SURFACE WAVE MODELLING USING SEISMIC GROUND RESPONSE ANALYSIS

E John MARSH[1] And Tam J LARKIN[2]

SUMMARY

This paper presents a study of surface wave characteristics using a two dimensional nonlinear seismic ground response analysis programme. The characteristics of both Love waves and Rayleigh waves are studied. Varying depths of alluvial basins, material properties and input motion strain levels are used in the analyses to study the properties of the surface waves in the nonlinear medium. The results of the nonlinear analyses are compared to the surface wave properties predicted by elastic wave theory.

Surface wave characteristics in the nonlinear medium generally agreed well with elastic theory. Modelling of wave modes, phase velocities and displacement profiles was encouraging. The close agreement of the analyses results to elastic theory is believed to be due to the nature of the surface wave propagation and the form of input motion adopted. Phase velocities in the higher strain analyses suggested some strain softening of the shear modulus in the nonlinear model. Further study of surface wave characteristics from earthquake input motions may be beneficial.

INTRODUCTION

A two dimensional nonlinear seismic ground response analysis programme, TENSA, has previously been used by the authors to investigate the behaviour of alluvial basins. The analysis includes both the out-of-plane (SH) and in-plane (PSV) two dimensional solution spaces for the total stress (undrained) case.

In both analyses the soil is modelled as a nonlinear hysteretic material using an incremental plasticity model for the stress - strain properties. The hysteresis loops are modelled using a hyperbolic model for the backbone curve. The data required for this rheological formulation is the low strain shear wave velocity, the low strain compression wave velocity, the density and the shear strength. Both programs are modified versions of the original programs written by Joyner (1975). More detailed information is available in Larkin and Marsh (1991), Marsh (1992) and Marsh et. al. (1995).

The purpose of this work is to study the characteristics of surface waves generated in an alluvial basin configuration using the two dimensional nonlinear analysis. The geometry of the basins, the material properties and the strain level of the input motion are all varied to study the behaviour of the surface waves. The results of the analyses are compared to the surface wave properties and behaviour predicted by elastic wave theory.

ALLUVIAL BASIN MODEL

The meshes used to model the alluvial basin configuration have a length to height (L/H) ratio of 8, and central depths of both 100 m and 500 m. The shallow basin has a nodal spacing of 10 m, and the deep basin a nodal spacing of 20 m. An example mesh is shown in Figure 1.

[1] *Senior Geotechnical Specialist, Beca Carter Hollings & Ferner Ltd, PO BOX 6345, Auckland, New Zealand*
[2] *Civil and Resource Engineering Department, University of Auckland, Private Bag, Auckland, New Zealand*

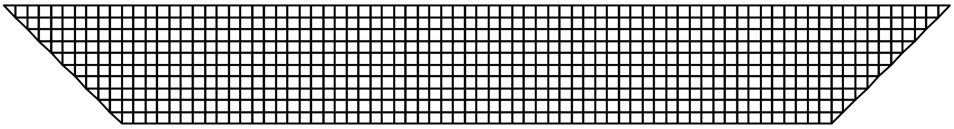


Figure 1 Example Basin Mesh Configuration

For material properties, the shear wave velocity in the underlying bedrock is 3000 m/s, and the compression wave velocity is 6870 m/s. In the alluvial deposit the shear wave velocities are constant throughout the basin, but are varied from 50 m/s to 400 m/s to investigate the generation of surface waves in uniform basins with different velocities. The ratio between the compression wave velocity and the shear wave velocity in the alluvium is 5.1 in all cases, corresponding to a Poisson's ratio of 0.48.

The input motion used is a simple displacement pulse input. This form of input motion is used so that the generation of surface waves can be studied easily without the complications of more detailed input motions. A record duration of 60 seconds is used to study the surface waves reflecting from side to side across the basins. Two levels of excitation are used to investigate the variation in surface waves developed at different levels of strain. This will take into account softening of the material shear modulus and varying levels of hysteretic damping. The high strain level is equivalent to a peak input acceleration of 2.4525 m/s^2 or 0.25g, and the low strain level used is equivalent to a peak input acceleration of 0.6131 m/s^2 or 0.0625g.The input motion is applied to the left hand sloping shoulder of the basin. This results in surface waves propagating initially from the left edge of the basin ground surface.

BACKGROUND TO SURFACE WAVES

Surface wave theories were developed by early authors, such as Lord Rayleigh and Love, in an attempt to explain large disturbances found in recorded seismograms which did not conform to the theories of either longitudinal or distortional waves. In an unbounded elastic solid the only two forms of waves possible are these two types of waves. However when solving the wave equation with the condition of a free surface, in-plane surface waves were found to exist which decayed rapidly with depth. These were termed Rayleigh waves. This theory could not however explain the transverse waves visible in the seismograms. It was postulated that these waves were a result of disturbances being trapped in a superficial layer in which the shear velocity is less than that of the underlying material. If the wave equation is solved for these conditions, it is found that there can exist a transverse surface wave in the superficial layer if the shear velocity there is less than that in the underlying material. These were termed Love waves.

Love Waves

To investigate Love waves, the elastic wave equation is solved for the out-of-plane component of motion. The boundary conditions applied are for zero stress at the free surface. The phase velocity of the Love wave is dependent on the wavenumber, and so Love waves are said to be dispersive. The phase velocity is not independent of frequency. The approximate solution for the phase velocity of the various Love wave modes is,

$$\frac{2\pi H}{\lambda}\left(\frac{c^2}{V_1^2} - 1\right)^{\frac{1}{2}} = \frac{\pi}{2}, \frac{3\pi}{2}, \frac{5\pi}{2} \ldots. \qquad (1)$$

where c is the phase velocity of the Love wave, V_1 is the shear wave velocity of the superficial layer, λ is the wavelength, and H is the depth of the layer. For example for the fundamental Love wave mode, upon rearranging to give the phase velocity of the Love wave in terms of the frequency of the Love wave f,

$$\frac{1}{c^2} = \frac{1}{V_1^2} - \frac{1}{16 H^2 f^2} \qquad (2)$$

Therefore for given values of V_1, H and f the possible existence of each Love wave mode can be determined. The variation with depth of the amplitude of the Love waves is dependent on the particular mode of the surface wave. There are no nodes in the variation within the layer for the fundamental mode, and the amplitudes decay with distance from the free surface. There is a single node in the variation with the first higher mode, and so on.

Rayleigh Waves

To investigate Rayleigh waves, the elastic wave equation is solved for in-plane horizontal and vertical components of motion for a semi-infinite medium bounded by a free surface. For a non-zero solution Rayleigh's function is derived for the phase velocity of the Rayleigh wave,

$$R(c) = \left(2 - \frac{c^2}{\beta^2}\right)^2 - 4\left(1 - \frac{c^2}{\alpha^2}\right)^{\frac{1}{2}}\left(1 - \frac{c^2}{\beta^2}\right)^{\frac{1}{2}} = 0 \tag{3}$$

where β is used here for the shear wave velocity of the elastic medium and α is the compression wave velocity. The phase velocity of the Rayleigh wave in this semi-infinite medium does not depend on the frequency of the wave, and so there is only one mode of Rayleigh waves and it is non-dispersive.

For any values of β and α Rayleigh's function can be solved and roots for c^2/β^2 found. However to be considered a surface wave the solutions for c also have to satisfy the requirement that the displacements corresponding to this value of c decay with distance from the free surface. For example if the material is incompressible, that is if $\nu = 0.5$, there is only one real root, which also satisfies the surface wave conditions, giving $c/\beta = 0.9553$. Alternatively if Poisson's relation is assumed to hold for the material, $c/\beta = 0.9194$. For the material properties used in the analyses presented in this chapter, $\alpha = 5.1\ \beta$, and the solution to Rayleigh's function for a surface wave gives a velocity ratio of $c = 0.95\beta$.

The surface displacements can be calculated for a surface particle. These give the well known retrograde elliptical motion associated with Rayleigh waves. The displacement of the Rayleigh wave decreases with depth, and there is a node where the sign of the horizontal displacement changes at a depth of 0.142 of the wavelength of the surface wave. In the vertical component of motion the displacement first increases to a maximum at a depth of 0.134 of the wavelength, and then decreases with depth.

All the above theory has been developed for a semi-infinite elastic half-space. For the case of a uniform alluvial basin overlying bedrock, the configuration is more readily compared to a single superficial layer on top of an elastic half-space, as it was treated for the case of transverse Love waves. With this configuration the in-plane elastic wave equation can be solved again. The phase velocity of the Rayleigh wave now depends on the wavenumber, and so the Rayleigh waves with this superficial layer are dispersive. Higher modes than the fundamental Rayleigh wave mode may also now exist. At the short period range of the dispersion curves the phase velocity of the fundamental mode approaches that of the simple Rayleigh wave velocity in the superficial layer. The phase velocity of all other modes approaches the shear wave velocity in the superficial layer.

RESULTS OF ANALYSES

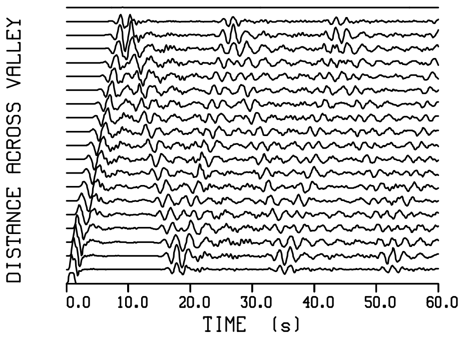
Love Wave Characteristics

The SH solution was used to investigate the properties of Love waves at varying strain levels in the nonlinear material. The alluvial basin acts as the low-velocity layer on top of the elastic bedrock half-space necessary for the generation of Love waves. Love waves are dispersive, and therefore their phase velocity depends on the frequency of the wave itself. The frequency of the Love waves produced by the nonlinear analyses was measured. Combining this with the material and physical properties enabled calculation of the theoretical phase velocity of the surface wave in an elastic medium. Higher modes of Love waves may also be predicted from elastic theory. The presence of these modes in the nonlinear solution can be ascertained from the phase velocities of the surface waves generated and the variation of the displacements with depth.

The results were obtained for both the shallow basin and the deep basin. Analyses were performed at the two strain levels. The results are displayed in the form of displacement time histories across the surface of the basin. The plots of the variation in displacement with depth can be used to determine the modal content at a particular moment in time. For each case the best estimate of the measured phase velocities, c_m, are compared with the velocities predicted by elastic theory, c, for the estimated wavelength of the surface wave. The surface waveforms may be a non-harmonic function, and the wavelength is calculated from the estimated peak-to-peak period of the motion. For the shallow case basins were studied with shear wave velocities of 50 m/s, 100 m/s and 200 m/s. With the deep basin velocities of 100 m/s, 200 m/s and 400 m/s were used.

Shallow Basin Analyses

The displacement time histories across the surface of the basin with a shear wave velocity of 100 m/s at the low strain level are displayed in Figure 2. The frequency of the surface wave is measured as 0.43 Hz, and only the fundamental Love wave mode is theoretically possible. The calculated phase velocity is 122 m/s, and the velocity measured from Figure 2 is 118 m/s. Only the fundamental mode is visible in the surface displacements. The variation in displacement with depth of the surface wave when it passes the centre of the basin is shown in Figure 3. The fundamental mode of vibration is evident.



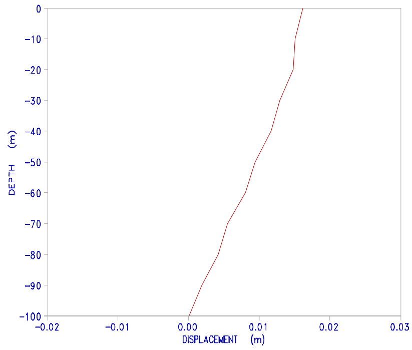


Figure 2 Surface Displacement Time Histories

Figure 3 Variation in Displacement with Depth

Similar results are obtained for the other shear wave velocities considered, and for the higher strain level. The displacement time histories for a shear wave velocity of 50 m/s at the low strain level are shown in three dimensional form in Figure 4.

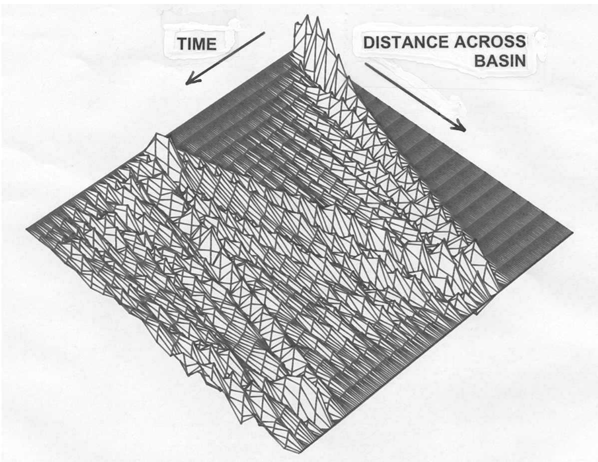


Figure 4 Surface Displacement Time Histories

The predicted and measured phase velocities for the fundamental Love waves in the shallow basin are presented below in Table 1 for both low strain and high strain analyses. All velocities are in m/s.

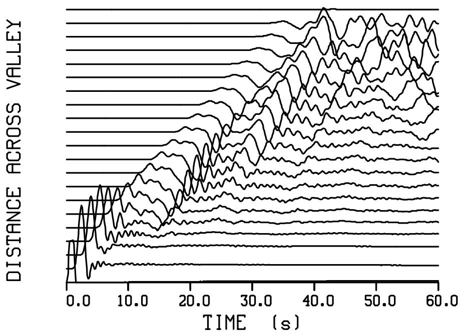
Table 1

	Low Strain			***High Strain***	
Basin V1	*Theoretical Cf*	*Measured Cmf*		*Theoretical Cf*	*Measured Cmf*
50	54	53		54	51
100	122	118		122	106
200	-	240		295	208

Deep Basin Analyses

The displacement time histories across the basin with a shear wave velocity in the deep basin of 100 m/s are displayed in Figure 5 for the high strain level. The measured frequency of the surface wave is 0.38 Hz. With this value and the physical and material properties, the fundamental and first and second higher Love wave modes are possible. The theoretical values of the three phase velocities are c_f = 101 m/s, c_1 = 109 m/s and c_2 = 133 m/s. The three modes are visible in Figure 5, and they give measured phase velocities corresponding to the theoretical values of 95 m/s, 118 m/s and 141 m/s respectively. The convergence of the modes is evident. The variation of displacement with depth of the three modes from the same analysis is shown in Figure 6 as the surface waves pass the centre of the basin at different times. The mode shapes of the first three modes of vibration are clear.

Similar results are obtained for the other shear wave velocities considered, and at both strain levels. The displacement time histories with a shear wave velocity of 100 m/s at the low strain level are shown in three dimensional representation in Figure 7. Further results of the Love wave analyses are presented in Marsh (1992).



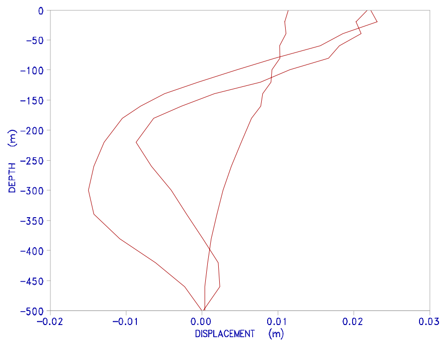


Figure 5 Surface Displacement Time Histories

Figure 6 Variation in Displacement with Depth

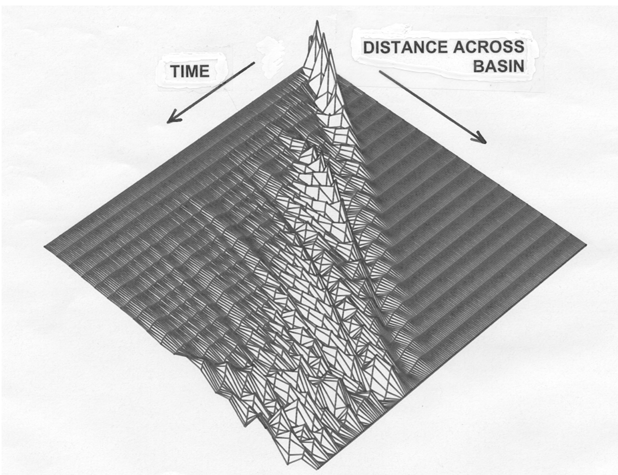


Figure 7 Surface Displacement Time Histories

The theoretical and measured phase velocities of the various surface wave modes for the deep basin are presented in Table 2 for both the low strain analyses and the high strain analyses. All velocities are in m/s.

Table 2

Theoretical				*Lo*	*Low Strain Analyses*				*High Strain Analyses*		
V1	*Cf*	*C1*	*C2*		*Cmf*	*Cm1*	*Cm2*		*Cmf*	*Cm1*	*Cm2*
100	101	109	133		98	112	138		95	118	141
200	207	326	-		200	290	-		199	253	-
400	470	-	-		450	-	-		419	-	-

Discussion of Results

The effect on the surface waves of varying the strain level of the input motion is not large. The initial displacements are larger with the high strain level, but these displacements rapidly decay and the form of the displacements is similar for both strain levels. The measured surface wave phase velocities at the two strain levels are similar, and agree well with elastic theory. With a nonlinear analysis the main effect of varying strain levels is the softening of the shear modulus of the material, and hence a reduction in the shear wave velocity used in the analysis. It appears that there is a general reduction in the phase velocities of the surface waves in the nonlinear analyses with the higher strain level. This implies that the reduction of the shear modulus in the nonlinear analysis may have an effect on the velocity of propagation of the surface waves. This is not always the case, and the effect is more noticeable with the higher values of the shear wave velocity in the alluvial basin. This is discussed in more detail in the conclusions section.

Rayleigh Wave Characteristics

The PSV solution was used to investigate the properties of Rayleigh waves in the nonlinear material. For an elastic half-space only a fundamental mode Rayleigh wave exists, and this wave is non-dispersive. With a single layer of material overlying a half-space, higher modes of Rayleigh waves are possible, and the waves are now dispersive. The velocities of the modes vary between the bounds of the non-dispersive Rayleigh wave velocity in the upper medium to the shear wave velocity in the half-space.

Results were obtained for the two basin depths and strain levels as before. The results are presented as displacement time histories across the surface of the basin. The variation in displacement with depth is investigated to check the horizontal and vertical attenuation of Rayleigh waves with depth. Polar plots of vertical displacements against horizontal displacements were used to examine the individual particle motion. The range of material properties used is the same as with the Love wave work presented in the previous section.

Shallow Basin Analyses

The horizontal and vertical displacement time histories across the basin with a shear wave velocity of 50 m/s and a compression wave velocity of 255 m/s are shown in Figures 8 and 9 for the low strain level. Both compression waves and Rayleigh waves are visible in the two plots. The Rayleigh waves are clearest in the plot of vertical displacements, as the vertical component of the Rayleigh wave is greater than the horizontal component. The measured phase velocity of the Rayleigh wave is 45 m/s, compared to the theoretical non-dispersive value of 47.5 m/s for a uniform half-space. The surface time histories are complicated by other reflected surface waves. For example Rayleigh waves are produced when the surface compression wave reflects from the boundary.

Higher modes of Rayleigh waves are not obvious, possibly because of the single frequency of the input motion. The phase velocity of the surface waves visible suggests that they are fundamental mode Rayleigh waves.

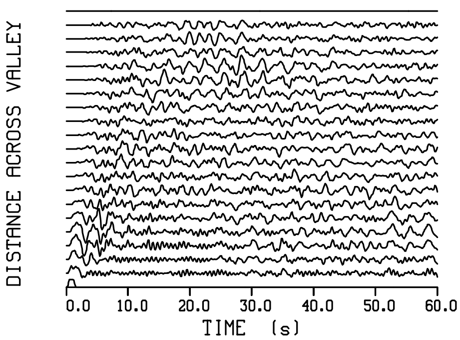


Figure 8 Horizontal Displacement Time Histories

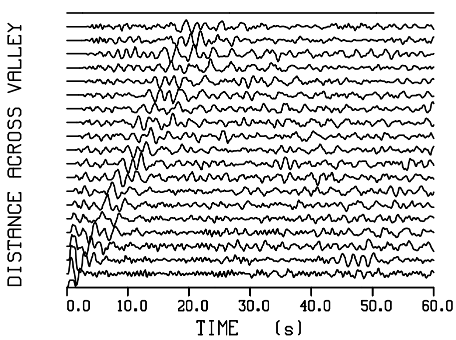


Figure 9 Vertical Displacement Time Histories

The variation in vertical displacement with depth as the surface wave passes the centre of the basin is shown in Figure 10. For a frequency of 0.38 Hz the peak vertical displacement should theoretically occur at a depth of 16m, and decrease with depth from there. The peak in Figure 10 is seen to occur at approximately 12 m, from where the amplitude decreases with distance from the free surface. This compares well with the elastic theory. Similar results are obtained for the other shear wave velocities considered, and for the higher strain level.

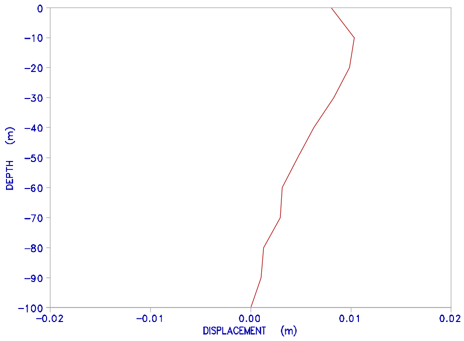


Figure 10 Variation in Vertical Displacement with Depth

Deep Basin Analyses

The horizontal and vertical surface displacement time histories with a shear wave velocity of 100 m/s and a compression wave velocity of 510 m/s for the deep basin are shown in Figures 11 and 12 respectively for the low strain level. The horizontal compression wave travelling across the surface of the basin is clear, with a measured velocity of 507 m/s. The Rayleigh wave is also evident, with a measured phase velocity of 94 m/s. This compares closely with the non-dispersive Rayleigh velocity of 95 m/s, and the shear wave velocity of 100 m/s in the alluvial material. A Rayleigh wave is also seen when the compression wave reflects from the boundary.

Similar to the shallow basin analyses, plots of the variation in horizontal and vertical displacement with depth agreed well with elastic theory. They displayed a peak displacement at a shallow depth below ground surface, decreasing in depth from there. Also the horizontal displacements were seen to change sign at shallow depth as predicted. Polar plots of the vertical and horizontal displacement were investigated to look at the particle motion of the waves. These confirmed the retrograde elliptical motion of the Rayleigh and the horizontal nature of the compression waves. Further results of the Rayleigh wave analyses are presented in Marsh (1992).

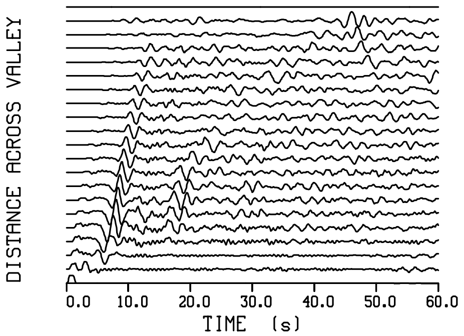


Figure 11 Horizontal Displacement Time Histories

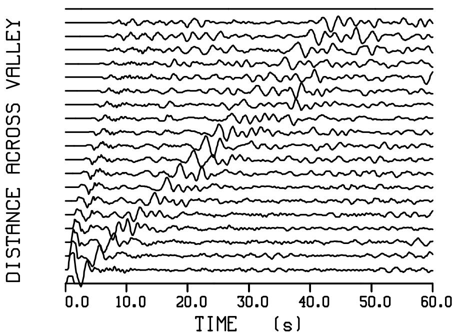


Figure 12 Vertical Displacement Time Histories

Values of the measured surface wave velocities for both depths of basin at both strain levels are presented in Table 3. The values all suggest, that the velocities of the surface waves are limited by the non-dispersive Rayleigh wave velocity and the shear wave velocity of the alluvial basin. All velocities are in m/s. V_1 refers to the shear wave velocity in the alluvium, and V_R to the non-dispersive Rayleigh wave velocity in the alluvium.

Table 3

		Shallow Basin		***Deep Basin***	
V1	*VR*	*Cm Low*	*Cm High*	*Cm Low*	*Cm High*
50	47.5	45	51	-	-
100	95	87	97	94	95
200	190	220	220	190	189
400	380	-	-	392	400

Discussion of results

As observed with the transverse Love waves, the effect of varying strain level on the characteristics of the Rayleigh waves is not large. In both cases the measured phase velocities of the surface waves are in close agreement with each other and with elastic theory. The modes of the Rayleigh waves are sometimes different with the higher strain level. It is clear from both sets of analyses that all the surface waves identified have phase velocities close to the velocities of the upper medium. In all cases the variation in displacement with depth, and the particle motions agreed well with elastic theory.

CONCLUSIONS

In general the Love wave characteristics in the nonlinear medium agreed well with elastic theory. At both strain levels the number of modes predicted by elastic theory existed in each case in the nonlinear analyses. The phase velocities were also in agreement with elastic theory, within the limitations of measuring their frequency from the time histories and using this value to calculate the phase velocities. The displacement profiles displayed the modes of vibration clearly. The reason for this may be that the propagation of the surface waves across the alluvial basin involves the first arrival of the wave front, and therefore the low strain elastic properties are appropriate. Also with the displacement pulse input, the response is only in the nonlinear range of the material for a short duration, and so the majority of the analysis is restricted to the linear range of behaviour. There may therefore be little strain-softening of the shear modulus. However it was apparent that the phase velocities of the Love waves in the nonlinear medium were reduced in the high strain analyses, suggesting that some reduction of the shear modulus at high strains may effect the phase velocity of the surface waves. This effect was most noticeable with the higher values of the shear wave velocity in the alluvial basin.

The characteristics of the Rayleigh waves in the nonlinear medium also agreed well with the elastic theory. At both strain levels only one mode of Rayleigh wave was detectable, and the phase velocity of these waves was close to the velocities corresponding to the upper medium. This may again be due to the propagation of the surface waves involving the low strain properties of the material. Some of the displacement profiles suggested that the surface waves were of the first higher mode of vibration. The retrograde elliptical particle motion of the Rayleigh waves was displayed. Compression waves were also seen to travel across the surface of the basin, and on reaching the edge of the basin reflect and create further Rayleigh waves. The effect of reduction of shear modulus on the propagation of Rayleigh waves was less apparent than that observed with the Love waves.

The modelling of surface waves was encouraging, and some effects of nonlinear strain softening were apparent. A study of recorded surface waves from earthquake input motions with the nonlinear analysis may provide better understanding of the effects of nonlinear strain softening behaviour on surface wave propagation.

REFERENCES

Joyner, W.B. (1975) *A method of calculating nonlinear seismic response in two dimensions,* Bulletin of the Seismological Society of America, Vol 65, 1337.

Larkin, T.J. and Marsh, E.J. (1991) *Two dimensional nonlinear site response analyses,* Proceedings Pacific Conference on Earthquake Engineering, Vol. 3, 217.

Marsh, E.J. (1992) *Two dimensional nonlinear seismic ground response studies,* Ph.D Thesis, University of Auckland.

Marsh, E.J., Larkin T.J., Haines, A.J. and Benites, R.A. (1995) *Comparison of linear and nonlinear seismic responses of two dimensional alluvial basins,* Bulletin of the Seismological Society of America, Vol. 85, No. 3, 874.