LIMIT DESIGN OF STRUCTURES TO RESIST EARTHQUAKES

by

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Abstract. Structures having small amounts of damping may develop stresses, when subjected to strong earthquake ground motion, that correspond to relatively large lateral loads, 50%g to 100%g having been observed. If plastic deformations are accepted, a safe design much less strong than required for 50%g can be made. The proposed method of limit-design is based on the energy input into a structure by recorded strong ground motions. When applied to an elevated water tank, limit-design shows that a 12.5%g design that permits stretching of the bracing rods has ample factor of safety during ground motion that would produce 40%g stresses in an elastic structure.

It has been observed that certain structures, as for example elevated water-tanks, that have been designed to resist earthquakes, have suffered damage when subjected to strong ground motion. Sometimes the anchor bolts have stretched or the bracing rods have undergone a permanent elongation. If one computes the motion that would be induced in such structures by recorded earthquake ground motion, it is found that very considerable amplitude of oscillations is experienced. The severity of the vibration is found to depend strongly upon the degree of energy dissipation possessed by the structure. If the energy dissipation is small, the maximum stresses may reach values equivalent to those produced by lateral forces equal to 50% or more of the weight of the structure. In most cases it would be quite costly to design for lateral forces of this magnitude, and it would probably be considered desirable to make a less strong structure and accept permanent deformations in the event of a severe earthquake. If this point of view is adopted, the design of the structure should be made on the basis of a plastic analysis or limit-design, that is, the factor of safety should be based on the ultimate capacity of the structure and not on the elastic limit. A relatively simple limit-design for earthquakes can be made if the properties of the structure are sufficiently well known. In many cases, such as elevated water-tanks, offshore oil-drilling platforms, etc., the structure is sufficiently simple so that all of its pertinent properties are known. In what follows, a method of limit design for such structures is presented.

Character of Earthquake Ground Motion. Since a structure is designed to resist an earthquake that will occur in the future, it is not possible to make an exact analysis, for the precise nature of the ground motion will not be known. It is logical to base the analysis and design on observation of past earthquakes and in particular to take the recorded ground motions of past earthquakes as samples of future earthquake ground motion. This, of course, introduces a certain degree of imprecision, since the ground motions of past earthquakes are not identical but differ both in intensity and in character. It is reasonable to take an intensity equal to that of the most intense of past earthquake ground motions as a basis for design, and to assume that the character of the ground motion will be the same as

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the average character of past strong earthquakes. To clarify what is meant by the character of earthquake ground motion, consider the simple linear oscillator shown in Fig. 1, and let its base be subjected to a horizontal acceleration identical with that recorded during an earthquake. For small values of damping, the shear force in the elastic support is given by

$$F = m\,\frac{2\pi}{T}\int_0^t a\,e^{-n\frac{2\pi}{T}(t-\tau)}\sin\frac{2\pi}{T}(t-\tau)\,d\tau \qquad (1)$$

m = mass of the oscillator

T = natural period of vibration

a = ground acceleration

n = fraction of critical damping

t = time at which F is evaluated

The character of the ground acceleration is exhibited by the integral appearing in Equation (1), that is, by

$$\int_0^t a\,e^{-n\frac{2\pi}{T}(t-\tau)}\sin\frac{2\pi}{T}(t-\tau)\,d\tau \qquad (2)$$

If this integral is evaluated for a particular ground acceleration (a), and a particular damping (n) and period (T), there is obtained an oscillatory response as shown in Figure (2). It is of special interest to observe the maximum value that occurs, for this determines the maximum shear force that is experienced. Let the maximum value of the integral be designated by S_v. If S_v is calculated for a range of periods (T) and for different values of damping (n), there will be obtained a good picture of the effect of the ground motion on oscillators in general. In Figs. 3, 4, 5 are shown plots of S_v as computed from recorded strong earthquake ground motion. If the values of S_v taken from these plots are multiplied by $2\pi/gT$ there is obtained the fraction of "g" acceleration that corresponds to the maximum F. The dimensions of S_v are feet per second, and for small damping S_v is the maximum velocity relative to the base attained by the mass m. The curves shown in Figs. 3, 4, 5 are called the velocity response spectra*. It will be noted that $\frac{1}{2}\,mS_v^2$ is the maximum kinetic energy attained by the mass (m) during the time of the ground motion.

The velocity spectra may be taken as measures of the intensity of earthquake ground motion for large values of S_v correspond to large stresses produced in a structure. Figs. 3, 4, 5 show that the spectra of strong earthquakes bear a general resemblance to each other but differ

*See the paper by D. E. Hudson, "Response Spectrum Techniques in Engineering Seismology," which appears elsewhere in this publication.

somewhat in the precise shapes of the curves. On the basis of the spectra of the strong ground motions recorded at El Centro, 1934; El Centro, 1940: Olympia, 1949; Taft, 1952, it is found that the average shapes of the spectrum curves can be represented as shown in Fig. 6. On the basis of present information, these average spectra may be taken to represent the average properties of future strong ground motions in the sense that they represent the best smooth-curve fit to the computed curves for past ground motions. To give an idea of the magnitude of S_v that can be expected, the ordinate S_o shown in Fig. 6 is listed for a number of past earthquake ground motions.

		S_o
El Centro, 1934,	EW	2.2
	NS	2.2
El Centro, 1940,	EW	2.8
	NS	3.0
Olympia, 1949,	EW	2.1
	NS	1.8
Taft, 1952,	EW	1.8
	NS	1.7

Maximum Energy in Structures. If the average spectra curves instead of having the shapes shown in Fig. 6 were straight, horizontal lines so that each $S_{v,n}$ were a constant for $0<T<3$, then the maximum energy, on the average, attained by an oscillator (Fig. 1) would be

$$E = \frac{1}{2} mS^2_{v,n}$$

and this would be independent of the period of vibration. It can also be shown that if an elastic structure has multiple modes of vibration, each of which has the same damping (n) and $S_{v,n}$ is a constant, the maximum energy, on the average, attained by the structure would be

$$E = \frac{1}{2} M S^2_{v,n} \tag{3}$$

M = total mass of the structure

This value of E is independent of the number and shapes of the modes of vibration, that is, E is independent of the size, shape and stiffness of the structure and depends only on the total mass and the damping. In this case, the maximum energy on the average would be the same for a ten-story building as for a two-story building if the total mass and the damping (n) were the same.

The foregoing is a simplifying viewpoint for considering the effects of ground motion. Although the actual, average spectrum curves (Fig. 6) are not horizontal lines, they are close to being so for small damping and for periods greater than approximately 0.4 seconds. Also, although not much is known about the damping in actual structures, there is indication that the damping (n) per mode may not differ much among the modes of a structure. So that one could say that structures whose first few modes have periods greater than 0.4 seconds will experience maximum energy on the average approximately equal to $\frac{1}{2} M S^2_{v,n}$. In particular, this will be the case for structures of the type of elevated water-tanks, offshore oil-drilling platforms, etc., where it is known that the damping is small ($n \approx 0.03$) and the periods relatively large. A damping $n = 0.03$ is about as small as is encountered* in structures so that $\frac{1}{2} M S^2_{v.03}$ may be taken as the maximum energy to be attained by a structure and if the structure has a damping $n > 0.03$ the maximum energy attained will be less.

Absorption of Energy. The effect of the ground motion is to feed energy into the structure. Some of the energy is dissipated through the damping and the remainder is stored in the structure in the form of kinetic energy of motion of the mass and in the form of strain energy of deformation of the structural members. If the energy input is sufficiently large, the structure cannot absorb it in elastic strain energy, for the stresses will exceed the elastic limit with consequent permanent deformations, or some part of the structure will fail. In either event, the effect is to dissipate energy. If the structure is designed so that permanent deformations can occur without failure of a member, then at any instant the sum of the kinetic energy plus strain energy plus energy dissipated through normal damping plus energy dissipated through permanent deformation will be equal to the total energy input. It thus follows that if a structure can absorb a large amount of energy through so-called plastic deformation, it will be able to withstand very intense ground motion without failing.

To determine exactly how much energy will be absorbed by plastic deformation it would be necessary to know the exact elastic and inelastic properties of the structure and the exact ground motion to which it will be subjected, and it would be necessary to make a detailed calculation of the behavior of the structure. In view of the uncertainty of the precise character of future ground motions and the large amount of labor required to make such calculations, it does not appear practical to attempt such an analysis when making a design. It is thought that the following simplified approach will have an accuracy sufficient for most design purposes.

If the structure behaves completely elastically, the maximum energy it will contain is

$$E_t = \frac{1}{2} M S^2_{vn}$$

If, however, the structure reaches the yield point when it contains an elastic energy $E_e < E_t$, it is proposed that the design be made on the basis that a plastic energy E_p must be absorbed where

$$E_p = E_t - E_e \tag{4}$$

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*See D. E. Hudson and G. W. Housner, Structural Vibrations Produced by Ground Motion, Proceedings A.S.C.E., Vol. 81, Paper 816, October, 1955.

Equation (4) contains two assumptions which will now be discussed. First, it assumes that the elastic energy E_e is entirely in the form of strain energy, that is, when E_e is such that the elastic limit is reached, all of the energy is in the form of strain energy. When the structure has only one degree of freedom, this is indeed the case, but when it has more than one mode of vibration, it is possible that some of E_e is in the form of kinetic energy. In this case Equation (4) will somewhat overestimate E_p. An exact determination of the amount of overestimation can be made only by a complete analysis, but on a basis of study of past earthquake ground motion it appears that the overestimation of the maximum possible E_p is of the order of 10%.

The second assumption implied by Equation (4) is that the energy input is the same when parts of the structure are stressed beyond the elastic limit as it would be if the structure behaved elastically and this again overestimates E_p. If the inelastic deformations do not have a major effect on the stiffness characteristics of the structure, it can be anticipated that, on the average, the overestimation of E_p is very small, but if the yielding has an effect of changing the force deflection diagram as shown in Fig. 7, the structure is effectively disconnected from the ground for small displacements and it can be expected that the energy input will be less than $\frac{1}{2} M S_{vn}^2$.

In view of the uncertainties introduced by assuming that future earthquakes will be essentially the same as past earthquakes and basing the analysis on their average properties, it is thought that the overestimations involved in using Equation (4) are not excessive for ordinary design purposes. For very special structures, it may be that a more complete analysis is justified.

Method of Limit-Design. In accordance with the foregoing discussion, the proposed method of limit-design is to proportion the structure so that it can absorb plastically an energy equal to

$$E_p = c\left(\frac{1}{2} M\, S_{v.03}^2 - E_e\right) \tag{5}$$

which for ground motion of the intensity of the El Centro, 1940 recorded motion is

$$\begin{aligned} E_p &= c\left(\frac{1}{2}\, M\, \bar{2}^2 - E_e\right) \\ &= c\left(\frac{w}{16} - E_e\right) \end{aligned} \tag{6}$$

where w is the total weight of the structure and c is the safety factor.

To illustrate the application of Equations (5) and (6), some special structures are considered below.

Catalytic Cracking Columns. During the 1952 Arvin-Tehachapi earthquake, the Paloma Oil Refinery was in the region of severe ground motion approximately 10 miles from the epicenter. A number of catalytic cracking columns at the refinery overstressed their anchor bolts*. The columns were essentially cylindrical, steel cantilevers anchored to heavy concrete foundations. After the earthquake the anchor bolts were found to have suffered permanent elongations. Details of the columns, anchor bolts and observed elongations are given in Table I. The last column in Table I is the equivalent static %g loading required to produce a yield point stress of 33,000 psi in the anchor bolts. It should be noted that this %g loading exceeds 100%g in one case and in all cases is much larger than 10%g.

TABLE I

Fractionator Column Properties

No.	Diam.	Height	Wall	Kips W't.	Anchor Bolts	Period	Measured Bolt Stretch	%g for 33 K.S.I. in Bolts
1	60"	60'	4.62"	211	16-1 1/4	0.32	1 1/2-1 3/4	28
2	66"	59'	0.44"	62	12-1 1/4	0.32	1/8-3/16	61
3	54"	68'	0.25"	36	16-1 1/4	0.42	1/16-1/8	92
4	78"	104'	1.00"	160	16-1 3/4	0.72	0-1/16	33
5	66"	97'	1.06"	123	16-1 3/4	0.70	0	37
6	36"	48'	0.38"	18	8-1 1/2	0.35	0-1/16	109
7	48"	97'	0.44"	57	16-1 3/4	1.00	0-1/16	57
8	78"	84'	2.25"	225	16-1 3/4	0.45	5/8-3/4	35
9	108"	80'	1.00"	223	20-1 1/4	0.29	1/2-5/8	37
10	60"	105'	0.88"	120	12-2	0.88	1/8-3/16	36
11	42"	54'	1.06"	39	8-1 1/2	0.36	3/8-1/2	52
12	42"	94'.	0.44"	46	12-2	1.05	0-1/8	63

If one takes as an estimate for the maximum energy attained by a column during the earthquake

$$E = \frac{1}{2} M S^2_{v.03}$$

and notes that when the column rocks on its base its c.g. is raised a distance $\delta_e/2$, the total energy available is

$$\frac{1}{2} M S^2_{v.03} - \frac{1}{2} W\delta_e$$

*See E. Bergman and N. Owen, Earthquake Damage Analyzed, Petroleum Refiner, March 1954.

The total plastic energy absorbed in stretching the bolts at 33,000 psi is 33,000 $A_b \delta_e$ where δ_e is the permanent elongation of the bolts. Table II shows the results of calculating δ_e on the basis that

$$33{,}000\ A_b\ \delta_e = \frac{1}{2}\ M\ S^2_{v.03} - \frac{1}{2}\ W\ \delta_e$$

$$\delta_e = \frac{\frac{1}{2}\ M\ S^2_{v.03}}{33000 A_b + \frac{1}{2} W} \tag{7}$$

In the calculations $S_{v.03}$ was taken to be 2.5 which corresponds approximately to the El Centro 1940 ground motion. These calculations are only approximate but the agreement between observed and computed bolt stretch as shown in Table II is not unreasonable. It will, however, be noted that the four columns that had observed bolt stretch of 1/2" or more have a computed bolt stretch appreciably smaller. Since ground motion very much more intense than that at El Centro 1940 would be required to produce sufficient energy to account for these four observed values and since there was no other evidence of such intense ground motions, it is thought that the observed values were not produced by a uniform stretching of the bolts but that there was some movement of the bolts in their holes.

The general agreement between observed and computed values of bolt stretch would indicate that the ground motion at the oil refinery was approximately the same as at El Centro 1940. It also gives some corroboration for the proposed method of limit-design.

TABLE II

Stretch of Anchor Bolts

No.	Observed	Computed
1	1 1/2 - 1 3/4	0.33
2	1/8 - 3/16	0.15
3	1/16 - 1/8	0.07
4	0 - 1/16	0.15
5	0	0.11
6	0 - 1/16	0.05
7	0 - 1/16	0.06
8	5/8 - 3/4	0.21
9	1/2 - 5/8	0.30
10	1/8 - 3/16	0.11
11	3/8 - 1/2	0.10
12	0 - 1/8	0.05

Elevated Water Tank. Consider an elevated water tank as shown in Fig. 8 having a weight of water plus tank equal to 250,000 lbs. If the tank is subjected to ground motion of the intensity of El Centro, 1940, the corresponding value of $S_{v.03}$ is 2 ft/sec. If the tank behaves elastically, the maximum expected energy of vibration (one component of motion) is

$$E = \frac{1}{2} \frac{250{,}000}{g} (2)^2 = 15{,}000 \text{ ft. lbs.} \tag{8}$$

and when this energy is entirely in the form of strain energy, the stresses correspond to a lateral load of

$$\frac{2\pi}{T} S_{v.03} = \frac{2\pi}{(32.2)(1)} (2)g = 0.40g \tag{9}$$

This is four times the usual $\frac{1}{10}$g load.

To simplify the computations it is assumed that the stress-strain diagram for the diagonal bracing rods is as shown in Fig. 9 with a yield stress of 33,000 psi and an ultimate strain $\varepsilon = 0.02$. When a rod reaches this strain, it will have absorbed plastically an energy equal to

$$\begin{aligned} E_p &= 33{,}000\ (0.02 - .0011) \\ &= 620 \text{ inch lbs per cubic inch} \\ &= 50 \text{ ft lbs per cubic inch} \end{aligned} \tag{10}$$

Suppose the tank had been designed for $\frac{1}{10}$g at 27,000 psi, then the maximum elastic strain energy in the structure would occur when the equivalent lateral load corresponded to

$$0.1 \times \frac{33{,}000}{27{,}000} g = 0.12g$$

or (11)

$$E_e = \left(\frac{0.12g}{0.4g}\right)^2 15{,}000 = 1{,}400 \text{ ft. lbs.}$$

This leaves 13,600 ft. lbs to be absorbed by plastic deformation. Suppose each of the diagonal rods has an area of 0.78 sq. in. and a length of 16 ft. If all 16 of the rods (two sides) are stretched equally, each rod must absorb

$$\frac{13{,}600}{16} = 850 \text{ ft. lbs.}$$

The volume of the rod is

$$16 \times 12 \times 0.78 = 150 \text{ cubic inches}$$

so that each cubic inch must absorb

$$\frac{850}{150} = 5.7 \text{ ft. lbs.}$$

This corresponds to a strain of

$$\frac{5.7}{50}(0.02) = 0.0023$$

which means a total permanent stretch of the rod of

$$0.0023 \times 16 \times 12 = 0.44 \text{ inches}$$

Since it cannot be insured that all rods will stretch equally, the design should be made on the basis of more severe conditions. For example, suppose only the four rods in the lowest panels to be stressed beyond the elastic limit and let them be such that they can absorb $2\frac{1}{2}$ times as much energy as the tank attains, that is

$$2.5\ (15,000) = 37,500 \text{ ft. lbs.}$$

Each rod must then be able to absorb

$$\frac{37,500}{4} = 9,400 \text{ ft. lbs.}$$

The rod will have a volume

$$= 16 \times 12 \times A = 192\ A \text{ cubic inches.}$$

So

$$192\ A\ 50 = 9,400$$

$$A = \frac{9,400}{192 \times 50} = 0.98 \text{ sq. in.}$$

If then the rods have this area, the tank will be able to undergo two earthquakes of intensity of El Centro, 1940, without failing even if all of the plastic deformation takes place in the lowest panel rods. It will be noted that this design corresponds to a

$$\frac{0.98}{0.78}(0.10)g = 0.125g \text{ design.}$$

It may be noted that for $S_v = 2$ the maximum energy attained by the tank corresponds to $\frac{1}{16}$ ft lbs for each pound weight of the tank. With a factor of safety of 2.5 this becomes approximately $\frac{1}{6}$ ft lb per lb of weight. In many cases it will be quite satisfactory to design so that the structure can absorb $\frac{1}{6}$ W ft lbs.

Discussion. It is seen from the foregoing that an elastic structure with small damping can be excited by earthquake ground motion into oscillations so violent as to produce stresses corresponding to lateral loads of 100%g or more. Even in the case of minor ground motion, building accelerations of 10%g have been measured*. It appears that ordinary structures would be stressed beyond 10%g if they had as much as 10% of critical damping and were subjected to strong ground motion. At first impression, it might seem that if this is the case, a 10%g building would fare rather badly during a strong earthquake, but there are a number of reasons why such buildings may survive an earthquake without visible distress. In some instances a 10%g building is actually much stronger than is implied by the nominal 10%g used in the design because of relatively strong, solid walls. In the case of most buildings, there may be appreciable energy dissipation when the building vibrates with relatively large amplitudes. Internal cracking, rubbing of surfaces, and even plaster cracking may absorb appreciable amounts of energy and thus reduce the maximum stresses. From the example of the elevated water tank, it is seen that stretching the 1" rods $\frac{1}{2}$" uses up essentially all of the energy of a structure weighing 250,000 lbs. If a building is designed so that very little energy is dissipated until some structural member reaches the failing point and that failing point corresponds to say 15%g, then it can be anticipated that failure will occur during strong ground motion. This can explain those observed cases where a clean, well-designed building has sustained local damage indicative of a relatively high %g loading.

The importance of damping in the ability of a building to ride through an earthquake can also explain those instances where a poorly designed, loose-jointed building that could not withstand a static horizontal load of 10%g did, nevertheless, survive the earthquake. Such structures apparently can dissipate a sufficient amount of energy so that the stresses never reach high values.

After an earthquake one often sees poorly designed structures that are badly cracked and deformed so that it appears surprising that they should have survived. However, it was precisely the cracking and deformation which, by dissipating energy, permitted the structure to survive. Another not uncommon example is the steel frame building with brick filler walls which are cracked by the earthquake, as was the case of the tower of the Los Angeles City Hall. It is just the energy dissipated by the cracking and grinding of the bricks that gives the structure its ability to resist the earthquake. Since there is a limit to the amount of energy a structure can absorb, the first earthquake to which it is subjected may use up most of its ability to dissipate energy without appreciable external evidence of distress, so that a subsequent earthquake may produce an apparently disproportionate amount of damage.

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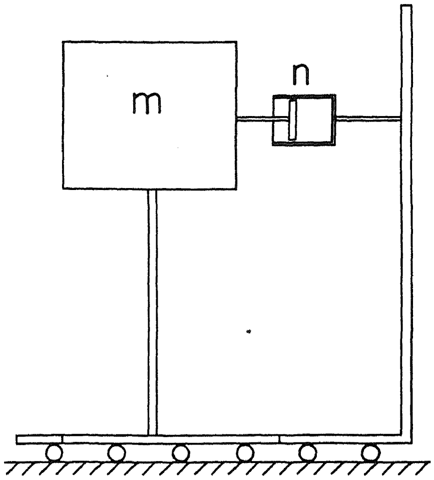
*D. Hudson, J. Alford, and G. Housner, Measured Response of a Structure to an Explosive Generated Ground Shock. Bull. Seism. Soc. Amer., Vol 44, No. 3, July 1954

Since the ability of a structure to resist earthquakes depends so largely upon its ability to absorb energy, it is of great importance to insure that it can do so. This can be done by designing so as to avoid the possibility of brittle failure, and conversely, designing so that relatively large inelastic deformations can occur before failure. In the case of elevated water tanks, for example, this means designing so that the diagonal bracing rods yield first, rather than the anchor bolts or columns. The end connections and the turnbuckles on the rods should be sufficiently strong so that the rod can elongate plastically without overstressing them. The rods should also be designed so that they can absorb several times the amount of energy put into the structure by one strong earthquake.



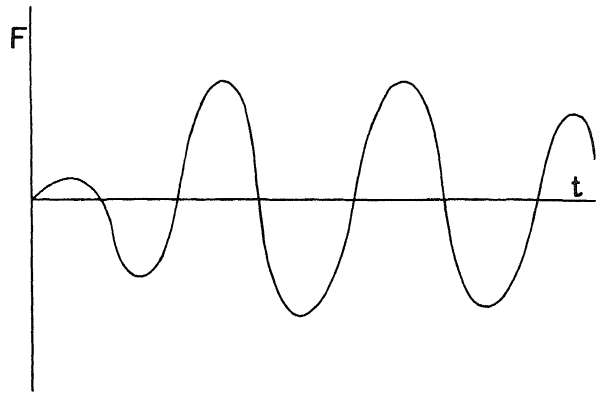


Fig. 1 Simple Oscillator

Fig. 2 Response of Oscillator

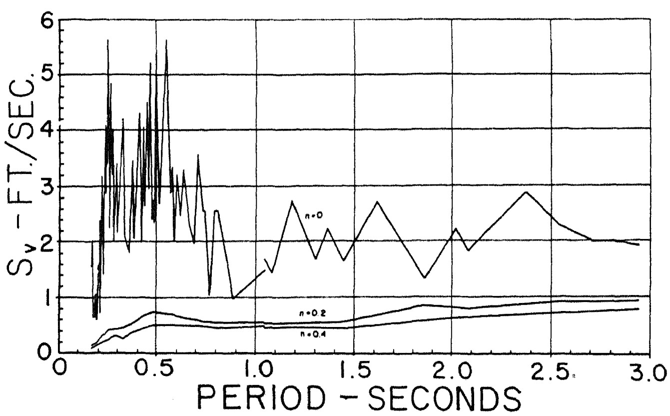


Fig. 3 Velocity Spectrum For Earthquake at El Centro, December 30, 1934, Component E-W

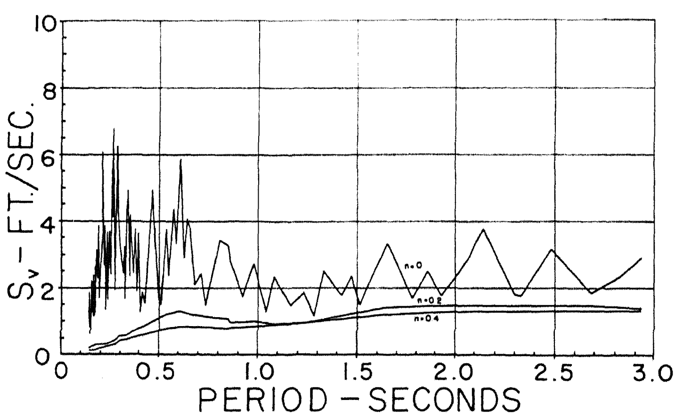


Fig. 4 Velocity Spectrum For Earthquake at Olympia, Washington, April 13, 1949 Component S80W

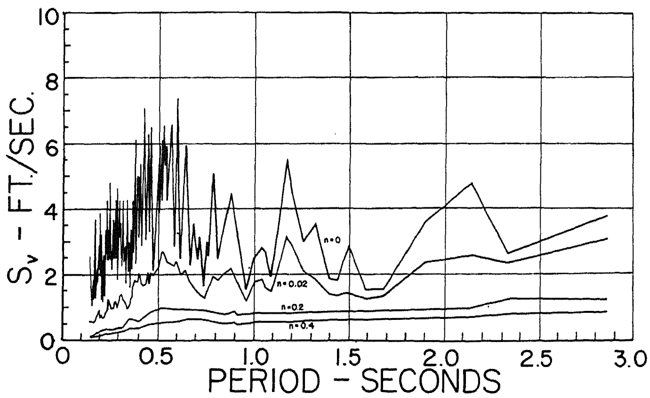


Fig. 5 Velocity Spectrum For Earthquake at El Centro, May 18, 1940 Component E-W

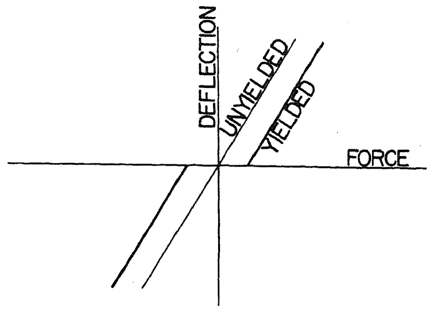


Fig. 7 Force-Deflection Diagram

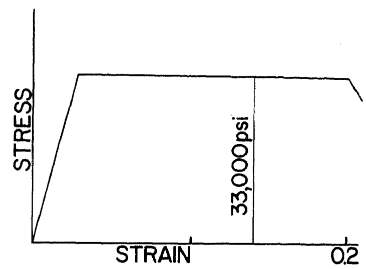


Fig. 9 Stress-Strain Diagram

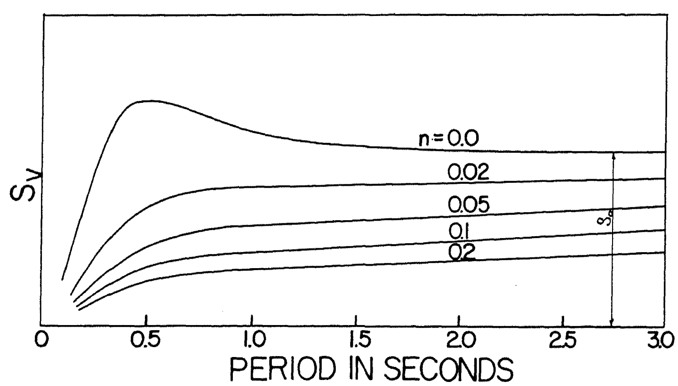


Fig. 6 Average Velocity Spectrum

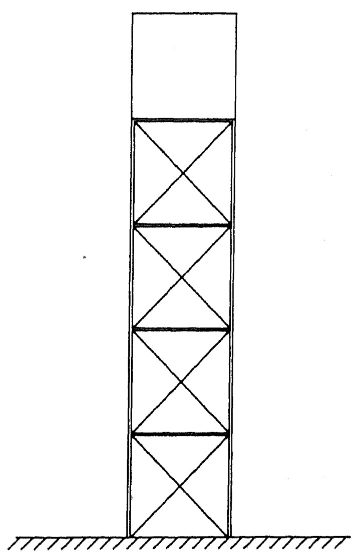


Fig. 8 Elevated Water Tank