

PERIOD DETERMINATIONS AND OTHER EARTHQUAKE STUDIES
OF A FIFTEEN-STORY BUILDING

by

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Man's quest for additional information on the often complex effects of earthquake motion on his structures has taken many approaches throughout the seismic areas of the world. In the United States, at least, this quest has also been subject to considerable variation in intensity depending upon the elapsed time since the most recent damaging earthquake and economical considerations. It is gratifying to see renewed interest in the problem.

The subject of this paper is typical of this more or less cyclical situation - it concerns an unpublished research effort undertaken over two decades ago, which was followed by occasional additional efforts, and which has recently come into renewed life in the technical literature. It is the purpose to herein link these various efforts together in one paper, to extend the investigation considerably beyond its original scope, and to present certain procedures, data and conclusions derived from considerations of many investigations. The common denominator of all the work in reference is one structure, a fifteen-story office building in San Francisco.

Earthquake research might be roughly divided into two categories: (1) the general, or broad, approach where all possible information is obtained from all possible sources, as for example determining and recording all damage (and lack of damage) following a destructive shock; and (2) a concentrated assault on a common objective, such as the single building which is the subject of this paper, a building which the writer believes may now be termed the seismological "guinea pig". The various research efforts which have been undertaken on this structure will be presented chronologically. All work will be mentioned for completeness, although some of the material which has been published elsewhere will be covered very briefly and with reference to the source of complete data. Following are the main subdivisions of this paper:

The Building
The Building Periods as Originally Recorded (1931)
The Original Mathematical Study (1933-34)
Wind Vibration Records (1934)
Earthquake Response (1934 to May 1956)
Dynamic Model Experiments (1934-1938)
Forced Vibration (1935)
Other Investigations (1952-1955)
Recent Investigation (1956)¹
General Conclusions

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ANALYSIS OF STRUCTURAL RESPONSE

THE BUILDING

The building is the Alexander Building, located at the southwest corner of Bush and Montgomery Streets, San Francisco. It was selected in 1933 for the initial investigation, a mathematical thesis by H. L. Hesselmeier and the writer in partial fulfillment of the requirements for the degree of Engineer at Stanford University. (1) Figure 1 shows the location of the building in relation to the shoreline of early San Francisco (contours from U. S. Coast Survey Map of 1853) and Figure 2 indicates the soil conditions of approximately 120 feet of alluvial deposits underlain by shale. The foundations of the Alexander Building are spread footings below the basement level, designed to a maximum bearing of 8000 pounds per square foot.

Figure 3 is a photograph of the Alexander Building and Figure 4 indicates the typical floor plan. There are fifteen stories plus an elevator equipment penthouse and a basement. The roof of the main building (top of the 15th story) is 197 feet above the street level. The ratio of height-to-width is 2.9 parallel to Montgomery Street and 3.3 parallel to Bush Street. The framing is of riveted structural steel with spandrel-to-column connections designed for moment. Exterior walls are face brick backed with unreinforced common brick. Exterior columns are fireproofed with brick masonry and interior columns, girders, and beams with concrete. The floors are reinforced concrete slabs on beams and the partitions are of hollow tile. There are two stair wells with concrete stairs running parallel to Montgomery Street.

The Alexander Building, which was erected in the 1920's, was selected for the original research because: (1) its first and second mode natural periods in each direction had been previously obtained instrumentally by Dr. Byerly and others (2); (2) it has a fairly slender height-to-width ratio with no setbacks; (3) it is rectangular in plan and almost free of adjoining buildings (there is only a four-story building adjoining one side); (4) the design drawings could be obtained; (5) the building owners gave consent; and (6) the walls, wall openings, and other features are such as to make the building fairly symmetrical in rigidity in one direction. It is noteworthy that few buildings in a city the size of San Francisco, and perhaps in most cities, comply with all of the above conditions. A further desirable factor was that the building is founded not on filled ground, or on piling, or on bedrock, but on a fairly deep bed of naturally deposited alluvial material. This permitted extensive considerations of the affect of ground yielding on periods.

It should be established here that the structural "strength" of the building was not a consideration either in the selection of the building for the original study, or in the subsequent studies which followed. Nothing herein or in this "guinea pig" effort is to imply that the building is or is not earthquake-resistant or stronger or weaker than other buildings. The building is of the fireproof steel-frame, Class "A" type and is typical of the better class of San Francisco office buildings erected in the 1920's. The brick walls were placed as non-structural "curtain" walls since no seismic code was in force in San Francisco at the time it or its many contemporaries was designed. In fact no local earthquake code provisions came into force for some two-and one-half decades after construction.

BLUME on Vibration Periods of a 15-story Building

However, provisions for wind forces in the steel frame provide substantial lateral strength. The building was as ideal as could be found for a comprehensive analysis of dynamic properties and the factors causing those properties. Little was it realized then that so many subsequent investigations would follow the original.

The building proper, or prototype, actually loses its identity in many phases of the research. For one example, the dynamic model has 16 stories, the lower one being a dynamic equivalent of the basement and the surrounding and underlying ground. Other factors have been varied in the mathematical work. With the existing structure as the starting point, certain dynamic properties have been assigned from time to time, often on a trial and error basis. It would seem more fitting therefore to refer to this seismological "guinea pig" not as the Alexander Building but perhaps as Building "A".

THE BUILDING PERIODS AS ORIGINALLY RECORDED (1931)

Byerly, Hester and Marshall obtained the natural periods of vibration of several San Francisco buildings in 1931 with a Hall Vibration Recorder. The results reported (2) for the Alexander Building are as follows:

In the east-west (parallel to Bush Street) direction:

Fundamental mode - 1.32 seconds \pm 0.02
Second mode - 0.41 seconds \pm 0.02

North-south (parallel to Montgomery Street) direction:

Fundamental mode - 1.23 seconds \pm 0.02
Second mode - 0.39 seconds \pm 0.02

The instrument for the above recordings was set on the 15th floor of the building.

THE ORIGINAL MATHEMATICAL STUDY (1933-34)

Professor Lydik S. Jacobsen suggested to Harry L. Hesselmeier and the writer at Stanford University in 1933 that they try to locate a suitable building for a thesis subject on structural dynamics in partial fulfillment of the requirements for the degree of Engineer. After considerable search, the Alexander Building was selected for this work for the reasons outlined above. The basic scope of the study was to develop methods of calculation and to determine the factors of an actual building which caused it to have certain (known) natural periods of vibration. The known items were the first and second mode periods of vibration in each translational direction, and to this was added the architectural and structural drawings of the building, as well as the geological information shown in Figure 2. The unknown items which constituted the objectives of the mathematical work included the following:

- (1) How should the fundamental and second mode periods of vibration of a multi-story building such as this be calculated solely from the drawings?

ANALYSIS OF STRUCTURAL RESPONSE

- (2) To what extent do the exterior curtain walls and the interior partitions enter into the dynamic rigidity of such a building?
- (3) Is flexure of the floor framing system appreciable in determining the rigidity of a story?
- (4) What is the effective modulus of elasticity of unreinforced brick work in conjunction with the building frame and fireproofing?
- (5) What are the effects of ground rotation and ground translation in the periods of vibration?
- (6) To what extent does flexure of the building as a whole, i.e. cantilever deflection, affect the period of vibration as compared to deflections due to "shear" translation of each story and ground yielding?

In working on the above problems it was realized, of course, that the results would apply directly only to this one building. However, due to the typical nature of the building selected it was felt that any results obtained and methods developed would have certain application (with judgment) to other structures.

Mass. The first, and the easiest step of all, was the computation of the weight of each story. It was decided early in the investigation that calculations on the assumption of hypothetical uniform cantilever beams would not be applicable to this study and that the building should be treated in as many masses as it had stories, namely 15 above the ground level, and that in each direction a story should be considered to have its own mass, spring factor, and moment of inertia and become an element in the vibrating system. There would be then, 15 modes of translational vibration in each horizontal direction; the periods of only the first and second modes in each direction were known at the time. A story was considered to extend from floor level to floor level. The eighth story, for example, was taken as beginning at the top of the eighth floor and extending to the top of the ninth floor. Since the weight of each floor system constituted a great part of the total weight of each story the center of gravity of each mass was assumed at floor level for convenience. The total height and the dead weight of each story are shown in Table I. The live loading in the building was negligible compared to the dead loads and was therefore neglected. The total weight to the street level is 13,848 kips and the average weight per cubic foot of building is 16.8 pounds.

Spring Factors. The next step was the computation of the spring factors or shear rigidities for each story. The spring factor, K , is defined herein as the ratio of a given lateral force to the "shear" deflection of the story produced by that force. In this paper the term "shear deflection" implies that the floors of the building remain horizontal and that the base of the story is fixed. Local bending and shear deflection of all vertical elements were considered in determining each story shear deflection. The floor systems of the building consist of concrete slabs with both steel and concrete beams and are sufficiently rigid to be effective

BLUME on Vibration Periods of a 15-story Building

horizontal diaphragms, able to distribute lateral forces to the various bents in any story and to the various other vertical elements such as pilasters, columns and partitions in proportion to their relative rigidities. The total rigidity of any story, in a given direction, therefore, was the sum of the spring factors of the various bents and walls, in that direction. All materials, including brick masonry and concrete fireproofing, were considered as participating in the rigidity of each story. Moreover, the deflection of all elements was computed, even though by observation some of them would be very rigid, as for example spandrel sections at floor lines. A considerable investigation was also made of the relative deflections of bents computed by the assumption of a rigid floor wherein flexure of the horizontal elements did not occur and by more accurate calculations wherein horizontal members were allowed to flex in proportion to their rigidity values. Although interior bents naturally showed more deflection where the floors were permitted to flex, it was found that the effect of such flexure on the total floor rigidity was negligible. Therefore the rigid floor method was adopted for the rest of the calculations.

Another investigation was made of the effect of the tile partitions, namely, as to whether or not they would enter into the rigidity under very small amplitudes such as those under which the periods were observed. It was found by trial and error that the assignment of even a low value of modulus of elasticity to tile partitions resulted in spring factors and computed periods that were so short as to be impossible of reconciliation with the actual known periods of vibration. Therefore, the rigidity of the movable tile partitions was not included in the spring factor of a story.

The most important single factor, beyond the physical dimensions, in computing the elastic constants of a story, was the determination of the actual moduli of elasticity and participation of the various materials. The modulus of steel was taken as 30 million p.s.i. and that of all concrete at 2 million p.s.i. It was found, however, that the modulus of the brick masonry, perhaps more accurately stated its effective modulus working in conjunction with the steel and concrete, not only presented a great uncertainty but was very important in the determination of the rigidity of each story. It was decided therefor to carry the modulus of the brick masonry, E_b , as one of the unknowns throughout the investigation. The shearing modulus of each material was taken as 0.4 of the flexural modulus of elasticity. Equivalent (transformed) areas were computed for all materials from their gross sections and were carried as equivalent steel areas with the geometrical distribution of the actual material.

The spring factor calculations were made with care and with full consideration of the relative rigidity of various structural elements sometimes functioning "in parallel" and sometimes "in series". For the direction parallel to Montgomery Street, which direction offered the greatest symmetry and constituted the principal consideration of the thesis, various techniques were employed and also various assumed participations of elements and moduli of brickwork were used, all on a trial and error basis.

ANALYSIS OF STRUCTURAL RESPONSE

Moments of Inertia. Even though flexure of the pilasters along the window bands and of the interior columns was allowed for in the spring factor calculations, the assumption was that the floors remained level. The spring factors, therefore, did not provide for flexure of the building as a whole, wherein vertical elements are loaded axially and the floors rotate. In order to evaluate this factor M/EI diagrams were plotted and the flexural deflections of the building for the fundamental mode were obtained by the method of moment areas. All sections of all materials cut by a plane through the window level of each story were calculated as contributing toward the moment of inertia. A check was made, however, as to the extent of error that might be involved in neglecting the portions of the walls above and below the windows; the error was found to be negligible. As for the computation of spring factors, transformed sections were developed in terms of steel area and the modulus of elasticity of the brick masonry was carried as a variable, E_p . For all stories of the building except the first and second the neutral axes coincide with the centerline of the building. On the first and second story levels the neutral axes are shifted slightly toward the wall adjacent to the neighboring building because that wall has no openings.

The Fundamental Mode. The calculation procedure used to determine the fundamental mode of vibration is developed quite readily from Lord Rayleigh's potential energy method, wherein it is assumed that the dynamic deflection curve is the same as the static deflection curve of the system. To obtain this deflection curve there is applied to each story a lateral force equal to the weight of that story. From this deflection curve the maximum potential energy of the system can be expressed with reference to the undeflected position. Assuming, then, that the lateral forces are suddenly removed, the potential energy of distortion will be transferred into kinetic energy of motion. When the building passes through its undeflected (normal) position the energy of motion is a maximum. By equating the maximum potential and kinetic energies of the building in the two positions, an expression is obtained for the frequency of vibration in terms of the story weights and static deflections. Although this procedure is not precise, comparisons with more accurate methods showed negligible error for Building "A". The formula obtained is:

$$N = \sqrt{\frac{\sum_1^n W_i \Delta_i}{\sum_1^n W_i \Delta_i^2}} \quad \text{-----} \quad (1)$$

and,
$$T_1 = \frac{60}{N} \quad \text{-----} \quad (2)$$

It should be noted that the spring factors and moments of inertia were assumed constant regardless of deflection or the amount of loading. This may not be the case in the prototype in the event of considerable

BLUME on Vibration Periods of a 15-story Building

motion. The deflections that result from the assumed lateral load may be composed of four parts: (1) shear (using the spring factors), (2) flexural bending of the building as a whole, (3) rotation of the structure in the ground and, (4) translation of the ground. With the characteristics of the building under consideration it was felt that all of these had to be evaluated. This resulted in the unfortunate condition of having more unknowns than equations to solve for same, unless higher modes of vibration were studied concurrently with the fundamental mode. Since the second mode was known, it was later brought into service.

The Second Mode of Vibration. The computation of the second period of vibration will be treated briefly here since the first and second modes had to be considered together in the evaluation of ground yielding and the modulus of elasticity of brick masonry. The development of a method of computation for the second (and higher) modes of vibration of a multi-mass vibrating system including the effects of shear translation, flexure of the structure as a whole, and ground yielding, included the following operations:

(1) A preliminary analysis was made of a uniform rod having the average properties of the Alexander Building for the purpose of determining the effect of flexure upon the second mode of vibration of such a rod. Rayleigh's method was again employed but only in an exploratory manner. Since, in the second mode, part of the structure is moving in the opposite direction to the remainder of the structure, some of the inertia forces will be in one direction while the rest will be opposite. It was necessary to determine the point of reversal of the inertia forces under various loading assumptions of opposite direction. The criterion for a solution was that the correct distribution of loading would produce a static deflection curve, both loops of which would have the same period of vibration. Flexural and shearing deflections caused by the loadings were computed and then added and these total deflections were used to calculate the frequencies of vibration of the resulting two loops. It was found for the second mode with the ground assumed rigid that the nodal point was at $.289L$ from the top of the "rod", where L represents the rod length. The consideration of flexure as well as shear increased the second period 18 percent over the amount for shear alone, or $T_{SF_2} = 1.18T_{S_2}$.

(2) Since the ground, or end condition, of the "rod" in (1) above was assumed infinitely rigid, the ratio, $T_{SF_2}/T_{S_2} = 1.18$, had to be re-evaluated for less than infinite ground rigidity. By ratio to previously developed T_{SF_1}/T_{S_1} values, an approximate further correction factor was developed as a function of E_b . Space herein does not permit detailed treatment of this subject, but the values of T_{SF_2}/T_{S_2} were finally taken as a straight line variation from 1.18 at $E_b = 260,000$ to 1.08 at $E_b = 600,000$.

(3) The above items (1) and (2) were merely to develop correction factors which would approximate the reduction of T_2 (observed) to T_{S_2} for various values of E_b . Thereafter, instead of using equivalent "rods", the discrete

ANALYSIS OF STRUCTURAL RESPONSE

values of mass and spring factor for each story were used in a general method applicable to any mode to determine T_{sex} . Reconciliation was then desired in this study not for the observed second period, but for that period reduced by the correction factor outlined above for flexure.

The procedure for calculating second (and higher) periods was developed and applied to a building, it is believed for the first time, from a device of Holzer (3) which had previously been applied to the torsional vibration of shafts with several masses(4). Space does not permit a complete treatment of the development or of the calculation procedure but application to a simple building is given elsewhere(5). A step-by-step tabular method using the equation:

$$\Delta_i = \Delta_{i-1} - \frac{1}{K_{i-1}} \sum_{j=1}^{i-1} m_j p^2 \Delta_j \text{ ----- (3)}$$

enables the calculation of the latter term for assumed values of the frequencies, p. When

$$-\sum_{j=1}^n m_j p^2 \Delta_j + K_n \Delta_n = 0 \text{ ----- (4)}$$

a correct value of p has been determined for the mode in question. Each time equation (4) is satisfied, starting with the slowest possible frequency, represents a modal solution. By properly combining K_n (the lowest story spring factor) with an assumed spring factor for the ground in translation, such ground yielding may also be considered in the same operation. Ground rotation for higher modes is accounted for by other procedures.

Reconciliation of Periods. Various calculations of T_1 and T_2 parallel to Montgomery Street were made using the above procedures and a considerable range of Eb and ground yielding values. Thus certain unknowns were either determined or isolated by comparison of such calculated periods to the observed natural periods. Time did not then permit more than a brief and approximate consideration of the Bush Street direction and periods higher than the 2nd were not known. The calculation of building periods from the drawings was found entirely feasible, although considerable exploratory and development work was necessary in this first comprehensive attempt. For this building, shear, flexural, and ground yielding deflections were all found to be factors in the fundamental period and also in the second period. In view of the more extended recent (1956) research, conclusions and data will be presented at the end of this paper.

Some of the thesis work was laborious in the absence of modern electronic calculators, but it was all extremely interesting. The writer is indebted to Professor Jacobsen for suggesting the thesis problem and for his enthusiastic cooperation and counsel during the work. Mr. Hessel-meyer was an ideal research partner and shares with the writer any credit (or criticism) of the initial effort on Building "A".

BLUME on Vibration Periods of a 15-story Building

WIND VIBRATION RECORDS (1934)

On October 17, 1934 vibration records were taken in the Alexander Building as part of the then intensive recording program of the U. S. Coast and Geodetic Survey, Seismological Division⁽⁶⁾. The motion, presumably induced by wind and/or traffic, was reported to have periods as follows:

In the east-west (parallel to Bush St.) direction

Fundamental	-	1.33 seconds
Other	-	0.40, 0.22 seconds

In the north-south (parallel to Montgomery St.) direction

Fundamental	-	1.25 seconds
Other	-	(0.80 torsion); 0.40; 0.22 seconds

EARTHQUAKE RESPONSE (1934 to May 1956)

Because of the increasing interest in the building as a "guinea pig", three strong motion instruments were installed, one in November 1934 in the basement, one in November 1934 at the roof level (penthouse floor), and one in October 1935 on the 11th floor. This was done by the U. S. Coast and Geodetic Survey under the direction of the late Franklin P. Ulrich⁽⁷⁾. The three 12-inch accelerographs are, at the time of this writing still in place, and are set to start and record automatically and with simultaneous time markings in the event of strong motion.

San Francisco (to May 1956) has experienced no major earthquake motion since this installation (in fact, not since April 1906), but one or more of these accelerographs has provided records at 9 different times from earthquake motion and once from a strong wind storm. The writer has recently analyzed the 9 earthquake responses (March 8, 1937, October 25, 1943, June 22, 1947, July 21, 1952, April 25, 1954, July 6, 1954, Aug. 23, 1954, December 16, 1954, and October 23, 1955 - the latter earthquake centered near Walnut Creek, and the previous 3 in Nevada) and found them most interesting. Motion in the first, 2nd and 3rd natural periods of the building can be plainly found and usually with several cycles. There is little evidence of vibration in periods other than those of the natural modes. Whether this would be true for stronger or more local earth movement is another question. Figure 5 indicates many of the responses in amplitude and period in the Montgomery Street direction and Figure 6 gives the same for Bush Street. The similarity of periods to those of forced vibration and wind is remarkable even though the amplitudes involved vary by almost 10,000 times in some cases. The value of obtaining simultaneous recordings under future extreme motion can not be overemphasized.

Following are the maximum accelerations and/or amplitudes found in these 9 earthquake records with the corresponding periods shown thus (0.46 etc.):

ANALYSIS OF STRUCTURAL RESPONSE

Earthquake Date	<u>Parallel to Montgomery St.</u>	<u>Parallel to Bush St.</u>
Oct.23,1955	Roof: 34.7cm/sec ² ; .186cm; (0.46)	24.9 cm/sec. ² ; 0.139cm;(0.47)
"	11th: 11.8cm/sec ² ; .046cm; (0.39)	7.8 cm/sec. ² ; 0.015cm;(0.28)
Oct.25,1943	Roof: 15.3cm/sec ² ; .024cm; (0.25)	12.0 cm/sec. ² ; 0.022cm; (0.27)
"	11th: 10.5cm/sec ² ; .01cm; (0.24)	6.3 cm/sec. ² ; 0.010cm;(0.25)
"	Bsmt: 6.3cm/sec ² ; .010cm; (0.25)	4.3 cm/sec. ² ; 0.07cm;(0.26)

It is to be noted that because of the small response, error in analyzing the records could be appreciable. The maximum acceleration recorded thus far is 34.7 cm/sec² or 3.5 % of gravity. The above earthquakes were minor but centered within 50 miles of the building. On Figure 7 is plotted simultaneous response for the 15th story (roof level) and the 10th story (11th floor) for the October 25, 1943 earthquake. The 10th story response is plotted negatively so as to illustrate the almost perfect third mode vibration of the building. Obviously this is not forced vibration (unless near resonance) but a natural vibration induced by previous ground disturbance. The strong motion instrumentation program is most valuable and should be continued indefinitely into the future.

DYNAMIC MODEL EXPERIMENTS (1934-1938)

Building "A" has also been the prototype for a dynamic model which was used in experiments on the Stanford University Vibration Laboratory shaking table. Professor Lydik S. Jacobsen, with the writer as his assistant, designed and constructed the dynamic model in 1934. The thesis previously outlined was used as the basis for the physical and dynamic values for each story. Later, Dr. Jacobsen with R. S. Ayre and E. P. Hollis calibrated and improved the model and, finally Jacobsen and Ayre conducted an extensive series of experiments and published a paper on this work⁽⁸⁾.

There is neither space nor reason to describe this work herein since it was so excellently and comprehensively covered in the original paper. It might be noted, however, that the model actually had 16 stories, the lower one of which could be varied in characteristics to simulate the combined base-ment and ground conditions of the prototype. Moreover, various first story rigidities and top story weights were substituted for various tests. The lineal scale was 1 to 50 and the model construction was such as to permit large deformations without damage and to record all motion accurately. The great importance of lower-story rigidities on the resulting shears and moments of the whole structure was demonstrated as well as the response of the higher modes. The reader is referred to the original paper for the details of construction, test procedures and results.

Two additional items should be mentioned here for completeness of this summary of Building "A". (1) The model was tested under a constant total time duration for ground motion in each frequency. This probably tended to accentuate the relative response of the higher modes as compared to the lower (more cycles), perhaps even more than the authors anticipated.

BLUME on Vibration Periods of a 15-story Building

(2) The weights and elastic constants for the model, after scale conversions, are not precisely those of the prototype, because of the original construction (the model's elastic constants were averaged for each direction so as to provide twofold symmetry since it had been found previously that the torsional coupling between the two directions made a comprehension of the experimental results very difficult) and also because the recent (1956) extended work indicates somewhat amended prototype data. These are not serious matters nor do they detract in any way from the valuable work done; they are mentioned herein only for completeness and reference in considering the model test results in a detailed quantitative manner.

FORCED VIBRATION (1935)

Under the U. S. Coast and Geodetic Survey Seismological Program directed in the field by the late Franklin P. Ulrich, the writer was in charge of the sub-party which conducted a very interesting series of forced vibration tests in the Alexander Building as well as in many other buildings and structures. The machine used, the so-called Building and Ground Vibrator, had been previously developed and constructed under the same program by the writer with Professor Jacobsen as Advisor (6) (9). Tests in the Alexander Building were conducted too late to be included in Special Publication No. 201(6) but they were reported on preliminary mimeographed sheets(10) prepared by the writer for issue by the U. S. Coast and Geodetic Survey. Certain of the test results will be presented here for convenience.

Altogether 23 different runs or tests with the vibrating machine were conducted, all late at night, from July 6 to July 16, 1935. In each case the vibrator was located in the penthouse (at the main building roof level). Simultaneous recordings of the actual building motion induced were taken at various floor levels and also at various locations on certain floors. The recording instruments were U.S.C. & G.S. vibration meters. Static magnifications (of instrument response) varied from 120 to 1600 and the total unbalance moment on the wheels of the vibrator was either 384 in. lb. or 768 in.lb. for various runs. The machine was driven up to a speed corresponding to about 0.14 second period and was allowed to decelerate after removal of the driving belt at top speed, to a stop. Records were taken during this entire time of deceleration. Excellent response curves were obtained, even on the basement floor and in the street alongside the building. From these records resonance curves were plotted (Figures 9 and 10). Obviously the basement floor and the street alongside the building were vibrating with the rest of the structure, all as caused by this small machine at roof level. The ground is not infinitely rigid! Figures 5 and 6 indicate the correlation of recorded periods between forced vibration of small amplitude from a small disturbing force at the roof level, and motion induced by wind and earthquake. Within the limits of error, it may be postulated for Building "A" that (1) such vibrator induced motion is an excellent duplication of tall building vibration from other causes including ground motion at least to the amplitudes shown, and (2) natural periods do not appreciably vary from 0.00001 inch to almost 0.1 inch single amplitude. Figure 8 shows the deflection of the building from forced vibration in the lowest three modes; the points are actual readings. The dash lines are calculated deflections from the most recent investigation. The unusually heavy penthouse and other construction at the top level as compared to the rest of the building and to other office buildings of larger plan size is

ANALYSIS OF STRUCTURAL RESPONSE

undoubtedly affecting the nodal points as well as the periods.

It has long been the writer's opinion that damping, both viscous and frictional, is extremely important in explaining why some buildings behave better than expected in actual earthquakes. The earthquake problem involves movement, energy, and work as well as forces, shears, and moments. Much more could be done with controlled forced vibration as one research approach. Even though the amplitudes are very small for the above tests, there is obviously damping in the building action. If one be permitted for the moment to investigate each resonance peak for damping, individually and regardless of mode or other considerations, the following average and approximate percentages of critical damping are obtained:

<u>Mode</u>	<u>Parallel to Montgomery St.</u>	<u>Parallel to Bush St.</u>
1st	2%	(no record)
2nd	4%	2%
3rd	4%	4%
4th	5%	4%

It might reasonably be expected that damping would increase with amplitude.

The forced vibration tests were extended to both translational directions of the building and also were planned to induce torsion. Following are the natural periods determined from this 1935 series of field experiments:

<u>Mode</u>	<u>Parallel to Montgomery St.</u>		<u>Parallel to Bush St.</u>		<u>Torsion</u>
	<u>T, sec</u>	<u>Ratio to T₁</u>	<u>T, sec</u>	<u>Ratio to T₁</u>	<u>T, sec.</u>
T ₁	1.27	1.00	1.37	1.00	0.84
T ₂	0.41	3.10	0.45	3.05	0.31
T ₃	0.24	5.29	0.26	5.27	-
T ₄	0.17	7.47	0.19	7.21	-

OTHER INVESTIGATIONS 1952-1955)

The writer is pleased to note the recent increased interest in seismological engineering and the use of modern high-speed calculators to perform calculations previously considered prohibitive in labor. In two cases, data from the original thesis investigation on Building "A" have been used in such work. Robinson and Rinne⁽¹¹⁾ have used certain physical factors of the building in a mathematical investigation and Clough⁽¹²⁾⁽¹³⁾ extended this work in 1955. In addition to these references to the Alexander Building, many others, direct or indirect, may be found in the technical literature ⁽¹⁴⁾⁽¹⁵⁾⁽¹⁶⁾⁽¹⁷⁾.

BLUME on Vibration Periods of a 15-story Building

RECENT INVESTIGATION (1956)

Recent activity and references to the unpublished thesis led the writer to the conclusion that the original work should be re-evaluated and extended in the light of increased knowledge of the actual building properties gained through forced vibration and earthquake records, and then be published. This effort led to a major extension of the original work and modification of certain physical properties suggested in the thesis. To those who may have previously used such Building "A" properties, apology is hereby tendered. It is hoped that the effects of such changes on the work of others will be minor and, in fact, might lead to increased knowledge from calculating the response of different hypothetical structures. Certainly the methods and qualitative conclusions of others will be unimpaired.

General. The re-evaluation of the original mathematical work on this building was undertaken by the writer as a "spare time" unsponsored effort in the expectation that it would be mainly a matter of also reconciling the third and fourth periods previously unknown. However, one thing led to another until finally that done amounted to an exhaustive test and an extension of all that had been done before: a reconciliation of not only the first, second, third, and fourth modes but also of such in the two directions of the building; of each building direction to the other; a more searching consideration of the ground characteristics as they apparently affected the building vibrational response in both horizontal directions, and of the stair rigidity in the Montgomery direction. I.S. Bjorklund of the writer's staff performed a considerable amount of the extensive period calculation work.

The basic approach was to take nothing for granted from the original work except the methods of period calculation which had been developed and the spring factors and moments of inertia for specific assumed conditions. It was decided to attempt to reconcile all four periods in the Montgomery direction and, if possible, all four periods in the Bush Street direction and then to reconcile these with each other and to see what values of the brick modulus and ground yielding and also other possible data might be indicated therefrom.

One important reason for wanting to also work in the area of the third and fourth modes of vibration previously unknown, was the fact, as shown by Jacobsen ⁽¹⁴⁾, that for a uniform cantilever rod the effects of flexure and ground yielding on the natural periods of vibration are less in the higher modes than in the first and second modes. Thus, the higher periods not only offered more data but also a means of reducing the effect of error in certain assumptions. As before, the modulus of elasticity of the brick masonry working in conjunction with the other structural materials was carried as an unknown, E_b . In addition to this, in the direction parallel to Montgomery Street the effect of the two stairwells and the stair systems in the building were included as another unknown. With reference to the ground conditions, a comprehensive search was made for all available boring and soil test data in the vicinity of the Alexander Building.

ANALYSIS OF STRUCTURAL RESPONSE

Known Data. The data known or found (within limits) by search, were as follows:

1. The 1st, 2nd, 3rd and 4th periods of vibration in each of the two translational directions, parallel to Montgomery Street and to Bush Street, were known from the forced vibration tests of 1935. (Figures 5, 6, 7, 8, 9 and 10).
2. More was known about the possible range of the natural periods of vibration as observed over many years: (a) they did not apparently vary to any degree with amplitude, nor (b) between stimuli such as wind, minor earthquake or forced vibration; the results of many period recordings and the slight errors therein led to a narrow banding or spread of values for the various periods under consideration (Figures 5 and 6).
3. The physical dimensions and materials of construction of the building.
4. The general type of soil under and about the building together with some test data on these and similar soils (Figures 1 and 2).

Unknown items were - to what extent, if any, do brick curtain walls, tile partitions, stairs, soil yielding, floor yielding, flexure of the structure as a whole, and possibly other factors affect the (known) 4 periods of vibration in each horizontal direction of this building? What are the probable dynamic properties of the building?

Criteria. It was postulated that:

1. The effective modulus of elasticity of the brick masonry and tile partitions should be essentially the same in the Bush Street direction as in the Montgomery Street direction.
2. The unit soil yielding characteristics both in translation and/or rotation should also be the same in the two directions, however the effects on the building would be different because of geometric layout.
3. Stairway contribution to rigidity, if any, would apply principally in the Montgomery Street direction because, as indicated on the plan of the building (Figure 4), both the inclined stairways and the longer stairwell walls are parallel to Montgomery Street.
4. Although the values of effective material moduli would result in different spring factors (K) and moments of inertia (I) for each story and also in each direction for each story, as determined from known dimensions, equal or closely similar values of such moduli would be expected to reconcile all 8 modes and both directions. Thus, independent procedures would be expected to conform to the above as well as to reconcile all known periods, or be re-evaluated. This was indeed a challenge!

BLUME on Vibration Periods of a 15-story Building

Dynamic Properties. The weights of each story were taken unchanged from the thesis study. The spring factor and moment of inertia values, where applicable, were also used but early trials at 4-period reconciliation showed that (1) more work had to be done in the Bush Street direction, (2) stair contribution to rigidity in the Montgomery direction had to be considered as more important, and (3) K's and I's for lower assumed values of E_b had to be calculated in both directions. As before, any appreciable dynamic participation of the tile partitions was found to be incompatible with the known periods of the building. Since all the brick masonry was assumed to function (for such small amplitudes as observed), a point of discontinuity was approached for very low values of E_b . The reasons for this are obvious when one considers the elevations of the building with its many windows (Figure 3). At what point does the brick cease to function at all insofar as fixing the top of pilasters is concerned? In this connection it should be recalled that the exterior steel columns are fireproofed with brick whereas the interior columns are fireproofed with concrete. It finally was assumed that at the 100,000 p.s.i. value of E_b the masonry would be functioning in transformed area (steel) in accordance with its own physical dimensions. At the zero value of E_b the exterior column lengths had to be increased to the steel and concrete spandrels. Smooth curves were drawn between K values thus calculated for E_b of 0 and 100,000 p.s.i. and were then extended to K values calculated for various E_b amounts over 100,000 p.s.i.; this was done for each story.

It is interesting to note that the narrower width of the Bush Street direction (almost 9 feet less than the Montgomery width) has greater spring factors for E_b above 100,000 p.s.i. This is due to the lesser number of windows and consequently greater width of brick pilasters and wall sections. On the other hand, the moments of inertia for corresponding values of E_b are less for flexure parallel to Bush Street. The ground conditions could not be considered infinitely rigid as evidenced by the forced vibration records made not only on the basement floor but on Bush Street (Figure 10); this characteristic would apply to both directions.

A great many combinations of K's and I's were calculated for different assumed E_b and stair values. The values finally determined as best and most consistently meeting all conditions, criteria and reconciliation of all periods are shown in Table I. In general the K's and I's are less and the ground more rigid than assumed from the original (thesis) study which basically reconciled only two modes in the Montgomery direction.

Calculation of the Fundamental Mode. As before, the first mode periods were calculated for various assumed values and conditions using the Rayleigh method and 15 discrete masses (Equations 1 and 2). Shear and flexural effects were computed separately and also in combination to obtain values of T_{S1} , T_{F1} , and T_{SF1} , respectively (the curves on Figures 11 and 12 indicate calculated values). The ground yielding problem was approached in several ways, often with approximate (trial) techniques which were later revised to more accurate methods in the area of "closing in". Jacobsen's treatment(14) of end conditions of a cantilever wherein consideration is

ANALYSIS OF STRUCTURAL RESPONSE

given to both rotation and translation in the ground was most useful. For various deflections of the building from either or both types of ground yielding and assumed range of soil values (such values were assumed after a careful consideration of the probable values for very short - time incremental loading) the Rayleigh method could again be employed or, with Dunkerley's empirical expression for the period of a complex system in terms of its isolated components⁽¹⁸⁾, rapid approximations could be obtained with the following:

$$T_{SFG1}^2 = T_{S1}^2 + T_{F1}^2 + T_{G1}^2 \text{ -----(5)}$$

$$\text{or } T_{SFG1}^2 = T_{SF1}^2 + T_{G1}^2 \text{ . -----(6)}$$

Final results, of course, were developed in the most accurate method feasible. In general, building rotation was found to be much more of a factor than translation in the ground. Rotation was assumed to be about the footing level.

Calculation of the Higher Mode Periods. The 2nd, 3rd, and 4th periods for shear deflections only were calculated by the general method used in the thesis (equations 3 and 4). However, before proceeding with trial frequencies, deductions were made from the observed periods to allow for flexure and ground yielding. Such corrections were difficult to develop. In the second mode, the thesis flexural correction was employed as well as another method which was used for all higher modes. In the latter, a hypothetical uniform beam was determined which had properties such that the ratio T_{SF1}/T_{S1} was the same as for the building. Then, by reference to Jacobsen's excellent paper (14), values of T_{SF}/T_S were determined for such a hypothetical beam. In the same manner, ratios of T_{SFG}/T_{SF} were obtained. It is important that the error involved in such assumptions becomes less with higher modes where shear is of major importance. Figures 11 and 12 indicate the calculated values of all periods (the sloping lines) and Figure 13 indicates the approximate relative influence of shear, flexure and ground yielding in all modes. It is apparent that with only the first and second modes known, the probable error is relatively large. It was intended to allow K and E_b values developed in the 4th and 3rd modes to govern over those of the 2nd and 1st modes in case of conflict; however, reconciliation of all modes was finally so good that no such favoritism was needed.

Reconciliation of Periods. Figure 11 for the Montgomery Street direction and Figure 12 for the Bush Street direction indicate the condensed results of the recent investigation and the reconciliation of 4 periods on each figure. The designation "K average" on Figures 11 and 12 should not be taken to infer that period calculations were made on the basis of averaging the spring factors for all stories in the building. This designation is used for convenience of plotting and reference; the discrete values of K , W and I for each story were actually employed except for determining certain minor corrective adjustments for flexure and ground yielding in the higher modes as outlined above. Period calculations were

BLUME on Vibration Periods of a 15-story Building

done in accordance with the methods described above and for 15-mass vibrating systems.

The four horizontal shaded bands on each figure indicate the range of observed periods of vibration for the building as recorded for many different conditions, times, floors, instruments and disturbing forces. The four arrows on the left indicate the forced vibration resonance peaks (see also Figures 5, 6, 9, and 10). The bands could be eliminated, of course, by merely taking average or other specific values; in fact such was done early in the investigation, but was eliminated as a procedure in order to more clearly show all conditions when it was found that certain results were rather sensitive to small variations in the higher periods. The sloping curves indicate calculated values of periods T_1 , T_2 , T_3 and T_4 for various combinations of shear, flexure and ground yielding. Each curve is plotted from the results of a great many period calculations.

For the fundamental mode, T_1 , is indicated a range of possible soil values, the bulk modulus, ϵ_v , under incremental vibration loading. The lowest acceptable value consistent with the determined soil conditions is 525, whereas a more probable and upper limiting value is shown at $\epsilon_v = 2000$. Note also the small change in T_1 from $\epsilon_v = 2000$ to $\epsilon_v = \infty$ (the T_{SF} line). The ground variations are thus confined to a fairly narrow band, especially for the higher modes. In order to consider all factors, another side investigation was conducted on the possible ratio of ϵ_v to ϵ_h . Ranges of this ratio, γ , from zero to 100 were considered not only in connection with the building periods and their reconciliation, but also in view of the actual soils and conditions involved. The building has "planting" in the ground (with the full basement story the foundations are approximately 20 feet below the ground surface); the building sidewall backfill has been in place for about three decades; and the undisturbed soil was considered worth 4 tons per square foot foundation design value many years ago. In view of these and other factors the probable value of γ would be expected to be in the order of 1. Other calculations indicated that γ ranges from zero to 10 made practically no difference in the periods for Building "A". Whether the actual γ value is 1 or 5 or even 10 would make no practical difference in Figure 11 or 12, so ϵ_h (or γ) is not shown thereon, even though considered throughout.

Vertical bands might also be constructed on Figures 11 and 12 to indicate ranges of K_{AVG} which would intersect horizontal bands for observed periods and diagonal lines for calculated T_{SFG} values. Actually a single value of $K_{AVG} = 10 \times 10^6$ can be seen as very closely intersecting the observed T and the calculated T_{SFG} values for all four modes of vibration on Figure 11. This then can be tentatively considered as a solution, if reconciliation is also possible in the Bush Street direction. It is noticed on Figure 12 that a value of K_{AVG} to reconcile all four modes is not as clearly defined as in the Montgomery Street direction although 10.2×10^6 does intersect all bands at values of ϵ_v corresponding to the Montgomery direction and also to each mode on Figure 12. The fact that there is more wall asymmetry in the Bush Street direction (at the lower

ANALYSIS OF STRUCTURAL RESPONSE

stories) and also that the calculations for the physical conditions and factors of the building could not be carried out with as complete detail as for the Montgomery Street direction because of lack of certain data, may be factors here. In other words, if a conflict should occur between the results for the two directions, the Montgomery values should be given more weight.

The selected K_{AVG} values of 10×10^6 (Montgomery) and 10.2×10^6 (Bush) do not in themselves signify reconciliation because of the fact that the stairs were found to have influence in the former direction only and because of the various story dimensions. In order to test the solutions, the postulation of E_b being common to both directions should be challenged. The common soil characteristics ϵ_v have already been shown. K_{AVG} of 10.2×10^6 (Bush) resolved to the discrete values for each story, means that E_b (Bush) is 58,000 p.s.i. If the same is done for the Montgomery direction and a previously calculated K contribution for the concrete stairs is allowed (4×10^6 for the 12' stories, and 3×10^6 for the 20' stories), then E_b (Montgomery) is 49,000 p.s.i., which is a close reconciliation. A single significant figure of 50,000 or 60,000 p.s.i. would seem acceptable as sufficiently close agreement. However, there is still some degree of uncertainty about the value of the stair contribution to story stiffness. The stair walls could not be considered effective because the floor framing is inadequate in stiffness to develop moment reversals in these walls. Only the diagonal stairs proper were finally assumed to function. As a trial, if the Bush Street E_b value of 58,000 were assigned to Montgomery Street also the stair stiffness factors would be reduced from 4×10^6 and 3×10^6 to 3.6×10^6 and 2.7×10^6 respectively. This variation of only 10% seems acceptable and indicates substantiation of the stair rigidities.

It may be said then, that within very small variations, considering the involved calculations, approximations and interpolations, the 4 modes in each direction are not only reconciled to each other but also each building direction is reconciled to the other, all with common values of ϵ_v and E_b as postulated above. The many and independent criteria seem to be satisfied.

Table I provides a summary of the most probable dynamic property values for each story in each direction and also a brief resume of other apparent values. The significant figures shown are for consistency, not to imply a corresponding degree of accuracy. Figure 13 indicates graphically the relative contribution of shear, flexure and ground yielding to each period and also the periods and their ratios. The ground yielding effect may be considered as approximately 90% rotation and 10% translation.

GENERAL CONCLUSIONS

The results of this recent effort, which are largely condensed onto Figures 11, 12, and 13 and Table I, together with the previous work reported herein, lead the writer to the following conclusions regarding the "guinea pig" building. No application of the results or conclusions to other buildings is intended although it is possible that some items could be of general interest with consideration of all the factors involved and judgment.

BLUME on Vibration Periods of a 15-story Building

(1) The very low value determined for the effective modulus of elasticity of the brick masonry working in conjunction with the steel and concrete indicates that the masonry is not fully participating in rigidity for the range of vibration amplitudes recorded thus far (Figures 5 and 6). Brick masonry per se is normally considered to have a flexural modulus of elasticity of many times the 50,000 to 60,000 p.s.i. value found necessary to reconcile all known periods and conditions. Complete and effective bond with the other materials is probably lacking and internal voids may also be a factor. It would be expected that modern reinforced grouted brick masonry, well tied and bonded to the framing and well placed, would materially shorten the natural periods of this, and similar, buildings.

(2) The tile partitions participate in little or no degree toward the rigidity of the building for the amplitudes of motion recorded.

(3) The fact that the brickwork still affects the periods, even though only in part of its total potential, is seen from a calculated fundamental period of vibration of 2.34 seconds assuming the mass of all brick and tile but neglecting all rigidity of same (1.33 seconds is the observed and reconciled corresponding period, Bush Street direction).

(4) The curtain wall materials may be expected to participate more after the framing has deflected some amount greater than recorded thus far and has, simultaneously, absorbed some energy. The walls would then tend to shorten the periods and increase damping and energy absorption. If cracking occurs with extreme earthquake motion, the periods would increase again, perhaps to longer values than the original. This probable increase in damping and energy absorption and the changes in period may be a clue as to the generally good record of similar framed structures in earthquakes.

(5) The stairs would be expected to offer considerable initial resistance in the north-south (Montgomery St.) direction; increased motion would lead to stair failure and greater participation of the walls, all with variations in natural periods.

(6) The floors can be considered as rigid plates with no appreciable error in calculating periods.

(7) The soil yielding contributes about 8% to the fundamental mode periods of this building. The other modes are progressively less affected by soil yielding. Rotation or rocking in the soil contributes far more to this effect than does translation in the soil, even though the vertical and horizontal soil deformation moduli be considered about equal for this site.

(8) Flexure of the building as a whole is an important part of its dynamic response, particularly in the fundamental and second modes. This structure, and undoubtedly others of even less height-to-width ratio, can not be called "shear" buildings wherein the floors remain level without considerable error in period and other calculations.

ANALYSIS OF STRUCTURAL RESPONSE

(9) The 3rd and 4th mode periods were necessary (with the 1st and 2nd) to adequately explore the dynamic properties of this building and the many factors that contribute to those properties. (This does not imply that these higher modes are as important insofar as response to ground motion and resulting maximum shears and moments are concerned - that is beyond the scope of this paper). Since these higher modes, as well as damping, can be determined reliably only by controlled forced vibration, that technique should be included in any major program of research.

(10) The fundamental and higher modes of vibration of buildings can be computed from the drawings with a degree of accuracy depending upon the knowledge of ground yielding and many other conditions that may be involved. An extension of this type of investigation into many additional buildings of various types seems very desirable.

(11) In view of the results of this effort and also the ratios of the known periods as compared to the relative amounts of shear and flexure found to be involved, it is questioned whether analogy to "average" uniform cantilevers alone can produce sufficiently accurate period determinations for this type of building. It would seem that the use of more rigorous methods including discrete stories is still indicated where accuracy is wanted, until more data are accrued. The known period ratios indicate a rod shear characteristic whereas flexure was found to be equally important with shear. The lumped masses of the floors creating a multi-mass vibrating system as well as wall openings, stairs and other conditions probably account for the variation from uniform rod period ratios.

(12) Periods obtained by wind vibration records, by forced vibration, and by actual (although minor) earthquake records show a good degree of correlation over a wide range of amplitudes.

(13) Detailed work such as has been done on this "guinea pig" building is hoped to ultimately increase our knowledge of what makes buildings respond as they do. The particular periods and other data are not only important in themselves, but they are also valuable indicators, often of many things including structural condition as well as dynamic response, if all data are known about the various component factors. It is the writer's hope and belief that periods, ratios of periods, damping factors, changes in periods and damping factors, and response to various (known) disturbing forces may some day be used to their full potential. Perhaps they might be utilized as indicators and symptoms (for structures) somewhat as medical doctors utilize pulse, blood pressure, metabolism, and temperature of the human body for the same purposes.

(14) In conclusion, may it be stated that the accurate recording of the response to earthquakes as well as the natural vibrational characteristics of structures is vital to research of this and many other types in engineering seismology. Continuous and not occasional effort is indicated.

BLUME on Vibration Periods of a 15-story Building

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ANALYSIS OF STRUCTURAL RESPONSE

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NOMENCLATURE

- i, j = Subscripts referring the main symbol to its appropriate story numbering from the top story.
- m = Mass of individual story, (Lb-Sec²/In).
- n = Subscript indicating the lowest story of the building numbering from the top story.
- p = Angular frequency, (Rad/Sec).
- W = Weight of individual story, (Lbs).
- E_b = Modulus of elasticity of brickwork, (lbs/In²).
- E_s = Modulus of elasticity of steel, (Lbs/In²).
- I = Moment of inertia of an equivalent steel area at an individual story, (In⁴).
- I_{AVG} = Average moment of inertia of equivalent steel areas (In⁴).
- K = Spring factor for shear deformation of individual story, (Lbs/In).
- K_{AVG} = Average spring factor for shear deformation, (Lbs/In).
- M = Bending moment, (In-Lbs).
- N = Frequency of vibration, (Cycles/Min).
- T_x = Observed period of vibration of the xth mode, (Sec).
- T_{S-x} = Computed period of xth mode considering shear deflections only, (Sec).
- T_{F-x} = Computed period of xth mode considering flexural deflections only (Sec).
- T_{SF-x} = Computed period of xth mode considering shear and flexure, (Sec).
- T_{SFG-x} = Computed period of xth mode considering shear, flexure and ground yielding, (Sec).
- γ = Ratio of ε_v to ε_h
- Δ = Displacement of individual story from its equilibrium position (Inches).
- ε_h = Deformation modulus of ground in horizontal direction, (Lbs/In³).
- ε_v = Deformation modulus of ground in vertical direction, (Lbs/In³).

BLUME on Vibration Periods of a 15-story Building

TABLE I

MOST PROBABLE DYNAMIC PROPERTY
VALUES FOR THE ALEXANDER BUILDING

(Weights from thesis, K's and I's from 1956 work)

Story From Bottom	n	Story Height (Ft.)	Weight (Lbs x 10 ⁶) W	For Motion Parallel to Montgomery Street*		For Motion Parallel to Bush Street*	
				K	I	K	I
15	1	20	1.550	4.9	41.0	3.8	22.9
14	2	12	0.749	6.8	41.0	4.3	22.9
13	3	12	0.809	8.6	46.5	7.6	27.5
12	4	12	0.850	8.7	46.8	10.2	27.2
11	5	12	0.869	8.1	50.0	8.8	29.7
10	6	12	0.841	8.1	50.0	8.8	29.7
9	7	12	0.841	9.3	61.1	9.4	38.3
8	8	12	0.847	9.3	61.1	9.4	38.3
7	9	12	0.856	10.6	70.2	10.0	44.9
6	10	12	0.865	10.6	70.2	10.0	44.9
5	11	12	0.863	12.6	75.5	14.3	48.8
4	12	12	0.886	12.6	75.5	14.3	48.8
3	13	12	0.918	17.2	88.6	19.0	58.7
2	14	12	0.931	14.2	92.9	16.2	63.2
1	15	20	1.173	8.2	109.8	6.5	75.8
AVERAGE		13.1	0.923	10.0	65.3	10.2	41.4

* All K values above are in pounds per inch x 10⁶ and all I values are in inch⁴ x 10⁶.

These values correspond to a modulus of elasticity of brickwork, E_b , between 50,000 and 60,000 Lbs/In², and reconcile 4 observed periods of vibration in each direction (and each direction to the other) when used in conjunction with a deformation modulus of the ground in the vertical direction, ϵ_v , of between 525 and 2000 Lbs/In³, and ratio of vertical to horizontal deformation moduli, γ , in the range 1 to 10. Since variations within these ranges of ϵ_v and γ have little effect on the results, $\epsilon_v = 1000$ Lbs/In³ and $\gamma = 1$, may be assumed as specific values for convenience in this case. Rotation constitutes about 90% of the total effect of ground yielding on the periods.

ANALYSIS OF STRUCTURAL RESPONSE

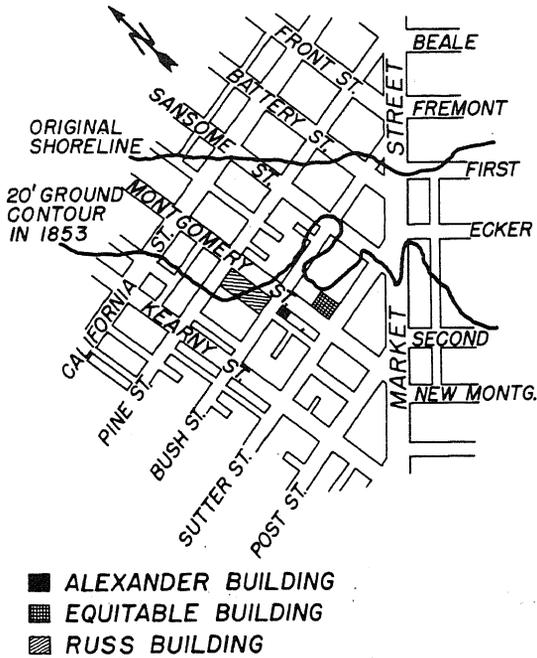


FIG. 1 - MAP SHOWING LOCATION OF THE ALEXANDER BUILDING

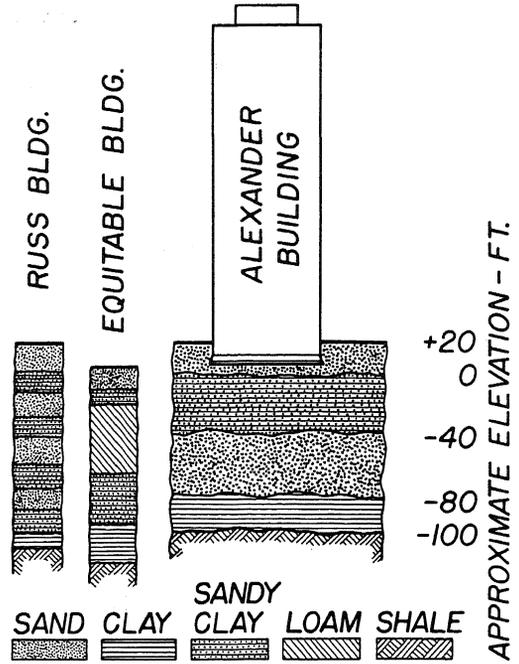


FIG. 2 - APPROX. SOIL CONDITIONS NEAR THE ALEXANDER BLDG.



FIG. 3 - PHOTO OF THE ALEXANDER BLDG.

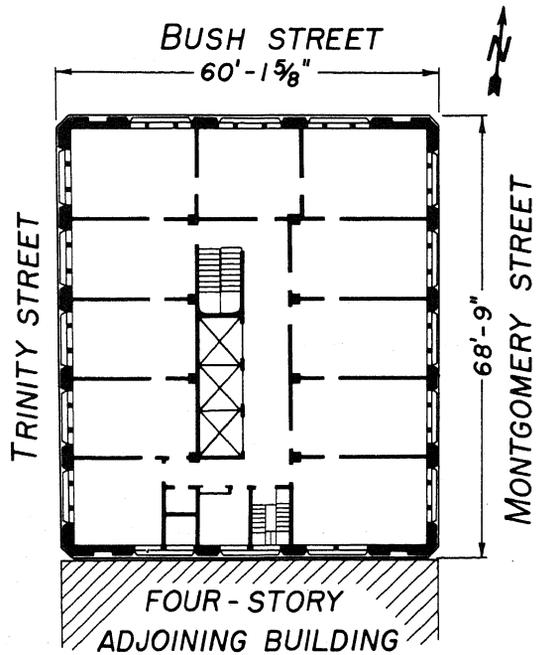


FIG. 4 - TYPICAL FLOOR PLAN OF THE ALEXANDER BLDG.

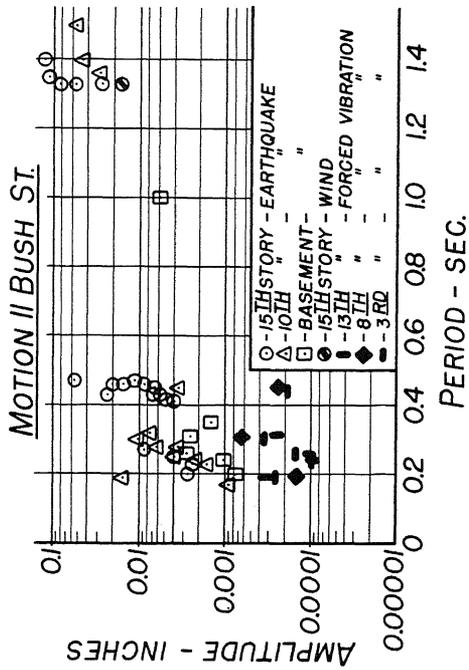


FIG. 6 - OBSERVED VIBRATION OF THE ALEXANDER BUILDING

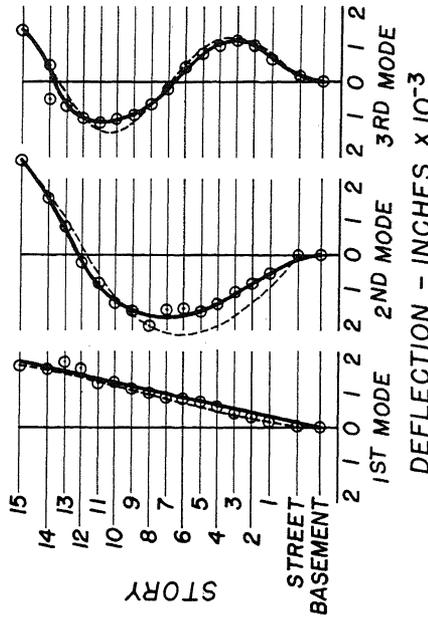


FIG. 8 - DEFLECTION CURVES FOR THE ALEXANDER BLDG; ○-OBSERVED, ----- CALCULATED

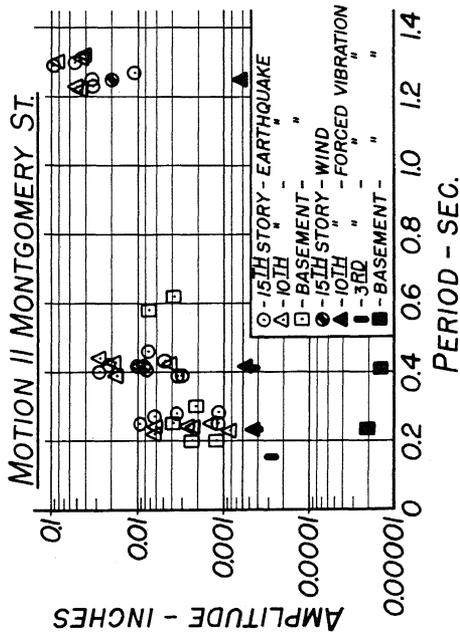


FIG. 5 - OBSERVED VIBRATION OF THE ALEXANDER BUILDING

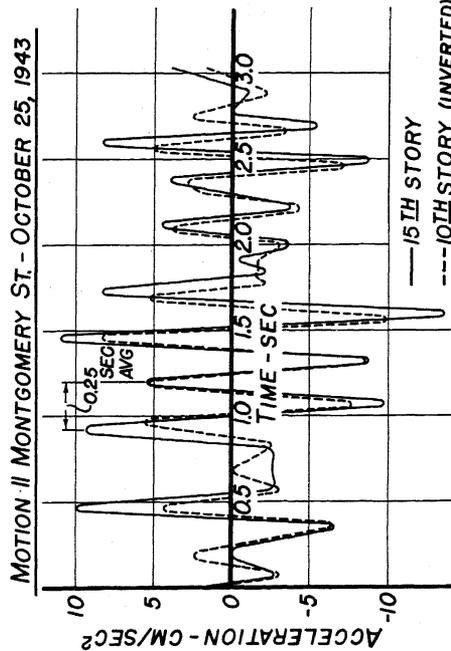


FIG. 7 - SIMULTANEOUS RESPONSE CURVES FOR THE ALEXANDER BLDG. DUE TO EARTHQUAKE MOTION

ANALYSIS OF STRUCTURAL RESPONSE

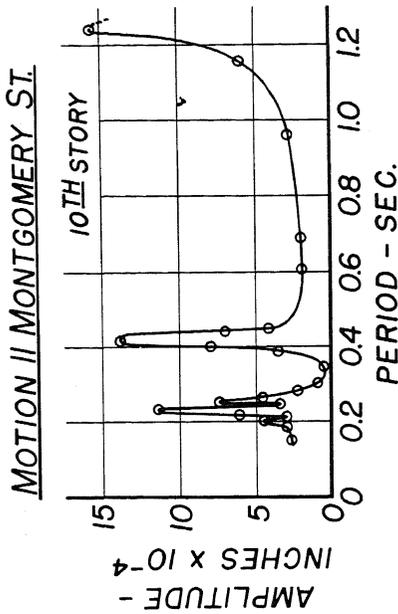


FIG. 9 - RESONANCE CURVE FOR FORCED VIBRATION OF THE ALEXANDER BLDG.

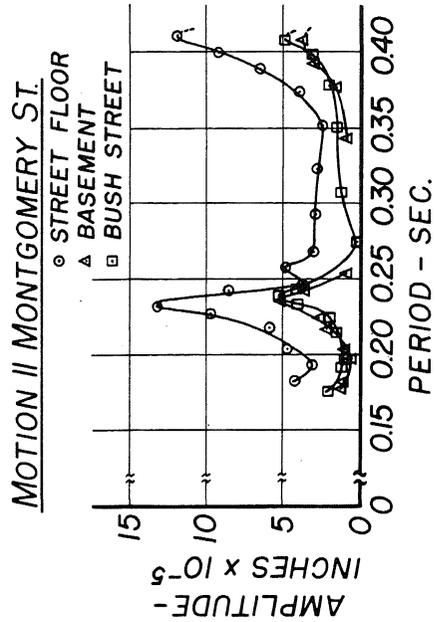


FIG. 10 - RESONANCE CURVES FOR FORCED VIBRATION OF THE ALEXANDER BLDG.

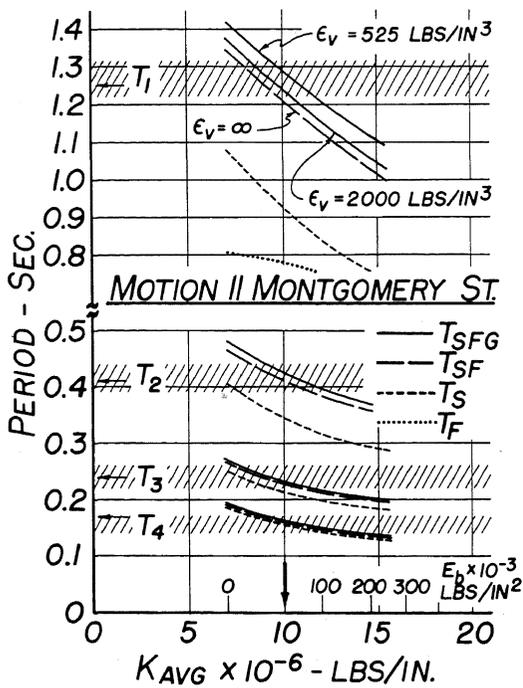


FIG. 11 - CALCULATED PERIODS OF THE ALEXANDER BLDG.

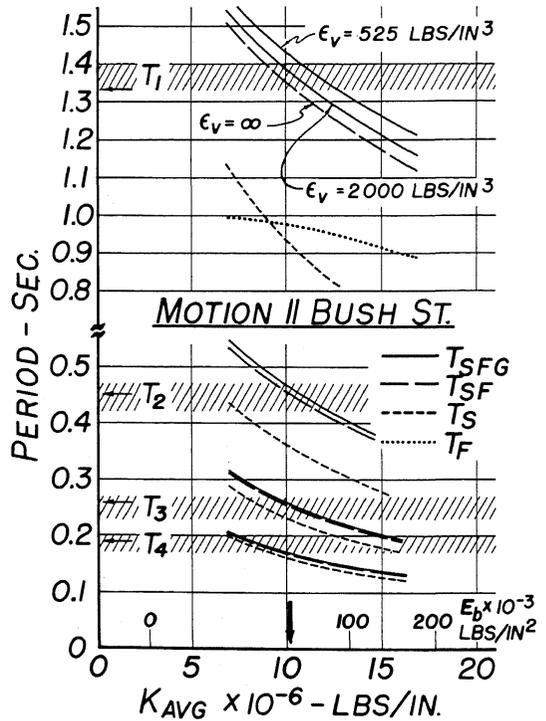
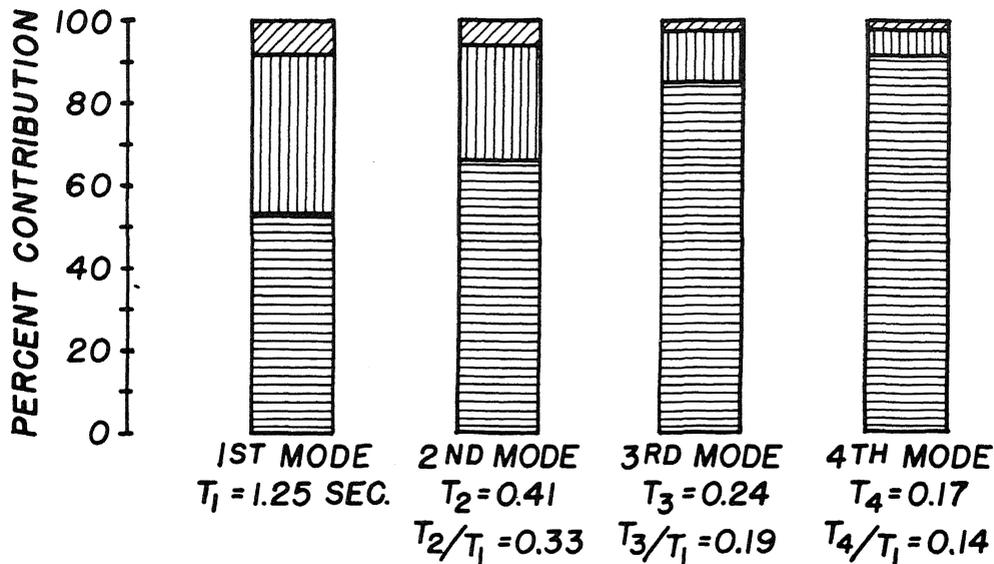


FIG. 12 - CALCULATED PERIODS OF THE ALEXANDER BLDG.

MOTION II MONTGOMERY ST.



MOTION II BUSH ST.

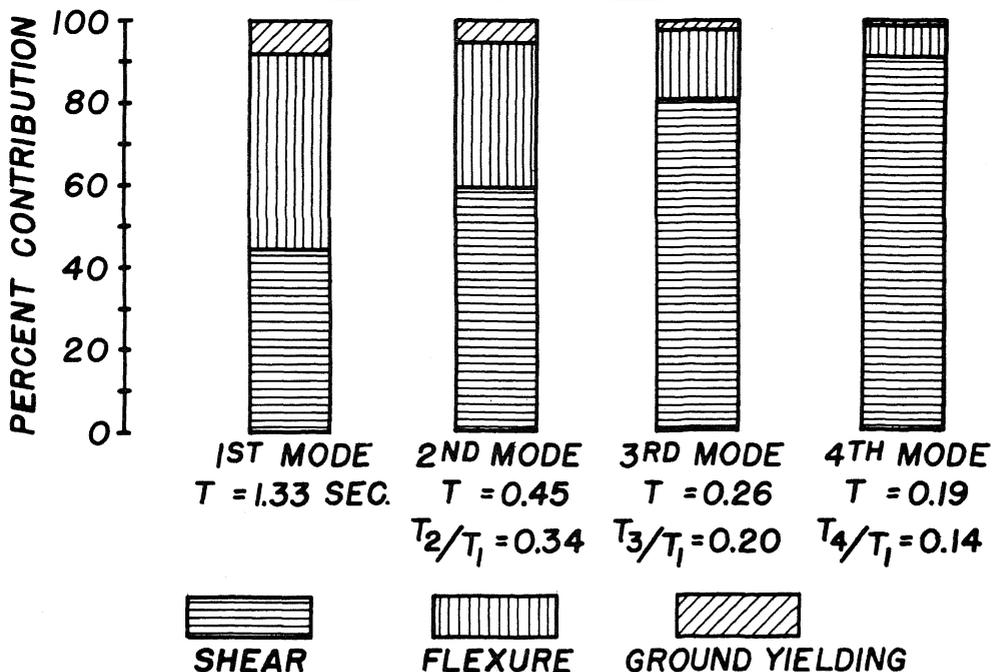


FIG. 13 - APPROXIMATE RELATIVE INFLUENCE OF SHEAR, FLEXURE & GROUND YIELDING ON THE NATURAL PERIODS OF THE ALEXANDER BUILDING.