REFINEMENTS TO THE NEWMARK SLIDING BLOCK MODEL

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SUMMARY

Newmark's original sliding block model for the seismic behaviour of geotechnical structures is investigated to see whether the inclusion of vertical and lateral excitation has significant effects on the longitudinal block displacement. The original sliding block model is modified by including a constant biassing force. Vertical effects are not discussed in detail. Lateral excitation is shown to produce a far more complex response than that due to longitudinal excitation alone. It is shown that the addition of lateral and vertical excitations can lead to large increases in block displacement for some combinations of the governing parameters.

INTRODUCTION

In the fifth Rankine Lecture, Nathan Newmark proposed a simple model for the behaviour of dams and embankments in earthquakes (Newmark 1965). The model assumed that a block rested on a horizontal plane. If the plane was subjected to an earthquake motion in one dimension, the block would slide if the plane acceleration was high enough to overcome friction. Newmark considered two cases: the block being free to slide in either direction, and the block being constrained to slide only in one direction. The latter case is the one appropriate to most applications.

The sliding-block model was used as the basis of a design method for gravity retaining structures by which a tradeoff could be made between static factor of safety and the ability of the retaining structure to move outwards in an earthquake (Richards and Elms 1979, Elms and Richards 1979). The effect of an earthquake would be to cause the structure to move by a finite and calculable amount. Whitman (1990) gives an overview of the basic approach and variations upon it.

One limitation of the original sliding block model is that it considers only the longitudinal component of earthquake excitation. In reality, earthquake motion occurs in three dimensions. The question addressed here is whether the inclusion of lateral and vertical components of earthquake motion would have any significant effect on the displacement predicted by the model. Though for practical reasons of physical constraint the lateral movement of most gravity retaining structures is not possible, nevertheless it could happen in some cases. Moreover, flexibility in the lateral direction could also allow relative movement between a wall, its backfill, and the surrounding soil.

The investigation has been restricted to the behaviour of a simple model. It assumes a block free to slide on a horizontal plane, but with friction constraining the movement. However, the model differs from Newmark's in two important respects. Firstly, the model is three-dimensional. Three components of excitation are considered, and the block can slide in any direction on the plane. The direction of the frictional force between plane and block is determined by their relative velocity. Secondly, it is assumed that a constant biassing force is applied to the block which gives it a tendency to move in the longitudinal direction. This corresponds to the outwards force acting on retaining structures due to the backfill, or the internal forces in an embankment ensuring that failure, if it occurs, will be downhill and outwards. Other than the biassing force, there is no restriction on the movement

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of the block. A significant consequence of this feature of the model is that it allows the possibility of passive failure.

The model was formulated in dimensionless terms as a simulation model so that its behaviour with regard to time could be tracked for various combinations of the input parameters. The paper concentrates on the effects of including lateral excitation only. However, as will be seen, both lateral and vertical motions can produce significant increases in longitudinal displacement.

BASIC MODEL

The basic model assumes that a block of mass m is free to slide on a horizontal plane or table as shown in Figure 1. The coefficient of friction between block and table is μ. No distinction is made between static and kinematic friction. A constant biassing force Pmg is applied in the negative x-direction to represent simplistically the effect of the backfill on a retaining wall, or the effect of gravity on an embankment, trying to push it out.

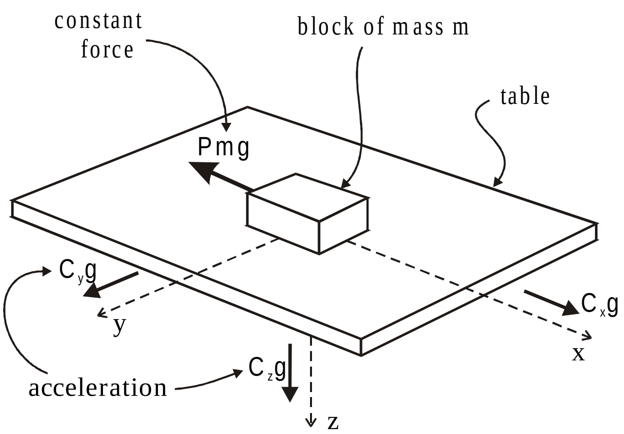

Figure 1 Three-dimensional sliding block model

The table has accelerations in the x, y and z directions of $C_x g$, $C_y g$ and $C_z g$ respectively, where C_x, C_y and C_z are acceleration coefficients. If the block is sliding relative to the table, its accelerations will in general be different except in the z-direction. Let the absolute block acceleration coefficients be k_x, k_y and k_z.

The axes are a right-handed set with the normal soil mechanics convention that z is downwards.

Let $$C_x = A\sin\pi T \tag{1}$$

where $T = 2t/T^*$, A = maximum acceleration coefficient in the x-direction, t = time, and T^* = period.

Let N = maximum value of k_x, if $C_y, C_z, k_y, k_z = 0$

In this situation, applying Newton's second law of motion in the x-direction (Figure 2),

$$\mu mg - Pmg = mNg$$

so that

$$N = \mu - P \tag{2}$$

This assumes that the table, accelerating in the x-direction, has started to move away from the block so that the relative velocity of the block with respect to the table is in the negative x-direction, resulting in a frictional force on the block in the x-direction, as shown in Figure 2. If, on the other hand, the relative velocity of the block with regard to the table happens to be in the positive x-direction due to the dynamic history of the situation, then the sign of the friction force is reversed and Eq. (2) would become

$$N = -\mu - P \tag{2A}$$

Though this would not be a normal situation (N is defined as a maximum), nevertheless the possibility of the reversal of the sign is raised here as it enters the discussion of the results below.

The ratio N/A was an important parameter in Newmark's original formulation.

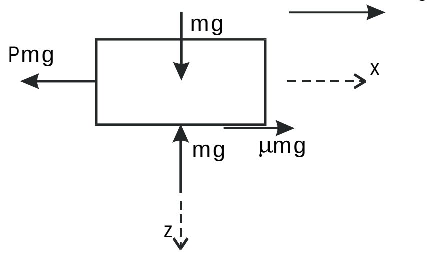

Figure 2 Free-body diagram - motion in x-direction

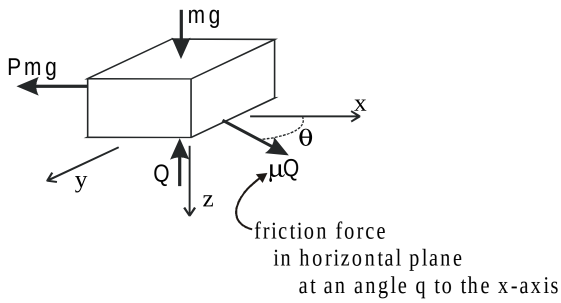

Figure 3 Free-body diagram : 3-D motion

Figure 3 shows a free body diagram of the block when it is sliding. Applying Newton's second law of motion vertically,

$$mg - Q = mk_z g$$

so

$$Q = (1 - k_z)mg \tag{3}$$

Horizontally, in the x- and y-directions,

$$\mu Q \cos\theta - Pmg = mk_x g \tag{4a}$$

$$\mu Q \sin\theta = mk_y g \tag{4b}$$

From Eqs. (3), (4a) and (4b),

$$\mu(1 - k_z)\cos\theta - P = k_x \tag{5a}$$

$$\mu(1 - k_z)\sin\theta = k_y \tag{5b}$$

Eliminating θ gives

$$\mu^2(1 - k_z)^2 = (P + k_x)^2 + k_y^2 \tag{6}$$

Equation (6) is perfectly general. However, there are two diffeent situations to consider: (a) when sliding just begins, and (b) whensliding continues. At the instant of initial sliding, the block and the table accelerations will be exactly the same. In that situation Eq. (6) becomes

$$\mu^2(1 - C_z)^2 = (P + C_x)^2 + C_y \tag{7}$$

In practice, there is a significant degree of correlation between earthquake motions in the N-S and E-W directions, though there is little with the vertical component. Assuming for simplicity that the table accelerations in the x- and y-directions are fully correlated, then we can write

$$C_y = RC_x \tag{8}$$

where R is a constant. Then

$$\mu^2(1 - C_z)^2 = (P + C_x)^2 + R^2 C_x^2 \tag{9}$$

with the solution

$$C_x = \frac{-P \pm \sqrt{\mu^2(1-C_z)^2(1+R^2) - R^2P^2}}{(1+R^2)} \tag{10}$$

The positive root represents the limit when the table and block are accelerating in the positive x-direction, that is, away from the direction of the biassing force Pmg. The negative root represents the limiting acceleration cofficient in the opposite direction. Computationally it is necessary to make a careful distinction between the two roots. Using an asterisk to indicate the special just-sliding nature of the values of C_x given by Eq. (10), we can write

$$C_{xA}^* = \frac{-P + \sqrt{\mu^2(1-C_z)^2(1+R^2) - R^2P^2}}{(1+R^2)} \tag{11}$$

$$C_{xB}^* = \frac{-P - \sqrt{\mu^2(1-C_z)^2(1+R^2) - R^2P^2}}{(1+R^2)} \tag{12}$$

Turning now to ongoing sliding, the direction of the frictional force on the block will be in a direction opposite to that of the relative velocity of the block with respect to the table.

Let the relative velocities of the block with respect to the table be v_{rx} and v_{ry} in the x- and y-directions, with the magnitude of the total relative velocity in the horizontal plane being

$$v_r = \sqrt{v_{rx}^2 + v_{ry}^2} \tag{13}$$

Thus the block acceleration coefficients in the x- and y-directions will be

$$k_x = -\left(\frac{v_{rx}}{v_r}\right)\mu(1-k_z) - P \tag{14}$$

$$k_y = -\left(\frac{v_{ry}}{v_r}\right)\mu(1-k_z) \tag{15}$$

The relative velocities are obtained by integrating (numerically) the acceleration differences, or rather, as the analysis is in dimensionless terms, the acceleration coefficient differences ($k_x - C_x$) and ($k_y - C_y$) in the x- and y-directions respectively.

The block behaves in the folowing manner. As the table acceleration is increased from zero, friction prevents the block from moving relative to the table until condition (11) is met. At this point, the block moves, a relative velocity develops and motion is governed by Eqs. (14) and (15). Relative motion will continue until the relative velocity v_r becomes zero (in practice, becomes less than a small number), when the block and table stick together again. They will continue to move together until one of the two conditions of Eqs. (11) and (12) are met, when relative motion resumes.

Clearly, if there is no lateral or vertical acceleration, the relative velocity will always return to zero at some point in the cycle, provided Eq. (12) does not come into play, which requires that ($P + \mu$) must be equal to or greater than A, or, from Eq. (2),

$$2\mu - N \geq A \tag{16}$$

However, if there is lateral excitation, there will be a component of velocity in the y-direction which will not in general be in phase with that in the x-direction. In such circumstances, v_r may never become zero and motion could for some combination of parameters continue indefinitely. Examples of this can be seen in the results below.

In the numerical computations, A is taken to be unity. The governing parameters are then N/A (following Newmark), μ and R. The value of the biassing force P is not independent of N/A and μ, through Eq. (2).

One other matter must be addresed. When the block initially begins to slide, the relative velocity v_r will be zero, so that its direction is indeterminate. This means there are computational problems with Eqs. (14) and (15) as to the appropriate values of the proportionality ratios (v_{rx}/v_r) and (v_{ry}/v_r). We can deal with the problem as follows.

Suppose, before the block starts sliding, that the frictional forces on the block in the x- and y-directions are F_x and F_y. Applying Newton's second law in the x- and y-directions leads to

$$F_x = mg(C_x + P) \tag{17}$$

$$F_y = mgC_y \tag{18}$$

Thus the proportions of frictional force in the x- and y-directions are

$$\frac{F_x}{\sqrt{F_x^2 + F_y^2}} \quad \text{and} \quad \frac{F_y}{\sqrt{F_x^2 + F_y^2}}$$

which can be written as

$$\frac{(C_x + P))}{\sqrt{(C_x + P)^2 + C_y^2}} \quad \text{and} \quad \frac{C_y}{\sqrt{(C_x + P)^2 + C_y^2}}$$

For initial sliding we can thus make the substitutions in Eqs. (14) and (15):

$$\frac{v_{rx}}{v_r} = \frac{-(C_x + P)}{\sqrt{(C_x + P)^2 + C_y^2}} \tag{19}$$

$$\frac{v_{ry}}{v_r} = \frac{-C_y}{\sqrt{(C_x + P)^2 + C_y^2}} \tag{20}$$

when $v_r = 0$. The negative sign is necessary because the frictional force will be in a direction opposite to the relative velocity.

RESULTS

The model was programmed on a computer and a number of simulations were run with different parameter values. Figure 4 can be considered the base case, with no lateral or vertical effects. The table acceleration coefficient C_x was simplified to a single sine wave, representing one pulse in an earthquake, with a maximum of A=1. N/A was set at 0.5, the coefficient of friction μ at 0.8, and there was no lateral or vertical acceleration. The biassing force coefficient P is not independent and is obtained from Eq. (1). Figure 4 shows the resulting motions of the block. The block acceleration (coefficient) k_x is the same as that of the table until C_x reaches a value of 0.5. At that point, k_x stays at its maximum value of 0.5 while the block begins to slide and the velocity v_x of the block relative to the table begins to grow. Its value is simply the integral of the difference between the two acceleration coefficients. It has a negative sign as it is in the negative x-direction. The block acceleration stays at a constant value while relative motion continues until C_x drops and the relative velocity v_x passes its peak and returns to zero. At this point the block sticks to the table and the block acceleration abruptly reverts to that of the table. As relative motion takes place, the relative displacement d_x between the block and the table increases, reaching a final value when the relative velocity becomes zero. This is the expected behaviour of the original Newmark sliding block model and has been more fully descibed elsewhere (Richards and Elms 1979, Elms and Richards 1979). As the model is dimensionless, units are not used. In any case, they would be unnecessary as what is sought is a relative not an absolute effect.

The introduction of lateral acceleration modifies the behaviour of the block. Figure 5 shows the behaviour with a moderate lateral excitation ratio of R=0.4. The behaviour in the x-direction is little affected, apart from a slight dip in the graph of kx, which is no longer a straight horizontal line. However, a relative velocity and displacement are introduced in the y-direction.

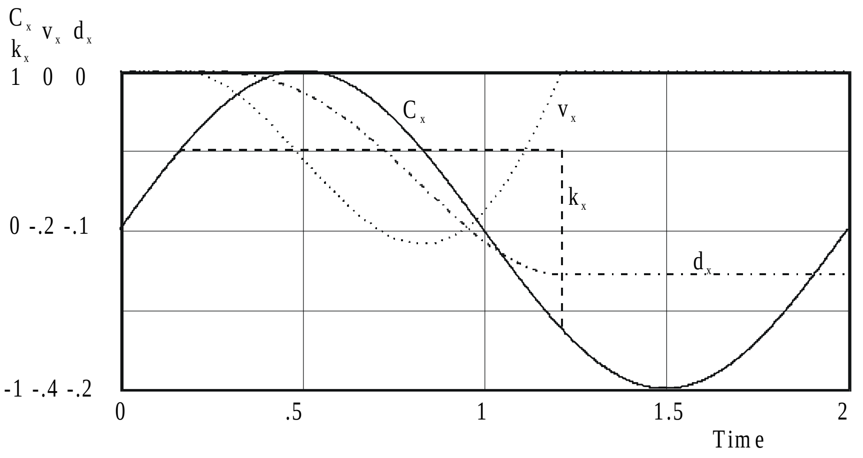

Figure 4 Motion in x-direction only; N/A=0.5; μ=0.8

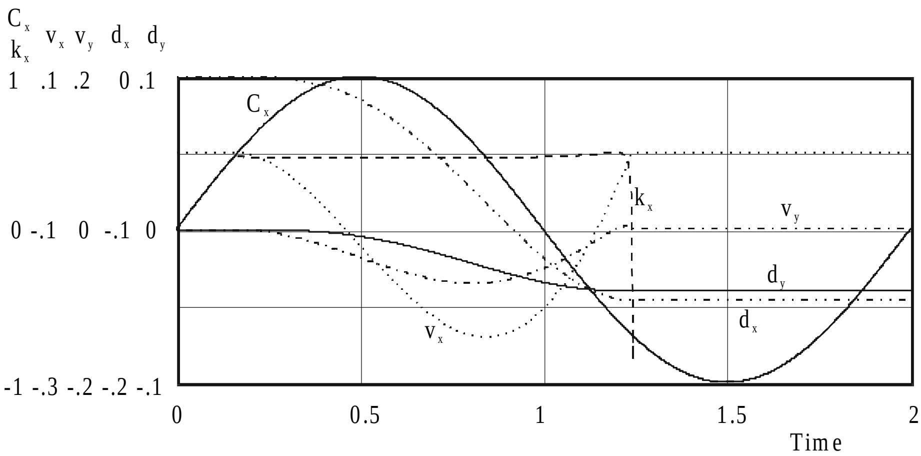

Figure 5 Motion including lateral acceleration: N/A=0.5; μ=0.8; R=0.4

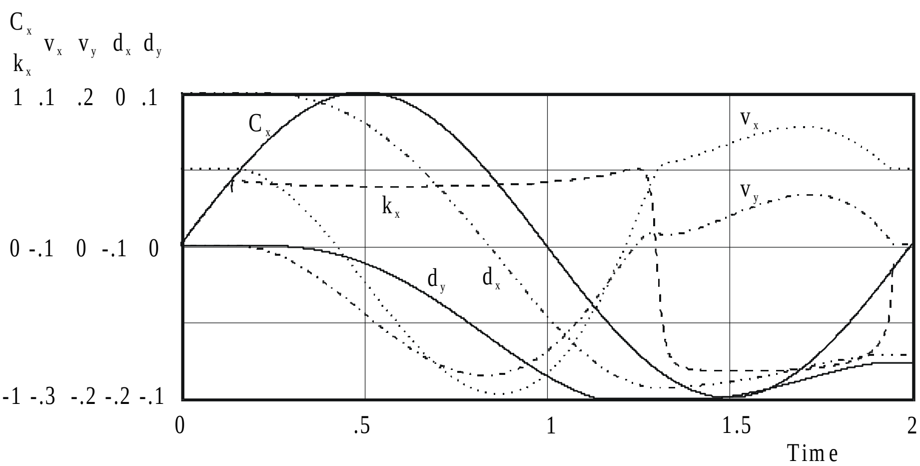

Figure 6 Motion including lateral acceleration: N/A=0.5; μ=0.8; R=0.8

The behaviour becomes significantly different if the lateral acceleration ratio is increased to R=0.8, as can be seen in Figure 6.

The key to the apparently strange behaviour is to see what happens to the x and y relative velocities. v_x changes sign and becomes positive for a while towards the end, rather than returning to zero. This means that the block is now moving faster than the table in the x-direction and is catching up with it a little – the relative displacement peaks at a maximum value before settling down. The block acceleration does not just snap down to the table acceleration as it does in the earlier cases, but leaves some relative acceleration, first positive then negative. At time-1.3 or so, k_x reaches a negative plateau of about –0.8, meaning that the friction force has reversed and the block is now catching up with the table, as it were. In effect it is the passive failure case. The change in direction of v_x means that the x-component of frictional force changes direction, though it is diminished in magnitude because some of it is used up by movement in the y-direction. To give an idea of the likely value of the passive acceleration plateau, consider that the biassing force ratio P is 0.3 and the coefficient of friction μ is 0.8. This means that the positive plateau of k_x will have a value of about 0.5, without the effect of lateral excitation. If, however, the lateral velocity v_y has the same magnitude as v_x, then the direction of frictional force would be at 45^0, so reducing the available frictional force in the x-direction by a factor of $\sqrt{2}$. If this is so, the passive acceleration coefficient k_x would have a value (remembering Eq. (2A)) of -0.707x0.8-0.3=-0.866. This is roughly the level of the plateau seen at about time = 1.5 in Figure 6. Following the negative plateau, k_x finally swings up to join the table acceleration and relative motion ceases.

Both v_x and v_y change direction and have a positive overswing towards the end of the cycle, before returning to zero. The components of relative displacement reach a peak before returning to final stability.

Thus the introduction of a substantial degree of lateral acceleration not only affects the final block displacement in the longitudinal direction, but also results in significantly different and more complex behaviour.

Turning now to the increase in final displacement due to the introduction of lateral excitation, Figure 7 is a typical graph of the effect of different levels of lateral excitation ratio R for different values of N/A. The percentage increase is greater for higher values of N/A, reaching almost 60%. However, the higher the value of N/A, the less the block will move. This must be borne in mind when thinking in terms of percent increase.

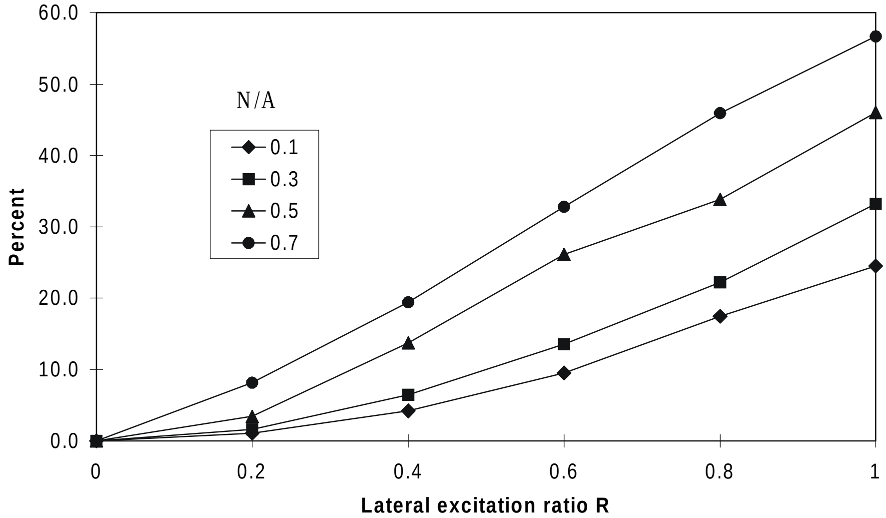

Figure 7 Percentage increase in displacement for μ=0.8 for different values of N/A

Figure 8 shows much the same results, this time with a fixed value of N/A while varying the friction coefficient μ. Once again the displacement increases up to a maximum of about 60% for high values of R. However, there is only sufficient space in this paper to show a few values. Some other combinations of the three parameters R, N/A and μ give more extreme results. For example, for the combination R=1.0, μ=1.0 and N/A =0.9, the final displacement is increased by nearly 700% over that of the equivalent R=0 case. As this is equivalent to a design case where very little displacement is expected or intended, the effect of lateral excitation is very significant indeed.

The behaviour of the block with vertical excitation included has been investigated with a number of simulation runs. The introduction of a varying vertical acceleration uncorrelated with the horizontal motion produces interestingly complex block behaviour. More to the point from a practical point of view, it can lead to significant displacement increases in the x-direction, in some cases greater than 100%. However, space limitations mean that the detailed results must be published elsewhere.

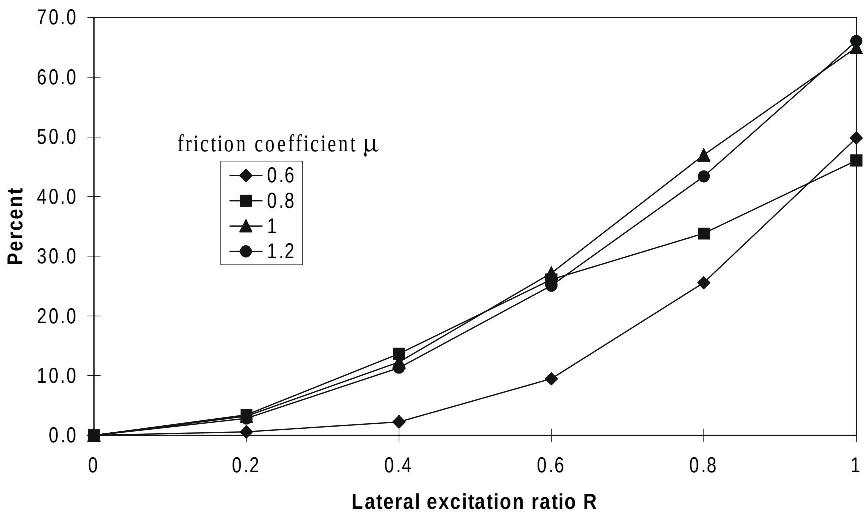

Figure 8 Percentage displacement increase for N/A=0.5, with different values of μ

CONCLUSIONS

There are two conclusions to be made. The first is that the force-biassed sliding block model shows complex behaviour when a fully correlated lateral earthquake excitation is added to that in the longitudinal direction. The second is that the addition of lateral excitation can increase the block displacements significantly. Vertical excitation can also increase block displacement, though this effect has not been emphasised here.

The work needs to be extended by running the model for a number of real and artificial earthquake motions, using the x-, y- and z-components, in order to obtain realistic and representative estimates of displacement for use in the design of geotechnical stuctures which can slide in earthquakes, following Richards and Elms (1979).

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