PHYSICAL PRINCIPLES OF SEISMIC METAMATERIALS FACED WITH EARTHQUAKE ENGINEERING CONSTRAINTS

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Abstract

In acoustics, optics or ultrasonics, microstructured media present, at specific frequency ranges, unconventional properties impossible to reach with homogeneous media. This open the possibility of manipulating the wave fields by judicious devices. In principle, the same ideas apply to seismic waves. This talk aims at analysing to what extent these physical concepts might match the earthquake engineering constraints.

Among metamaterials, one distinguishes, (i) phononic crystals, the specificities of which arise from interferences at wavelengths of the order of the size of the period and (ii) inner resonance media that present band-gaps at large wavelengths. We investigate three examples where such situations may appear.

- *Reinforced soils by piles array*: Such massifs are constituted by weak soils and stiff slender piles. Under transverse motion their response couples bending and shear, while under axial motion inner resonances may occurs. It appears that these effects are significant when the soil/piles mechanical contrast is quite strong and the pile array is very dense.

- *Substitution of soils by inner resonance media:* The feasibility of this option is considered on the basis of inner resonance composites concept. In the range of seismic frequencies, lower significantly the surface motion requires huge volumes of substitution together with an additional mass difficult to sustain by usual soils.

- *Site-City Interaction*: The cities can be idealized as a distribution of oscillators representing the buildings. For large wavelengths with respect to the building basis, the city behaves as a surface impedance, the properties of which are inherited from that of the oscillators. An analysis performed with realistic values for the soils, structures, and seismic frequencies, enables to estimate the perturbation induced by the city. It shows that, except for very dense cities lying on very soft soils (e.g. Mexico city), the actual effect of the « urban layer » is weak.

These realistic examples show that metamaterial effects can be activated under seismic motions, yet in rather specific configurations. The large size of wavelengths makes that their manipulation by metamaterial devices is very challenging. These statements are alleviated at higher frequency range (as for instance train or traffic induced vibrations), or if metamaterials belong to the structure itself which would reduce drastically number of uncertainties.

Keywords: Metamaterials ; Site-City interaction ; Reinforced soils ; Inner resonance media ; Physics of waves

1. Introduction

In acoustics, optics or ultrasonics, microstructured media, usually referred as metamaterials, present, at specific frequency ranges, unconventional properties impossible to reach with homogeneous media. This open the possibility of manipulating the wave fields by judicious devices. In principle, the same ideas apply to seismic waves. This work aims at analysing to what extent these concepts might match the earthquake engineering constraints.

The paper is organized as follows. Some key aspects of earthquake engineering are first recalled in Section 2. The Section 3 is devoted to the physics of metamaterials, that can be split into, (i) phononic crystals, the specificities of which arise from interferences at wavelengths of the order of the size of the period and (ii) inner resonance media that present band-gaps at large wavelengths. Then three examples where such situations may appear are investigated. The shear dynamics of reinforced soils by piles array in addressed in Section 4. The feasibility of the substitution of soils by inner resonance media is considered in Section 5, and the phenomena of Site-City Interaction are analysed in Section 6. Finally, the learning from these examples for practical applications of metamaterial effects in earthquake engineering are discussed.

2. Key features of earthquake engineering

Meanwhile the phenomena and engineering science involved in acoustics, optics, ultrasonics and seismic motions are all related to the physics of wave propagation, the seismic waves and the earthquake engineering present some specificities that deserves to be recalled.

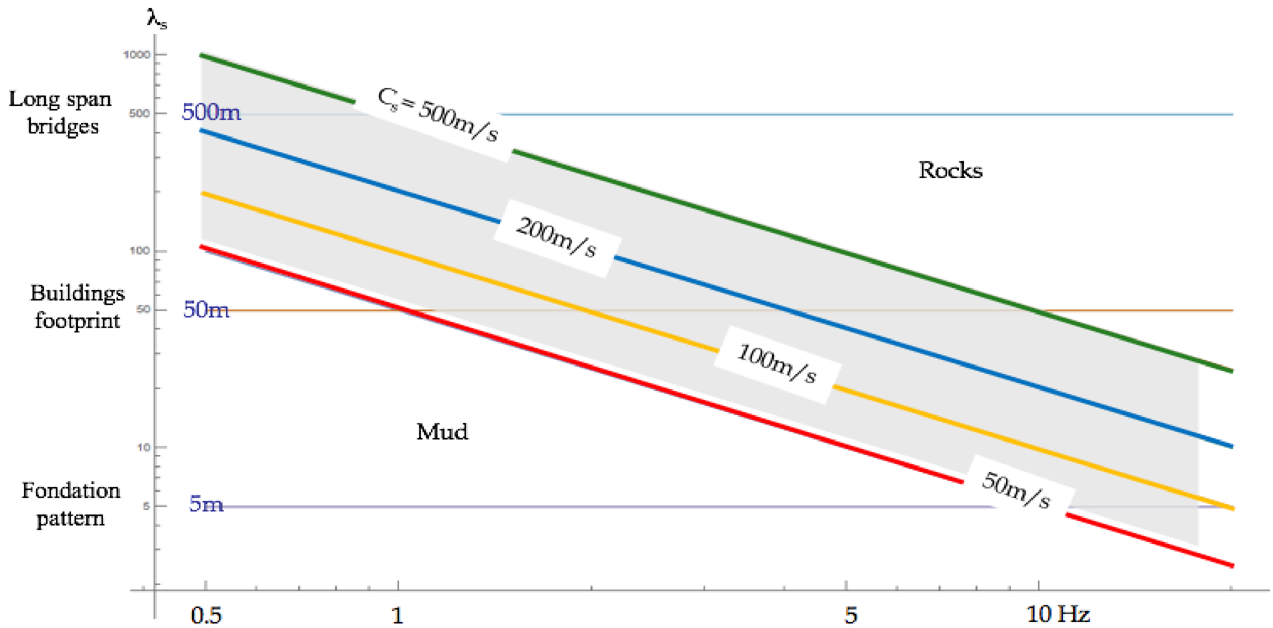

Fig. 1 The frequency range of interest for earthquake engineering and the wavelengths according to the shear waves velocity of soils. For comparison, the typical sizes encountered in civil engineering are indicated.

2.1 Typical orders of magnitude

A first specificity concerns the order of magnitude of the key physical parameters (see Fig.1). Indeed, because of the usual design of the structures, the critical frequency domain where they suffer dynamic amplification, lies in between say 0.5Hz to 10Hz in most cases. Thus the frequency range of interest in earthquake engineering is much lower and narrow than the spectrum considered in acoustics [100hz -10kHz], ultrasonics [10kHz, 1Mhz], and optics [100THz, 1000THz]. Consequently, according to the shear wave velocity C_s of soils or rocks encountered on the earth surface, whose extreme values are respectively 50m/s to 2500m/s, the seismic wavelengths λ_s ranges form 5m (soft soil at 10Hz) to 5km (rocks at 0.5Hz). For a soil of medium velocity (200m/s) and a frequency of 2Hz, the wavelength is of 100m. It must be emphasized that:

- Such metric to kilometric wavelengths are much larger than those of involved in acoustics or optics. They define the typical scale of the addressed seismic phenomena. The physical domains to be considered in earthquake engineering are at least in proportion of this scale, and are therefore rather large compared to the human scale.

- These wavelengths are comparable to current dimensions in civil engineering. As a matter of fact, 5m is a typical size of a foundation array, 50m corresponds to the classical footprint of a building, 500m to few kilometre is the length of conventional long span bridges.

2.2 Partial information and knowledge

A second specificity results form the fact the seismic motions propagates through natural substratum. Consequently, contrarily to the "clean" physics prevailing in acoustics or optics, the seismic waves propagate in domains for which only partial information and knowledge are available. In fact, the geological formation is a complex domain, that usually presents heterogeneity at different scales, the geometry of which is not precisely known. Their complex rheological behaviour (specially under strong motion) is reduced to a few mechanical parameters, each of them having its own uncertainty. In addition, the seismic source radiates a complex field not precisely known.

Thus it clearly appears that, independently of their real scientific interest, the modelling of seismic waves based on "pure" physics assumptions can only provide a rough picture of the reality. This situation strongly departs from acoustics or optics where the physics and the parameters are accurately known.

2.3 Critical societal issue

Compared to optics and acoustics, the third specificity lies in the fact that seismic events are a critical societal issue. Indeed, the tremendous societal and economical issues related to earthquakes, constrains very strongly the practice of earthquake engineering. It imposes a robust design insuring the effectiveness and the and reliability of the earthquake protection.

3. The two classes of metamaterials

The term metamaterial refer to artificial materials made of two or more constituents that are organized according a regular pattern which is generally spatially periodic. It is now well recognized that, due to the specific architecture of their microstructure, they may have unconventional dynamic properties impossible to reach with homogeneous media. This subject receives now a great interest and a wide amount of works are devoted it (see e.g. the monograph [1]). According to the ratio between the wavelength and the characteristic size of the pattern, two distinct physical principles yield to such atypical features. This enables to split the metamaterials in two categories, namely that of the phononic crystals and that of the inner resonant material, as detailed here-below and sketched in Fig. 2.

3.1 Phononic crystals

The unconventional dynamic behaviour of phononic crystals arises at sufficiently high frequency where the wavelengths becomes of the order or smaller than the size l of the period. The modification of the classical wave propagation though a large number comes from Bragg interferences at period scale, that leads, by multiple replications, to a totally reconstructed wave field which drastically departs from the original homogeneous fields. The phononic crystal viewed as a whole is a highly dispersive media that may presents band-gaps (by destructive interferences), negative index, … at specific frequencies ranges.

By principle, these unusual properties require a strict periodicity of the microstructure. The nature of the specific effects and their frequency range are very sensitive to the geometry of the periodic pattern. Since the interferences vanish in a homogeneous media, the difference of mechanical properties of the constituents (particularly the impedance) plays also a role but the effect can be significant even with moderated contrast. Note in addition that due to the complexity of the phenomenon its quantitative description needs to go through numerical simulations.

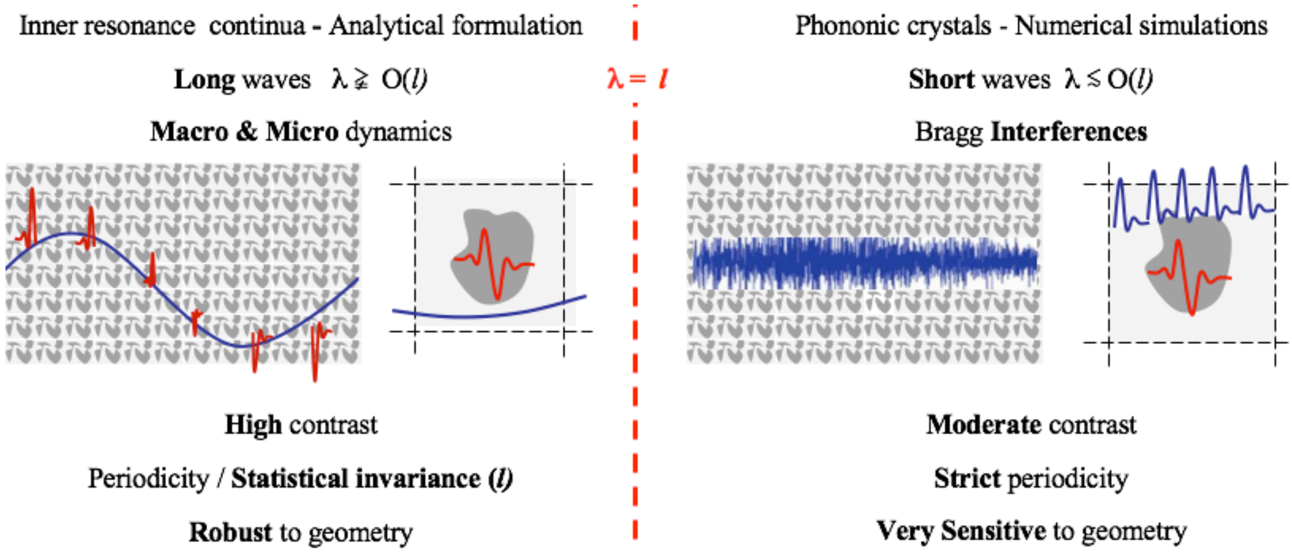

Fig. 2 The main differences between inner resonance media and phonic crystals of characteristic size *l*

3.2 Inner resonance of highly contrasted media

The inner resonance media, are characterized by the fact that they experience a "co-dynamics" regime. This means that dynamic phenomena co-exist at both the micro-scale of the Representative Elementary Volume (REV) and at the macro-scale of the large wavelength. Such a regime is impossible in (nearly-)homogeneous media and can occur only in highly contrasted heterogeneous materials. The dynamic behaviour departs from the usual dynamics by the apparent mass that is frequency dependent and can be negative in the frequency range related to the inner-resonance frequency. Furthermore, the high contrast may also lead to a generalized continua description.

Unlike phonic crystals, inner resonance media can either be periodic or simply presents a statistical invariance with an REV. Hence their properties are robust with regards to the geometry. Furthermore, their can be handle theoretically by taking benefit of existence of large wave length compared to the REV size submitted to the local resonance. When this condition of "scale separation" is fulfilled, the classical framework of two scale asymptotic homogenisation method [2], enables to establish the mean behaviour in a quasi-analytical form corresponding to a generalized (i.e. non-Cauchy) continuum.

Note that the type of behaviour depends on the ratio l/λ_s and therefore on the frequency. Hence, for a given metamaterial of period size l, the low frequency behaviour can be governed by a generalized continua or inner resonance description, while at higher frequency, the behaviour will be that of a phonic crystal. According to this classification, we address hereafter three practical configurations corresponding to metamaterials in the field of earthquake engineering.

4. Soft soils reinforced by an array of stiff piles

In presence of weak soils, a standard design of foundations consists in reinforcing the soft soil by a regular (generally periodic) array of stiff and slender piles. Hence, the resulting massif is a highly contrasted composite. In practice the characteristic size of the pile array is of about $l \approx 5$ - 10m. According to Fig.1, in the frequency range of earthquake engineering [0.5-10Hz] the wavelengths are larger than the period even for very weak soils. Thus the behaviour of the massif falls in the category of inner resonance generalized media. Note however that for higher frequencies a modelling as a phonic crystal would be needed.

4.1 Shear-bending coupling

Let us focus on the behaviour of the reinforced massif under transverse motions when $\lambda \geq l$. The soil matrix works as usual in shear, while the embedded piles may behave as beam, provided that the soil is sufficiently soft. The effective coupling between shear in the soil and bending in the piles arises when both contribute to the balance of the inertia. A simple dimensional analysis enables to formulate this condition explicitly.

Consider a massif of thickness H undergoing a transverse displacement of magnitude O(U) resulting in a shear deformation O(U/H) and a curvature $O(U/H^2)$. The shear force T_s in the soil is estimated as:

$$T_s = S\mu_s.O(U/H)$$

Where μ_s is the shear modulus of the soil, S is the soil surface of the period. Now the shear force T_p in the pile is related to the bending moment M_p by $T_p = O(M_p/H)$ with $M_p = EI.O(U/H^2)$ where EI is the pile bending stiffness with $I = O(d^4)$ for a Thus:

$$T_p = EI.O(U/H^3)$$

Consequently, the condition for the soil and pile shear forces to be of the same order reads:

$$S\mu_s = EI/H^2 \quad \text{i.e.} \quad K = S\mu_s H^2/EI = \mu_s/E.O(l^2/d^2)(H^2/d2) = O(1)$$

Thus a significant coupling is expected when the soil/piles mechanical contrast is quite strong and the pile array is very dense. This occurs when the dimensionless number K= O(1) meaning that the piles modulus is of the order of the square of the slender ratio (of the pile) compared to the shear modulus of the soil.

4.2 Second gradient effective media

The long wavelengths behavior a such highly contrasted composite can be established by means of the asymptotic homogenization method. It has been shown in [3] that the reinforced massif presents a non-conventional second gradient shear behavior driven by following shear/bending governing equation (ρ stands for the mean density of the reinforced massif):

$$-EIU_{,xxxx} + \mu_s SU_{,xx} + \rho U_{,tt} = 0$$

Accordingly, the shear wave propagation is dispersive but does not presents band gaps. Following the physical intuition, as $K \to \infty$ the shear effect dominates and one recover the classical non dispersive shear waves, while oppositely as $K \to 0$ the shear effect vanishes and one recover the classical dispersion of bending waves. The transition from bending to shear behaviour is illustrated on Fig. 3. The latter displays the influence of the K, on (i) the ratio of the three first eigen frequencies and (ii) on the two first modes shapes for a reinforced soil layer clamped on the bottom and free on the top.

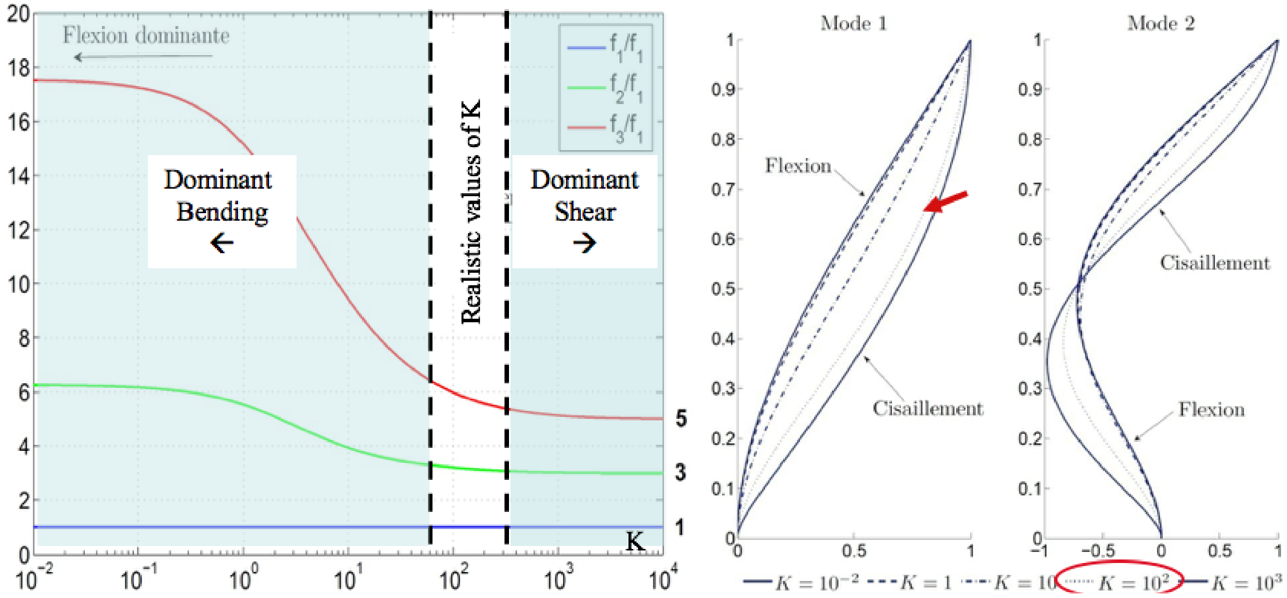

Fig. 3 Left : Nature of the reinforced soil behavior according to the ratio K with the range of realistic values. Right : The two first mode shapes of a reinforced soil layer according to the K values.

4.2 Shear-bending effect in earthquake engineering

Laboratory experiments performed on a shaking table (at Bristol University in the frame of EU Series project) [4] have confirmed the validity of this modelling for analogue reinforced materials that match the condition K = O(1). The question is now to investigate the relevancy of the modelling for earthquake engineering. For this purpose, the value of K has been determined on 19 cases of pile foundations (for building or bridge) documented in the literature. The outcome of this survey is that in practice K lies in between 60 and 300: smaller values of K would correspond to too weak soils for setting structures, larger values K would correspond to soils sufficiently strong that do not need reinforcements. This interval of realistic K reported on Fig. 3 shows that for real pile foundations the effect of actually exists but has a relatively small influence.

Let us mention that, due to the high contrast, this metamaterial is strongly anisotropic and that under compression inner resonance effects arises [3]. The physical principle of the latter is described in the next section.

5. Substitution of soils by inner resonance media

In mechanics, the first study dedicated to inner-resonance media was developed in 1985 by Auriault and Bonnet [5]. Physically the principle is very close to the tuned mass damper well known in structural dynamics, except that the tuned mass damper is here periodically distributed in the stiff matrix [6]. From this pioneer paper, many laboratory experiments have been performed and have confirmed the validity of this concept for metamaterial design [7, 8, 9]. In this section, we examine the feasibility for earthquake engineering by considering the substitution of the soil by a layer made of inner resonance media.

5.1 Physical principle

Consider a two phase highly contrasted periodic media made of a stiff and connected matrix with soft inclusion. In the frequency range such that the wavelength λ_I in inclusion is of the order of the size l of the period, this constituent experiences a local resonant regime. Now due to the much higher stiffness of the matrix, its wavelength λ_M at this frequency is much larger than that in the inclusion, and in turn than the period. This defines precisely the condition of a "co-dynamic" regime in the metamaterial. Assuming that the density of both constituents are of the same order, expressing that $\lambda_I \approx l$ and $\lambda_M >> l$ imposes that the following strong contrast condition for the stiffnesses a_M and a_I:

$$a_I/a_M = l/\lambda_M << 1$$

5.2 Unconventional dynamics

Making use of the homogenization method the macroscopic behaviour of such inner resonance media reads as follows [6] in harmonic regime ($\exp(i\omega t)$) and assuming macro-isotropy ($e(U)$ stands for the strain tensor):

$$\mathrm{div}\,(A:e(U)) = -\rho^*(\omega)\omega^2 U\ ; \qquad \rho^*(\omega) = \rho + \rho_I.\ \Sigma_J\ \alpha_J(\omega)\ ; \qquad \alpha_J(\omega) = -\langle\Phi_J\rangle\langle\Phi_J\rangle / (1-(\omega_J^2/\omega^2))$$

The macroscopic elastic tensor A is the effective elastic tensor of the stiff matrix where the soft inclusions have a vanishing stiffness. The effective frequency dependent density $\rho^*(\omega)$ is the mean density corrected the effects of the inner resonance encapsulated in the functions $\alpha_J(\omega)$ associated to each mode $\{\Phi_J , \omega_J\}$of the soft inclusion. The expression of $\alpha_J(\omega)$ implies that the waves are dispersive and that frequency band-gaps occur around the resonance frequencies of the inclusion, even at large wavelengths.

5.3 Simple illustrative design

Let us explore a few possible designs for earthquake engineering purposes. In this view, consider a cubic periodic media, with a size of the period lying in between 1 to 5m, which is lower that the wavelengths (see Fig.1) so that the inner resonance modelling applies. For simplicity, we design the inclusion in such a way that it presents mostly a single resonance. This situation would correspond a sphere of mass M hanged from its centre to the stiff matrix by an elastic bar that acts as a spring of rigidity k. In that case, the effective density simply reads (ξ stands for the loss factor)

$$\rho^*(\omega) = \rho + \rho_I.\ \alpha(\omega) \qquad \alpha(\omega) = (\omega_0^2/\omega^2 + 2i\xi\omega_0/\omega - 1)^{-1} \qquad \omega_0 = \sqrt{k/M}$$

In addition, as the elastic tensor A is that of the matrix where inclusions are replaced by voids, simple estimates can be obtained by the self consistent approach [10].

These elements enable to assess simply the effective parameters of the four following metamaterials M_1, M_2, M_3, M_4, whose the periodic cell is illustrated on Fig.4. In order to ease comparisons, in all the cases, (i) the resonant frequency is fixed at 2Hz which can be achieved by adjusting the bar spring k, and (ii) the matrix and the resonating inclusion occupies respectively 2/3 and 1/3 of the volume of the cubic period. For M_1 and M_2 the matrix is made of concrete, the inclusion is sphere made of steel (M_1) or of concrete (M_2). For M_3 and M_4 the matrix is made of soil and the inclusion is an embedded cube made of the cell M_1 (respectively M_2) for the materials M_3 (respectively M_4).

The shear modulus of the constituents are $\mu_{concrete} = 2.10^{10}$Pa, $\mu_{soil} = 6.10^7$Pa, and their densities are $\rho_{steel} = 7.510^3$ kg/m^3, $\rho_{concrete} = 2.510^3$ kg/m^3, $\rho_{soil} = 1.610^3$ kg/m^3. The self consistent estimate of the effective shear modulus of M_1 and M_2 is $\mu_{concrete}/2 = 10^{10}$Pa (concrete with 1/3 of void), while the effective shear modulus of M_3 and M_4 is $2\mu_{soil} = 1.210^8$Pa (soil with 1/3 of stiff inclusion).

Thus, all the effective parameters are determined and allow to perform calculations of wave propagation. This has been done in the configurations of layers of metamaterials M_1, M_2, M_3, M_4 of 10m, 20m, 30m high, lying on a rock-substratum of density 2200 kg/m^3 and shear velocity 1000m/s. To have a first assessment of the efficiency of the metamaterials layer, the top surface frequency response under the incidence of a vertical SH wave has been calculated. The results are displayed on Fig. 4.

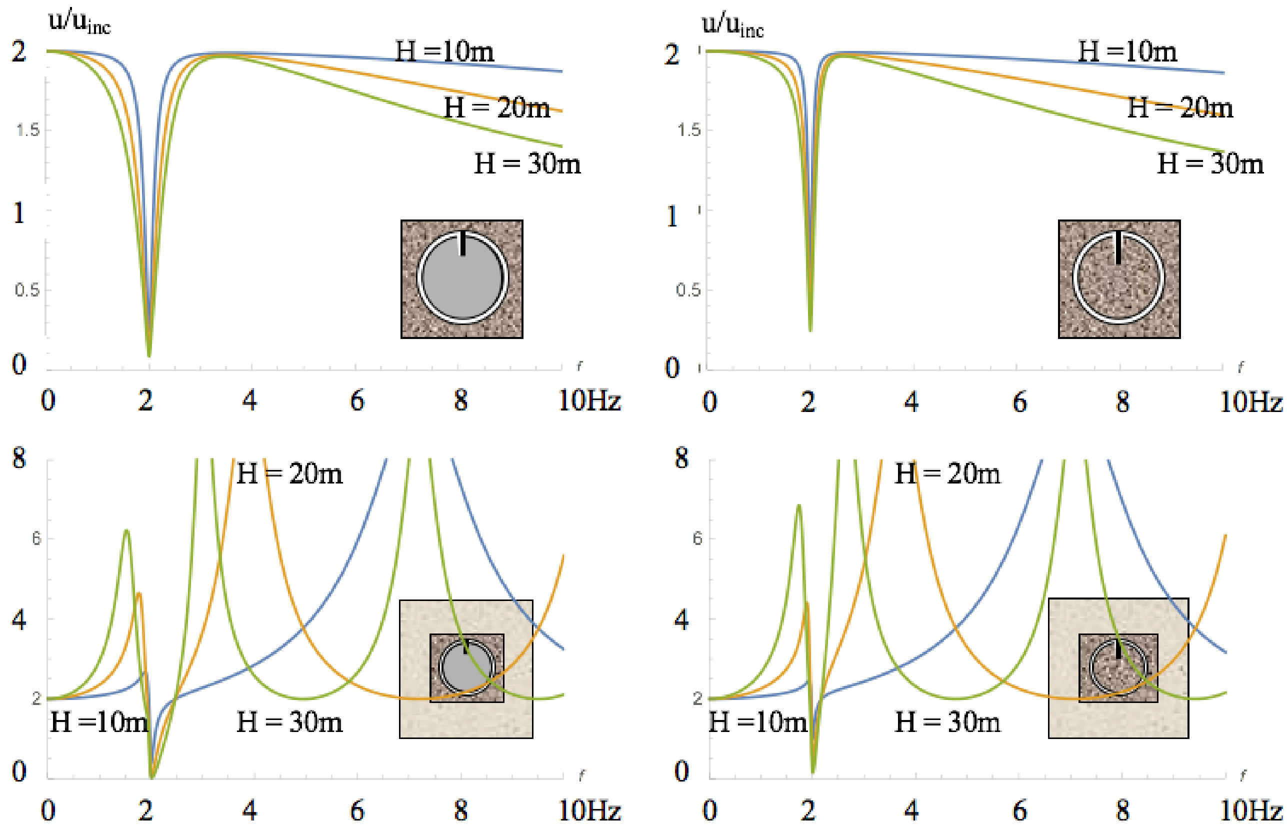

Fig. 4 – Frequency response of inner resonance layers M_1, M_2, M_3, M_4 of 10m, 20m, 30m high to a vertically incident SH wave. The four devices have a inner frequency at 2Hz. Top : concrete matrix with resonating hanged sphere (left M_1 ; right M_2). Bottom: soil matrix with embedded resonating devices M_1, M_2.

5.3 Theoretical results versus prospective earthquake engineering applications

The Fig. 4 clearly highlights that the inner resonance at 2Hz leads to vanish the top surface motion in the vicinity of this frequency. It also shows that the heaviest the mass is, the largest is the frequency range of decreased motion. As for M_1 and M_2 layers, a reduction is also observed at higher frequencies, because the metamaterial impedance is largest than that of the substratum. Conversely the M_3 and M_4 layers have lower impedance and meanwhile their response is reduced at 2Hz it remains significantly amplified at the layer resonances.

Now from a practical point of view these theoretical results have to be faced with the earthquake engineering constrains together with the realization and economical issues. A first point is that lowering significantly the surface motion requires huge volumes of substitution together with an additional mass. Indeed, media M_1, M_2, M_3 are very heavy to heavy (ρ_{M1} = 4170, ρ_{M2} =2500, ρ_{M3} = 2455 kg/m^3) and the own static stability of the layer may itself be difficult to ensure. The second point concerns the reliablity of the design that should be able to sustain several strong motions while keeping its properties. In addition, for the long term perenniality one must ensure that the concrete would not be damaged, that the soil will remains in linear behaviour, that the spring does not yield, that no corrosion might degrade the behaviour, etc… Thus, meanwhile the physical principles are attractive their practical implementation seems very challenging.

6. Site-City Interaction

The site-city interaction effect has been introduced in [11] by Wirgin and Bard (1996) to explain some features (beatings, long duration) of the seismic motion recorded in Mexico City during the 1985 Michoacan earthquake. The leading idea is that the buildings set in vibration re-radiate the energy in their surroundings. Modelling of the seismic response in presence of multiple interactions between resonating structures through the soil is extremely complex even when the structure and soil behaviours are assumed linear. However, the fact that numerous cities presents a regular urban pattern, of a size smaller than the seismic wavelengths at sufficiently low frequency, enables to significantly reduces the complexity of the problem by idealizing the city as a metasurface.

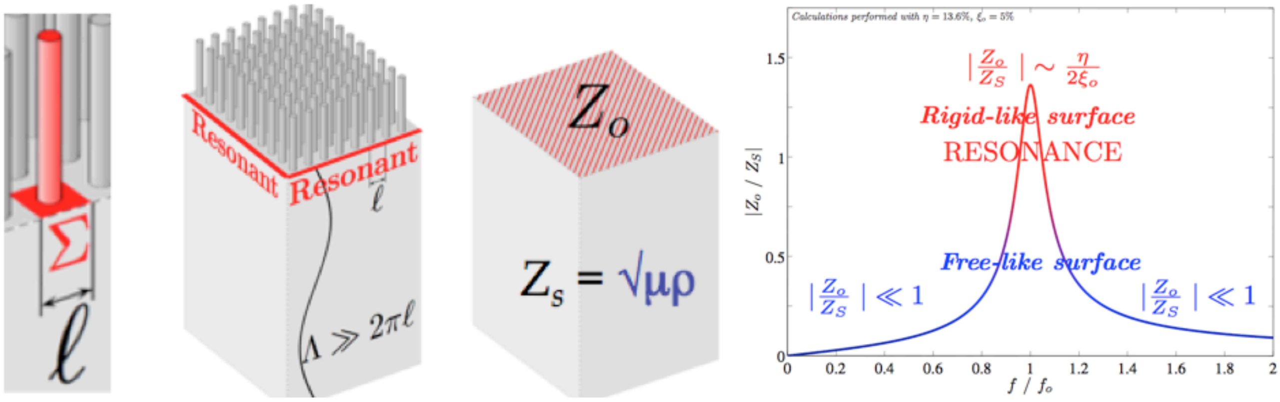

Fig. 5 – Modelisation of an idealized city. From left to right: Building on the period surface ; Periodic distribution of resonators ; Equivalent impedance representation ; Equivalent impedance versus frequency.

6.1 Idealizing cities as metasurfaces

To investigate the effect of the city, the latter is idealized as a periodic distribution of oscillators on the top surface [12]. The oscillators set in motion by a long wavelength induces a quasi-periodic surface forces distribution on the surface of the substratum. The model, is derived from a 2D homogenization method where a boundary layer at the soil-city interface Γ describes the near fields radiated by each building and their multiple interactions. This leads to the equivalent boundary conditions at the macro-scale expressed in terms of surface impedance equivalent to "the resonant metasurface". Hence, in harmonic regime, the description of the substratum covered by the city is given by (i) the standard elastodynamic equation for the substratum:

$$\mathrm{div}\,(A_s : e(U)) = -\rho_s \omega^2 U$$

and (ii) the boundary condition on Γ (of normal n) expressed in term of an equivalent impedance matrix $[Z_0(\omega)]$ of the «city»:

$$(A_{sl} : e(U)).n - [Z_0(\omega)]\, i\omega U = 0$$

The city impedance is directly inherited from the oscillator properties. Reducing for simplicity the oscillator to a single degree of freedom of mass m, eigen frequency ω_0 and damping factor $\xi << 1$, the expression of the impedance in the direction of oscillation reads, where Σ is the area of the period:

$$Z_0(\omega) = (m\omega_0/\Sigma).(i\omega/\omega_0 + 2\xi\omega^2/\omega_0^2).(1 - 2i\xi\omega/\omega_0 - \omega^2/\omega_0^2)^{-1}$$

The unconventional character of the city impedance holds on its variation according to frequency. As $\omega \to 0$, then $Z_0 \approx im\omega/\Sigma \to 0$, and as $\omega \to \infty$, then $Z_0 \approx (m\omega_0/\Sigma).\xi << (m\omega_0/\Sigma)$. Hence the condition of free surface are almost satisfied at low and high frequency. Conversely, at $\omega = \omega_0$ then Z_0 reach its maximum value g and we have $Z_0(\omega_0) = (m\omega_0/\Sigma)/(2\xi)$ which means that the city impedance can be very large (infinite for $\xi = 0$). In that case the city behaves as a stiff surface that lowers the top surface motion.

With this long wavelength modelling it is possible to study the perturbation of the waves field induced by the city. The main features of the phenomena – reduction of motion, atypical wave refraction, frequency range of efficiency, characteristic time of response - were studied in detail in [], []. The key parameter that determines the influence of the resonating surface is the dimensionless impedance ratio of the substratum, $Z_s = \rho_s C_s$ and of the city at the building resonance $Z_0(\omega_0)$. In fact, it is more convenient to introduce following the parameter η which is independent of the damping ratio which is generally less accurately known:

$$\eta = \xi Z_0(\omega_0)/Z_s = (m\omega_0/S)/(\rho_s C_s)$$

Furthermore, shaking table experiments (at Bristol University in the frame of EU Series project) realized on an analogue set-up has confirmed the relevancy of the above modelling [13]. As an example the Fig. 5 shows that the significant change of the frequency response of a layer with and without resonant surface is actually clearly observed and accurately described by the equivalent impedance modelling.

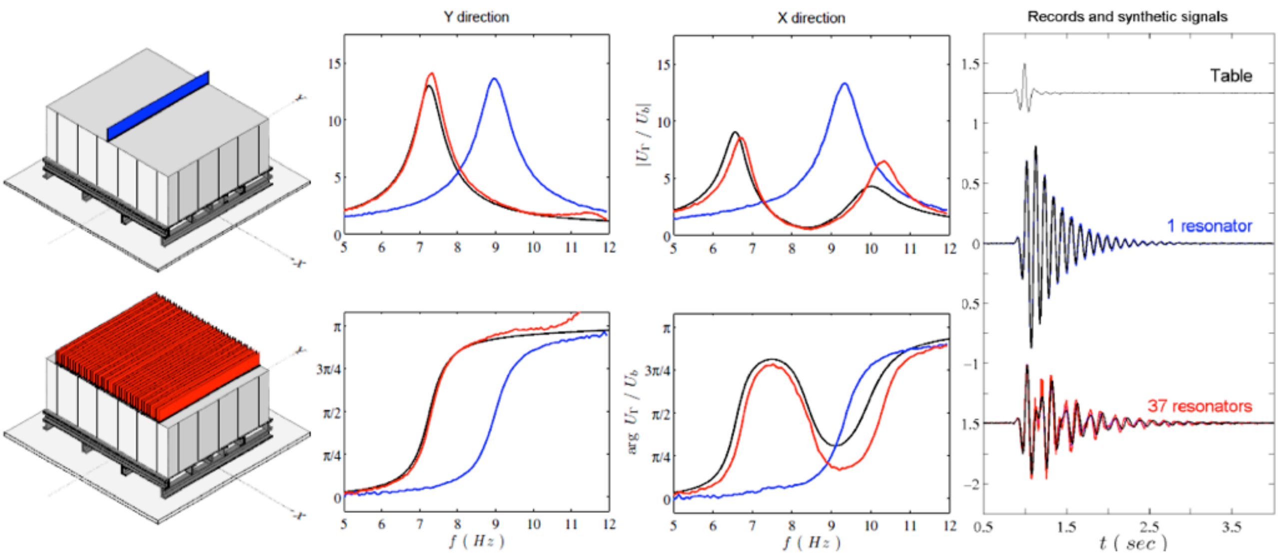

Fig. 6 – Evidence of the metasurface effect on a layer submitted to a vertically incident SH wave. Experimental top surface response in X(blue line) and Y (red line) directions in presence of a single X-resonator and 35 X-resonators. Same responses calculated by the equivalent impedance model (black line).

6.2 From idealized cities to real cities

Let us first mention that l = 50m is a realistic characteristic size of the statistically regular urban pattern. Hence, referring to Fig. 1, the above impedance modelling is limited to low frequency, namely below 1Hz for weak soils (C_s = 50m/s), and below 5Hz for stiffer soils (C_s = 200m/s). Note also that cities are statistically regular but not periodic. Consequently, at higher frequency the cities cannot be described as a phonic crystal. Indeed, the properties of the latter results from accurate interference arrays that a non-periodic geometry avoid to build-up.

The parameter η provides a simple criterion for assessing the actual influence of cities. It can be assessed by means of reasonably well established empirical relations for the building characteristics. Considering a building of high H and footprint S, and using the usual estimates of density $\rho_b \approx 250\text{kg/m}^3$ then the mass is given by $\rho_b HS$. Furthermore, due to the general principles of design used for modern buildings, the fundamental period is well correlated to the high (expressed in meter) by the relation $f_0 = \omega_0/2\pi = 30/H$ (Hz). Consequently, η can be estimated as:

$$\eta = (m\omega_0/S)/(\rho_s C_s) = 2\pi(S/\Sigma)\,(\rho_b/\rho_s)(30/C_s)$$

The ratio S/Σ describes the concentration of the city, and ranges between 0.1 for loose cities to 0.5 for very dense cities. The expression of η highlights that the effect of the city is magnified, when the city is denser, the and substratum softer.

Applying this expression to the Mexico city that lies on a exceptionally soft and light soil, (C_s = 65m/s, ρ_s = 1400kg/m^3), and to Nice (France) whose the soil properties are more usual (C_s=250m/s, ρ_s = 1800kg/m^3) yields the following estimates.

$$\eta_{Mexico} = 0.26 \quad ; \quad \eta_{Nice} = 0.05$$

where a concentration of S/Σ= 0.5 is taken. With a realistic damping value of $\xi = 5.10^{-2}$, the maximum impedance ratio at the building resonance is are η_{Mexico}/ξ = 5.2 and η_{Nice}/ξ = 1. This results indicate that the city effect may be actually observed especially in favourable configurations such as Mexico. Nevertheless, one should be aware that these values are over estimated as they assume that all the buildings resonate at the same frequency and are very densely distributed. If the spectrum of resonant frequency is wider, the impedance ratio will be weakened. Hence one may assess that for moderately dense city, lying on a soil of reasonably good quality $\eta \leq 10^{-2}$ and $\eta_{Mexico}/\xi \leq 0.2$, so that the effect actually exists but is of limited significance.

7. Conclusion

To conclude, metamaterial effects in elastic materials are physically sound and proven experimentally. Obviously the considered configurations are over simplified compared to the reality. However, such analysis based on up-scaling, provide a first level of information on the phenomenon that occurs in the linear elastic range. This enables to identify the key parameters that governs complex phenomena as the pile group effect or multi-building interaction usually present in practice. These examples show that metamaterial effects can be activated under seismic motions, yet in rather specific configurations.

However, the large size of seismic wavelengths makes that the manipulation of the wave field by metamaterial devices is very challenging. In particular, the functionality of the device under tremors as to be insured, its long term perennality must be proven and the economical issue related to the huge dimensions of the device need also to be seriously considered.

One should also be aware the real configurations match only imperfectly the assumptions of linear behaviour, homogeneous soils and periodic distributions considered here. These disregarded phenomena are likely to be alter the expected functionality of the systems. The irreducible uncertainty of the involved parameters (mechanical, geometrical, seismic, ...) is also a serious limitation for a robust design.

These statements are alleviated at higher frequency range (as for instance train or traffic induced vibrations), where the wavelengths are shorter and then easier to controlled.

As for earthquake engineering an alternative approach consists in introducing metamaterials in the structure itself [14] which would reduce drastically number of uncertainties.

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