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AN EFFICIENT INELASTICITY-SEPARATED SEISMIC RESPONSE ANALYSIS METHOD FOR ENGINEERING STRUCTUR

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Abstract

Nonlinear analysis is inevitable in evaluating the structural performance and often require time-consuming computation, especially for large-scale structures. This study propose a efficient nonlinear analysis method for seismic performance evaluation of engineering structure based on the inelasticity separated finite element method (IS-FEM), which is developed in recently years through generalizing the displacement decomposition concept in force analogy method (FAM) and have the characteristic of high efficiency and the potential of wide applicability. Within the framework of the proposed method, the nonlinear material strain is firstly decomposed into two parts, which are linear-elastic strain and inelastic strain. Then, the interpolation scheme is adopted to model the inelastic strain field of element approximately and the governing equation with the characteristic of inelasticity-separation is derived by using the principle of virtual work. Because the inelastic degrees of freedom representing the local nonlinear behavior of structure are separated from the global system, the structural stiffness matrix can keep in linear elastic state during analysis and this lead to the fact that the Woodbury formula can be used to implement efficient nonlinear analysis. By incorporating the governing equation of IS-FEM with the equation of motion and using the Newmark average acceleration method for integration, the inelasticity-separated dynamic governing equation is developed, in which the value of time increment is required for constructing the effective elastic stiffness. Because in such case the achievement of high efficient computation of Woodbury formula in each step require the effective elastic stiffness keeping unchanged as much as possible, the time increment cannot change optionally during analysis and this indicate that the computational cost of the whole analysis process may be huge. To take the efficiency advantage of Woodbury formula in seismic response analysis, an adaptive dynamic analysis strategy (ADAS) is finally proposed. The main idea of the ADAS is that several time increments and the corresponding coefficient matrixes required in Woodbury formula are predefined before analysis, and in each incremental step the appropriate time interval and the corresponding coefficient matrixes are determined adaptively according to the instantaneous inelastic state of global structure. The proposed ADAS not only can meet the requirement of Woodbury formula per step, but also be helpful for improving the efficiency of whole dynamic analysis process.

Keywords: Inelasticity-separated finite element method; nonlinear analysis; Woodbury formula; computational efficiency

1. Introduction

Evaluating the nonlinear seismic performance of engineering structures is of great important in performance-based earthquake engineering [1-2]. There are many methods developed for performing nonlinear seismic performance evaluation of structures, such as static pushover analysis method and nonlinear dynamic analysis method. The nonlinear dynamic analysis method is known as the most accurate method because it can simulate the actual response of structure under earthquake excitation. However. Because of the huge computation cost required for the nonlinear dynamic analysis, it is hard to apply this method widely. Thus, developing advanced nonlinear seismic response analysis method that can improve computational efficiency still attract many attention.

To analyze the nonlinear response of engineering structure which is dependent on the loading path, an incremental step-by-step approach should be employed and iteration usually cannot be avoided. This lead to the fact that the global stiffness matrix should be updated and re-factorized repeatedly. With the increase of the scale of the problem, the dimension of structural stiffness matrix increase and the updating and re-factorization of the global stiffness matrix become a computational expensive process [3]. To reduce the computational cost of nonlinear analysis process, many high efficient methods have been proposed [4-8], such as the sparse solution algorithm of system of equations, parallel computation technique etc. By decomposing the total displacement of each structural member according to the corresponding initial elastic stiffness and formulating the inelastic displacement of each member as additional degrees of freedom (DOFs), Wong et al. [9] successfully applied the force analogy method (FAM) which was original proposed by Lin [11] in civil structures for implementing high efficient seismic response analysis. The high efficiency of the FAM stem from the fact that it can depict the global structural nonlinear behavior through unchanging stiffness matrix during analysis [9,10]. Considering that the nonlinearity generally occur at some local regions of structure, many researchers aim to achieve high efficient nonlinear analysis by taking advantage of the local nonlinearity characteristic. The structural reanalysis, which intend to efficiently calculate the response of structures with local modification, provide a typical way for solving the local nonlinearity problem. Based on the concept of displacement decomposition in FAM and the fundamental theory of finite element method (FEM), Li et al. [12-14] proposed a novel efficient nonlinear analysis method for local nonlinearity problem, which is called inelasticity-separated finite element method (IS-FEM). Because this method begin at the material level and was developed based on FEM, it can be used to implement refined simulation and has the advantage of wide applicability. Although the basic framework of IS-FEM for nonlinear iterative solution and the method of solving the equation of motion have been presented by Li et al. [12,13], The seismic response method based on the IS-FEM still should be investigated.

In this paper, the basic theory of IS-FEM is firstly explained. Then, by introducing the inelasticity-separated governing equation into equation of motion and employing Newmark average acceleration method, the inelasticity-separated seismic response analysis method is established. Because the Woodbury formula is adopted as the solver, the updating and re-factorization of large dimension global stiffness matrix can be avoided per iteration and the main computational effort only focuses on a small dimension matrix representing local nonlinearity. Considering that in each step, the achievement of high efficient computation of Woodbury formula require the effective elastic stiffness, which is related to value of time interval, keeping unchanged as much as possible. The time interval cannot change optionally during analysis and this indicate that although the high efficiency can be achieved in each iteration, the computational cost of the whole analysis process may be huge because the minimum value of time interval generally should be adopted in such case. To make full use of the efficiency advantage of Woodbury formula, an adaptive dynamic analysis strategy (ADAS) is proposed for implement high efficient seismic response analysis. Finally, a numerical example is presented to illustrate the application of the proposed method.

2. Basic theory of IS-FEM

By decomposing the strain of the nonlinear material into two parts, linear-elastic strain and inelastic strain, the IS-FEM can keep the structural global stiffness matrix unchanged throughout the whole analysis process and express the local nonlinear behavior through separated manner. Fig. 1 illustrates the strain decomposition process of uniaxial stress-strain relation of a nonlinear material. It can be seen that according to initial material modulus E_e, the total strain ε at point Q can be decomposed as:

$$\varepsilon = \varepsilon' + \varepsilon'' \tag{1}$$

where ε' represent linear-elastic strain, and ε'' represent inelastic strain. Additionaly, the stress σ at point Q can be expressed as:

$$\sigma = E_e \varepsilon' = E_e(\varepsilon - \varepsilon'') \tag{2}$$

For the general case of multiaxial strain-stress relations, the strain decomposition equation (Eq. (1)) and the stress calculation equation (Eq. (2)) can be generalized accordingly by replaceing the strain, stress and elastic modulus by the corresponding vector and matrix forms.

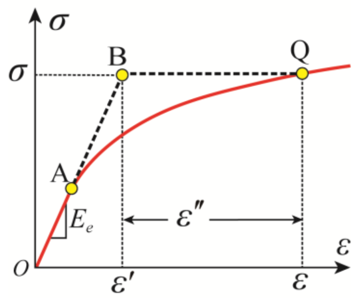

Fig. 1 Strain decomposition of uniaxial stress-strain relation of nonlinear material

Then, the inelastic strain field of a element can be modeled by interpolation scheme in which some collocation points should be predefined for interpolation. Considering that the nonlinearity generally occur within some local regions and incremental solution scheme should be used for analysis, it can be inferred that in a certain step, the incremental inelastic strains at most collocation points will be equal to zero and nonzero inelastic strain increments only occur in the collocation points that are located in local nonlinear regions. Based on the principle of virtual work, the incremental inelasticity-separated governing equaiotn can be constructed in which only the collocation points with nonzero incremental inelastic strain are considered. The governing equation can be written as:

$$\begin{bmatrix} \mathbf{K}_e & \mathbf{K}' \\ \mathbf{K}'^{\mathrm{T}} & \mathbf{K}''_p \end{bmatrix} \begin{bmatrix} \Delta\mathbf{X} \\ -\Delta\mathbf{E}''_{pr} \end{bmatrix} = \begin{bmatrix} \Delta\mathbf{F} \\ \mathbf{0} \end{bmatrix} \tag{3}$$

where the matrix $\mathbf{K}_e$ with dimension $n \times n$ is the initial elastic stiffness matrix of global system and n represent the total number of displacment DOFs of structure; $\Delta\mathbf{F}$ and $\Delta\mathbf{X}$ are the vectors of incremental applied load and nodal displacement respectively; $\Delta\mathbf{E}''_{pr}$ (with dimension $m \times 1$) is the vector that is assembled by the nonzero incremental inelastic strains, each term in this vector represent an inelastic degree-of-freedom (IDOF) and m represents the number of IDOFs; $\mathbf{K}'$ (with dimension $n \times m$) and $\mathbf{K}''_p$ (with dimension $m \times m$) are the coefficient matrices related to local nonlinearity. When adopting the full Newton-Raphson iteration scheme, the matrices $\mathbf{K}'$, $\mathbf{K}''_p$ and the number of IDOFs (i.e. m) should be updated in each iteration. For the local nonlinearity problem, the number of IDOFs (i.e. m) will be much smaller than the dimension of global stiffness matrix (i.e. $m<<n$) and the Woodbury formula can be employted to solve Eq. (3) efficiently:

$$\underbrace{\Delta\mathbf{X}}_{n\times 1} = (\underbrace{\mathbf{K}_e^{-1}}_{n\times n} + \underbrace{\mathbf{K}_e^{-1}}_{n\times n}\underbrace{\mathbf{K}'}_{n\times m}\underbrace{(\mathbf{K}''_p - \mathbf{K}'^{\mathrm{T}}\mathbf{K}_e^{-1}\mathbf{K}')^{-1}}_{m\times m}\underbrace{\mathbf{K}'^{\mathrm{T}}}_{m\times n}\underbrace{\mathbf{K}_e^{-1}}_{n\times n})\underbrace{\Delta\mathbf{F}}_{n\times 1} \tag{4}$$

in which

$$\mathbf{K}_{inf} = \mathbf{K}'^{\mathrm{T}}\mathbf{K}_e^{-1}\mathbf{K}'$$

In Eq. (4), the matrix $(\mathbf{K}''_p - \mathbf{K}_{inf})$ (with dimension $m\times m$) is the Schur complement of elastic stiffness matrrix $\mathbf{K}_e$. Because the elastic stiffness matrix of global structure can keep unchanged during analysis, it only require to be factored once at the beginning of analysis. Thus, the main computational cost of Woodbury formula is invested in the factrization of the small dimension Schur complement matrix rather that the large dimension global tangent stiffness matrix of structure. Accoring to the above disscussion, it can be seen that the high efficiency of Woodbury formula in IS-FEM mainly stem from the local nonlinearity property of structure and the invariable characteristic of elastic stiffness matrix.

3. Seismic response analysis method based on IS-FEM

3.1. Equation of motion

For the structure subjected to ground motion excitation, the incremental equation of motion can be expressed as follows:

$$\mathbf{M}\Delta\ddot{\mathbf{X}} + \mathbf{C}\Delta\dot{\mathbf{X}} + \Delta\mathbf{F} = -\mathbf{M}\boldsymbol{\iota}\Delta\ddot{x}_g \tag{5}$$

where **M** and **C** are the mass matrix and damping matrix respectively. In this study, the damping matrix is calculated based on the Rayleigh damping. Furthermore, in Eq. (5), the vectors $\Delta\ddot{\mathbf{X}}$ and $\Delta\dot{\mathbf{X}}$ denote the vectors of relative acceleration and velocity increment, respectively; the vector $\boldsymbol{\iota}$ is the influence coefficient vector; $\Delta\ddot{x}_g = \ddot{x}_g(t_k) - \ddot{x}_g(t_{k-1})$ represents the difference between the ground motion acceleration at steps k and k-1, and $\ddot{x}_g(t)$ denote the ground motion acceleration time history. By substituting the first equation of Eq. (3) (i.e $\mathbf{K}_e\Delta\mathbf{X} - \mathbf{K}'\Delta\mathbf{E}''_{pr} = \Delta\mathbf{F}$) into Eq. (5), the inelasticity-separated equaiton of motion can be established as follows:

$$\mathbf{M}\Delta\ddot{\mathbf{X}} + \mathbf{C}\Delta\dot{\mathbf{X}} + \mathbf{K}_e\Delta\mathbf{X} = -\mathbf{M}\boldsymbol{\iota}\Delta\ddot{x}_g + \mathbf{K}'\Delta\mathbf{E}''_{pr} \tag{6}$$

It can be seen from Eq. (6) that the global stiffness matrix in the left side of the equation can remain elastic state and the effect of local nonlinear behavior is considered through the fictitious forces $\mathbf{K}'\Delta\mathbf{E}''_{pr}$ appeared in the right side of the equaiton.

3.2. Adaptive solution scheme

By treating the term $-\mathbf{M}\boldsymbol{\iota}\Delta\ddot{x}_g + \mathbf{K}'\Delta\mathbf{E}''_{pr}$ in the right side of the Eq. (6) as the extermal excitation applied to the elastic structure and introducing the Newmark average acceleration method for integration, the following equation can be established:

$$\bar{\mathbf{K}}_e\Delta\mathbf{X} = \Delta\bar{\mathbf{F}} + \mathbf{K}'\Delta\mathbf{E}''_{pr} \tag{7}$$

in which $\bar{\mathbf{K}}_e$ and $\Delta\bar{\mathbf{F}}$ denote effective elastic stiffness matrix of structure and the effective incremental load vector respectively. The equations for calculating $\bar{\mathbf{K}}_e$ and $\Delta\bar{\mathbf{F}}$ are as follows:

$$\bar{\mathbf{K}}_e = \mathbf{K}_e + \frac{2}{\Delta t}\mathbf{C} + \frac{4}{(\Delta t)^2}\mathbf{M} \tag{8}$$

$$\Delta\bar{\mathbf{F}} = -\mathbf{M}\boldsymbol{\iota}\Delta\ddot{x}_g + \left(\frac{4}{\Delta t}\mathbf{M} + 2\mathbf{C}\right)\dot{\mathbf{X}}(t_{k-1}) + 2\mathbf{M}\ddot{\mathbf{X}}(t_{k-1}) \tag{9}$$

where Δt denote the time interval for analysis; $\dot{\mathbf{X}}(t_{k-1})$ and $\ddot{\mathbf{X}}(t_{k-1})$ are the vectors of relative acceleration and velocity of structure at step k-1 respectively. By integrating the Eq. (7) with the second equation of Eq. (3) (i.e $\mathbf{K}'^{\mathrm{T}}\Delta\mathbf{X}-\mathbf{K}''_p\Delta\mathbf{E}''_{pr}=\mathbf{0}$), the dynamic governing equation with the form of inelasticity-separation can be estblished as follows:

$$\begin{bmatrix} \bar{\mathbf{K}}_e & \mathbf{K}' \\ \mathbf{K}'^{\mathrm{T}} & \mathbf{K}''_p \end{bmatrix}\begin{bmatrix} \Delta\mathbf{X} \\ -\Delta\mathbf{E}''_{pr} \end{bmatrix}=\begin{bmatrix} \Delta\bar{\mathbf{F}} \\ \mathbf{0} \end{bmatrix} \tag{10}$$

The Woodbury solution formula for dynamic problem can be obtained accordingly.

$$\Delta\mathbf{X}=\left(\bar{\mathbf{K}}_e^{-1}+\bar{\mathbf{K}}_e^{-1}\mathbf{K}'(\mathbf{K}''_p-\bar{\mathbf{K}}_{inf})^{-1}\mathbf{K}'^{\mathrm{T}}\bar{\mathbf{K}}_e^{-1}\right)\Delta\mathbf{F} \tag{11}$$

where

$$\bar{\mathbf{K}}_{inf}=\mathbf{K}'^{\mathrm{T}}\bar{\mathbf{K}}_e^{-1}\mathbf{K}'$$

It can be seen from Eqs. (8) and (11) that the effective elastic stiffness matrix $\bar{\mathbf{K}}_e$ and coefficient matrix $\bar{\mathbf{K}}_{inf}$ are related to time interval Δt. Thus, to achieve the high efficiency of the Woodbury formula (Eq. (11)) in each step, the time interval should be defined by a constant value. However, for the seismic response analysis focused on the study, the instantaneous intensity of the ground motion exciation is changing in real time and in most steps, the instantaneous intensity of the ground motion exciation will be in a relatively lower level. Fig. 2 present the schematic representation of a typical ground motion acceleration time history curve. It can be seen that the the large instantaneous ground motion intensity only occur in a very small time section. If the constant time interval is defined for seismic response analysis, a relatively small value of time interval that can guarantee iteration convergence in high intensity region should be seleted. In such case, although the high efficiency advantage of Woodbury formula can be achieved in each step, the computational cost of the whole analysis process may be huge because the use of small time interval may increase the number of incremental step greatly such that the IS-FEM is hard to be used for performing high efficient seismic response analysis.

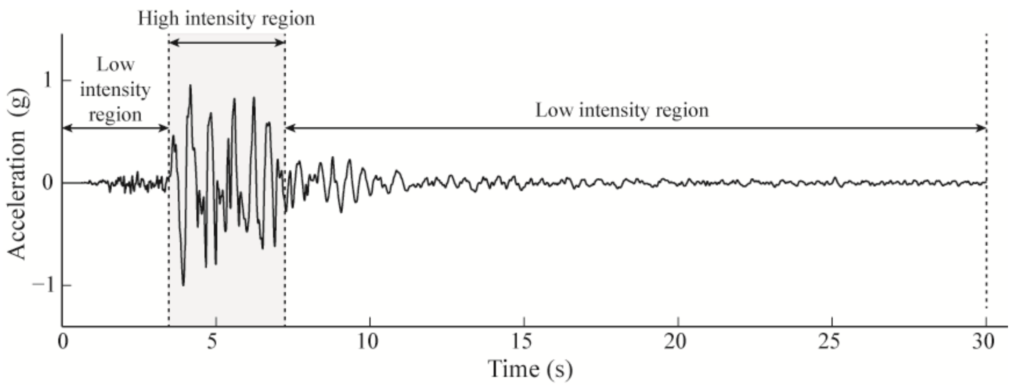

Fig. 2 Typical ground motion time history curve

To improve the computational performance of the IS-FEM in seismic response analysis, an adaptive dynamic analysis strategy (ADAS) that can select the time interval adaptively during analysis on the premise of maintaining the high efficiency of Woodbury formula is proposed in this subsection. Firstly, at the beginning of analysis, several time intervals are predefined and the corresponding effective elastic stiffness are calculated. Assuming that there are c time intervals are predefined, the corresponding effective elastic stiffness can be calculated as follows:

$$\begin{aligned}
\left(\bar{\mathbf{K}}_e\right)_{\Delta t_1} &= \mathbf{K}_e + \frac{2}{\Delta t_1}\mathbf{C} + \frac{4}{\left(\Delta t_1\right)^2}\mathbf{M} \\
\left(\bar{\mathbf{K}}_e\right)_{\Delta t_2} &= \mathbf{K}_e + \frac{2}{\Delta t_2}\mathbf{C} + \frac{4}{\left(\Delta t_2\right)^2}\mathbf{M} \\
&\vdots \\
\left(\bar{\mathbf{K}}_e\right)_{\Delta t_c} &= \mathbf{K}_e + \frac{2}{\Delta t_c}\mathbf{C} + \frac{4}{\left(\Delta t_c\right)^2}\mathbf{M}
\end{aligned} \tag{12}$$

where Δt_1, Δt_2, …, Δt_c represent the predefinded time intervals and they follow the following relation:

$$\Delta t\_1 > \Delta t\_2 > \cdots > \Delta t\_c \tag{13}$$

The effective elastic stiffness matrices calculated in Eq. (12) can be factorized before analysis. The factorized matrices can be used for the subsequent analysis. In this study, the LDLT method is adopted for factorization. Thus, there are

$$\begin{aligned}
\left(\bar{\mathbf{K}}_e\right)_{\Delta t_1} &= \bar{\mathbf{L}}_{\Delta t_1}\bar{\mathbf{D}}_{\Delta t_1}\bar{\mathbf{L}}_{\Delta t_1}^{\mathrm{T}} \\
\left(\bar{\mathbf{K}}_e\right)_{\Delta t_2} &= \bar{\mathbf{L}}_{\Delta t_2}\bar{\mathbf{D}}_{\Delta t_2}\bar{\mathbf{L}}_{\Delta t_2}^{\mathrm{T}} \\
&\vdots \\
\left(\bar{\mathbf{K}}_e\right)_{\Delta t_c} &= \bar{\mathbf{L}}_{\Delta t_c}\bar{\mathbf{D}}_{\Delta t_c}\bar{\mathbf{L}}_{\Delta t_c}^{\mathrm{T}}
\end{aligned} \tag{14}$$

where $\bar{\mathbf{L}}_{\Delta t\_i}$ and $\bar{\mathbf{D}}_{\Delta t\_i}$ are the lower triangular matrix and diagonal matrix factorized from the i-th effective elastic stiffness matrix $(\bar{\mathbf{K}}_e)_{\Delta t\_i}$ $(1 \le i \le c)$. Considering that in each iteraiton, the matrix $\bar{\mathbf{K}}_{inf}$ in Woodbury formula can be obtained directly from the matrix $\bar{\mathbf{K}}_{INF}$, which denote the matrix $\bar{\mathbf{K}}_{inf}$ corresponding to the special case of nonlineariy occuring in whole domain of strucutrue [14], the matrix $\bar{\mathbf{K}}_{INF}$ corresponding to various time intervals also can be calucluated before analysis:

$$\begin{aligned}
\left(\bar{\mathbf{K}}_{INF}\right)_{\Delta t_1} &= \hat{\mathbf{K}}'^{\mathrm{T}}\left(\bar{\mathbf{K}}_e\right)_{\Delta t_1}^{-1}\hat{\mathbf{K}}' \\
\left(\bar{\mathbf{K}}_{INF}\right)_{\Delta t_2} &= \hat{\mathbf{K}}'^{\mathrm{T}}\left(\bar{\mathbf{K}}_e\right)_{\Delta t_2}^{-1}\hat{\mathbf{K}}' \\
&\vdots \\
\left(\bar{\mathbf{K}}_{INF}\right)_{\Delta t_c} &= \hat{\mathbf{K}}'^{\mathrm{T}}\left(\bar{\mathbf{K}}_e\right)_{\Delta t_c}^{-1}\hat{\mathbf{K}}'
\end{aligned} \tag{15}$$

where $\hat{\mathbf{K}}'$ represent the matrix $\mathbf{K}'$ correponding the case of global nonlineariy.

Then, duing analysis, the appropriate time interval for a given incremental step can be determined adaptively according to the current nonlinear state of global structure, and the correponeding matrices calculated before analysis (i.e. the matrices obtained by Eqs. (14) and (15)) can be used directly to implement high efficent calculation of Woodbury formula. To achieve the idea of adaptive analysis presented above, a adaptive time interval selection scheme should be established. In this study, the number of iteration required for each incremental step is adopted as the indicator to select the appropriate time interval. Assuming that the time interval of k-th incremental step is $\Delta t_k = \Delta t\_i$ $(1 \le i \le c)$ and the number of iteration for convergence in this step is $N_{iter,k}$, the time interval of the next incremental step (i.e. $(k+1)$-th incremental step) can be determined adaptively through the following conditional judgment statements.

① If $N_{iter,k} \le N_0$, in which the value of N_0 denote the threshold value of iteration number definded before analysis, and $max\{N_{iter,k-u+1}, \ldots, N_{iter,k}\} \le N_0$, which means the iteraiton number of u continuous steps are less than or equal to N_0, then $\Delta t_{k+1} = \Delta t\_(i\text{-}1)$ (i should be greater than 1);

② If $N_{iter,k} \le N_0$ and $max\{N_{iter,k-u+1}$, ... , $N_{iter,k}\} > N_0$, then $\Delta t_{k+1}=\Delta t\_i$;

③ If $N_0<N_{iter,k}<N_{max}$, in which the value of N_{max} is the predefined maximum iteration number, then $\Delta t_{k+1}=\Delta t\_(i+1)$ (i should be less than c);

④ If $N_{iter,k}=N_{max}$, which indicates that the iteration is divergency in k-th incremental step, then the k-th step should be restarted by increasing the time interval (i.e. $\Delta t_k=\Delta t\_(i+1)$ where i should be less than c).

It can be seen from the above presentation that the proposed ADAS should use three predefined parameters (i.e. N_0, u, N_{max}) and the value of N_0 should be less than N_{max}. For the threshold value N_0, it is suggested that this value can be determined by the equation $N_0= N_{max}/2$. Fig. 3 presents the schematic flowchart of the proposed seismic response analysis method. It can be seen the the although the proposed method can increase the computational complexity of preprocessing process to some extent, the computation efficency of Woodbury formula in nonlinear analysis stage can maintain in a very high level. Because for a given structrual model the preprocessing process only require to perform once, the high effiecency advantage of the proposed method will become very significant when multiple earquake excitations and ground motion intensities are considered, such as incremental dynamic analyses.

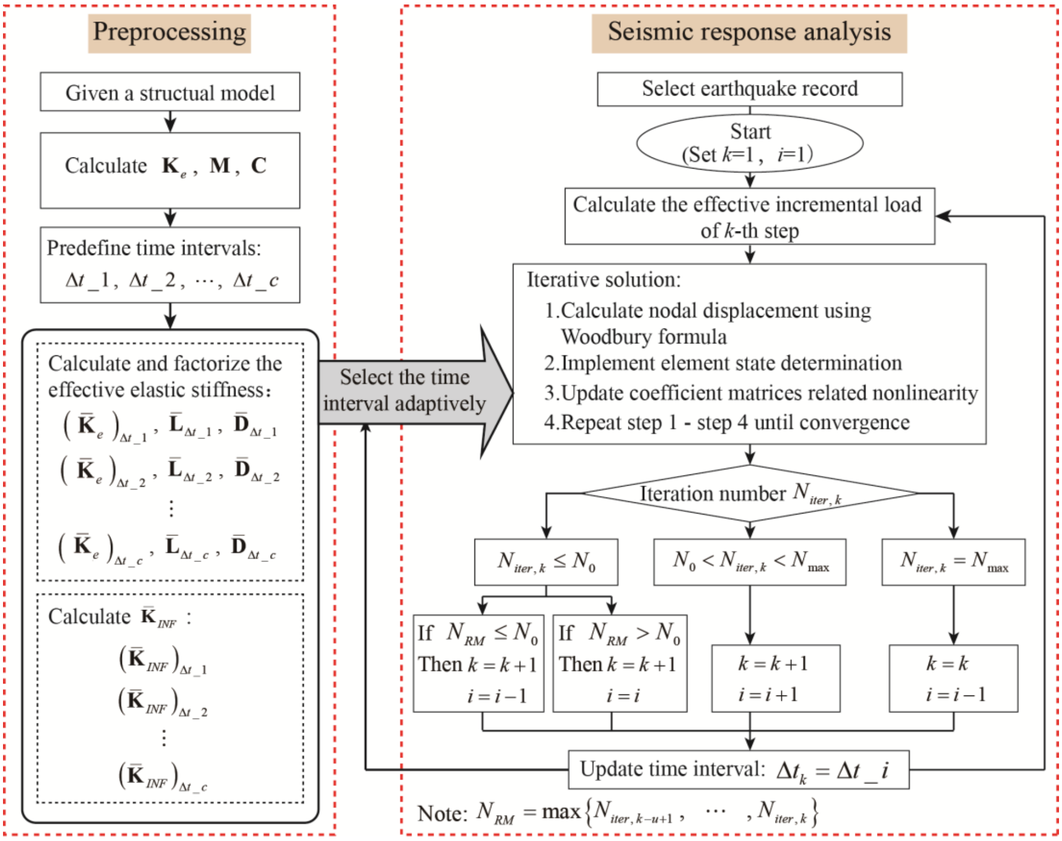

Fig. 3 Solution flowchart

4. Numerical example

An eight-story three-bay reinforced concrete frame structure shown in Fig. 4 is used to implement nonlinear seismic response analysis for illustrating the application of the proposed method. The structure was designed according to the Chinese code (GB 50011-2010) [16]. The beam and column members are modeled by the inelasticity-separated fiber beam-column element developed by Li et al. [13] and each member is modeled by three elements. The beam column joints are simulated by the model proposed by Yu et al. [15]. The total number of nodal DOFs of the structural model is 1464. The fundamental period of the structure is 1.3 s. The ground motion record from the 1995 Kobe earthquake is selected as the excitation and the peak ground accelerations (PGAs) is scaled to 0.5 g. Three time intervals, which are Δt_1=0.01, Δt_2=0.001 and Δt_3=0.0001, was defined before analysis and the corresponding matrices required for Woodbury formula are calculated based on Eqs. (12), (14) and (15). The maximum iteration number N_{max} and the threshold value N_0 are taken as 10 and 5 respectively. Additionally, the improved Woodbury formula developed by Yu et al. [14] is adopted in the example for reduce the computational cost further. Fig. 5 shows the displacement history responses of the top floor. Fig. 6 shows the time interval with respect to the incremental step. It can be seen that the smaller time intervals are only used in minority steps and the total number of incremental step can maintain in a relatively small level. Thus, the proposed method can achieve the balance between the requirement of Woodbury formula for high efficient computation in each iteration and the high efficiency of whole seismic response analysis process.

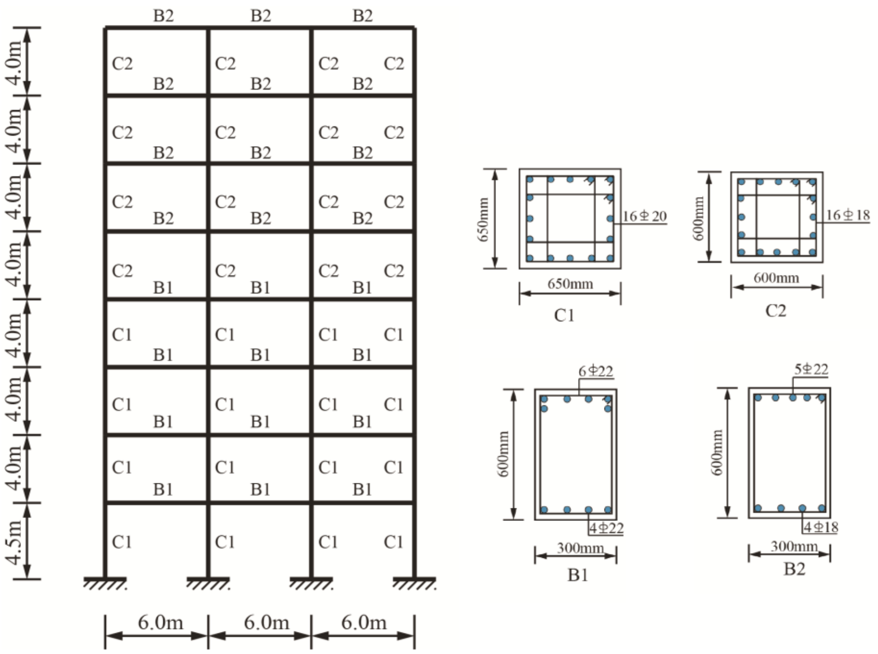

Fig. 4 Frame structure model

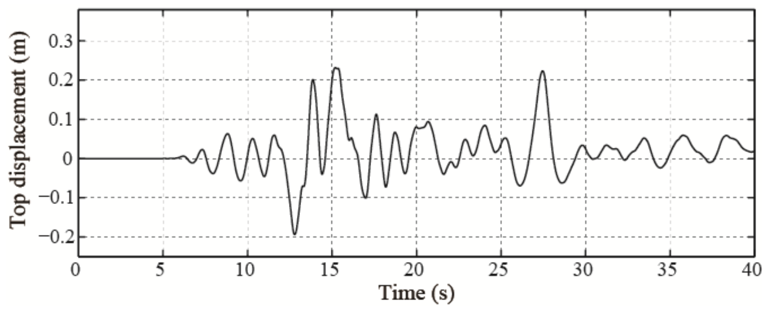

Fig. 5 Displacement history response of top floor

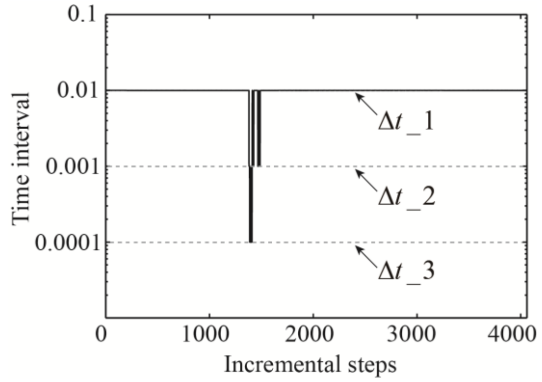

Fig. 6 History of time interval

5. Conclusion

In this study, a novel seismic response analysis method is proposed based on IS-FEM. By decomposing the strain of nonlinear material into linear-elastic part and inelastic part and formulating the local nonlinearity behavior through inelasticity-separated manner, the IS-FEM can use the Woodbury formula to improve the computational efficiency significantly. To extend the IS-FEM for use in seismic response analysis, the Newmark method is adopted to establish the inelasticity-separated dynamic governing equation and the corresponding Woodbury solution formula. Because in the dynamic Woodbury formula, the effective elastic stiffness matrix and the coefficient matrix $\bar{\mathbf{K}}_{inf}$ are related to the value time interval and the requirement of Woodbury formula for high efficient calculation is that the updating of effective elastic stiffness matrix should be avoided, the time interval cannot change optionally during analysis. In such case, although the use of Woodbury can achieve high efficient computation in each step, the computational cost of the whole analysis process may be huge because a very small time interval generally should be adopted. To improve the computation performance of IS-FEM in dynamic analysis and reduce the computational cost further, an adaptive dynamic analysis strategy (ADAS) is proposed. The proposed ADAS should determine several time increments and calculate the corresponding coefficient matrixes required in Woodbury formula before analysis. Thus, appropriate time interval can be selected adaptively in each step according to the instantaneous inelastic state of global structure. Because the use the proposed ADAS allow for changing the time interval adaptively during analysis on the premise of maintaining the high efficiency advantage of Woodbury formula, it is helpful for improving the computational efficiency of whole analysis process.

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