SIMPLIFIED METHODS FOR SEISMIC RELIABILITY ANALYSIS

Edmund D BOOTH[1] And Michael J BAKER[2]

SUMMARY

The reliability of two concrete shear walls, one squat and one slender, was calculated using a simplified method based on, and slightly extended from, Cornell (1996), which greatly reduces computational effort. A more rigorous method of calculation based on first order reliability analysis (Thoft-Christensen & Baker 1982) was found to agree with the simplified results from the extended Cornell method to within 30%. This is sufficiently accurate for most purposes and makes the Cornell method attractive for applications in the nuclear industry, and more generally for investigating reliability issues, including calibrating partial factors for seismic codes, and for investigating unusual or important structures under seismic loading. A particular application of the method is for deciding whether particular failure modes triggered by seismic intensities beyond the design basis ('cliff edge effects') contribute significantly to the probability of failure. Examples of such failure modes may be associated with liquefaction of foundation soils, P-delta instability in structures and (as investigated in this study) uneven plastic strain distribution in plastic hinges.

INTRODUCTION

Cornell (1996) has published a method for calculating seismic performance reliability. The method provides a simple and direct way of calculating reliability; it assumes that the seismic hazard curve follows a logarithmic distribution, and that the structural resistance (fragility) is lognormally distributed. Under these conditions, Cornell shows that the annual probability of failure p_f is given by

$$p_f = p_m e^{1/2(k_1\delta_r)^2} \qquad (1)$$

where

p_m = probability of occurrence of earthquake motions necessary to achieve the limit state in question, given *mean* structural properties.

k_1= slope of the annual probability of exceedence ground motion hazard curve. This implies that the return period T corresponding to a ground motion intensity x is assumed to be of the form:

$$T = k_0 x^{k_1} \qquad (2)$$

where k_0 is a constant not affecting equation (1).

δ_r = coefficient of variation of the structural properties, which therefore describes the intrinsic uncertainty in structural resistance. δ_r also allows for the uncertainty in frequency content of the input ground motion. Where the structural properties are well established, and the response is well controlled, a value of δ_r = 0.4 would usually be conservative, according to Cornell (1996).

[1] *Consulting Engineer, 67 Orford Road, Walthamstow, London E17 9NJ, UK*
[2] *Professor of Safety Engineering, University of Aberdeen, Fraser Noble Building, Kings College, Aberdeen, Scotland AB24 3*

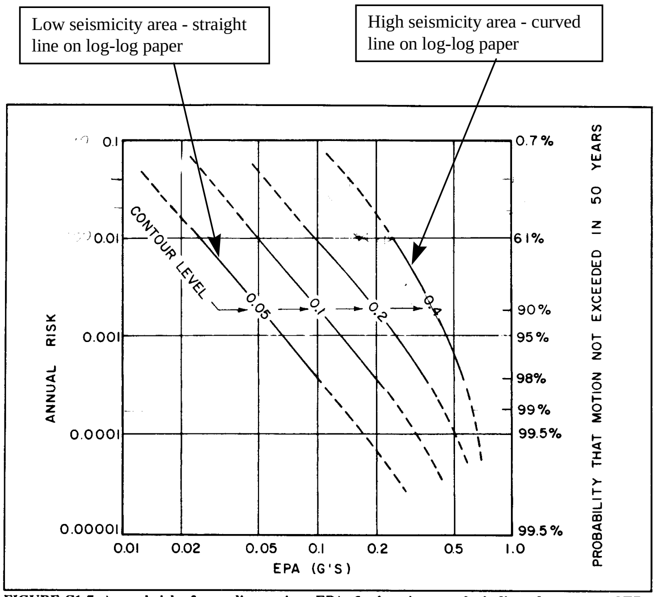

FIGURE C1-7 Annual risk of exceeding various EPAs for locations on the indicated contours of EPA in Figure C1-3.

Figure 1: Hazard curves for high and low seismicity areas (from NEHRP 1991)

A typical value of k_1 for a high seismicity region is 4, while for the UK (an area of low seismicity) it is about 2.8. As can be seen from Figure C1-7 of NEHRP (reproduced here as Figure 1), a logarithmic approximation for the hazard curve is a good fit for low seismicity areas, but somewhat less good for high seismicity areas. From equation (1), and assuming δ_r = 0.4, the quotient p_f / p_m is 3.60 in an area of high seismicity (k_1 = 4) and only 1.87 in an area of low seismicity (k_1 = 2.8). Therefore, using the Cornell approximation, quite good estimates of failure probability can be obtained from calculations with all structural properties set to their mean values. More rigorous methods involve considering response with structural properties varied from their mean values; this variation is accounted for in the Cornell method by the term δ_r in equation (1). If the limit state of interest is one of failure, it is likely that non-linear response will be involved. Therefore, using the Cornell method, the computational effort can be expended on investigating non-linear behaviour under extreme ground motions using only one structural model with mean properties, rather than on performing analyses using a number of structural models with varying properties.

As discussed above, it is often sufficient to take δ_r = 0.4 for the purposes of evaluating equation (1). Where a more rigorous justification is required, or where there are very large uncertainties in structural properties, then a specific calculation for δ_r is needed. This can be done by calculating δ_r from the individual statistical variances of the structural properties. In the case where all the properties are lognormally distributed, and statistically independent, and the ground motion intensity at failure is given by a simple multiplicative law, it can be shown that δ_r equals the SRSS (square root sum of squares) of the individual COV's (coefficients of variations) of the structural components. That is:

If Intensity at failure, $I_f = X * S_1 * S_2 * \ldots\ldots * S_N$

Then $\delta_r = \{ [(COV)_{S1}]^2 + [(COV)_{S2}]^2 + \ldots\ldots + [(COV)_{SN}]^2 \}^{1/2}$ (3)

Where X = a constant

S_1 = first structural property

S_2 = second structural property

etc.

$(COV)_{S1}$= Coefficient of variation of first structural property

$(COV)_{S2}$= Coefficient of variation of second structural property

etc.

In the more usual case, however, the intensity at failure will not follow a simple multiplicative law but will be much more sensitive to changes in some of the structural properties than in others. It is proposed that a *weighted* SRSS sum of the coefficients of variation should be used to calculate δ_r, where the weighting factors are equal to the partial derivatives of the failure intensity with respect to each structural properties, as follows.

$$\delta_r = \{ [a_1(COV)_{S1}]^2 + [a_2(COV)_{S2}]^2 + \ldots\ldots\ldots + [a_N(COV)_{SN}]^2\}^{1/2} \quad (4)$$

Where $$a_1 = \frac{\partial(Ln(I_f))}{\partial Ln(S_1)} \text{ etc} \quad (5)$$

The point at which these derivatives are calculated is discussed in the next paragraph. The partial derivatives a_R can be obtained by recalculating the ground motion intensity at failure with a small change in each of the structural properties in turn. This requires a new structural model with properties differing from the mean values. However, since only the sensitivity to change is required, a very simple model is likely to suffice; an equivalent linear model may be sufficient. In this study, simple equivalent linear models were used to calculate the derivatives a_R and hence δ_r, with results that appear quite adequate even for large values of δ_r. The method used is now described in more detail.

Initially, the derivatives a_R were calculated with the structural properties S_R set at their *mean* values $S_{R,mean}$. Strictly, they should be calculated at the *design* point (at which failure is most likely to occur), which for each variable is a constant number *Z* of standard deviations from the mean value. A simple but approximate iterative technique, easily implemented with a spreadsheet, was employed to allow for this; by trial and error, the value of *Z* was found which yielded the same value of failure probability p_f calculated by two methods: *firstly* from using equations (4), (5) and (1), and *secondly* from the failure probability associated with the ground motions producing a response corresponding to structural properties $S_{R,\,mean} + Z * (COV)_{SR}$ in the equivalent linear structural model.

It is instructive to calculate the relative contribution to overall uncertainty of each parameter. For the R^{th} parameter, it follows directly from equation (4) that this can be defined as α_R where

$$\alpha_R = [a_R(COV)_{SR}] / \delta_r \quad (6)$$

In the rigorous RELY analysis, the α_R values are calculated separately for each return period of motions considered. In the approximate Cornell analysis, only one return period of motions is considered, and so only one set of α_R values are calculated. It may be noted that it follows directly from equations (4) and (6) that the SRSS of the α_R values equals 1.

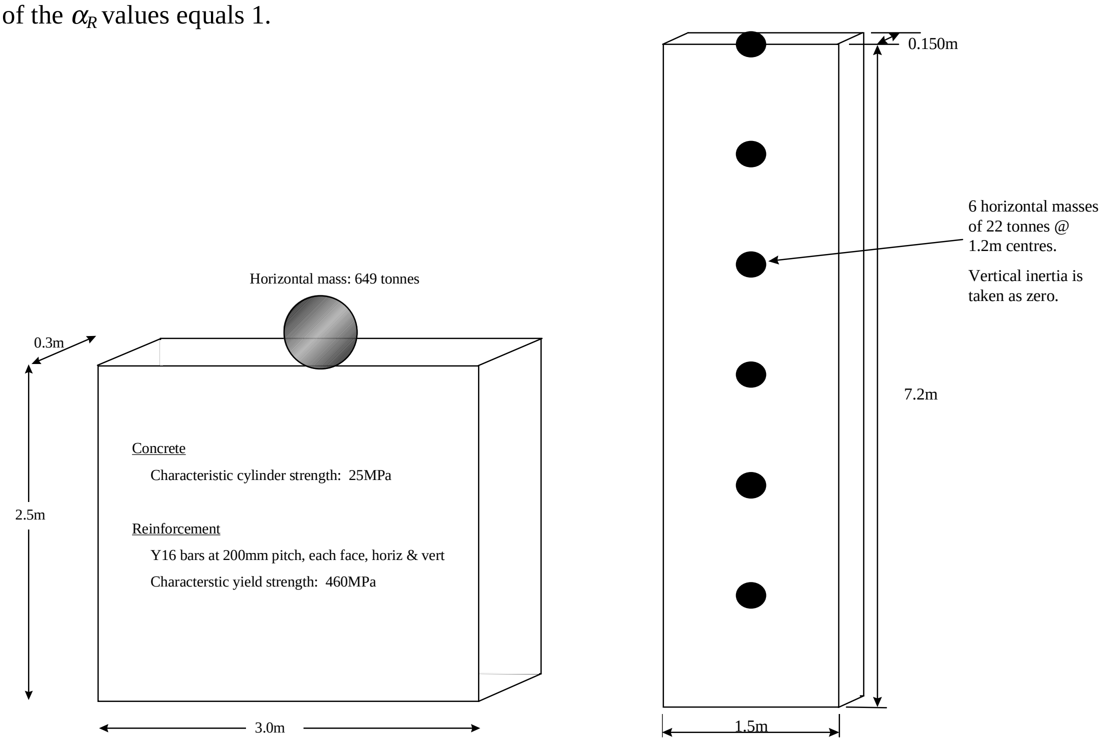

Figure 2: Geometry of squat and slender shear walls studied

RELIABILITY ANALYSIS FOR SQUAT SHEAR WALL

The theory outlined above was tested on two simple structures (Figure 2). Within the confines of this paper, full details of the analysis have not been provided; further information can be obtained from the authors.

Figure 2 shows the squat shear wall analysed for the study. Using the program *Oasys* DYNA-3D (1994), a model of the shear wall was subjected to various intensities of earthquake loading, intended as representative of UK earthquake motions. The computer model consisted of a single degree of freedom mass spring system, with the spring characterised by a non-linear model developed in a research contract for Health & Safety Executive (Ove Arup & Partners, 1994).

The annual probability of failure under seismic loading was initially found by a response surface technique. This involved carrying out a large number of non-linear dynamic analyses using DYNA-3D to construct regression equations for the response of the wall as a function of its structural properties; these regression equations were then used to produce 'response surfaces' corresponding to the failure strength of the wall. Using the specialist structural reliability package RELY (Ramachandran & Baker 1982), the annual probability of failure was calculated as $p_f = 2.9 \times 10^{-6}$. Only one earthquake time history was used, so the effect of uncertainty in ground motion characteristics was excluded. Six contributors to structural uncertainty were considered, as listed in Table 1.

Using the same DYNA-3D model, the annual exceedence probability p_m of the motions needed to cause failure with the structural parameters set at their mean (expected) values was calculated; it was found to be 1.5×10^{-6}. As a check, and to enable the later calculation of the derivatives a_R in equation (4), the wall was modelled as an equivalent elastic-perfectly plastic system, which gave $p_m = 2.2 \times 10^{-6}$ (ie 50% greater than the 'accurate' value from DYNA-3D). Given the approximate nature of the analysis, the match was considered good. The sensitivity of the structure to changes in the governing parameters was calculated by hand, using the same simple equivalent linear SDOF model, in order to calculated the derivatives a_R. From equation (4), δ_r was calculated as 0.30, and hence the Cornell increase factor p_f / p_m from equation (1) was 1.4. Therefore, the annual failure probability p_f was estimated to be $1.4 \ast 1.5 \times 10^{-6} = 2.1. \times 10^{-6}$ - ie 30% less than the more accurate estimate from RELY.

The relative contribution of each of the structural parameters to the overall structural variability also compared quite well with the values reported from the original RELY analysis (see Table 1). However, it does appear that the contribution of the uncertainty or 'error' term in the shear strength equation may have been underestimated in the approximate analysis.

In this case, therefore, a value of seismic reliability within 30% of the 'correct' answer was produced with the aid of the Cornell method. Where seismic reliability is being compared with other failure sources, an accuracy to a factor of 2 or 3 may well be quite sufficient. These findings lend support to the usefulness of the Cornell method.

Table 1: Squat shear wall - relative contribution of parameters to failure

		Return period (years)			
		5E+04	2E+05	1E+06	
R	Contributor to uncertainty	α_R (equation 6) RELY (rigorous) analysis			α_R (equation 6) Cornell (approx.) analysis
1	Effective hysteretic damping in model	-0.20	-0.45	-0.81	0.73
2	Error term in the shear strength equation	-0.31	-0.52	-0.44	0.26
3	Shear strength degradation on cyclic loading	0.87	0.56	0.27	0.43
4	Regression uncertainty in response surface	0.37	0.40	0.22	0.46
5	steel yield strength, f_y	-0.12	-0.21	-0.18	0.11
6	concrete cylinder strength f_c	-0.04	-0.06	0.05	0.04

RELIABILITY ANALYSIS FOR SLENDER SHEAR WALL

A similar procedure was adopted for the slender shear wall shown in Figure 2, which was modelled using the fibre element in DRAIN-2DX (Prakash et al, 1993). The shear wall was based on a wall tested by Goodsir & Paulay (1985), and the experimental results they report were used to validate the DRAIN model. The model was used to generate regression equations for the peak steel strain corresponding to a given intensity of ground motion; the results are shown in Figure 3. This shows the interesting feature that the peak steel strain rises very rapidly above a certain ground motion intensity; this can be seen where the regression lines become vertical for normalised ground motion intensities between 2½ and 3½. This is a sort of 'cliff edge' effect. The reason was that once the steel strain in the lowest fibre element of the wall exceeded a certain value, it became so lacking in stiffness that all additional strain became concentrated in this element; the strain value at which this occurred depended on the shape of steel stress strain curve that was assumed. In part, this reflects a real phenomenon, and many seismic codes specify a minimum degree of strain hardening to prevent it occurring at too low a value of local curvature ductility. However, it might be expected that in the DRAIN analysis, the phenomenon of strain concentration would become more pronounced, if the thickness of the lowest fibre element were reduced, and this was found in practice to be the case. Thus the results became highly model dependent, with apparently more refined models (ie those with narrow fibre elements) producing less realistic results. In reality, bond slip averages the steel strain over a length of concrete, effectively implying that plane sections no longer remain plane. By contrast, the DRAIN fibre element assumes that the steel remains perfectly bonded to the concrete. An interesting way of resolving this problem in DRAIN might be to use the 'pull-out' element that is provided. This was not tried for this study; instead, a shape of steel stress strain graph and thickness of fibre element were chosen to give reasonable agreement with the Goodsir & Paulay (1985) experimental results. Some uncertainty in the shape of the steel stress strain graph was assumed, corresponding to the three shapes ('low, 'best estimate' and 'high') shown in Figure 3.

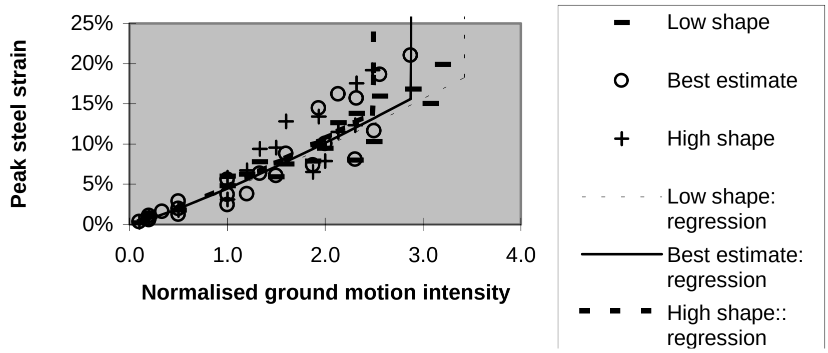

Figure 3: Regression analysis for slender shear wall

Table 2 Comparison of Cornell and RELY estimates for seismic reliability of slender shear wall

Contribution to failure probability	RELY analysis	Cornell approximation
Mode 1	5.3E-07	4.9E-07
Mode 2	1.4E-07	1.4E-07
TOTAL	**6.7E-07**	**6.3E-07**

Table 3: Slender shear wall - relative contribution of parameters to mode 1 failure (prior to 'cliff edge')

	Return period					
	1.0E+03	1.0E+04	1.0E+05	1.0E+06	2.0E+06	
Uncertainty Contributor	α_R (equation 6) RELY (rigorous) analysis					α_R (equation 6) Cornell (approx.) analysis
Fracture strain	0.89	0.90	0.90	0.90	0.90	0.91
Regression error term	0.44	0.43	0.43	0.43	0.43	0.41
Shape factor	0.07	0.07	0.07	0.07	0.07	0.06
Steel yield strength	-0.09	-0.09	-0.09	-0.09	-0.09	0.09

Table 4: Slender shear wall - relative contribution of parameters to mode 2 failure (vertical 'cliff edge')

	Return period					
	1.0E+03	1.0E+04	1.0E+05	1.0E+06	1.0E+07	
Uncertainty Contributor	α_R (equation 6) RELY (rigorous) analysis					α_R (equation 6) Cornell (approx.) analysis
Regression error term	-0.73	-0.72	-0.68	-0.57	-0.23	0.27
Shape factor	0.14	0.14	0.17	0.30	0.79	0.76
Steel yield strength	-0.67	-0.68	-0.71	-0.77	-0.56	0.59

The mean steel strain corresponding to fracture was assumed to equal 10%, with upper and lower five percentile values of 20% and 5% respectively. Failure at values above and below the critical 'cliff edge' point shown in Figure 3 was treated as belonging to two different failure modes, and the failure probability was summed for each separately. The results from the two analyses are shown in Table 2. Tables 3 & 4 show the relative contributions of the variables to overall failure probability. It can be seen that agreement between the rigorous (RELY) and approximate (Cornell) method is good.

EFFECT OF DIFFERENT TIME HISTORIES

The main work was done considering only one ground motion time history for each structure, which was scaled to produce a variety of loading intensities. A very limited study was done on the effect of the ground motion frequency characteristics by repeating some of the slender shear wall analysis using the same input time history run in reverse order. This had very little effect on failure mode 1, but a more significant effect on failure mode 2, where the associated failure probability increased by about 50%.

'CLIFF EDGE EFFECTS'

As previously discussed, a significant aspect of the analytical work was the appearance of a sudden change in response in the slender shear wall , at a critical earthquake intensity, above which the incremental plastic strain in the steel became concentrated at the bottom of the plastic hinge region. The question then arose as to whether this behaviour might be associated with a 'cliff edge' effect. The answer depends partly on what this is taken to mean.

A cliff edge clearly exists in the sense that there is a significant change in structural response affecting resistance, which occurs at a critical ground motion intensity and an associated probability of occurrence. Both approximate and rigorous methods were able to allow for this rapid change in response by assuming that two separate failure modes (modes 1 & 2) were involved. However, a more obvious definition of a 'cliff edge' would relate to the cumulative probability of failure function (CPF) curve, shown for the slender shear wall from the rigorous analysis in Figure 4. A cliff edge by this definition would involve a sudden increase in the slope of this curve at a certain return period, which (because the contribution of failure mode 2 is relatively small) did not occur.

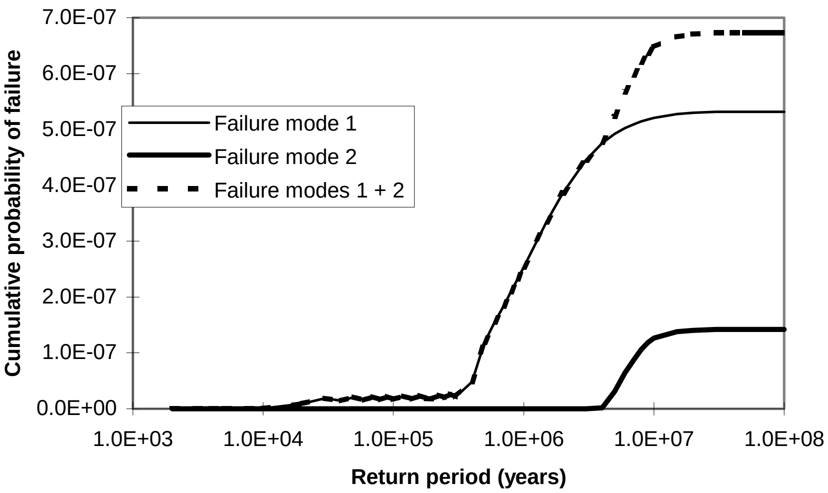

Figure 4: Cumulative probability function for slender shear wall

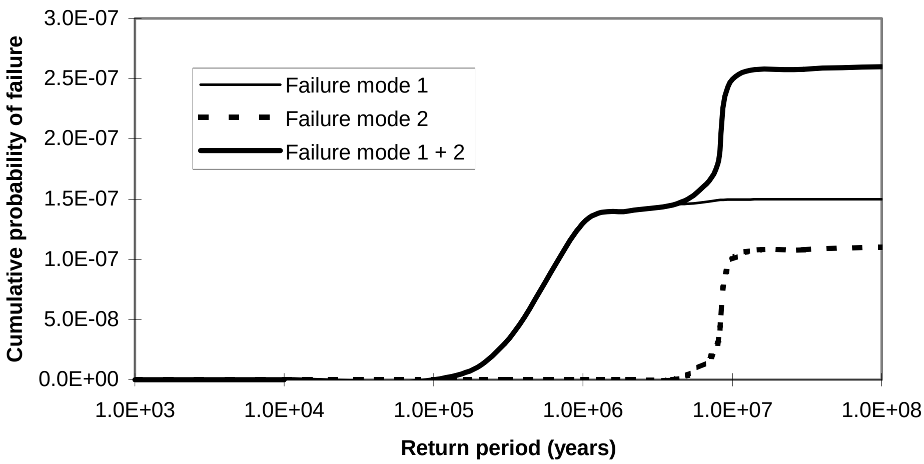

Figure 5: Cumulative probability function for hypothetical structure

However, were the CPF for failure mode 1 to be modified to the hypothetical case shown in Figure 5, then a 'cliff edge' by this definition would indeed have occurred

The practical value of identifying such a cliff edge is less clear (and it may be noted would not be revealed by the Cornell method, since this does not involve direct calculation of the CPF as an intermediate stage). What really matters is that all failure modes which make a significant contribution to failure should be identified. Analyses which do not consider intensities of motion sufficiently large to trigger all such failure modes are clearly deficient. The present case of strain distribution in plastic hinges is one possible example. Others might be critical intensities which triggered liquefaction in soils, P-delta instabilities in structures or brittle failures of any type. The approximate Cornell method is highly suitable, both for establishing whether a particular failure mode is significant or not (ie whether it contributes significantly to overall failure probability) and for checking out design measures intended to ensure that a particular failure mode is not significant. In most cases, failure modes with an annual probability of failure less than 10^{-7} can be neglected; in the context of the nuclear industry, this is likely to be the case for failures leading to large releases of radioactivity under the UK Health & Safety Executive's Safety Assessment Principles (HSE, 1991). In Eurocode 1 (CEN 1996), the threshold annual probability of failure is 10^{-6}. It may then be observed that a 'significant' failure mode cannot be associated with a return period greater than 10^{7} (for HSE) or 10^{6} (for EC1). Failure modes triggered by ground motion intensities with longer return periods can therefore be neglected.

CONCLUSIONS

Cornell's approximate method for calculating structural reliability under seismic loading has been validated for two simple structures. The method shows great promise, particularly for areas of low seismicity (such as the UK) where it will be most accurate. The advantage of the method is that it can be based on results from the analysis of a single structural model, with structural properties set at their mean (best estimate) properties. More rigorous methods require a much larger number of analyses with the structural properties set at different values. Use of Cornell therefore means that computational effort can be directed towards modelling the true non-linear behaviour of a structure, which is likely to be important if ultimate failure is the limit state under consideration.

On 'cliff edge effects' (ie sudden changes in response for loading intensities beyond the design basis), it was concluded that rather than identifying cliff edges, it is more important to ensure that all significant modes of failure are accounted for in analysis. Cornell's method is highly suitable for establishing this. In particular, the ground motion intensity considered in design and analysis must be great enough to trigger all such modes of failure, and Cornell can be used both for checking whether this is the case, and for investigating design measures intended to prevent potentially troublesome failure modes from becoming significant.

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