistent for the three typologies, indicating that it is not necessarily dependent on period. The relatively good agreement between the two SA methodologies, however, is also noteworthy. The SA-RVT approach generally predicts higher losses, which can easily be accounted for by its marginally higher standard deviations compared to Boore *et al.* (2014) and higher median motion at short-to-intermediate periods for small-to-moderate magnitudes. The differences between the SA- and FAS-based approaches are likely consequences of both the poorer efficiency of FAS as a predictor intensity measure of structural damage and the higher variability of the FAS ground motion model itself, which is approximately 0.1 log units greater in the frequency range considered here than the equivalent variability for the response spectrum GMMs.

5. Discussion and Considerations for Application

The comparisons of the fragility functions when using the Fourier amplitude spectrum in place of the conventional SA approaches clearly show that Fourier amplitude is a less efficient predictor of structural response, and that when propagating this to the seismic risk analysis this is likely to result in higher losses at shorter return periods. This result is not necessarily unexpected per se, as one would intuitively assume that a measure of a peak elastic response of a damped oscillator under forced vibration would likely be a better predictor of structural response than the spectrum of the free ground motion. Indeed, from random vibration theory (Eq. 1), we recognise that the response of a SDOF oscillator is itself dependent on the entire range of frequencies motion and not just that of the natural frequency of the oscillator itself. It is also interesting to note that scalar intensity measures accounting for multiple frequencies in the Fourier domain, e.g. AvgFAS, not only fail to improve on the efficiency of FAS itself, but in many cases they are notably poorer. One does not preclude the possibility that better predictors could be found derived directly from the FAS, but it would remain challenging to ensure sufficiency in terms of source, path and site characteristics.

Does this mean to say that direct use of the FAS in seismic hazard and risk studies offers no benefit over the conventional approaches and is limited only to the role of applying adjustments to the median value of ground motion models? We would argue that this is not the case. As highlighted in section 2, for the purposes of hazard-consistent selection and scaling of ground motion records for nonlinear-time history analysis, a complete process can be followed in which the characterisation of the response spectrum is entirely absent from the construction of the target spectrum. Adopting the conditional spectra approach, which is becoming more widely used in engineering practice, it is easy to see how a set of time histories can be generated directly from the Fourier target spectra [3], which may supplement the suite of available time-histories or otherwise capture characteristics of local ground motion not well constrained by observed strong motion records but inferable from weak motions. In regions of low seismicity this could be of potentially great importance.

Even where direct use of Fourier spectra may be sub-optimal in the seismic risk calculations, the results shown in Fig. 6 do, however, suggest that better agreement between Fourier and response spectrum approaches can be found even without requiring a response spectrum GMM, via the application of random vibration theory. This may be in fact the most appropriate means by which the benefits of the Fourier spectra in terms of its adjustment to different source, path and site characteristics and the scenario independence in terms of linear site amplification, can be reconciled with the improved efficiency that comes with the use of response spectra. We acknowledge that the direct integration of RVT into the risk calculation process results in a slower computation of the input ground motions, as it requires calculation of a FAS for a range of oscillator frequencies, in addition to the execution of two numerical integrals, in order to obtain the resulting peak SDOF response. This particular cost can be minimized with efficient implementation of the process into existing seismic hazard codes and may, in any case be tolerable depending on the application. The example calculations shown here generated complete Fourier spectra and SDOF response for more than 166,000 rupture scenarios and 12 GMM logic tree branches in only a matter of minutes using the OpenQuake-engine, which, though slower than direct use of SA, still suggests an achievable level of practicality. Furthermore, if one were to choose to adopt an event-based approach to seismic hazard and risk analysis, i.e. one in which the probability of exceeding a given level of ground shaking or loss is determined from the generation of repeated synthetic catalogues of ruptures and corresponding ground motions, there can be additional benefits. For example, one can sample the uncertainty in ground motion using a multivariate Gaussian distribution in order to generate the Fourier spectra to be input into the RVT framework that explicitly captures the cross-correlation between spectral periods. For heterogeneous building portfolios this may yet make the Fourier approach advantageous when compared to current SA approaches.

6. Conclusions

This analysis aimed to demonstrate the application of characterising ground motions exclusively in terms of Fourier amplitude spectra for PSHA, and the potential implications of doing so for the calculation of seismic risk. As a means of representing the intensity of ground shaking, and for the purpose of adapting the ground motion models to different source, path and site conditions, the direct use of Fourier amplitudes can offer benefits such as better constraint of the target spectrum for nonlinear time history analysis [3]. Its efficiency in predicting structural response, even when extended to multi-frequency dependent metrics, is notably poorer than that of response spectra, especially at shorter periods. This inefficiency would ultimately translate to higher losses in the risk domain. The analyses presented here are not exhaustive, and it remains to be seen whether any improvement or deterioration in efficiency would be achieved if attempting to derive fragility functions using more detailed dynamic analysis procedures compared to the equivalent inelastic SDOF approach. From the theory underpinning the definition of the response spectrum, there are few reasons to believe that this would be the case. A more promising pathway toward direct usage of FAS in seismic hazard analysis, and one that appears to produce losses comparable to those achieved with conventional approaches, is to adopt FAS GMMs as the basis for modelling the ground motion, then transform the resulting motions to SA via the application of random vibration theory. Though computationally costlier, the advantages of regional adaptability, scenario independence of linear site amplification, and the direct integration of uncertainties in the seismological properties of the region within the input ground motions, may provide sufficient benefits to outweigh the costs for some applications. Inevitably, further exploration is needed to understand the benefits and limitations more fully, but this investigation suggests the possibility of a greater role for FAS in the future of PSHA and PSRA.

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8. References

[1] Bora SS, Scherbaum F, Kuehn N, Stafford PJ (2014): Fourier spectral- and duration models for the generation of response spectra adjustable to different source-, path- and site conditions. *Bulletin of Earthquake Engineering*, **12**, 467-493.

[2] Bayless J, Abrahamson NA (2019): Summary of the BA18 Ground-Motion Model for Fourier Amplitude Spectra for Crustal Earthquakes in California. *Bulletin of the Seismological Society of America*, **109** (5), 2088-2105.

[3] Stafford P (2017): Interfrequency Correlations among Fourier Spectral Ordinates and Implications for Stochastic Ground Motion-Simulation. *Bulletin of the Seismological Society of America*, **107** (6), 2774 - 2791.

[4] Campbell KW (2003): Prediction of Strong Ground Motion Using the Hybrid Empirical Method and Its Use in the Development of Ground-Motion (Attenuation) Relations in Eastern North America, *Bulletin of the Seismological Society of America*, **93** (3), 1012-1033.

[5] Cotton F, Scherbaum F, Bommer JJ, Bungum H (2006): Criteria for selecting and adjusting ground motion models for specific target regions: Application to Central Europe rock sites, *Journal of Seismology,* **10**, 137-156.

[6] Bora SS, Scherbaum F, Kuehn N, Stafford P (2016): On the Relationship between Fourier and Response Spectra: Implications for the Adjustment of Empirical Ground-Motion Prediction Equations (GMPEs). *Bulletin of the Seismological Society of America*, **106** (3), 1235-1253.

[7] Stafford PJ, Rodriguez-Marek A, Edwards B, Kruiver PP, Bommer JJ (2017): Scenario Dependence of Linear Site-Effect Factors for Short-Period Response Spectral Ordinates. *Bulletin of the Seismological Society of America*, **107** (6), 2859-2872.

[8] Bora SS, Cotton F, Scherbaum F (2019): NGA-West2 Empirical Fourier and Duration Models to Generate Adjustable Response Spectra. *Earthquake Spectra*, **35** (1), 61-93.

[9] Yenier E, Atkinson GM (2015): An Equivalent Point-Source Model for Stochastic Simulation of Earthquake Ground Motions in California. *Bulletin of the Seismological Society of America*, **105** (3), 1435-1455.

[10] PEER (2015): NGA-East Median Ground-Motion Models for the Central and Eastern North America Region. *Technical Report PEER 2015/04*, Pacific Earthquake Engineering Research, Berkeley, USA.

[11] Woessner J, Danciu L, Giardini D, Crowley H, Cotton F, Grünthal G, Valensise G, Arvidsson R, Basili R, Demircioglu M, Hiemer S, Meletti C, Musson R, Rovida A, Sesetyan K, Stucchi M, and The SHARE Consortium (2015): The 2013 European Seismic Hazard Model: key components and results. *Bulletin of Earthquake Engineering*, **13**, 3553-3596.

[12] Kottke A, Abrahamson AA, Boore DM, Bozorgnia Y, Goulet C, Hollenback J, Kishida T, Der Kiureghian A, Ktendidou O-J, Kuehn N, Rathje EM, Silva WJ, Thompson E, Wang X (2018): Selection of Random Vibration Procedures for the NGA-East Project. *Technical Report PEER 2018/05*, Pacific Earthquake Engineering Research, Berkeley, USA.

[13] Atkinson GA, Bommer JJ, Abrahamson NA (2014): Alternative approaches to modeling epistemic uncertainty in ground motions in probabilistic seismic hazard analysis. *Seismological Research Letters*, **85** (6),1141-1144.

[14] Baker, J (2011): Conditional mean spectrum: Tool for ground motion selection. *Journal of Structural Engineering*, **137** (3), 322-331.

[15] Lin T, Harmsen SC, Baker JW, Luco N (2013): Conditional Spectrum Computation Incorporating Multiple Causal Earthquakes and Ground-Motion Prediction Models. *Bulletin of the Seismological Society of America*, **103** (2A), 1103-1116 .

[16] Boore DM (2003): Simulation of ground motion using the stochastic method. *Pure and Applied Geophysics*, **160** (3 - 4), 636-676.

[17] Martins L, Silva V (2018): A global database of vulnerability models for seismic risk assessment. *16th European Conference on Earthquake Engineering*, Thessaloniki, Greece.

[18] Bal IE, Bommer JJ, Stafford PJ, Crowley H, Pinho R (2010): The influence of geographical resolution of urban exposure data in an earthquake loss model for Istanbul. *Earthquake Spectra*, **26** (3), 619-634.

[19] Villar-Vega M, Silva V, Crowley H, Yepes C, Tarque N, Acevedo AB, Hube MA, Gustavo CD, María HS (2017): Development of a fragility model for the residential building stock in South America. *Earthquake Spectra*, **33** (2), 581-604.

[20] Silva V, Yepes-Estrada C, Dabbeek J, Martins L, Brzev S (2018): GED4ALL - Global Exposure Database for Multi-Hazard Risk Analysis – Multi-Hazard Exposure Taxonomy, *GEM Technical Report 2018-01*, GEM Foundation, Pavia, Italy.

[21] Kohrangi M, Bazzurro P, Vamvatsikos D (2016): Vector and scalar IMs in structural response estimation, part II: building demand assessment. *Earthquake Spectra*, **32** (3), 1525-1543.

[22] Konno K, Ohmachi T (1998): Ground-motion characteristics estimated from spectral ratio between horizontal and vertical components of microtremor. *Bulletin of the Seismological Society of America*, **88** (1), 228-241.

[23] Boore DM, Stewart JP, Seyhan E, Atkinson GM (2014): NGA-West 2 equations for predicting PGA, PGV and 5 % damped PSA for shallow crustal earthquakes. *Earthquake Spectra*, **30** (3), 1057-1085.

[24] Silva V, Crowley H, Pagani M, Monelli D, Pinho R (2014): Development of the OpenQuake engine, the Global Earthquake Model's open-source software for seismic risk assessment, *Natural Hazards,* **72** (3), 1409 – 1427.

[25] Romao X, Castro JM, Pereira N, Crowley H, Silva V, Martins L, Rodrigues D. (2019). European physical vulnerability models, *SERA Deliverable 26.5 available at*: http://eu-risk.eucentre.it/wp-content/uploads/2019/08/SERA_D26.5_Physical_Vulnerability.pdf

