NON-LINEAR MOMENT-ROTATION RELATIONSHIP AT THE BASE OF SHEAR WALLS

Aurelio GHERSI[1], Maria Rossella MASSIMINO[2] And Michele MAUGERI[3]

SUMMARY

In multi-storey buildings shear walls are often included to resist against seismic loads, due to their efficiency and their low cost. Because of the high stiffness of these structural elements, the soil flexibility cannot be neglected. It plays a fundamental role in the displacement response of the shear wall and could change significantly the behaviour of this element in a building structure.

In this paper the interaction of a single shear wall with a significant soil volume is examined. The whole soil-foundation-shear wall system is analysed through the new finite element code SOFIA. In this way it is possible to evaluate the effects of the basement rotation of the shear wall.

The shear wall is then submitted to dead and live vertical loads and to simplified pseudo-static horizontal forces at different levels. An incremental load procedure allows the non-linear behaviour of the subsoil to be considered.

The soil-structure interaction is analysed through a parametric study that allows the separation of the effects caused by the foundation dimensions, the soil properties and the stress and strain levels. In particular, for three different sand deposits the Young's soil modulus is considered constant or linearly variable with the depth. At the same time the analyses are performed with both linear and non-linear soil constitutive laws.

INTRODUCTION

When shear walls are considered, fixed base schematisations give significantly approximated results as regards the displacement response of the structures. In relation to the dimension and the shape of the shear wall and to the soil condition, the foundation rotation can play a not-negligible role in the reliable design of the structure. This aspect becomes greater when wind and/or earthquake forces occur, considering that the shear walls are designed in multi-storey buildings to absorb lateral forces, while the frame in which they are included is designed to absorb vertical loads. Moreover, of course, columns resist lateral forces, but their contribution, when there are shear walls, is very limited.

In this paper the interaction of a single shear wall with a significant subsoil volume is investigated by means of the finite element code SOFIA [Massimino, 1999], simulating the earthquake by means of the pseudo-static procedure [Italian Seismic Code, 1996]. This approach has some disadvantages: first of all the interaction with the other plane frames linked to the single shear wall, is neglected, then the shear wall displacements are overvalued. Secondly, the pseudo-static procedure does not take into account the real dynamic aspect of the problem, i. e. the wave propagation in the soil and the response of the structure submitted to a dynamic input coming from the subsoil. On the other hand ,the presented approach allows us to focus on the main aspect of the base flexibility of shear walls. Moreover, as in the framework of the ICONS research program [Li Destri et al, 1999], in different civil engineering laboratories single shear walls or structures similar to a single shear wall are tested under cyclic pseudo-static or dynamic loads. Finally, with the present approach the soil and the structure are considered as parts of a unitary system.

[1] *Professor of Structural Eng, Univ. of Catania, Viale Andrea Doria 6, 95125 Catania, Italy, aghersi@isc.ing.unict.it*
[2] *Ph.D. of Geotechnical Eng, Univ. of Catania, Viale Andrea Doria 6, 95125 Catania, Italy, sar@mail.pandorasicilia.it*
[3] *Professor of Soil Mechanics, Univ. of Catania, Viale Andrea Doria 6, 95125 Catania, Italy, mmaugeri@isfa.ing.unict.it*

In particular, the numerical analyses are performed considering both linear and non-linear constitutive laws for the soil. The static linear–elastic rocking stiffness K_r of the foundation obtained through the SOFIA code is compared with those proposed by Borowicka [Borowicka, 1943; Richart et al., 1970; Toutanji, 1997] and by Gazetas [Gazetas, 1991]. To take into account the soil non-linearity, the M-ϕ relationships are evaluated by means of the incremental procedure employed in the SOFIA code. Because the rocking stiffness K_r is not a specific soil property, but also depends on the foundation dimensions and on the boundary conditions of the examined soil-structure system, several numerical analyses with different geometrical and mechanical conditions are performed.

DESCRIPTION OF THE SOIL-STRUCTURE SYSTEM

A single shear wall with four different foundation dimensions is analysed. The shear wall is 12.00 m high, 4.00 m wide and 0.30 m thick. The foundation is 2.00 m high and 6.00 m wide. The thicknesses of the considered foundations are 1.50, 2.00, 3.00 and 6.00 m, to investigate the effects of the L/B ratio (Fig. 1). The whole structure is of elastic-linear reinforced concrete with a Young's modulus equal to 28500 N/mm^2 and a Poisson's ratio equal to 0.28. The structure rests symmetrically on a sand deposit 25.00 m high and 42.00 m large, so that the boundaries of the numerical schematisation are far enough from the structure not to disturb it significantly. Moreover, an embedment of 2.00 m is considered in some analyses.

The soil is subdivided by means of isoparametric quadratic plane elements with nine Gauss integration points for each element, while the structure is subdivided by means of the typical monodimensional elements [Ghersi et al., 1999].

The floor systems, infinitely rigid in their own plane, are simulated by means of vertical and horizontal forces applied at every 3.00 m on the shear wall, as it is possible to see in Fig. 1. The vertical forces represent the dead and live loads on the structure, coming from a possible floor system of 5.00x5.00 m at each elevation. The horizontal forces represent the seismic actions and are computed by means of the pseudo-static approach, considering the soil coefficient ε = 1.3 [Italian Seismic code, 1996] and the ductility coefficient for the main resting elements β = 1.2. The other coefficients, which regard the analysis typology and the building functionality, are fixed equal to one. To compute the seismic weight, the live loads are reduced by 67 %, as suggested by the Italian Seismic code [Italian Seismic code, 1996] for residential civil buildings.

The seismic coefficient C, correlated to the seismic zone class, is computed so that any up-lifting is approximately avoided by the eccentricity of the total load. The design vertical and horizontal forces applied on the structure for L/B = 4 are reported in table 1, where q_F is the spread vertical load due to the foundation weight and $F_{F.v.}$ accounts for the weight of the shear wall up to the first elevation.

In the case of L/B = 3, 2 and 1, the typical design loads applied on the structure are calibrated to have the same contact pressure at the soil-foundation interface of the case of L/B = 4, to avoid the effect of the stress level combined with the effect of L/B ratio.

All the cases considered involve values of the seismic coefficient in the range of 0.19 < C < 0.77. Only the minor value of C is related to a realistic load condition on the structure in respect to the Italian Seismic code [Italian Seismic code, 1996], because for L/B < 4 the design loads are amplified not to change the soil-foundation contact pressure, as previously explained.

Each analysis is performed for shallow foundation without embedment and for foundation with embedment. In this last situation the analyses are performed in two different phases: firstly the excavation to locate the foundation is simulated; secondly, the whole structure, subjected to the vertical and horizontal loads, is added.

Table 1 – Vertical and horizontal forces for *L/B* = 4

Elevation	***Vertical forces [kN]***	***Horizontal forces [kN]***
I	*252.5*	*27.33*
II	*252.5*	*54.66*
III	*252.5*	*81.98*
IV	*162.5*	*86.85*
Foundation: q_F = 75 kN/m; $F_{F.v.}$ = 90 kN		

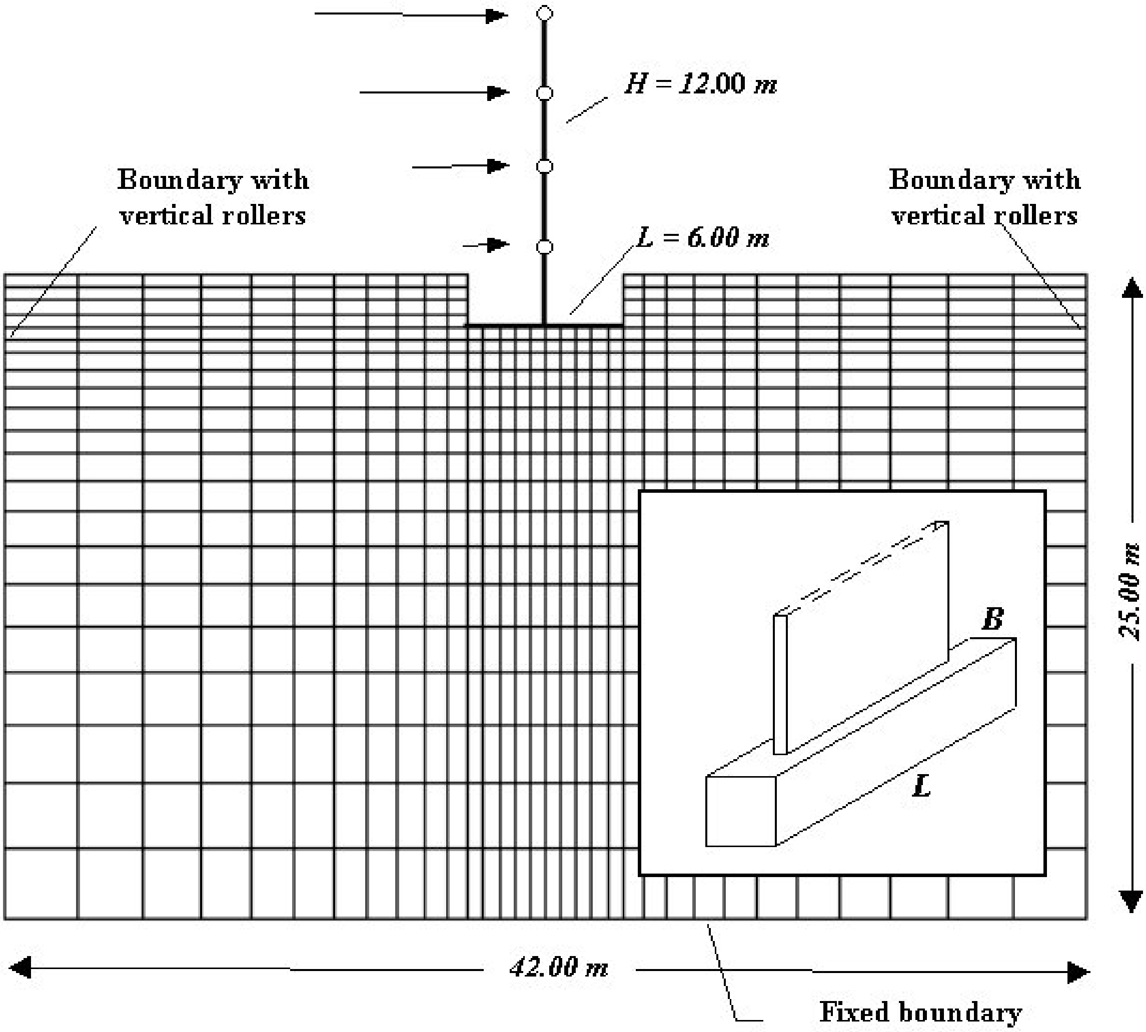

Fig. 1 – Soil-structure system

SOIL CHARACTERISATION

For the soil below the shear wall different sand deposits are considered. Firstly a homogeneous soil is considered, secondly a non-homogeneous soil with stiffness linearly increasing with depth from a zero value at the free surface is considered [Gibson, 1967]. For both the two above cases the following values of the relative densities are fixed: D_R = 40 %, 65 % and 90 %. The other soil parameters used in the SOFIA code are reported in table 2. In particular, the values of the shear resistance angle ϕ' are computed by means of the D_R - ϕ' correlation suggested by Schmertmann [Schmertmann, 1978]. For the Gibson model the slope of the $E(y) = a \cdot y$ relationship is fixed in accordance with the indication reported in table 3, developed for the Ticino sand and for the Wokksund sand [Lancellotta, 1993].

To use soil with comparable stiffness, the value of the Young modulus for the homogeneous soil model is established as a 1/3 of the maximum value of the Young moduli of the Gibson [Gibson, 1967] model inside the

Table 2 – Geotechnical properties of the soil

D_R [%]	ν	*OCR*	*c'* [kPa]	γ[kN/m^3]	ϕ' [°]	*Homogeneous soil*	*Gibson soil*
						E[MPa]	*a* [kN/m^3]
40	0.33	1.00	0.00	16.0	37	92	11497
65	0.33	1.00	0.00	18.5	40	135	16916
90	0.33	1.00	0.00	21.0	43	165	20614

Table 3 – Young modulus values for the Ticino and the Mokksund sand [in Lancellotta, 1993]

D_R [%]	$(\sigma'_a + 2\sigma'_3)/3$ [kPa]	E' [MPa]
40	100	120
40	300	210
65	100	160
65	300	260
90	100	180
90	300	300

significantly influenced area. This area extends to the depth where the ratio $\Delta\sigma_v/p$ is more than or equal to 0.1, being $\Delta\sigma_v$ the increment of the vertical stress due to the average soil-foundation contact pressure *p*.

In particular, the SOFIA code considers a hyperbolic stress-strain relationship for loading and elastic unloading-reloading with the same modulus of the initial Young Modulus [Massimino & Maugeri, 1999].

Moreover, due to numerical problems, for each integration point the minimum value of the Young modulus is fixed equal to 5% of the initial value.

MOMENT – ROTATION CORRELATION

General considerations

Because of the high stiffness of the shear wall, the foundation rotation can play a significant role in the real behaviour of this structural element and then on the behaviour of the whole structure into which it is put. In the dynamic field this aspect is often analysed by means of a dynamic rocking spring and a dashpot. In the present paper, only the static rocking stiffness is analysed in linear and non-linear soil conditions. The analyses allow the developing of the evaluation of the static rocking stiffness through the moment-rotation correlation for the simpler system shown in Fig. 2. However, for a more careful analysis, all the coupled and uncoupled foundation stiffnesses could be considered instead of only the rocking stiffness. In point of the fact that the rocking is associated with horizontal and vertical displacements.

Firstly, the numerical analyses are performed in linear soil conditions, to evaluate the agreement between the results of the SOFIA code with the expressions of the elastic linear K_r reported in literature. Secondly, the non-linear analysis is performed.

Moreover, even if the analyses are developed in plane-strain conditions, the tridimensionality of the soil-structure system is taken into account modifying the Young's soil moduli by means of an approximated procedure [Massimino, 1999], that comes from the comparison of the Boussinesq [Boussinesq, 1885] solutions for equally wide *L* rectangular and strip foundations.

Elastic-linear analysis

The rocking stiffness K_r, obtained by means of the numerical analyses, is compared to the most reliable expressions reported in the geotechnical literature for elastic-linear and homogeneous subsoil. The first expression links K_r to the properties of the soil and the foundation dimensions as reported below for circular foundations [Borowicka, 1943]:

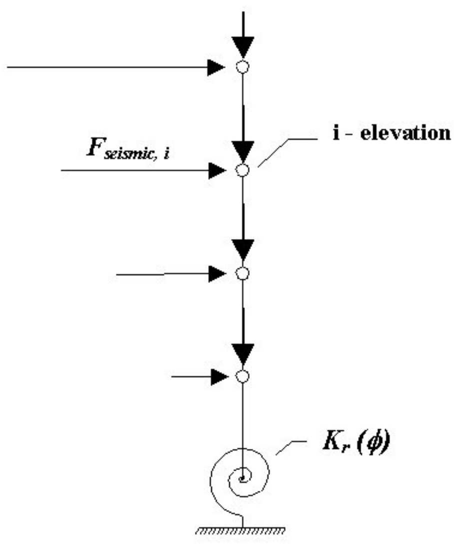

Fig. 2 – Simplified soil-structure schematization

$$K_r = \frac{8 \cdot G \cdot r^3}{3 \cdot (1 - \nu)} \qquad (1)$$

in which *G* and ν are respectively the shear modulus and the Poisson's ratio of the soil and *r* is the radius of the circular foundation. The shear modulus can be related to the Young's modulus used in the SOFIA code, by means of the well-known expression derived from the theory of elasticity for homogeneous and isotropic soil.

For rectangular foundations of *BxL* dimensions it is possible to define the following equivalent radius [Richart et al., 1970]:

$$r = \sqrt[4]{\frac{B \cdot L^3}{3 \cdot \pi}} \tag{2}$$

being the moment applied in the direction of L.

More recently, Gazetas and his co-workers have developed different expressions to compute dynamic impedance functions for various foundation geometries, boundary conditions and mechanical soil properties. In particular, for a rectangular foundation on a homogeneous halfspace the following expression of the static rocking stiffness is suggested [Gazetas, 1991]:

$$K_r = \frac{G}{1-\nu} \cdot I_L^{0.75} \cdot \left[3 \cdot \left(\frac{L}{B} \right)^{0.15} \right] \tag{3}$$

being I_L the moment of inertia of the foundation-soil contact surface around the lateral axis.
For square foundation the expression (3) becomes:

$$K_r = \frac{3.6GB^3}{1-\nu} \tag{4}$$

The comparison between the numerical results and the theoretical results obtained using the above relationships is reported in table 4. In every case it is possible to see some agreement between numerical and theoretical results. The divergence between numerical and theoretical results is greater for a rectangular foundation than for a square foundation, due to the fact that the foundation is rotated along the major side, instead of along the minor side. Then, the approximated procedure to take into account the tridimensionality of the foundation, affects the results much more. However, an error of 15 %, or even slightly more, is acceptable [Gazetas, 1991]. Comparing the theoretical results, it is also possible to note that the rocking stiffness given by Gazetas [Gazetas, 1991] is higher than that suggested by Borowicka [Borowicka, 1943] for rectangular foundations, while it is smaller for square foundations.

Table 4 – Elastic-linear rocking stiffness for homogeneous sand soil

	D_r [%]	**K_r [MNm]**		
		Numerical results	*Borowicka [1943]*	*Gazetas [1991]*
L/B = 4	40	1592	1913	2211
	65	2310	2815	3253
	90	2823	3431	3965
L/B = 3	40	1910	2374	2627
	65	2810	3493	3866
	90	3456	4257	4711
L/B = 2	40	2590	3218	3351
	65	3762	4735	4931
	90	4618	5770	6009
L/B = 1	40	4652	5411	4912
	65	6808	7963	7228
	90	8286	9704	8809

Non-linear analysis

To analyse both the effects of the L/B ratio and of the stress level, the analyses are also performed in non-linear conditions.

As regards the typical design load condition (L/B = 4), Fig. 3 shows the effects of the soil characteristics on the $M(\phi)$ relationship. Fig. 4 shows the same effects for L/B = 1. In all the figures it is possible to see the changing of the rocking stiffness with the increasing of the applied moment. This changing becomes more evident with the decrease of the relative density of the soil, i. e. with the decrease of the stiffness and strength of the soil. Moreover, the rotations are only just a little greater considering a Gibson model [Gibson, 1967] than considering a homogeneous model. These results come from the fixing of the initial Young's modulus, for the homogeneous soil, approximately equal to 1/3 of the maximum value of the Gibson *E(y)* relationship in the significant soil volume. This is due to the greater sensitivity to the applied loads of the strata nearer to the foundation. Then, to have a comparable deformation of the two homogeneous and non homogeneous soil models, it is necessary to consider a weight average value of the Young moduli of the Gibson model and not a simple average value in the significant soil volume.

Fig. 5 reports the M/ϕ - ϕ diagrams for L/B = 4 in both the hypothesis of a homogeneous soil model and of a Gibson [Gibson, 1967] soil model. In this picture it is easier to note the degradation of the static rocking stiffness, starting from low overturning moments. More precisely, for the homogeneous soil model the most significant degradation of the rocking stiffness occurs in the first steps. While for the Gibson soil model, because of the embedment considered, during the first steps the rocking stiffness is quite constant, then decreases with the increase of the rotation. Moreover, in both the homogeneous and Gibson soil models, approaching the highest rotation levels the decrease of the rocking stiffness is quite less evident. In any case, the above results underline once more how it is more reliable to use a non-linear constitutive law for the soil even for load conditions far from the collapse. Comparing, finally, the M/ϕ - ϕ curves of the two homogeneous and Gibson soils, it is possible to note a substantial overlapping for D_r = 40 %, with the exception of the lowest rotation levels. For D_r = 65 and 90 % the homogeneous and Gibson curves have the same trend respectively, but the Gibson model gives smaller values of the rocking stiffness. This gap increases as the relative density D_r also increases, but decreases with the increasing of the rotation level.

DISPLACEMENT RESPONSE AT THE TOP OF THE SHEAR WALL

The possibility of foundation rotations of a shear wall can significantly change the displacements of the shear wall, as recently clearly pointed out by Kumar and Prakash [Kumar & Prakash, 1999] and then the behaviour of the whole structure into which the shear wall is put.

In Fig. 6 the ratio between the maximum horizontal displacement of the shear wall u_{max}, that is reached at the top elevation, and the maximum shear stress T_{max}, that is reached at the base of the shear wall, is plotted versus the L/B ratio. In particular, in Fig. 6, where only the homogeneous soil model is considered, the effects of the relative density and the soil non-linearity are hightlighted. To better understand the results shown in Fig. 6, it must be pointed out that the soil-foundation contact pressure is the same for the different L/B ratio, while the loads applied on the shear wall are opportunely modified. The u_{max}/T_{max} ratio increases with the decreasing of the relative density. In the hypothesis of elastic-linear soil the maximum value of the above ratio is 10 times bigger than that in absence of rotation. Considering the soil non-linearity the u_{max}/T_{max} ratio increases much more and achieves a value 5 times bigger than the value reached in the case of elastic-linear analysis and about 50 times bigger than the value reached in the case of fixed base shear wall.

Moreover, in table 5, the maximum horizontal displacement u_{max} of the shear wall resting on the sand deposit is compared to that of the fixed base shear wall. The results are reported for the typical Italian [Italian Seismic code, 1996] design loads, i. e. for L/B = 4, for both the homogeneous and the Gibson soil model [Gibson, 1967] and considering also the soil non-linearity. In every case a strong divergence between the results of the two fixed and not-fixed base configurations can be noted. Once more in the best situation, reached in the hypothesis of a homogeneous subsoil with an elastic-linear constitutive law, the maximum horizontal displacement of the not-fixed base shear wall is 10 times the maximum horizontal displacement of the fixed base shear wall. In the worst situation, reached in the hypothesis of a Gibson soil model with a non-linear constitutive law, the maximum horizontal displacement of the not-fixed base shear wall is more than 50 times the maximum horizontal displacement of the fixed base shear wall.

CONCLUSION

Considering the important role of the foundation rotation of shear wall structures, the behaviour of a single shear wall resting on sand deposits is investigated by means of the finite element code SOFIA. In particular, the M/ϕ ratio and the displacement response of the structure is investigated for different foundation dimensions and geotechnical conditions, taking into account the soil non-linearity.

In the simple elastic-linear soil condition the comparison between the numerical static rocking stiffness $K_r=M/\phi$ and the theoretical ones shows some agreement, reaching a minimum error of 5 % for square footing. In any case, it is possible to note a not-negligible increase of the rocking stiffness with the increasing of the relative density and with the decreasing of the L/B ratio, in the hypothesis of the same contact pressure at the soil-foundation interface.

This aspect is amplified in non-linear soil conditions.. In this last case, however, it is important to emphasise the

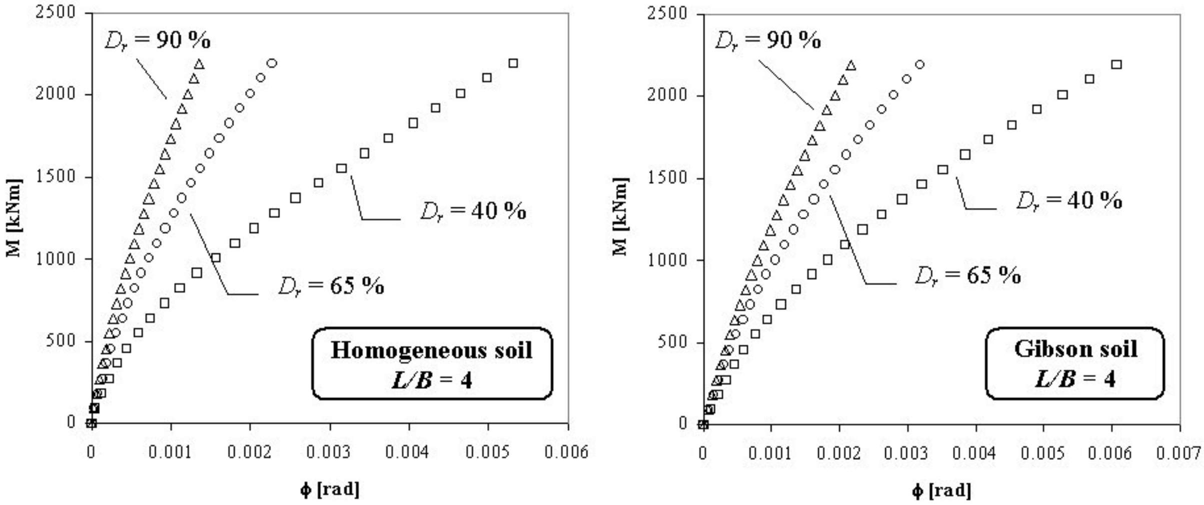

Fig. 3 – Moment-rotation diagrams developed by means of the SOFIA code for *L/B* = 4

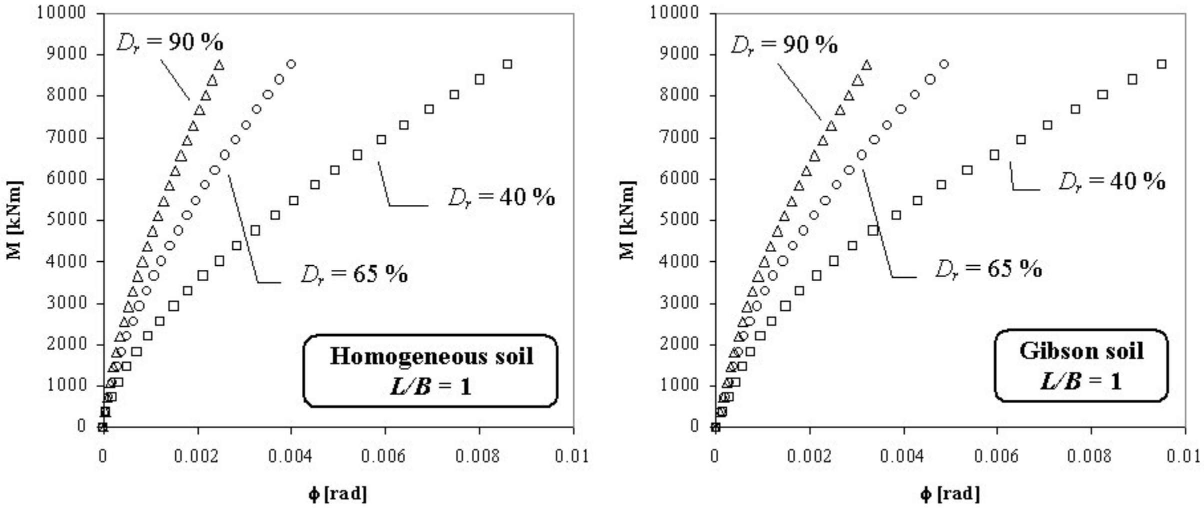

Fig. 4 – Moment-rotation diagrams developed by means of the SOFIA code for *L/B* = 1

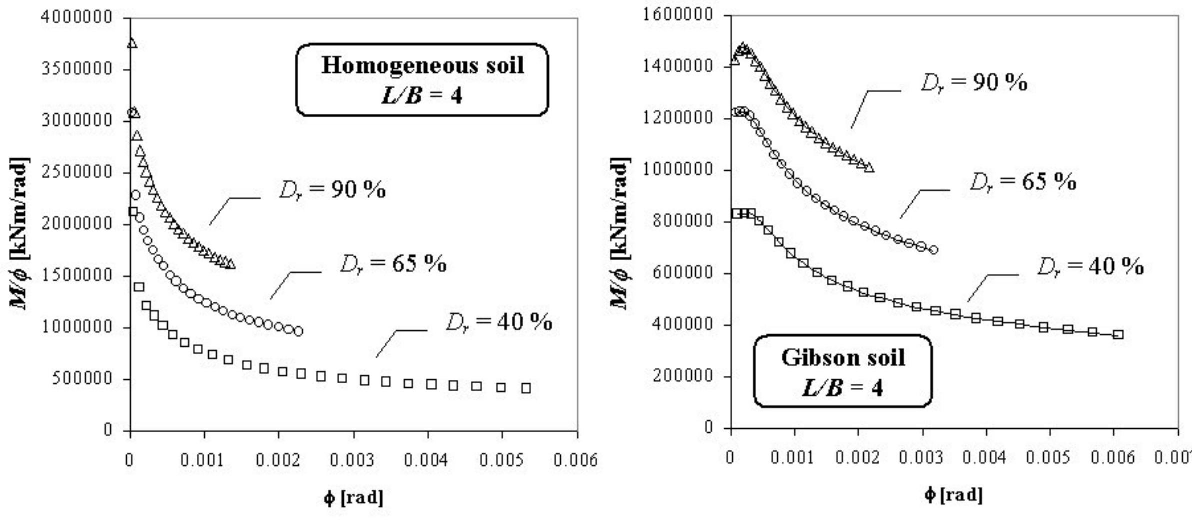

Fig. 5 – Numerical degradation of the rocking stiffness (*L/B* = 4)

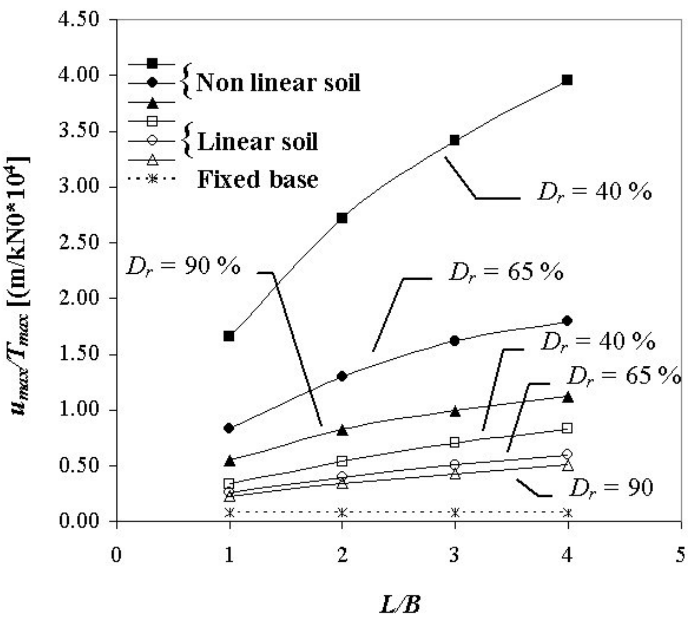

Fig. 6 – The u_{max}/T_{max} ratio versus the L/B ratio

Table 4 – Horizontal displacement at the top elevation (L/B = 4)

		D_r *[%]*	***u_{max} [mm]***
	Fixed base	*/*	*2.00*
Homogeneous soil	***Elastic-linear analysis***	*40*	*20.77*
		65	*15.05*
		90	*12.76*
	Non-linear analysis	*40*	*99.37*
		65	*45.05*
		90	*28.11*
Gibson soil	***Elastic-linear analysis***	*40*	*48.37*
		65	*33.56*
		90	*27.91*
	Non-linear analysis	*40*	*113.61*
		65	*60.09*
		90	*41.31*

evident degradation of the rocking stiffness with the foundation rotation level. The more evident the above degradation the smaller the relative density, as the soil deformability is higher, and cannot be neglected for a more realistic analysis of the soil-shear wall interaction. This interaction modifies significantly the displacement response of the shear wall if compared with the fixed base schematisation. More precisely, comparing the shear wall resting on sand deposit with the fixed base shear wall, it is possible to find that the maximum horizontal displacement at the top elevation is amplified 10 times, considering an elastic-linear behaviour of the soil, and 50 times taking into account the soil non-linearity.

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