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## SEISMIC LOSS ESTIMATION MODEL FOR MEXICO CITY

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#### SUMMARY

A loss estimation model for Mexico City is presented. The approach is designed for insurance purposes, in order to compute pure premiums and probable maximum losses in portfolios of buildings. Some of its main features are the following:

Earthquake motions are characterised in terms of acceleration response spectra, so both intensity and frequency content are accounted for. The model uses response spectra at a firm site as a reference point to compute response spectra at soft sites. The reference spectra are computed using semi-empirical spectral attenuation laws, some of which were derived from accelerographic information recorded in Mexico during the past 35 years. Response spectrum transfer functions are used to compute response spectra at lakebed-zone accelerometric stations. For sites other than accelerometric stations, an original interpolation procedure is employed.

From the acceleration spectra and general structural characteristics, the maximum inter-storey drift ratio is estimated using a simplified model, which consists on a combination of flexural and shear vertical cantilever beams. The approach uses displacement-based vulnerability functions to estimate the damage ratio from the maximum inter-storey drift. Based on this inter-story drift, a probability density function is obtained to compute the expected annual loss and the probable maximum loss of the structure.

#### INTRODUCTION

In recent years, especially after the devastating earthquake of September 19, 1985, there has been a great advance in many aspects of earthquake engineering in Mexico. In particular, soil effects in Mexico City have been thoroughly investigated, new methods for the evaluation of structural response and damage have been developed, and several attenuation studies have been carried out with the use of the several sate-of-the-art strong-motion networks installed in the country. This has given rise, among other things, to the development of a new and more precise method of loss evaluation for insurance purposes. This paper describes this method, some of whose most relevant aspects are the following:

1) New hazard maps that include recent tectonic and seismological knowledge in Mexico, as well as site-specific attenuation relations, one for each of the different types of earthquakes that affect Mexico City; 2) an expert procedure to characterise buildings, which infers structural parameters (predominant period, structural type, etc.) using the level of information furnished by the user, be it rough or very detailed, or anything in between; 3) a new approach to characterise earthquake actions, in which the intensity measure is the maximum inelastic interstory drift suffered by the structure. This implies that, to evaluate losses, each structure is rigorously analysed (although with a simplified method); 4) intensity-damage curves in which damage is a function of maximum inelastic inter-story drift.

The method is oriented to determine two quantities of interest for insurance companies: the basic net premium and the probable maximum loss. In what follows we will describe the most relevant aspects of this approach.

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#### SEISMIC HAZARD AND GROUND MOTION ESTIMATION

Figure 1 shows the main seismic sources that affect Mexico City. Large earthquakes in Mexico (Ms>7.0) occurring along the Pacific coast are caused by the subduction of the oceanic plates, Cocos and Rivera, beneath the North American plate. The limit of the Cocos and Rivera plates is poorly known, but is estimated that it reaches the coast of Mexico close to Manzanillo (19.1°N, 104.3°W). The Jalisco 1932 (Ms 8.2) earthquake is the largest one occurred in Mexico in the present century, and the Michoacán, 1985 (Ms 8.1) event also belongs to this kind of earthquakes. Great earthquakes also occur in the continent, at depths of about 60 km. These earthquakes exhibit normal fault mechanisms that reflect the breaking of the subducted oceanic lithosphere (Singh et al., 1985). Although these normal fault events are relatively infrequent, it is well known that they can cause heavy damage. Some examples of this type of earthquakes took place in Oaxaca (January 15, 1931, Ms 7.8), Orizaba (August 23, 1973, Ms 7.3) and Huajuapan de Leon (October 24, 1980, mB 7.0).

Even more infrequent are earthquakes that occur in the Trans-Mexican Volcanic Belt (TMVB). Depending on their location, these events may cause considerably heavy damage to some human settlements. Two examples are the Jalapa earthquake of January 3, 1920 (Ms 6.4) and the Acambay earthquake of November 19, 1912 (Ms 7.0). There is also what has been called background seismicity, consisting on earthquakes with M<6, whose origin can not be clearly associated to any geologic structure.



# Figure 1. Main seismic sources that affect Mexico City. Left frame: shallow events of the subduction zone (SZ), Trans-Mexican Volcanic Belt (TMVB) and diffuse seismicity zone (DS). Right frame: intermediate depth earthquakes.

#### **Firm ground**

Three different attenuation relations have been used to evaluate hazard at firm sites in Mexico City. For the coastal earthquakes, we use the relations derived by Reyes (1999), constructed using about 25 accelerograms recorded at station CU (firm ground in Mexico City) from 1962 to 1997, generated by subduction events. Attenuation of the intermediate-depth events is evaluated with the relation presented by Rosenblueth et al (1989), which was derived using recordings obtained at the same station from earthquakes of this kind. Finally, for the TMBV and the area of diffuse seismicity, we use the relation by Sadigh (1995). All these attenuation laws were constructed for spectral pseudoaccelerations (5% damping) at several structural periods, T, so, given magnitude and distance, the full response spectrum can be computed.

Hazard curves, given in terms of exceedance rates for spectral accelerations, v(Sa), were computed using the standard Esteva-Cornell approach (see Esteva, 1970), for several structural periods between 0 and 5 sec. For the case of coastal events, computed hazard curves were later compared with empirical hazard curves (see Ordaz and Reyes, 1999). Figure 2 shows one of these comparisons (for T=0), which is satisfactory, giving confidence in the hazard estimations for Mexico City.

#### Soft soil

The effect of soil type on the amplitude and nature of seismic waves has long ago been recognised as crucial for the estimation of seismic hazard. This is particularly important in Mexico City, where amplifications due to local geology are enormous. Because of this, the following section is dedicated to describe the soil amplification model of the city.



Figure 2. Comparison of computed and observed hazard in Mexico City. v(Sa) is the exceedance rate of spectral acceleration Sa.

As it was mentioned before, strong motion is estimated in terms of response spectra so, given the magnitude and the distance, it is possible to estimate, with semiempirical regressions, the response spectra (2 components) at station CU, chosen as a reference site (Reyes, 1999). It is assumed that the motion at the reference site is a measure of the seismic input motion at the hard sites of the city.

To characterise the response at accelerometric sites at the lakebed zone of the city, average empirical spectral ratios are employed. These ratios may be interpreted as the transfer functions (ETF) between each instrumented site and the reference station. The ETF can only be estimated at accelerometric sites. However, in general, ETF are required to estimate motions at sites that not necessarily are instrumented. This is solved applying an

(1)

interpolation procedure with the following basis: 1) periods of ETF at the instrumented sites are normalised with respect to their dominant periods; which were obtained with microtremor measurements, geotechnical information and strong motion data (Reinoso and Lermo, 1991); 2) the normalised ETF are used in a 2-dimensional interpolation scheme to obtain the normalised ETF at an arbitrary site; and 3) this ETF is renormalised with respect to the dominant period at the selected site. With this procedure, the 2- or 3-dimensional effects are included because the ETF were obtained from real earthquake data.

Once the ETF has been obtained for the desired site, it is multiplied by the response spectrum at the reference site. Note that this implies soil linearity, which seems a valid assumption at least for earthquakes as large as the September 19, 1985 event. Note also that, if soil linearity is assumed, then the exceedance rate of acceleration Sa(T) at soft soil, vs(Sa(T)), can easily be computed if exceedance rates at the reference hard site, v(.) are known:

$$v_{s}\left(Sa(T)\right) = v\left(\frac{Sa(T)}{ETF(T)}\right)$$

where ETF(T) is the value of the ETF for structural period T. The validity of this approach to estimate ground motions at Mexico City has been thoroughly tested (see, for instance, Pérez-Rocha, 1999). Figure 3 shows a recent example, with estimated and observed response spectra for the June 15, 1999 Mw=7 Tehuacán earthquake.





#### **VULNERABILITY ANALYSIS**

Structural vulnerability is the relation between the intensity of the seismic motion and the damage level. As it was explained in the previous section, in this approach intensity is initially given in terms of spectral acceleration. The parameter employed to estimate the damage level in the structure is the maximum inter-storey drift, which is defined as the ratio between the relative displacement of two continuous levels divided between the inter-storey height. There are an important number of studies that conclude that this parameter of structural response is the one that best correlates with structural damage (Bertero et al., 1991; Moehle, 1996; Miranda, 1997; Priestley, 1997; Sozen, 1997). Contrary to most systems that base the estimation of damage level on the Modified Mercalli Intensity, the method presented here is based on a parameter with excellent correlation with structural damage produced during seismic motion.

The maximum inter-storey drift of any structure is estimated from the spectral acceleration as:

$$\gamma_i = \frac{\beta_1 \beta_2 \beta_3 \beta_4 \left(\eta N^{\rho}\right)^2}{4\pi^2 \text{ Nh}} S_a(T)$$
<sup>(2)</sup>

 $\beta$ 1 is the ratio between the maximum lateral displacement at the upper level of the structure and the spectral displacement, considering linear-elastic behaviour. This factor depends on the structural type and the number of storeys of the structure. It is computed from the solution of the differential equation that governs the behaviour of a coupled system of a continuous shear beam with a continuous flexion beam subjected to lateral load with a load distribution that varies with height of the structure. The degree of participation of lateral flexion and shear strains in the system is a function of parameter  $\alpha$ h that depends on the structural type. For instance, in a building structured with flexible frames (without structural concrete walls or bracing) lateral shear strains govern, while in a building structured with reinforced concrete structural walls flexural strains govern. More information on how this parameter is estimated can be found in Miranda (1997), where it is shown that this approach produces very good estimations of the maximum lateral displacements.

 $\beta$ 2 is the ratio between the maximum storey drift and the global distortion of the structure, which is defined as the maximum lateral displacement at the upper level divided by its total height.  $\beta$ 2 depends on the degree of participation of lateral shear and flexure deformations, and on the structural type. It takes into account the fact that lateral strains in any structure during an earthquake are not uniformly distributed along the building height, but there is a trend to concentrate larger inter-storey drifts at certain levels (see Miranda, 1997).

 $\beta$ 3 is the ratio between maximum lateral displacement with inelastic behaviour and the maximum lateral elastic displacement. This factor depends on the displacement ductility demand, the fundamental period of the structure and the soil type where the structure stands. It is calculated with functions that have been calibrated with statistical studies of relations between maximum lateral displacements of one degree of freedom oscillators with inelastic behaviour and their elastic counterparts, when submitted to hundreds of accelerograms recorded over different soil types during more than 25 earthquakes occurred world-wide. For structures over soft soils,  $\beta$ 3 does not depend on the fundamental period of vibration of the structure but on the relation between this period and the dominant soil period. For more information about this parameter see Miranda (1993) and Miranda (1997). The global ductility demand in the structure is estimated with the spectral acceleration associated to the fundamental period of the structure, and the reduction factor of lateral forces. Lateral strength of the structure depends on the location of the structure and its age, which classifies the structures in terms of code characteristics at the time when the structure was constructed.

 $\beta$ 4 is the ratio between elastic and inelastic  $\beta$ 2 factors. This factor takes into account the fact that the lateral force distribution with the structure height is not the same for elastic or inelastic behaviour. For the inelastic behaviour a large force concentration is produced. This factor depends on the number of storeys and the level of inelastic deformation of the structure, which is measured in terms of the displacement ductility demand. For more information about this factor see Miranda (1997).

 $\eta$  and  $\rho$  are factors to estimate the fundamental period of the structure from the number of storeys, N:  $T = \eta N^{\rho}$ . These factors depend on the location of the structure, its structural type, soil type, and year of construction. They take into account that lateral stiffness of structures located at high seismicity zones is higher than that of structures at low seismicity zones. It also considers that structures over soft soils are more flexible than structures over hard soils due to the flexibility of the foundation. These parameters have been calibrated with analytical models, experimental measurements and different considerations according to the changes in the requirements of evolving codes. It must be noted that, recognising the uncertainty in the estimation of T, it is assigned uncertainty, which is later propagated throughout the analysis.

h is the height of each storey of the structure, which depends on structural type, geographic location and date of construction.

Sa(T) is the spectral acceleration, which depends on the fundamental period of vibration, the damping of the structure and the seismic hazard at the site.

Once the maximum inter-storey drift of the building is obtained, its seismic vulnerability may be incremented by the presence of some factors. Among these penalty factors: irregularities, both in elevation and plane, pounding of neighbour structures, existence of previous damage without repair, short columns, etc.

The expected gross damage of a structure, given a maximum storey distortion is calculated with:

$$E(\beta | \gamma_{i}) = 1 - \exp\left[\ln 0.5 \left(\frac{\gamma_{i}}{\gamma_{0}}\right)^{\varepsilon}\right]$$
(3)

where  $\beta$  is the gross damage,  $\gamma 0$  and  $\gamma i$  are parameters of structural vulnerability that depend on the structural system and the date of construction, and E(.) stands for expected value. Note that, by definition,  $\beta$ , the ratio between repair cost and total cost lies between 0 and 1.

#### PROBABILISTIC LOSS MODELLING

As it will be shown later, the probability density function of the less is required for several computations. This probability density is assumed to be Beta, with the expected value given in equation 3. There is little information to determine the variance of  $\beta$  given  $\gamma_{i,j}$ ,  $\sigma_{2}(\beta|\gamma_{i})$  It is known, however, that when the expected value of the damage is null, so is the dispersion. Similarly, when the expected damage is 1, the dispersion is null. To fix the variation of the variance with the expected value, we analysed results of ATC-13 (ATC, 1985) and performed simulations with simple structures. In view of this, we fixed the variation of the variance as follows:

$$\sigma_{\beta}^{2}(\beta | \gamma_{i}) = Q \left( E(\beta | \gamma_{i}) \right)^{r-1} \left( 1 - E(\beta | \gamma_{i}) \right)^{s-1}$$
(4)
$$Q = \frac{V_{\max}}{D_{0}^{r-1} \left( 1 - D_{0} \right)^{s-1}} \quad \text{and} \quad s = \frac{r-1}{D_{0}} - r + 2$$

where

Vmax, D0 and r depend on structural type. Once  $E(\beta | \gamma i)$  and  $\sigma 2(\beta | \gamma i)$  have been determined, the probability density function of  $\beta$  is completely fixed.

#### Effect of deductible and limit in an individual building

and

Up to here, we have discussed the estimation of the gross loss,  $\beta$ . However, it is needed to estimate the net loss, βN, which results after applying deductible and limit. Let D and L be the deductible and limit, respectively, given as a fraction of the exposed value. The net loss is defined in the following form:

$$\beta_{N} = \begin{cases} 0, \text{if } \beta < D \\ \beta - D, \text{if } D < \beta < L \\ L - D, \text{if } \beta > L \end{cases}$$
(5)

Under these conditions, the probability density of  $\beta N|\gamma$  adopts the following form:

$$Pr(B_N = 0) = Ba(D, A, B)$$

$$Pr(B_N < \beta_N) = Ba(\beta_N + D, A, B)$$

$$Pr(B_N = L - D) = 1 - Ba(L, A, B)$$
(6)

where A and B are the canonical parameters of the Beta distribution of  $\beta$ , given by:

$$A = \frac{1 - E(\beta | \gamma_{i}) - E(\beta | \gamma_{i}) C^{2}(\beta | \gamma_{i})}{C^{2}(\beta | \gamma_{i})}, B = A \left[\frac{1 - E(\beta | \gamma_{i})}{E(\beta | \gamma_{i})}\right]$$
(7)

 $C2(\beta|\gamma i)=\sigma 2(\beta|\gamma i)/E2(\beta|\gamma i)$  is the coefficient of variation of  $\beta|\gamma i$ , and Ba(x;A,B) is the Beta cumulative function. Statistical moments of  $\beta N|\gamma i$ , like its expected value and variance, can be computed from the expressions given above.

#### **Computation of pure premium**

Pure premium, PP, is defined as the expected value of the annual loss, so it can be computed as follows:

$$PP = \int_{0}^{0} -\frac{dv(Sa)}{dSa} E(\beta \mid \gamma_{i}(Sa)) dSa$$
(8)

#### **Computation of PML**

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Probable maximum loss (PML) in a portfolio of buildings is a measure of the largest loss that can be suffered during and individual event. In this model, it is defined as the loss that would take place, on average, every 1500 years, that is, a loss that has an exceedance rate of 1/1500 per year. In view of this, exceedance rates of the net losses in the portfolio,  $\beta$ (PN) must be computed. If a the j-th seismic source generates an earthquake, the net loss for the portfolio, PNj, will be given by:

$$P_{Nj} = \sum_{i} V_i \beta_{Nji} \tag{9}$$

where Vi is the value of the i-th structure,  $\beta$ Nij is the net loss in structure i, if an earthquake with given characteristics occurs at source j, and the sum comprises all structures in the portfolio. Computation of the exact probability density function of PNj is difficult. In this model, it is assumed that the quantity PNj/ $\Sigma$ iVi is also beta-distributed. The expected value of PNj can easily be computed from equation 9:

$$E(P_{Nj}) = \sum_{i} V_i E(\beta_{Nji} | \gamma_{ij})$$
<sup>(10)</sup>

where  $\gamma i j$  is the maximum inter-storey drift experienced by structure i if an earthquake of known magnitude takes place at source j. However, to compute the variance of PNj, correlation among losses must be accounted for. In this model, it is assumed that, given the occurrence of an earthquake, all losses have a coefficient of correlation of 0.3. This value has been selected after analysing real building portfolios and calibrating he effect of alternative choices for this coefficient.

Once the expected value and variance of PNj are computed, the exceedance rates of PN can be computed with:

$$\mu(P_N) = \sum_{j} \int_{M_0}^{M_u} -\frac{d\lambda_j(M)}{dM} \Pr(P_{Nj} > P_N | M) dM$$
(11)

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where  $\lambda j(M)$  is the magnitude exceedance rate at source j, and the sum comprises all seismic sources. Once this curve has been computed, PML values can be determined.

#### SELECTED RESULTS AND OTHER INTERESTING FEATURES

Based on the theoretical lines given above, a computer system was constructed to be used by insurance companies. Some results and interesting features follow.

Figure 4 shows pure premiums and PML values computed for several locations in Mexico City, for the same building (15 floors, office building, reinforced concrete frame, built in 1980, no irregularities, without possibility of pounding, no previous damage). It can be noticed that both PP and PML vary widely along the city, reflecting the large differences in soil conditions that are found in Mexico City.



# Figure 4. Pure premiums (left frame) and PML values (right frame) computed for several locations in Mexico City, for the same building

Several options to locate a building are offered. They are, from the most accurate to the least: geographical coordinates, zip code, state and county codes, and CRESTA zone. The user can select the location mode that is available, and the system uses the most accurate provided.

The system includes an expert procedure to characterise buildings, which infers structural parameters (predominant period, structural type, etc.) using the level of information furnished by the user, be it rough or very detailed, or anything in between. Instead of, for instance, giving directly the structural type of the building, the user can answer simple questions, which guide the system in inferring the structural type.

The system contemplates a wide variety of deductible-limit schemes, including policies that cover more than one building and that have overall liability limits

#### DISCUSSION AND CONCLUSIONS

An approach to evaluate earthquake losses in Mexico City has been presented. It includes some innovative aspects, like full-fledged probabilistic computations, characterisation of seismic intensity via the maximum interstorey drift, and vulnerability relations, which are partly analytical and partly empirical.

A computer system has been created on these lines, which has been adopted by the Mexican Government to evaluate the risk that insurance companies operating in Mexico are facing, depending on the specific characteristics of their portfolios. This has brought changes in the regulations of the insurance industry; the most important is, perhaps, that there is not a general PML for the Mexican industry anymore, but a different PML for each company.

In this paper, we have presented considerations and results only for Mexico City. However, the system has been expanded to the whole country. The basic techniques remain unchanged, although site effects for other locations are evaluated with far less accuracy than for Mexico City.

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