MODELING THE NONLINEAR BEHAVIOR OF RC BEAM-COLUMN JOINT USING INELASTICITY SEPARATED FINITE ELEMENT METHOD

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Abstract

Nonlinear response analysis is of great important in seismic performance evaluation and design of engineering structure. For the reinforced concrete (RC) frame structure, the primary members for resisting seismic load are beam, column and beam-column joint. In most available analysis programs, the joint was generally assumed as a rigid panel for simplicity. Although the rigid joint assumption may be acceptable in some cases, however, it have been demonstrated by many researchers that the effect of joint deformation on structural response cannot be ignored, both for the ductility structures design according to the current seismic code and the structures with non-seismic joint. In this study, a new method for seismic response analysis of RC frame structure considering the effect of beam-column joint is developed based on the inelasticity-separated finite element method (IS-FEM), which is an accurate and efficient algorithm for structural nonlinear analysis. In the proposed method, an improved inelasticity-separated fiber beam-column element is firstly proposed by assigning additional bond slip inelastic degrees of freedom to the member ends and incorporating them into the framework of inelasticity-separated fiber element formulation. Then, to model the concrete core of joint region and depict its nonlinear shear behavior, an inelasticity-separated shear panel element is proposed and a local inelastic mechanism of shear force versus inelastic shear strain is established successively. By assembling the improved inelasticity-separated fiber element and shear panel element into the global system, the inelasticity-separated RC frame model that can consider the major components contribution to the beam column joint deformation, including the bond slip deformation of beam longitudinal reinforcement in the joint and joint distortion, is established. Because the global stiffness matrix keep unchanged during analysis and in each iteration the modified stiffness representing local nonlinear behavior can be treated at the low rank perturbation of initial elastic stiffness, the Woodbury formula can be used to solve the governing equation efficiently. Thus, the efficiency advantage of IS-FEM still retain. Finally, the proposed method is verified against experimental data and the results demonstrate its exactness.

Keywords: Inelasticity-Separated Finite Element Method; Bond Slip Deformation; Joint Shear Distortion; Woodbury Formula.

1. Introduction

Nonlinear seismic response analyses are of great importance for the determination of the structural seismic performance. To improve the seismic performance of engineering structures, the nonlinear analysis method that can simulate the structural seismic response accurately and efficiently always attract many attention.

In traditional methods of analyzing the nonlinear behavior of RC frame structures, the beam-column joints are usually ignored or assumed to remain rigid [1]. However, many investigations have shown that beam-column joints may exhibit remarkable nonlinearity under seismic excitation [1-3]. For the older RC frames, many researchers suggested modeling the nonlinear behavior of beam-column joints explicitly because the beam-column joints in those structures usually have no shear reinforcements [1-2]. Additionally, for the newly structures, although the beam-column joints generally have seismic reinforcement detailing, many studies also shown that using the structural model considering the joint deformation can improve the accuracy level of the simulation results [1,4]. Therefore, developing an appropriate beam-column joint model is essential for evaluating the seismic performance of RC frame structures accurately. To date, several analytical models have been proposed for simulating the nonlinear behavior of beam-column joints [5-10]. For example, Lowes and Altoontash [5] proposed a super element, which consists of a joint shear panel, four interface-shear springs and eight bar-slip springs, to model the inelastic response of RC beam-column joint. Although this model can provide accurate representation for a joint, it requires some detailed information that is currently difficult to obtain [7]. To overcome the limitation of the above model, Altoontash [9] subsequently developed a simplified joint model, which consists of a joint shear panel and four zero-length bond-slip rotation springs. Furthermore, some researchers have simulated the beam-column joint through continuum-type elements combined with refined discretization [10].

The inelasticity-separated finite element method (IS-FEM) proposed by Li et al. [11-13] is developed based on the displacement decomposition concept in force analogy method (FAM) [17] and have the ability of achieving efficient and accurate simulation of structural nonlinear behavior. Because the local nonlinear behavior in IS-FEM can be captured by a series of small-dimensional coefficient matrices separated from the global system, and the global stiffness matrix of structure can keep unchanged throughout the analysis process, the Woodbury formula can be adopted as the solver to improve the computation efficiency [11-16]. More recently, Li et al. [12] proposed inelasticity-separated fiber beam-column element (IS-Fiber element) in which the section deformation is decomposed by linear-elastic and inelastic parts. However, the IS-Fiber element only can be used to establish the structural model without beam-column joints. To make full use of the high efficiency advantage of the IS-FEM for analyzing the nonlinear response of RC frame structures, further investigation that can consider the simulation of beam-column joint is still required.

In this study, a novel method for the simulation of nonlinear behavior of beam-column joint is proposed based on the IS-FEM. Firstly, an improved inelasticity-separated fiber beam column element (improved IS-Fiber element) that is derived by assigning two inelastic rotation hinges (RHs) to the ends of the existing IS-Fiber element is proposed. The RHs are used to depict the bond-slip nonlinear behavior at beam ends and the corresponding rotational degrees of freedom (RDOFs) are formulated through inelasticity-separated manner. Secondly, by using an inelastic sliding hinge (SH) to represent the shear inelastic behavior of the RC beam-column joint core and by establishing the corresponding inelasticity-separated governing equation, a shear panel element that can model the failure of joint region is developed based on the framework of IS-FEM. Because the Woodbury formula is employed as the solver, the proposed method can implement high efficient nonlinear analysis. The exactness and the availability of the proposed method is finally demonstrated by comparing the simulation results with the experimental data.

2. Background of IS-FEM

The IS-FEM is developed by extending the displacement decomposition concept in FAM to the framework of finite element method, as presented above. The fundamental principle of the IS-FEM is separating the material nonlinearity occurring in some local regions from global structure and depicting the separated nonlinear strain or deformation through additional inelastic degrees of freedom (IDOFs). Because the structural stiffness matrix can keep unchanged during analysis and the Woodbury formula is used as the solver, the IS-FEM can achieve high efficient nonlinear analysis. To illustrate the derivation of the structural governing equation based on the idea of inelasticity-separation, a plane frame shown in Figure 1 is taken as the example. Assuming that a beam-column element exhibits inelastic bending behavior in a certain step, the section curvature should be decomposed into linear elastic and inelastic parts according to the initial elastic bending stiffness of section firstly. In such case, the section moment can be evaluated by multiplying the unchanged elastic bending stiffness by the linear elastic section curvature, as illustrated in Figure 1. Then, by using interpolation method to depict the inelastic section curvature field along the element and adopting the principle of virtual work and incremental solution scheme, the inelasticity-separated governing equation of structure can be obtained as follows:

$$\begin{bmatrix} \mathbf{K}_e & \mathbf{K}' \\ \mathbf{K}'^{\mathrm{T}} & \mathbf{K}''_{in} \end{bmatrix} \begin{bmatrix} \Delta\mathbf{X} \\ -\Delta\mathbf{E}''_{in} \end{bmatrix} = \begin{bmatrix} \Delta\mathbf{F} \\ \mathbf{0} \end{bmatrix} \tag{1}$$

where $\mathbf{K}_e$ represents the structural elastic stiffness matrix with dimension $n\times n$ and n denotes the number of nodal degrees of freedom (DOFs) of the structure; $\Delta\mathbf{F}$ and $\Delta\mathbf{X}$ are the vectors of nodal loads increment and nodal displacements increment, respectively; The matrix $\mathbf{K}''_{in}$ with dimension $m\times m$ is the coefficient matrix related to instantaneous tangent stiffness of the material in local nonlinearity regions and m denote the number of IDOFs. $\mathbf{K}'$ is the $n\times m$ matrix that relate the m IDOFs and n nodal DOFs; $\Delta\mathbf{E}''_{in}$ is the vector formulated by assembling the inelastic deformation increment measured at the inelastic deformation interpolation points of the elements located at the regions exhibiting incremental inelastic deformation. Considering that the nonlinearity generally occur at some local regions, the value of m can maintain in a very small level and $m<<n$. This allow for the Woodbury formula available for achieving high efficient calculation:

$$\underset{n\times 1}{\Delta\mathbf{X}} = (\underset{n\times n}{\mathbf{K}_e^{-1}} + \underset{n\times n}{\mathbf{K}_e^{-1}}\underset{n\times m}{\mathbf{K}'}\underbrace{(\mathbf{K}''_{in} - \mathbf{K}_{\mathrm{inf}})^{-1}}_{m\times m}\underset{m\times n}{\mathbf{K}'^{\mathrm{T}}}\underset{n\times n}{\mathbf{K}_e^{-1}})\underset{n\times 1}{\Delta\mathbf{F}} \tag{2}$$

where

$$\mathbf{K}_{\mathrm{inf}} = \mathbf{K}'^{\mathrm{T}}\mathbf{K}_e^{-1}\mathbf{K}'$$

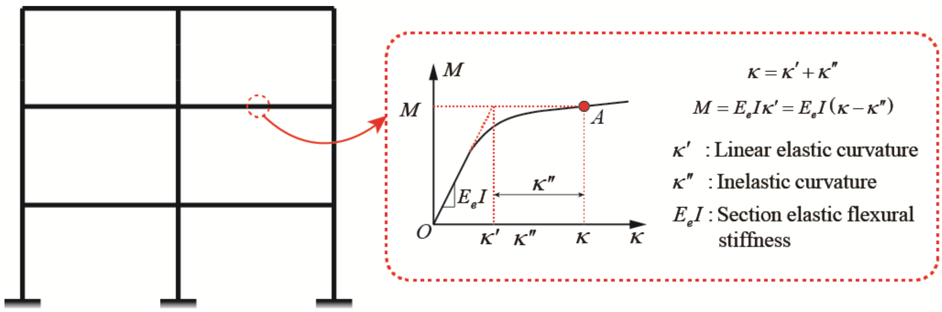

Figure 1 Frame structure example

In Eq. (2), the $m\times m$ matrix $(\mathbf{K}''_{in} - \mathbf{K}_{\mathrm{inf}})$ is called the Schur complement matrix. Because the elastic stiffness matrix $\mathbf{K}_e$ can keep unchanged during analysis, it only requires factorization once at the beginning of the analysis. Thus, in each iteration step, the use of the Woodbury formula only need to factorize a small-dimensional Schur complement matrix (with dimensions *m*×*m*) rather than the large-dimensional global stiffness matrix (with dimensions *n*×*n*), such that the computational efficiency can be improved significantly.

3. Development of beam-column joint model using IS-FEM

Because the primary inelastic response mechanisms in joint are the inelastic shear deformation of the beam-column joint zone and the bond-slip of the beam longitudinal reinforcements in the joint, simulating these two inelastic response mechanisms is the key of developing appropriate RC beam-column joint model. The proposed strategy for modeling the effect of beam-column joint is illustrated in Figure 2. It can be seen that the proposed method uses two RHs assigned to the beam ends to simulate the nonlinear rotation deformation caused by bond-slip of longitudinal reinforcements in joint and integrates them into the formulation of existing IS-Fiber element. This newly formulated element is called improved IS-Fiber element in this paper. Additionaly, a inelasticity-separated shear panel element (IS-Shear element) is developed to depcit the inelastic shear response of joint core. The detailed derivation of the improved IS-Fiber element and the IS-Shear element are presened in the following of the section. The hysteretic model proposed by Lowes et al. [10] is adopted as a base to define the moment versus inelastic rotation relationship of the RHs and the shear force versus shear deformation relationship of the proposed IS-Shear element.

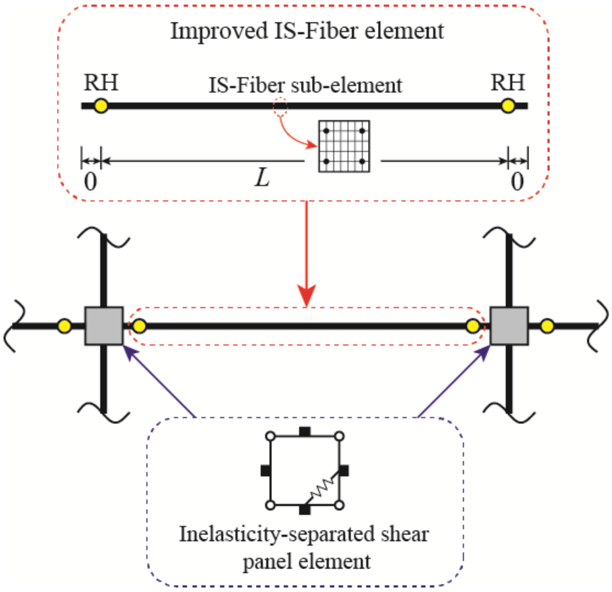

Figure 2 Proposed modeling strategy for beam-column joint

3.1. Improved IS-Fiber beam-column element

Figure 3(a) shows the schematic representation of the improved IS-Fiber element. The points A_1 and A_2 are the external nodes of the element, and the two RHs (i.e., RH#1 and RH#2) are located at the internal nodes P_1 and P_2, respectively. The segments A_1P_1 and A_2P_2 are zero length. Denoting the rotations of the external nodes A_1 and A_2 as θ_{z1} and θ_{z2}, respectively, the corresponding nodal displacements vector can be expressed as follows:

$$\mathbf{a} = \begin{bmatrix} u_{x1} & u_{y1} & \theta_{z1} & u_{x2} & u_{y2} & \theta_{z2} \end{bmatrix}^{\mathrm{T}} \tag{3}$$

where u_{y1}, u_{x1} and u_{y2}, u_{x2} represent the vertical and horizontal displacements of nodes A_1 and A_2,. Because the improved IS-Fiber element proposed in this study is constructed by connecting a IS-Fiber sub-element and two RHs components in series, it can be split into three isolated components as illustrated in Figure 3(b).

According the the geometrical relationship between the nodal displacements of the improved IS-Fiber element and the IS-Fiber sub-element, the vector of nodal displacements of the IS-Fiber sub-element, denoted as $\mathbf{a}_f$, can be expressed by:

$$\mathbf{a}_f = \begin{bmatrix} u_{x1} & u_{y1} & \bar{\theta}_{z1} & u_{x2} & u_{y2} & \bar{\theta}_{z2} \end{bmatrix}^{\mathrm{T}} \tag{4}$$

in which

$$\bar{\theta}_{zi} = \theta_{zi} - \theta''_{zi} \qquad i = 1, 2 \tag{5}$$

In Eq. (5), θ''_{z1} and θ''_{z2} are the relative rotation deformation of the two RHs (RH#1 and RH#2). Based on the research of Li et al. [15], the governing equation of the IS-Fiber su-belement can be written as follows:

$$\begin{bmatrix} \mathbf{k}_{R,e} & \mathbf{k}'_R \\ \mathbf{k}'^{\mathrm{T}}_R & \mathbf{k}''_{R,in} \end{bmatrix} \begin{bmatrix} \Delta\mathbf{a}_f \\ -\Delta\bar{\mathbf{d}}''_{in} \end{bmatrix} = \begin{bmatrix} \Delta\mathbf{f}_f \\ \mathbf{0} \end{bmatrix} \tag{6}$$

where $\mathbf{k}_{R,e}$ is the initial elastic stiffness matrix of the IS-Fiber sub-element; $\mathbf{k}'_R$ and $\mathbf{k}''_{R,in}$ represent the coefficient matrices related to the nonlinearity; $\Delta\bar{\mathbf{d}}''_{in}$ is formulated by assembling the incremental inelastic section deformations measured at the predicted inelastic interpolation points with nonzero inelastic deformation increments; and the vector $\Delta\mathbf{f}_f$ involves the loads applied to the nodes of this sub-element when it is isolated. Based on the condition of equilibrium, the following relation is hold:

$$\Delta\mathbf{f} = \Delta\mathbf{f}_f \tag{7}$$

where $\Delta\mathbf{f}$ denotes the incremental applied load acting on the external nodes of the improved IS-Fiber element (i.e., the nodes A1 and A2).

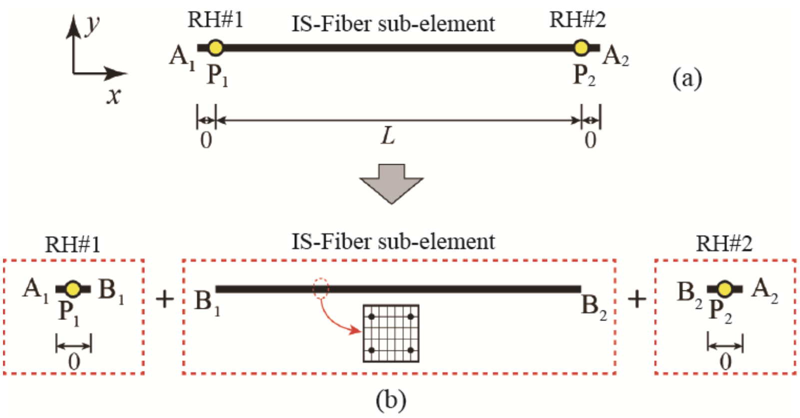

Figure 3 Inelasticity-separated frame element considering the joint

The RHs in this study is assumed to be activated during analysis once the moment demand exceeds the corresponding capacity. Thus, they will only affect the nonlinear response of the structure. This lead to the fact that the nonlinear behavior generated from the RHs can be depicted by inelasticity-separated manner, such that they can be incorporated into the framework of the IS-FEM. Thus, it is assumed that the slippage of longitudinal reinforcements in the beam end section do not occur before section cracking. In such case, the rotation deformation of a RH will be zero when its moment demand does not exceed the section cracking moment. To meet the assumption presented above, the original hysteretic model of Lowes is modified slightly to define the relationship of moment versus rotation deformation (M-θ'') of the RH. Figure 4 shown the proposed monotonic M-θ'' relationship of the RH, in which M_c denotes the cracking moment of the beam end section and M_y and θ''_y define the section yield point. The parameters for determining the monotonic M-θ'' relationship can be obtained by using the corresponding theories and empirical equations [5, 6].

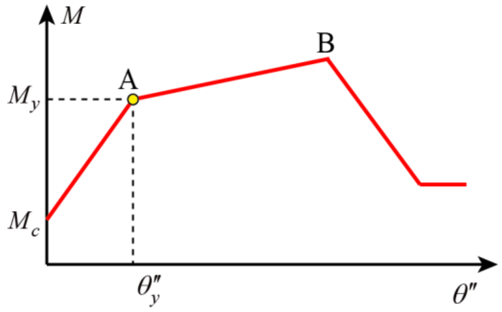

Figure 4 M-θ'' relationship of the proposed RH

When the moment of the two RHs exceed the corresponding section cracking moment in a certain step, the rotation deformation increments of the RHs will be nonzero and the following relation can be obtained by substituting Eq. (5) into (6) and considering Eq. (7):

$$\begin{bmatrix} \mathbf{k}_{R,e} & \mathbf{k}'_R & \mathbf{k}'_\theta \\ \mathbf{k}'^{\mathrm{T}}_R & \mathbf{k}''_{R,in} & \mathbf{k}''_\theta \end{bmatrix} \begin{bmatrix} \Delta\mathbf{a} \\ -\Delta\bar{\mathbf{d}}''_{in} \\ -\Delta\boldsymbol{\theta}'' \end{bmatrix} = \begin{bmatrix} \Delta\mathbf{f} \\ \mathbf{0} \end{bmatrix} \tag{8}$$

where $\Delta\boldsymbol{\theta}'' = [\Delta\theta''_{z1} \quad \Delta\theta''_{z2}]^{\mathrm{T}}$. The matrices $\mathbf{k}'_\theta$ and $\mathbf{k}''_\theta$ in Eq. (8) are constructed by collecting the third and sixth columns of the matrices $\mathbf{k}_{R,e}$ and $\mathbf{k}'^{\mathrm{T}}_R$. Considering the equilibrium condition of the RHs and the corresponding M-θ'' relationship, the inelasticity-separated govering equaiotn of the improved IS-Fiber element proposed in this study can be obtained as follows:

$$\begin{bmatrix} \mathbf{k}_{R,e} & \mathbf{k}'_{R\theta} \\ \mathbf{k}'^{\mathrm{T}}_{R\theta} & \mathbf{k}''_{R\theta,in} \end{bmatrix} \begin{bmatrix} \Delta\mathbf{a} \\ -\Delta\boldsymbol{\Lambda}_{in} \end{bmatrix} = \begin{bmatrix} \mathbf{k}_{R,e} & \mathbf{k}'_R & \mathbf{k}'_\theta \\ \mathbf{k}'^{\mathrm{T}}_R & \mathbf{k}''_{R,in} & \mathbf{k}''_\theta \\ \mathbf{k}'^{\mathrm{T}}_\theta & \mathbf{k}''^{\mathrm{T}}_\theta & \mathbf{k}''_{\theta,in} \end{bmatrix} \begin{bmatrix} \Delta\mathbf{a} \\ -\Delta\bar{\mathbf{d}}''_{in} \\ -\Delta\boldsymbol{\theta}'' \end{bmatrix} = \begin{bmatrix} \Delta\mathbf{f} \\ \mathbf{0} \\ \mathbf{0} \end{bmatrix} \tag{9}$$

where

$$\mathbf{k}''_{\theta,in} = \mathbf{k}''_{\theta\theta} + \mathbf{k}_{\theta t} \tag{10}$$

The matrix $\mathbf{k}''_{\theta\theta}$ in the above equation is formed by collecting the the third and sixth rows of matrix $\mathbf{k}'_\theta$, and the matrix $\mathbf{k}''_{\theta,in}$ can be calculated by using the following relation:

$$\mathbf{k}_{\theta t} = \begin{bmatrix} k_{\theta t,z1} & 0 \\ 0 & k_{\theta t,z2} \end{bmatrix} \tag{11}$$

3.2. Inelasticity-separated shear panel element

Figure 5 shown the proposed shear panel element for modeling the shear behavior of joint core. As can be seen from this figure, the four rigid rods around the perimeter of the joint provide the dispacement constraint of the points A, B, C, and D and the inelastic axial component (AC) which consist of a elastic spring and sliding hinge (SH) is used to resist the shear force applied to the joint shear panel. By performing geometry analysis and force analysis for the element and considering small displacement assumption, the relation bewteen the shear force f_s of the element and axial force f_{AC} of the AC and the relation between shear deformation γ_s of the element and the axial displacement u_{AC} of the AC can be established as follows:

$$u_{AC} = l_{AC}\left(\gamma_s \sin\alpha_0 \cos\alpha_0\right) \tag{12}$$

$$f_{AC} = \frac{2 f_s}{\cos\alpha_0} \tag{13}$$

where the angles α_0 is illustrated in Figure 5 and. l_{AC} denote the initial length of the AC.

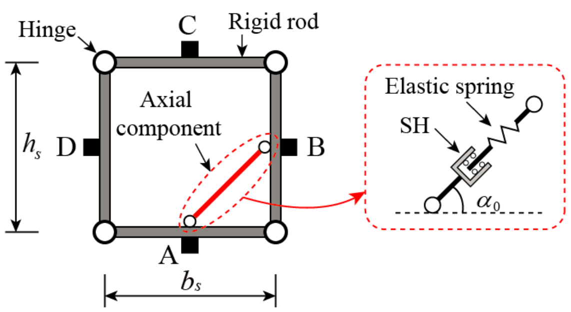

Figure 5 Proposed shear panel element for the joint region

Based on the principle of the deformation decomposition concept in IS-FEM and considering the use of incremental solution scheme, the relative displacement increment Δu_{AC} of the AC in each incremental step can be decomposed into a linear elastic part and an inelastic part, as shown in Figure 6. The displacement decomposition equaiton as follows:

$$\Delta u_{ALC} = \Delta u'_{ALC} + \Delta u''_{ALC} \tag{14}$$

in which $\Delta u'_{ALC}$ denotes the linear elastic displacement part which is generated by the spring; $\Delta u''_{ALC}$ denotes the inelastic displacement part which is contributed by the SH. The linear elastic displacement $\Delta u'_{ALC}$ can be calculated by using the following relation:

$$\Delta u'_{ALC} = k_{s,e}^{-1} \Delta f_{ALC} \tag{15}$$

where $k_{s,e}$ denotes the stiffness of the elastic spring. Figure 6 illustrates the incremental displacement decomposition process for the force-displacement relationship of the AC. By substituting Eq. (15) into (14), the following equation can be obtained:

$$\Delta f_{AC} = k_{s,e}\left(\Delta u_{AC} - \Delta u''_{AC}\right) \tag{16}$$

Moreover, in a certain step, the incremental axial force and displacement relation also can be expressed by

$$\Delta f_{AC} = k_{s,t} \Delta u_{AC} \tag{17}$$

where $k_{s,t}$ is the tangent stiffness of the force-displacement relationship of the AC. Substituting Eqs. (15) and (17) into (16), the incremental axial force and inelastic displacement relation of the SH can be expressed as follows:

$$\Delta f_{ALC} = \frac{k_{s,t} k_{s,e}}{(k_{s,e} - k_{s,t})} \Delta u''_{ALC} \tag{18}$$

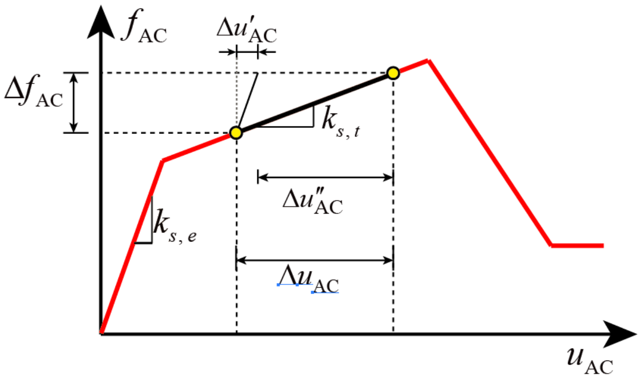

Figure 6 Incremental deformation decomposition of the force-displacement of the AC

Based on Eqs. (12), (13), (17), (18) and considering the constraint imposed by the rigid rods, the inelasticity-separated govering equation of the proposed joint shear panel element can be established as follows:

$$\begin{bmatrix} \mathbf{k}_{J,e} & \mathbf{k}'_J \\ \mathbf{k}'^{T}_J & \mathbf{k}''_{s,t} \end{bmatrix} \begin{bmatrix} \Delta \mathbf{u}_J \\ -\Delta \mathbf{u}''_s \end{bmatrix} = \begin{bmatrix} \Delta \mathbf{f}_J \\ 0 \end{bmatrix} \tag{19}$$

where $\mathbf{k}_{J,e}$ represents the elastic stiffness of the element; $\Delta\mathbf{u}_J$ and $\Delta\mathbf{f}_J$ are the incremental nodal displacement vector and incremental applied load of the element, respectively; $\mathbf{k}'_J$ and $\mathbf{k}''_{s,t}$ are the coefficient matrices related to nonlineariry generated by SH.

4. Verification

To verify the accuracy of the proposed method, two beam-column joint subassemblages, which are called specimens J1 and J2, were selected from Shafaei et al. [1]. The beam-column joint in specimen J1 represents the seismic joint with sufficient shear reinforcements and the specimen J2 has no shear reinforcement in the joint region. For the simulation of the inelastic behavior beam and column members, the modified Kent-Park model proposed by Li et al. [12] is used to model the inelastic behavior of concrete fibers, and the model developed by Dodd and Cooke [18] is adopted to define the hysteretic rule of the reinforcement fibers. Figure 7 and Figure 8 show the comparisons between the experimental data and the simulation results for the two specimens, respectively. As can be seen from the figures, the results obtained by the proposed method agree well with the experimental data, and this indicate that the proposed method have the ability of predicting the effect of beam-column joint on the hysteretic response RC frame structure accurately.

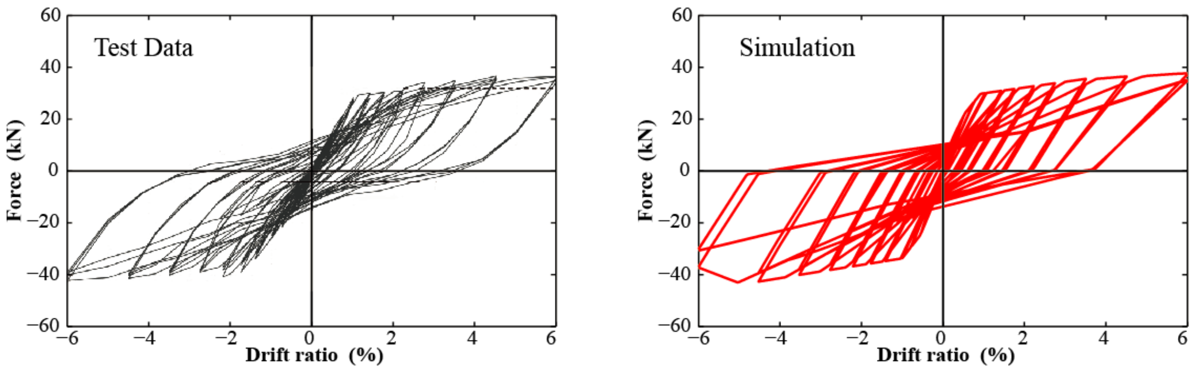

Figure 7 Comparison of the hysteretic behaviors for specimen J1

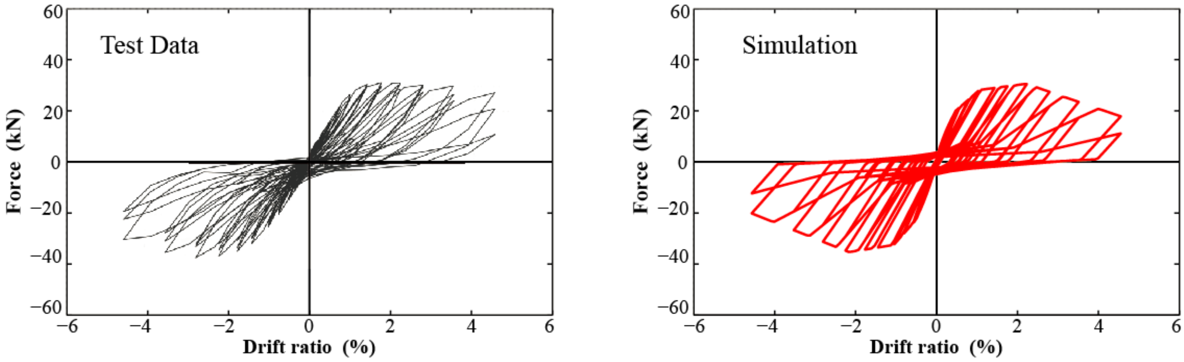

Figure 8 Comparison of the hysteretic behaviors for specimen J2

5. Conclusion

In this study, two new tpye of elements, improved IS-Fiber element and inelasticity-separated joint shear panel element, are developed based on the concept of inelasticity-separation to implement the simulation of nonlinear behavior of beam-column joint using IS-FEM, thus extending the application of IS-FEM. The verification against the experimental data shows that the proposed method can accurately capture the nonlinear hysteretic response of the beam-column joint. Thus, the proposed method is helpful for providing high accuracy prediction of the seismic performce of RC frame structure. Because the global stiffness matrix of the structure can remains unchanged throughout analysis, and the coefficient matrices related to local nonlinearity can be separated from the global system. The proposed method can take full advantage of the Woodbury formula to implement high efficient computation.

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