MODELLING OF HARDENING AND DEGRADATION BEHAVIOR OF CLAYS AND SANDS DURING CYCLIC LOADING

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SUMMARY

It is demonstrated that the stress-strain response in both undrained and drained cyclic loading conditions is of the hardening type until the shear strain is smaller than the so called volumetric threshold. On the contrary, beyond this limit, degradation phenomena occur. In this paper, the possibility of modelling the hardening and degradation behaviour of clays and sands with a simple law is examined. In particular, the case of one-dimensional loading is considered. Experimental data on reconstituted Toyoura sand specimens and undisturbed specimens of three Italian clays (Pisa, Augusta and Catania) have been used.

INTRODUCTION

It is commonly accepted that the stress-strain behaviour of soils is quasi-elastic at very small strain (i.e.,< 0.001 %). It is also commonly accepted that undrained cyclic loading beyond the elastic limit produces degradation of the stress-strain properties, while, in the case of drained cyclic loading, as for dry sands, hardening of the stress-strain relationship has been observed. The experimental results of Figures 1 (reconstituted dry Toyoura sand) and 2 (undisturbed Augusta clay) clearly confirm that the assumption of elastic behaviour at small strains is acceptable. In fact, at small strains, the secant shear modulus G_s, from monotonic loading torsional shear tests (MLTS), and the equivalent shear modulus G_{eq}, from cyclic loading torsional shear tests (CLTST), are practically the same regardless of type of loading and strain rate. On the contrary, at larger strains, systematically results $G_s < G_{eq}$. This experimental fact is observed for dry Toyoura sand, as well as for Augusta clay tested in undrained conditions. Therefore, drained or undrained cyclic loading produces hardening of the stress-strain relationship at least for the strain interval reproduced in Figures 1 and 2. This latter observation contradicts what is commonly believed about the effects of undrained cyclic loading.

However, when a certain limit strain is exceeded the stress-strain relation in CLTST becomes unstable and degradation phenomena are observed. This limit strain has been called volumetric threshold shear strain (Dobry et al. 1982, Vucetic 1994) γ_t^v the values of which are at least one order of magnitude higher than the elastic limit (Vucetic 1994, Dobry et al. 1982, Lo Presti 1989). Moreover, the values of γ_t^v can be influenced by factors like: creep, moderate cyclic loading, overconsolidation ratio or prestressing, direction of the perturbing stress path and strain rate of the perturbing stress path (Tatsuoka et al. 1997).

In this paper the possibility of modelling the hardening $(\gamma \le \gamma_t^v)$ and degradation $(\gamma > \gamma_t^v)$ behaviour of clays and sands with a simple law is examined. In particular, the case of one-dimensional loading is considered. Experimental data on reconstituted Toyoura sand specimens and undisturbed specimens of three Italian clays (Pisa, Augusta and Catania) have been used.

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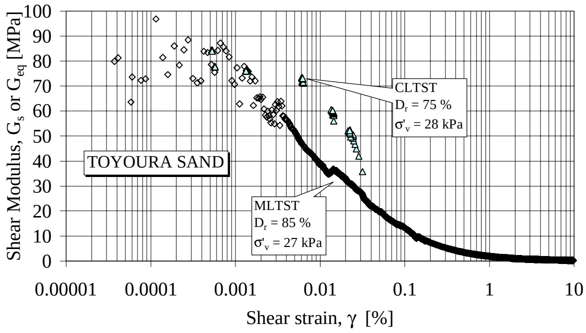


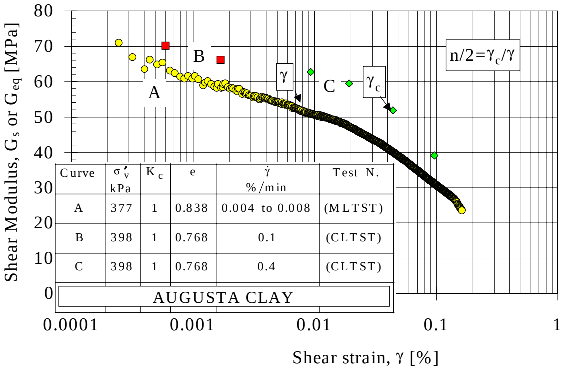
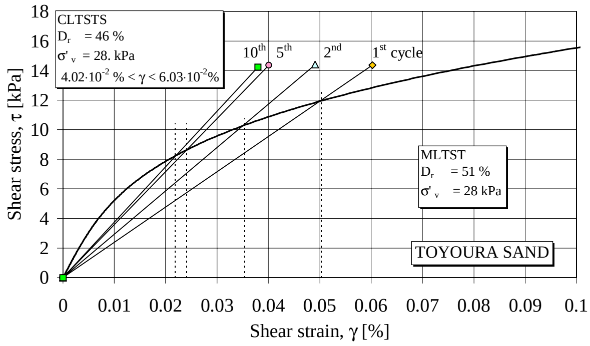


Figure 1. Shear modulus of Toyoura sand from MLTST and CLTST (Ionescu, 1999).

Figure 2. Shear modulus of Augusta clay from MLTST and CLTST.

USE OF MODIFIED SECOND MASING RULE

It is quite common to use simple one-dimensional non-linear models for the analysis of seismic response of soil deposits. These models use a quasi-linear relationship to describe the first-loading stress-strain curve, also called skeleton curve or backbone curve, and represent the unload-reload cycles using the Masing rules (Masing 1926). The second Masing rule implicitly assumes that, for a given strain level, the secant shear modulus (G_s) from first loading curve and the unload-reload shear modulus (G_{eq}) are coincident. This last consideration is valid only at very small strains (< 0.001 %) where G_s values practically coincide with G_{eq}. In this case the second Masing rule is verified, whilst, at larger strains, systematically results $G_s < G_{eq}$ which devalues the second Masing rule. Similar results were observed for various sands and clays (Lo Presti et al. 1997).



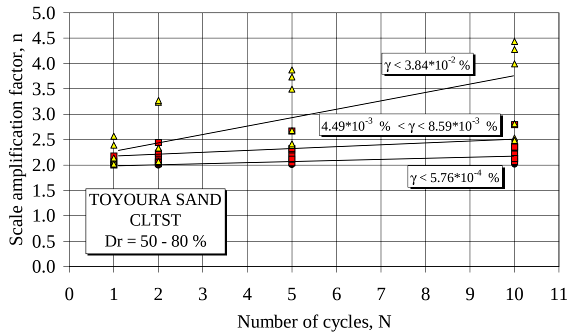


Figure 3. Determination of n parameter for Toyoura sand.

Figure 4. n values of Toyoura sand.

In order to overcome some of the intrinsic limits of Masing rules, various modifications of the second rule has been proposed (see Idriss et al. 1978 and Tatsuoka et al. 1993). The proposal by Tatsuoka et al. (1993) continues to assume that unload and reload branches have the same shape as the backbone curve but with scales of stress and strain enlarged by a factor not necessarily equal to 2. The scale amplification factor n is computed by means of the following expression:

$$\gamma = \frac{\gamma_c}{n/2} \tag{1}$$

where γ is the shear strain level at which the secant shear modulus is equal to $G_{eq} = \gamma_c/\gamma_c$.

Examples of computation of the parameter n are shown in Figures 2 and 3. Figure 3 refers to Toyoura sand (Ionescu 1999). The n values have been obtained by comparison of monotonic and cyclic test results obtained on duplicated specimens. It is worthwhile to notice that, in the case of cyclic tests on Toyoura sand, a new specimen was used for each shear stress level (Ionescu 1999), so that the cyclic stress-strain response is not influenced by the previous strain history. In Figure 4 the n values of reconstituted dry Toyoura sand are reported as a function of strain level and number of loading cycles. It is possible to notice that n=2 regardless of number of loading cycles (N), till the strain level is close to the elastic limit. At larger strains n increases with N up to values equal to 4.5. Similar n values were obtained regardless of relative density (50 to 80 %) and effective consolidation pressure (25 to 150 kPa).

Of course, it was not possible to use a different specimen for each imposed shear stress level in the case of undisturbed clays. In this case, the usual multistage procedure was followed. The specimen experienced increasing cyclic shear stress levels (τ_c). For each stress level, 30 loading cycles were applied to the specimen. It was not possible to observe a variation of the n values with N or γ. The n values were on average equal to 5 for γ greater than the elastic limit. The obtained values are reported in Table 1 with the main characteristics of the tested clays.

Table 1. Characteristics and n values of Augusta, Catania and Pisa clays.

Site	PI	e	σ'_c [kPa]	c' [kPa]	ϕ'	n
Augusta	38	0.768 - 0.838	377 - 398	35	17	4 to 6
Catania	29	0.582	246	43	24	4 to 6
Pisa	21	1.023	138	0	26	4 to 6

When the volumetric threshold shear strain is exceeded, the unload-reload cycles become unstable and degradation phenomena of material occur. The maximum applied strain, in the case of sand specimens (Outer Diameter = 70 mm, Inner Diameter 50 mm, Height = 140 mm), was of about 0.04 % and no material degradation was observed. For Pisa clay specimens (Diameter 50 mm, Height =100 mm) the maximum observed shear strain was less of 0.06 % with no degradation occurrence. For Augusta and Catania clay specimens (Outer Diameter = 50 mm, Inner Diameter 30 mm, Height = 100 mm), the onset of degradation phenomena occurred at strain level of 0.1 %. Degradation was more severe in the case of Augusta clay (Cavallaro 1997), even though, according to Vucetic and Dobry (1988) a greater degradation would have occurred for the Catania clay having a lower plasticity index (Cavallaro et al. 1999). As a consequence of degradation, the n parameter decreased with an increase of N. As the test was performed under stress control, the shear strain also increased with N. In the case of Augusta clay n decreased from an initial value of about 6 to values lower than 2 and the shear strain increased from 0.1 to about 0.2 after about 30 cycles. The n values obtained for Augusta, and Catania clays are reported in Table 2.

USE OF DEGRADATION (HARDENING) PARAMETER

Another way to consider degradation or hardening phenomena is to introduce a degradation or hardening parameter. Idriss et al. (1978) proposed a non-linear model which take into account the degradation behaviour of clays. The model was based on the results of laboratory cyclic test under controlled-strain loading conditions and the R-O equation was used to fit the initial undegraded backbone curve. The soil degradation was represented in terms of the ratio of the equivalent shear modulus of the N^{th} cycle $(G_{eq})_N$, to the equivalent shear modulus of the first cycle $(G_{eq})_1$ (the so called undegraded modulus), at the same strain. This ratio is defined as the degradation index δ, and for a given controlled-strain test, the plot of the log of the degradation index versus logarithm of number of loading cycles has a linear relationship. The degradation index is related to the number of loading cycles N, by:

$$\delta = \frac{G_{eqN}}{G_{eq1}} = \frac{(\tau_{cN}/\gamma_c)}{(\tau_{c1}/\gamma_c)} = \frac{\tau_{cN}}{\tau_{c1}} = N^{-t} \qquad (2)$$

Thus the degradation index δ, for a given strain value, is calculated as the ratio of the "measured stress" τ_{cN} of the N^{th} cycle divided by the determined peak stress ι_{c1} of the 1^{st} cycle that is inferred from the initial backbone curve at that strain amplitude. It is worthwhile to notice that in the work by Idriss et al. (1978) the undegraded backbone curve was not determined from monotonic loading tests, as is proposed by the authors, but from the first undegraded cycle. In principle, the degradation index could also be defined in term of damping ratio increase with N.

Table 2. Degradation parameters and n values for Augusta and Catania clays.

AUGUSTA CLAY

Cycles	γ [%]	ι[kPa]	δ	t	n'	n
1 to 2	0.1003	38.28	0.981	0.026	5.89	6
3 to 6	0.1063	38.86	0.971	0.016	5.83	6
7 to 8	0.1077	38.82	0.946	0.026	5.68	5.9
9 to 13	0.1082	38.34	0.912	0.035	5.48	5.6
14 to 18	0.1112	38.41	0.893	0.039	5.36	5.1
19 to 22	0.1125	37.81	0.859	0.049	5.15	4.5
23 to 24	0.1153	37.54	0.816	0.063	4.89	4.1
25	0.1217	38.03	0.792	0.072	4.75	3.8
26	0.1279	38.16	0.763	0.082	4.58	3.2
27	0.1323	37.56	0.736	0.092	4.42	2.6
28	0.1453	37.50	0.669	0.120	4.02	1.7
29	0.1974	16.03	0.225	0.441	1.35	1

CATANIA CLAY

Cycles	γ[%]	ι[kPa]	δ	t	n'	n
1	0.0929	25.61	0.985	0.021	5.91	5.9
2 to 3	0.0941	25.26	0.953	0.043	5.71	5.7
4 to 9	0.1001	25.65	0.916	0.039	5.49	5.6
10 to 21	0.1032	25.8	0.889	0.038	5.33	5.4
22 to 28	0.1058	25.66	0.855	0.046	5.13	5.2

In eq. (2) t is the slope of logγ-logN plot and represents a degradation parameter. Practically t describes the negative slope of the relationship:

$$t = -\frac{\log\delta}{\log N} \tag{3}$$

The values of the parameter t, increased as the strain increased and a reasonably unique relation between the degradation parameter t, and strain level has been found to exist (Idriss et al. 1978, Vucetic and Dobry 1988, Vucetic 1991).

The above described approach is not directly applicable to controlled-stress loading conditions. Lin and Chen (1991) have proposed an extension of the Idriss et al. (1978) non-linear model to the case of stress-controlled tests. The degradation index δ_N, on any cycle N, can be calculated using the undegraded backbone curve fitted by R-O equation and the stress and measured strain on the given cycle, by:

$$\delta_N = \frac{\tau_c}{\tau_{R\text{-}O\text{-}\gamma_1}} \tag{4}$$

where ι_c is the "applied stress" and $\iota_{R\text{-}O\text{-}\gamma 1}$ is the stress determined on the R-O initial undegraded backbone curve for the γ_N strain level of N^{th} cycle. Thus the degradation index, δ_N at each strain value, could be calculated as the ratio of the applied peak stress of the cycle divided by the peak stress determined from the undegraded backbone curve.

According to Lin and Chen (1991), for a controlled stress test, the measured strain varies with number of cycles thus the eq. (2) does not directly apply. However, if the number of cycle N in eq. (2) is replaced by an equivalent number of cycles N', which is the number of cycles of uniform strain amplitude required to produce the degradation index δ_N, at a strain amplitude equal to the actual amplitude on the $N+1^{th}$ cycle the eq, (2) can be expressed as:

$$\delta_{N+1} = (\delta_N^{-\frac{1}{t}} + 1)^{-t} \tag{5}$$

Thus, the degradation parameter t can be calculated for each cycle in a controlled stress test, with an iterative procedure for the full range of strains encountered in the test.

The values of degradation parameter t were computed step by step using eq. (5) in the case of Augusta and Catania clays (Figure 5). It is possible to notice that the same trend of t with γ was obtained for both clays. The obtained values are much greater than those available in litterature (Vucetic and Dobry 1988, Vucetic 1991).

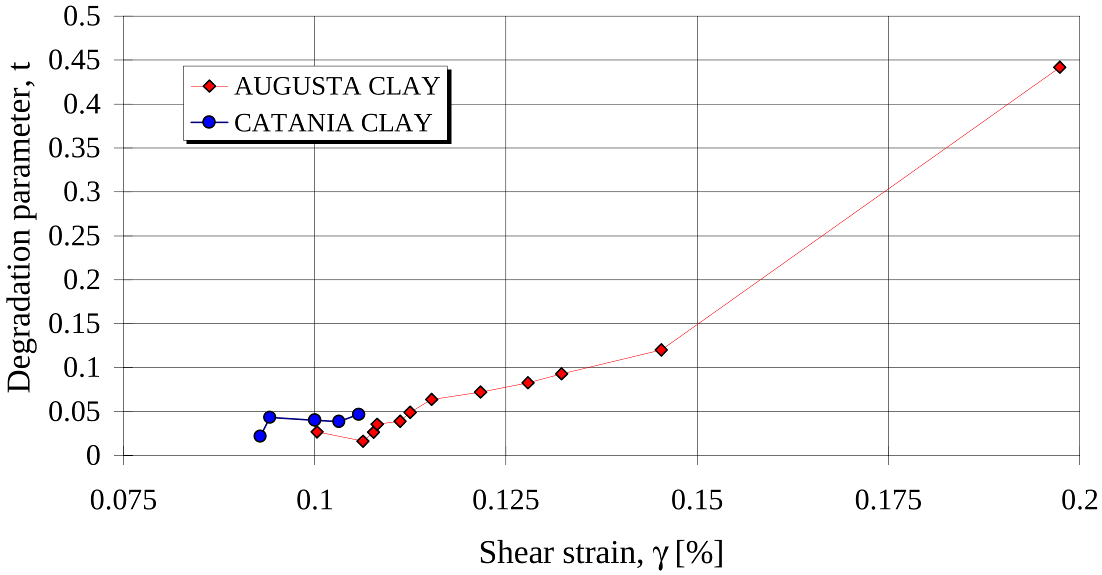


Figure 5. Degradation parameter t of Augusta and Catania Clay.

The same model can be applied, in principle, to the hardening type behaviour. In this case, the parameter δ is a hardening parameter and the exponent t in eqs. (2) and (5) has a positive sign. The hardening parameter t was computed step by step using eq. (5) in the case of Toyoura sand results. The hardening parameter can be also computed as shown in Figures 6 where the ratios G_{eqN}/G_{eq1} and D_N/D_1 are plotted vs. log(N). After a certain number of loading cycles the shear strain is not changing too much and it is possible by means of a regression analysis to find a $G_{eq1(corr)}$, i.e. the equivalent shear modulus at first cycle for that given shear strain level. Using the $G_{eq1(corr)}$ it is possible to compute the t parameter as shown.

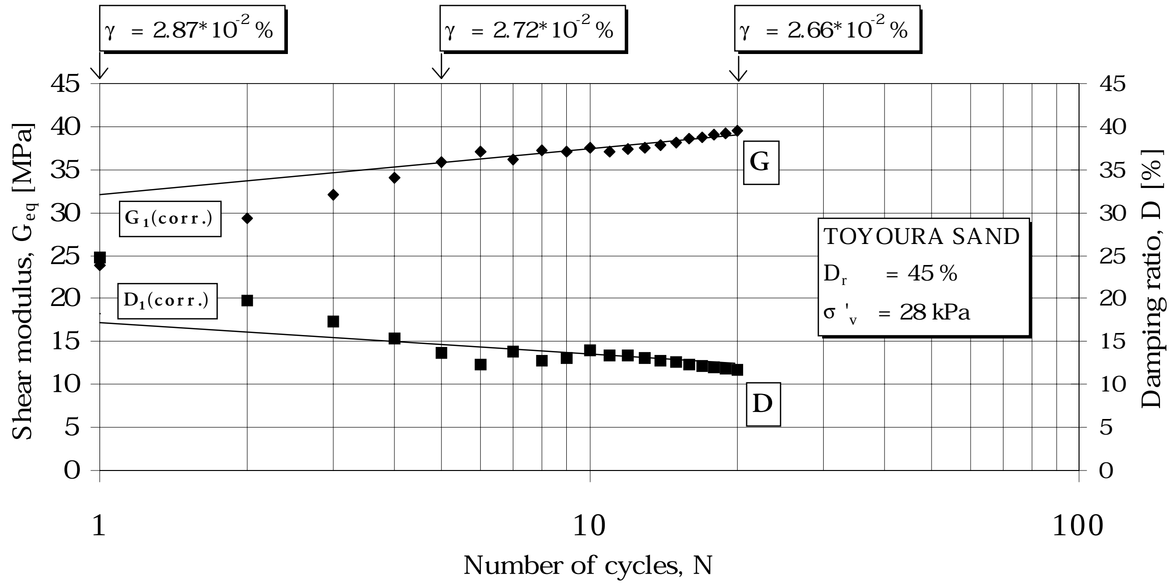


Figure 6. Determination of t parameter.

The t values evaluated in the case of Toyoura sand by means of both described methods are plotted in Figure 7. It is possible to see that the Lin and Chen (1991) method give greater values. Those obtained assuming constant shear strain level are more in agreement to those available in literature (Vucetic and Dobry 1988, Vucetic 1991).

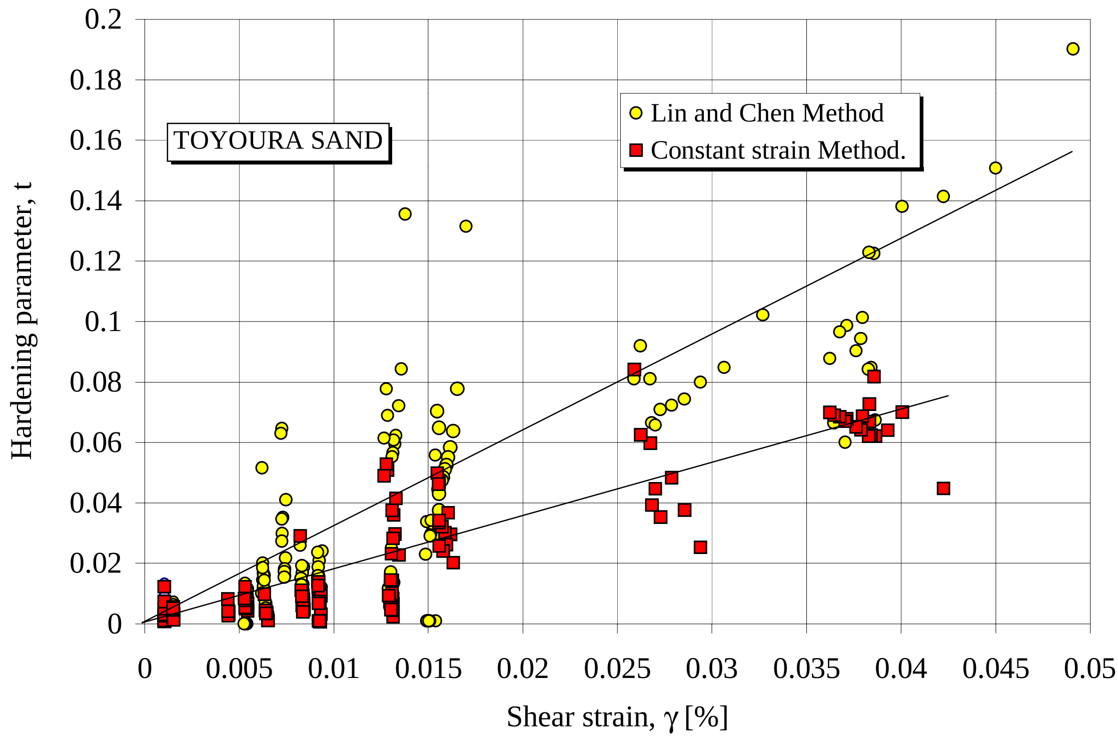


Figure 7. Hardening parameter t of Toyoura sand.

COMPARISON BETWEEN THE TWO APPROACHES

In Table 2 are also shown the n' values evaluated with the method proposed by Idriss et al. (1978) and Lin and Chen (1991) by means of the following relation:

$$n' = n_o \cdot \delta$$

where $n_o = 6$ is the initial scale amplification factor for stable cycle and δ is the degradation index at any cycle. There are little differences between n and n'. Some scatter is observed for the last cycles where the degradation phenomena caused high stress decrease as the shear strain increased. Determining t values with the Lin and Chen method seems to lead to a certain overestimate of the degradation parameter. Therefore it seems preferable to use the method based on the modified 2^{nd} Masing Rule by considering an appropriate variation of n with a varying time history.

CONCLUSIONS

In this paper the possibility of modelling the hardening and degradation behaviour of clays and sands in the case of one-dimensional loading conditions is examined.

The results showed that the modified second Masing rule can be conveniently used to model the hardening or degradation behaviour of sands and clays The scale amplification factor n assumed values ranging from 4.5 to 6 until the cycles are stable while n reduces to values smaller than 2 when the degradation phenomena occur.

The degradation or hardening parameter t (Idriss et al. 1978, Lin and Chen 1991) should be assessed from strain controlled tests. However, in the case of hardening behaviour it is possible to determine t from stress controlled test because the cycles become very soon stable, i.e. $\gamma \approx \mathrm{const}$. In the case of softening behaviour, cycles are unstable, i.e. γ continues to increases. In this case t can be computed from stress controlled test only through the method proposed by Lin and Chen (1991) which seems to overestimate the degradation parameter values.

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