A STUDY ON IDENTIFING METHOD OF NONLINEAR DYNAMIC SOIL RESISTANCE AGAINST PILES

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SUMMARY

Pile foundations were severely damaged during the 1995 Hyogoken Nanbu Earthquakes. It is well known that these damages were attributed to nonlinear soil-pile foundation interaction. It has become important in designing of pile foundations to evaluate the soil resistance against piles during earthquakes or to excitations applied at the pile top. It is desired to develop a method to evaluate the soil resistance against piles based on measurement on piles. However, as measured results include inevitably some measurement errors, the errors must be removed in order to estimate properly soil resistance against piles. This paper describes an improving method to identify with accuracy the dynamic soil resistance against piles combining a smoothing procedure and a penalty matrix. The displacement and the nonlinear dynamic soil reaction are shown to be evaluated based on bending moments measured on pile nodes. The validity of the proposed method is studied through the numerical examples.

INTRODUCTION

Pile foundations were severely damaged during the 1995 Hyogoken Nanbu Earthquakes. It is well known that these damages were attributed to nonlinear soil-pile foundation interaction. Therefore, it has become important in designing of pile foundations to estimate properly the nonlinear soil resistance against piles during earthquakes or to excitations applied at pile top.

The dynamic soil resistance against piles can be observed directly by earth pressure gauges installed along the piles. Such measurement, however, has not been carried out as far as the authors know. The measurement on piles is usually conducted by means of strain gauges or sometimes accelerometers at some points on the piles. The measurement by strain gauges gives the information of bending moments along the piles. We are, therefore, obliged to estimate the dynamic soil resistance indirectly from those observations. In the observed data, some measurement errors are included inevitably, and it becomes essential to minimize effects of the errors in estimation of the dynamic soil resistance.

The objective of this paper is to study a method to estimate the nonlinear dynamic soil resistance against piles when excited by lateral forces at the top of piles on an assumption that the bending moments are known a priori at some points along the piles. The problem considered here belongs to identification of nonlinear systems. The modeling and identification of nonlinear systems have been recognized to be important problems in the structural dynamic fields and have been studied extensively. The identification methods to determine such nonlinear systems are classified into two categories according to the methods; one is parametric method and the other is nonparametric method. Some studies of the parametric method have been presented, for example, using the extended Kalman filter [Takimoto and Hoshiya (1997)] and by use of the recursive prediction error method [Sato and Takei (1997)].

The parametric method needs in advance to make a model with the parameters governing the nonlinear system. As the system behavior is unclear in the soil-pile system, it is difficult to model the system. On the other hand,

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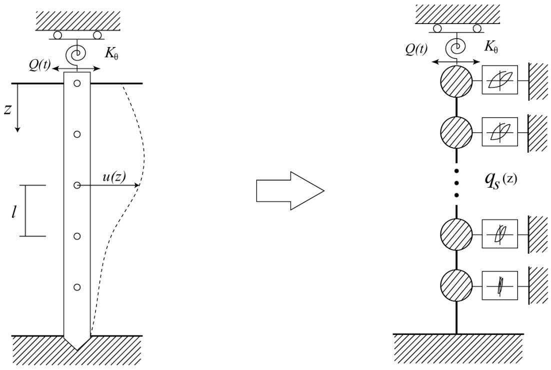

(a) A Single Pile and Soil System (o indicates the observation points of bending moments) (b) Lumped Mass System

Fig. 1 Soil-pile system and analysis model

the nonparametric method does not need to model the system. In the method, the nonlinear system is described as functions of the displacement and the force. Some methods have been proposed by using different series such as power series [Toussi and Yao (1983)], Chebyshev series [Masri and Caughey (1979)] or scaling function [Kitada (1998)]. Furthermore, a neural network method was also presented [Masri, Chassiakos and Caughey (1993)]. As the nonparametric methods are very sensitive to the measured errors, in order to identify the system parameters accurately these errors must be removed.

The measured data include inevitably some observation errors, which diminish the accuracy of identified result. Because of the measurement errors included in the bending moments, the identified displacement and soil resistance tend to oscillate around the exact value.

In this paper, smoothing of data with rectangle filter and regularization with a penalty matrix are introduced in order to improve the estimation result. The validity of the proposed method is investigated through the numerical examples.

2. STATEMENT OF PROBLEM

The soil-pile foundation system considered in this paper is illustrated in Fig. 1(a). The model consists of a single elastic pile surrounded by nonlinear soil media. The pile is assumed to be uniform along the axis and divided into equal length. The effect of inertia moment of a pile is assumed to be neglected. The pile head is supported by an elastic rotational spring. By changing the rotational spring constant, it is possible to treat from fixed to hinge end supports. The external lateral forces are applied at pile top.

The aim of this paper is to estimate the nonlinear dynamic soil resistance against pile using measured moments, which are assumed to be known a priori at some points along the pile. As for the data of bending moments at nodal points of the pile, instead of using observed data the numerically evaluated results are used in this paper. The procedure corresponds to a forward analysis. In the forward analysis, the displacement-soil resistance relationships are assumed to be a bi-linear model or Wen's model, and force-displacement relationships at nodes may be obtained numerically according to the presumed nonlinearity. Thus obtained bending moments are used as the measured data in the backward analyses. To estimate the soil resistance using measured data, the system is modeled as shown in Fig. 1(b), which is composed of equally spaced lumped masses and linear springs representing the pile stiffness. The nonlinear springs representing the soil reactions are assumed to act on the discretized nodes. The soil-pile interconnecting spring is attached to the lumped mass. The characteristics of the soil springs may differ from each point. In estimation of the soil reaction, all parameters of the pile are assumed to be known a priori. The displacement of mass is determined from the observed bending moments at each time steps. The soil resistances are found from the displacement-moment relationships and equilibrium relations.

IDENTIFICATION PROCEDURE

The identification procedure of the soil-pile system will be outlined in this chapter, in which the unknown soil resistance against pile is denoted by $\{q_s(t)\}$. Pile head-excitation $Q(t)$ and the bending moments $\{\overline{M}\}$ are assumed to be known by the measurement at each time.

Basic Equations

The equation of motion for soil-pile system can be written as follows.

$$\{Q_P\} = \frac{6}{l^2} K_P [K_{st}]\{u\} + Dl[e]\{q_s\} + m[e]\{\ddot{u}\} \tag{1}$$

where $\{Q_P\}$ = external forces applied at nodes, K_P = pile stiffness $\left(= \frac{E_P I_P}{l}\right)$ with E_P = Young's modulus of pile, I_P = second moment of area for the pile and l = element length of the pile. Moreover, in Eq. (1) $\{u\}$ = pile displacement, $[K_{st}]$ = pile stiffness matrix, which may be given by

$$[K_{st}] = [A_P]^{-1}[B_u] \tag{2}$$

where $[A_P]$ and $[B_u]$ can be given as follows for a pin-end condition at the bottom,

$$[A_P] = \begin{pmatrix} \frac{a+b+1}{b} & 1 & & & & \\ \frac{2(2b-1)}{b} & 4 & 1 & & & \\ & 1 & 4 & 1 & & \\ & & \ddots & \ddots & \ddots & \\ & & & 1 & 4 & 1 \\ & & & & 1 & 4 \end{pmatrix} \tag{3}$$

$$[B_u] = \begin{pmatrix} \frac{2c(1+a)}{b} - 3b & -1+3b-\frac{2c(1+a)}{b} & 1 & & & & & \\ -\frac{4c}{b} & 3+\frac{4c}{b} & -4 & 1 & & & & \\ 1 & -4 & 6 & -4 & 1 & & & \\ & & & \ddots & & & & \\ & & & & \ddots & & & \\ & & & 1 & -4 & 6 & -4 & 1 \\ & & & & 1 & -4 & 6 & -4 \\ & & & & & 1 & -4 & 5 \end{pmatrix} \tag{4}$$

with parameters

$$a = 2 - \frac{2}{K_\theta / K_P + 4}, \quad b = 1 - \frac{2}{K_\theta / K_P + 4}, \quad c = 1 - \frac{3}{K_\theta / K_P + 4} \tag{5}$$

in which K_θ = rotational spring stiffness attached at the pile top. The change of K_θ enables to represent from fixed to pin-end supports.

Finally, $m[e]$ appearing in Eq. (1) is a diagonal mass matrix defined by

$$m[e] = diag(m/2, m, \cdots, m) \tag{6}$$

with $[e] = diag(1/2, 1, \cdots, 1)$ and m = mass of pile at node $\left(= \frac{\pi D^2}{4}\frac{l\gamma}{g}\right)$ (γ = density of pile, D = diameter of pile).

The resultant forces of soil resistance, R_i, may be evaluated from the normal contact stress between the pile and the soil q_s(z). The soil resistance at i-th node may be expressed with the following formula at i-th node.

$$R_i = D\left[\int_{z_i - l/2}^{z_i + l/2} q_s(z)\, dz\right] \approx Dl q_s(z_i) \tag{7}$$

where z_i = the coordinate of i-th measurement point, D = pile diameter, l = the interval length of measurement point.

In the forward analyses, two nonlinear displacement-soil stress models are considered, one is the bi-linear model and the other is the Wen's model. [Minai and Suzuki(1989)]

Nonlinear Soil model

Bi-Linear Model

An expression representing restoring force characteristic of the elasto-plastic was presented by Minai and Suzuki(1989). The expression was extended in this paper so as to enable to express the bilinear type characteristic with the second stiffness and used in numerical calculation. With use of the proposed expression for bi-linear nonlinear model, the normal contact stress, q_s, may be expressed by

$$\dot{q}_s(z_i) = \left[1 - \frac{1}{2}\mu\{\mathrm{sgn}(\dot{u}(z_i)q_s(z_i)) + 1\}U(|q_s(z_i) - \mu E_s u| - q_Y)\right]E_s\dot{u}(z_i) \tag{8}$$

where E_s = the initial stiffness, $(1-\mu)\,E_s$ = the second stiffness.
The overdot denotes the time derivative, and | | represents the absolute function. The sgn( ) means the signum function.

$$\mathrm{sgn}(x) = \begin{cases} 1 & \text{if } x > 0 \\ -1 & \text{if } x < 0 \\ 0 & \text{otherwise} \end{cases} \tag{9}$$

$U(x)$ denotes the unit step function as defined by

$$U(x) = \begin{cases} 1 & \text{if } x > 0 \\ 0 & \text{otherwise} \end{cases} \tag{10}$$

<u>Wen's Model</u>

$$\dot{q}_s(z_i) = \left[1 - \{\beta\,\mathrm{sgn}(\dot{u}(z_i)q_s(z_i)) + \gamma\}|q_s(z_i)|^{\alpha}\right]E_s\,\dot{u}(z_i) \tag{11}$$

where E_s = the initial stiffness, α, β and γ are profile variables expressing the nolinearity of soil.

The bending moments at the nodes that are used as the observed data can be obtained by the following equation.

$$\{M\} = -\frac{1}{6}[c_P]\left(\{Q_P\} + \frac{6K_P}{l^2}[D_u]\{u\}\right) \tag{12}$$

where $\{Q_P\}$ is defined in Eq.(1) and

$$[c_P] = \mathrm{diag}\left(\frac{2(2b-1)}{b}, 1, \cdots, 1\right), \qquad [D_u] = \begin{pmatrix} 1 & -1 & & & & \\ -1 & 2 & -1 & & & \\ & -1 & 2 & -1 & & \\ & & \ddots & \ddots & \ddots & \\ & & & -1 & 2 & -1 \\ & & & & -1 & 2 \end{pmatrix} \tag{13}$$

NUMERICAL CORRECTION PROCEDURE FOR CONTAMINATED DATA

The displacements are estimated from measured bending moments $\{\overline{M}\}$ using Eq.(12).

$$\{u\} = \frac{l^2}{6K_P}[D_u]^{-1}\left(-\frac{6}{l}[c_p]^{-1}\{\overline{M}\} - \{Q_s\}\right) \tag{14}$$

The soil reactions at time step k can be estimated from Eqs.(1) and (14).

$$\{q_s\}_k = \frac{1}{Dl}[e]^{-1}\left(\{Q_s\}_k - \frac{6}{l^2}K_P[K_{st}]\{u\}_k - m[e]\{\ddot{u}\}_k\right) \tag{15}$$

where the acceleration $\{\ddot{u}\}_k$ can be approximated by the central finite difference.

$$\{\ddot{u}\}_k \approx \frac{1}{\Delta t}\left(\{u\}_{k+1} - 2\{u\}_k + \{u\}_{k-1}\right) \tag{16}$$

The measurement errors have a great influence on estimation results. Therefore, this error must be removed in order to make the identification effective. These errors cause fluctuation of results around the exact value. To eliminate such measured errors, following two steps are considered. Firstly, the smoothing method using the rectangle filter is considered. If width of the smoothing is chosen for discretized $2d$+1 intervals, smoothing procedure will be given by the following formula.

$$\{M\}_{k\,smooth} = \frac{1}{2d+1}\sum_{i=k-d}^{k+d}\{M\}_i \tag{17}$$

Secondary, the regularization method using a penalty matrix is considered. The penalty matrix chosen in this paper shows minimizing of the integration of second derivatives. This matrix is given as follows. [Okamoto and Musha(1992)]

$$
[P]=\begin{pmatrix}
1 & -2 & 1 & & & & & & \\
-2 & 5 & -4 & 1 & & & & & \\
 & \cdots & & & & & & & \\
 & & 1 & -4 & 6 & -4 & 1 & & \\
 & & & & & \cdots & & & \\
 & & & & & 1 & -4 & 5 & -2 \\
 & & & & & & 1 & -2 & 1
\end{pmatrix}
\tag{18}
$$

Using this penalty matrix, the soil reaction, $\{q_{s\,\mathrm{crct}}\}$, can be corrected by following equation

$$
\{q_{s\,\mathrm{crct}}\}_k = \left(I+\lambda^2[P]\right)^{-1}\{q_s\}_k \tag{19}
$$

where λ is an arbitrary positive real number as regularizing parameter.

NUMERICAL EXAMPLES

The parameters used in numerical examples are shown in Table 1. As for the data of bending moments at nodal points of the pile, instead of using observed data, used are the numerically evaluated results that are obtained with combining Eqs. (1) and (12) and integrating Eq. (1) numerically with use of 4th order Runge-Kutta method. The soil-pile system is assumed to be subjected to a sinusoidal excitation applied at the pile top. In the numerical evaluations, the displacement-soil resistance relationships are assumed to be a bi-linear model or Wen's model. Time step was set to 1/10000 sec in the numerical integrations. The computed results of the displacement-reaction force relationships under the assumed nonlinearity model for soil reaction will be referred to original data.

To obtain the displacement of the pile and the soil resistance, time histories of measured data are sampled at every 1/1000 sec. The pile displacements and soil reactions are estimated using the bending moments by the proposed procedure. In the first numerical example, the observation errors of the bending moment are not considered. For this special case, the smoothing procedure for the given data and regularization with the penalty matrix are not necessary. The estimated results of displacements, resistance of soil and force-displacement relationships at the second node from the top are shown in Fig. 2a (bilinear model) and Fig. 2b (Wen's model). The estimated displacement and soil resistance show good agreement to the original data, and cannot be distinguished in the figures.

The measured data usually include the inevitably measurement errors. In the second numerical examples the effects of errors of the bending moments are studied. To consider the effect of the measurement errors, 1% artificial errors are introduced in the computed bending moments. The results obtained without any correction procedures are shown in Fig. 3. It may be seen that for the displacement of the pile the estimated results show good agreement to the original data. However, the estimated soil resistances fluctuate around the original data. To remove these fluctuations, smoothing procedure for the original data with rectangle filtering and regularization using the penalty matrix are introduced in the next numerical examples. The results obtained by the correction procedures described earlier are shown in Fig. 4a (bilinear model) and Fig. 4b (Wen's model). It may be seen that by using the proposed method, the identified results could be improved remarkably. Also, the displacement-resistance relationships are reproduced even if measured bending moments include the artificial errors. In Fig. 5, the estimated results of the displacement-soil resistance relationships at each node are compared with the original data that are obtained by the forward analyses. In the numerical computation, the same parameters shown in Table 1 are used. It should be noted from the figure that satisfactory results can be obtained by combining the smoothing procedure and the regularization using the penalty matrix even if measurement errors were included in the observed data and nonlinearity of soil is strong.

Table 1. Parameters used in the numerical calculations

Pile	Soil	External Force
Diameter: 1.2 m Length: 20 m Young's Modulus: 2.0 x 10^6tf/m^2 Element Number: 5 Pile Top: Fixed support Density: 2.4 tf/m^3	Bilinear Model: Yield Force: q_Y = 1tf/m^2 1st Tangent Modulus: 600 tf/m^3 2nd Tangent Modulus: 120 tf/m^3 Wen's Model: α=3.0; β=0.5; γ=0.4; E_s=600 tf/m^3	Amplitude: 40tf Frequency: 0.5 Hz

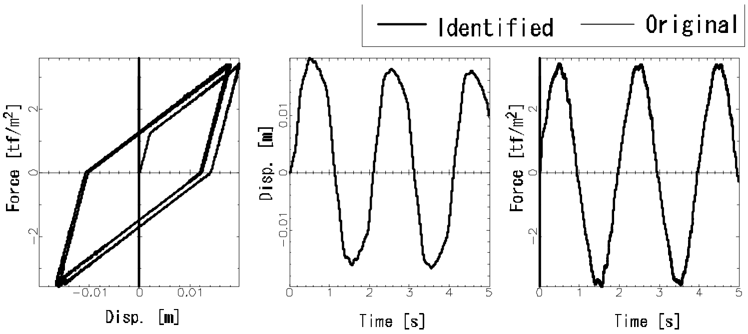

Fig. 2a Estimated results at 2nd node without measurement errors (Bi-linear model)

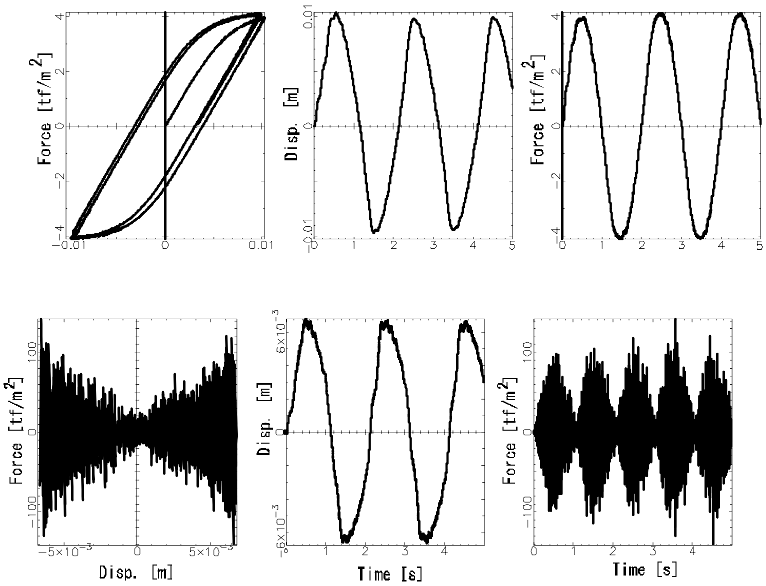

Fig. 3 Results for data including measurement errors

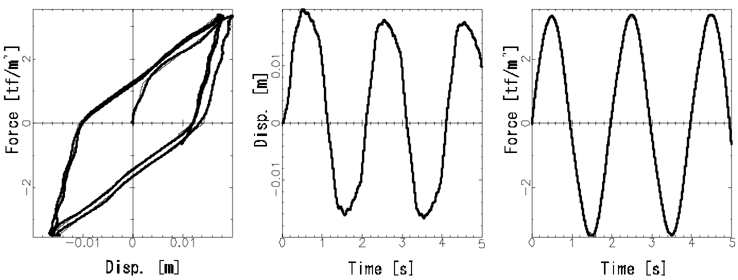

Fig. 4a. Improved results at 2nd node (Bi-linear model)

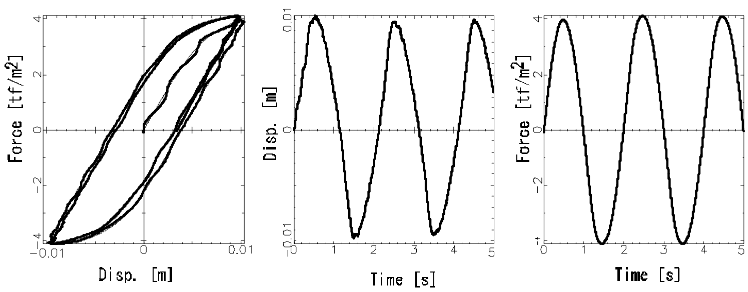

Fig. 4b Improved results at 2nd node (Wen's model)

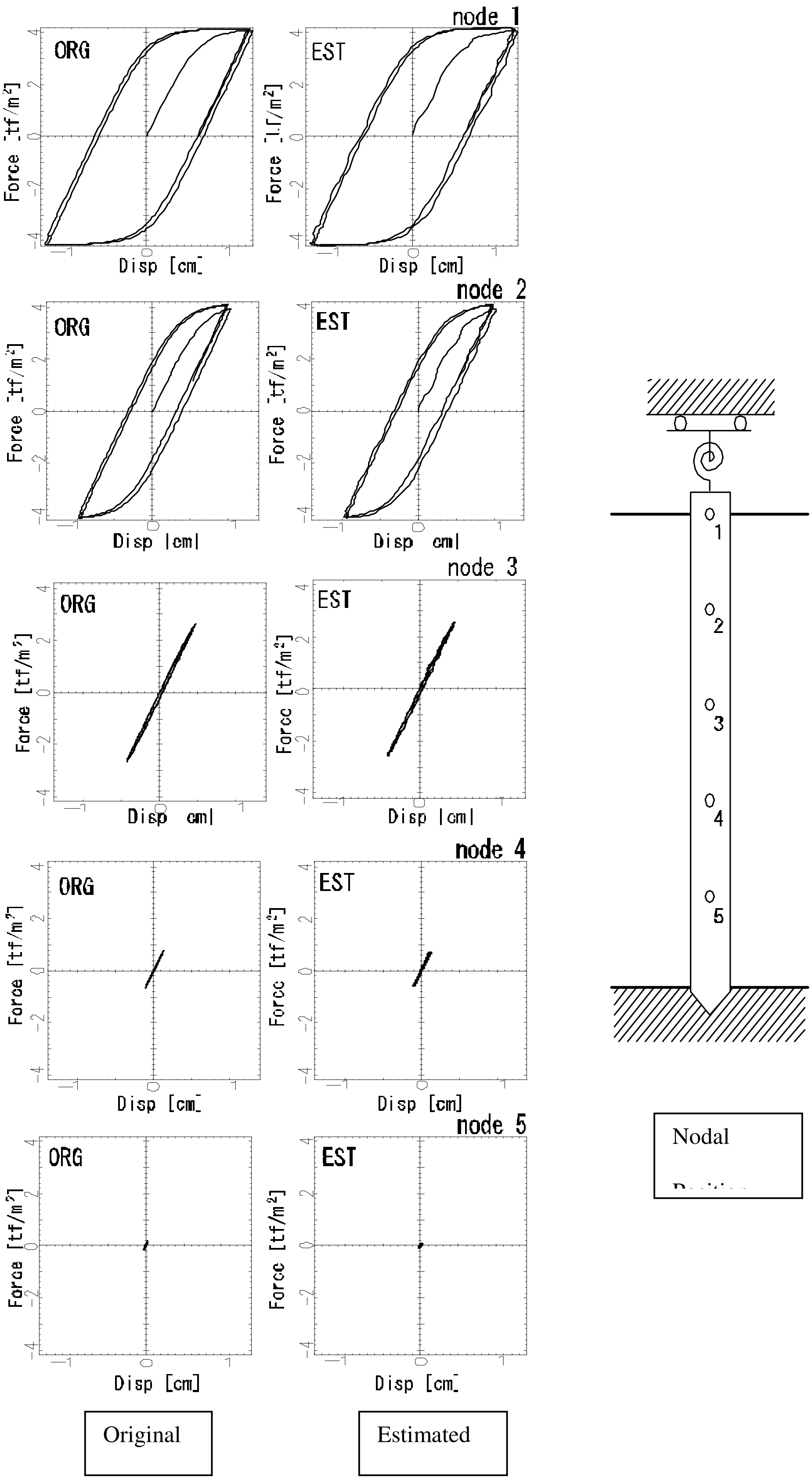

Fig. 5 Comparison of estimation results to original data at each node.

CONCLUSIONS

In this paper an estimation method for the nonlinear dynamic soil resistance against piles when excited by lateral forces at the top of piles was presented. The method is combined with two procedures; smoothing of time-history data for appropriate intervals and regularization method using a penalty matrix.

The validity of the proposed method was studied through the numerical examples, which have shown that the proposed method gave satisfactory results even when the measurement errors were included in the observed data and the nonlinearity of soil is strong.

REFERENCES

1. Argoul, P., Jezequel, L., (1989), "Improvement of Nonparametric Identification Procedure Used in Nonlinear Dynamics", *J. Appl. Mech*., *Vol. 56*, *September*, pp697-703
2. Benedettini, F., Capecchi, D., Vestroni, F., (1995), "Identification Hysteretic Oscillators under Earthquake Loading by Nonparametric Model", *J. Eng. Mech*, ASCE, *Vol. 121, No. 5,* May, pp606-612
3. Kitada, Y., (1998), "Identification of Nonlinear Structural Dynamic Systems Using Wavelets", *J. Struct. Constr. Eng*., AIJ, *No. 504, February,* pp43-48.
4. Masri, S. F., Caughey, T. K., (1979), "A Nonparametric Identification Technique for Nonlinear Dynamic Problems", *J. Appl. Mech*., ASME, *Vol. 46*, *June*, pp433-447.
5. Masri, S. F., Sassi, H., Caughey, T. K., (1982), "Nonparametric Identification of Nearly Arbitrary Nonlinear Systems", *J. Appl. Mech.,* ASME*, Vol. 49, September,* pp619-628.
6. Masri, S. F., Miller, R. K., Saud, A. F., Caughey, T. K., (1987), "Identification of Nonlinear Vibrating Structures: Part I & II", *J. Appl. Mech*., ASME,*Vol. 54, December*, pp918-929.
8. Minai, R., Suzuki, Y., (1989), "Nonlinear Wave Equation of Hystereteic Media", *Proc. of the Japan-China (Taipei) Joint Seminar on Natural Hazard Mitigation*, June, pp203-216.
7. Masri, S. F., Chassiakos, A. G., Caughey, T. K., (1993), "Identification of Nonlinear Dynamic Systems Using Neural Networks", *J. Appl. Mech.,* ASME, *Vol. 60*, *March*, pp123-133.
9. Okamoto, Y., Musha, T., (1992), *Inverse Problems and the Analysis Methods*, Ohm Publishing Co., Tokyo, Japan (in Japanese).
10. Sato, T., Takei, K. (1997), "A Recursive Non-Stationary Structural Identification with Fading Memory", *Proc. of JSCE*, *No. 577/I-41, October*, pp65-73 (in Japanese)
11. Smyth, A. W., Masri, S. F., Chassiakos, A. G., Caughey T. K.,(1999), "On-line Parametric Identification of MDOF Nonlinear Hysteretic Systems", J. Eng. Mech., ASCE, Vol. 125, No. 2, February, pp133-142.
12. Takimoto, M., Hoshiya, M. (1997), "Identification of Nonlinear Structures by Kalman Filter", *Proc. of JSCE*, *No. 556/I-38, January,* pp179-187 (in Japanese).
13. Toussi, S., Yao, T. P.,(1983), "Hysteresis Identification of Existing Structures", *J. Eng. Mech.,* ASCE*, Vol. 109, No. 5, October*, pp1189-1202.