TILTING FAILURE OF RETAINING WALLS INCLUDING P-DELTA EFFECT AND APPLICATION TO KOBE WALLS

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SUMMARY

The purpose of the research described in this paper is to generalize the Richards and Elms (1979) procedure for prediction of seismically induced permanent displacement of retaining walls for the case of mixed sliding and rotation modes. A simplified mathematical model and modified computational method based on the work of Siddharthan et al (1992) is presented. In the model presented in this paper seismic reduction of bearing capacity is included in the Siddharthan approach; and the P-Δ effect, and corresponding magnification of displacements is also considered in the analysis. Initially, the model is applied to a simple problem to demonstrate the concepts and application. Finally, as a check on the validity of the analysis, retaining walls located in Kobe, Japan are investigated using an available time history of acceleration record from the Hyogoken-Nambu Earthquake as input. The results from the calculation are in good agreement with the observed behavior of the Kobe walls.

INTRODUCTION

Seismic Lateral Thrusts

It is assumed that there is full mobilization of the shear strength of the backfill by wall movement sufficient to induce active earth pressures. For walls that fail by sliding and/or rotation with respect to the base the Mononabe-Okabe equations are employed to compute the thrust from lateral earth pressure as:

$$P_{AE} = \frac{1}{2} K_{AE} \gamma H^2 (1 - k_v) \tag{1}$$

where

$$K_{AE} = \frac{\cos^2(\varphi - \theta - \beta)}{\cos\theta \cos^2\alpha \cos(\delta - \alpha + \theta)]^2 \left[1 + \sqrt{\dfrac{\sin(\varphi + \delta)\sin(\varphi - \theta - i)}{\cos(\delta + \alpha + \theta)\cos(i - \alpha)}}\right]^2} \tag{2}$$

and $\theta = \tan^{-1}(k_h/(1-k_v))$, γ = unit weight of the soil, k_h = the constant horizontal acceleration /g, and k_v = vertical acceleration/g, ϕ is inclination of backfill, and other terms are defined as shown in Figure 1. A number of methods exist to determine the line of action of the active thrust. However, for the sake of simplicity, the

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assumption adopted by Siddharthan et al (1992) that the seismic active thrust acts at 0.5H above the base will be used unless stated otherwise.

Vertical Equilibrium (Bearing Capacity)

In this paper the general formulation for seismic bearing capacity described by Shi and Richards (1995) is applied to gravity retaining walls. The seismic degradation of bearing capacity is shown to primarily depend on two factors related to earthquake acceleration and lateral thrust applied to retaining wall systems: (a) shear tractions at the foundation-soil interface and (b) the inertia driven lateral body forces within the foundation soil. For simplicity, a "Coulomb-type" of failure mechanism is considered consisting of an active wedge directly beneath the retaining wall and a passive wedge that provides lateral restraint with a hypothetical friction angle between them of $\phi/2$. With the basic mechanism established the seismic reduction of bearing capacity can be derived from equilibrium. The corresponding seismic bearing capacity equation is expressed as:

$$p_{LE} = cN_{cE} + qN_{qE} + \frac{1}{2}\gamma BN_{\gamma E} \tag{3}$$

Here c is the soil cohesion, q is the surcharge, and B is the width of the footing. The bearing capacity factors N_{cE}, N_{qE}, and $N_{\gamma E}$ are dependent on the foundation-soil friction angle, ϕ_b, and the friction factor, f, which describes the shear traction applied to the foundation soil as:

$$f = \frac{S}{k_h P_v} \tag{4}$$

where S = resultant of shear traction and P_v = the vertical component of the reaction at the base of the wall. Shi and Richards (1995) presents charts depicting ratios of N_{cE}/N_{cS}, N_{qE}/N_{qS}, and $N_{\gamma E}/N_{\gamma S}$ where the "S" subscript represents the bearing capacity factors with S and k_h equal to zero.

COUPLED EQUATIONS OF MOTION FOR SLIDING AND TILTING

Based on the work of Nadim (1980), Siddharthan et al (1992) proposed a method for investigating both the sliding and tilting modes of rigid wall deformation. Siddharthan's research developed a rigid- plastic model leading to two equilibrium equations. If there is no rotation, his fundamental equilibrium equations uncouple and reduce to the Richards-Elms method (Richards and Elms, 1979). If only the rotation motion is considered and the center of rotation is located at the toe of the retaining wall, Siddharthan's moment equilibrium equation reduces to the method of Steedman and Zeng (1996).

The response of the wall is given in terms of wall translation, x, and rotation, θ, about the center of rotation (CR), which is assumed to be located along the base of the wall. Based on the free-body-diagram shown in Figure 1 the equations for different modes of motion can be written as:

1. Sliding (horizontal equilibrium):

$$\frac{W}{g}(\ddot{x}) + \left[\frac{WR\sin(\phi_b+\eta)}{(g)\cos(\phi_b)}\right]\ddot{\theta} + \frac{W}{g}R\dot{\theta}^2\cos\eta = Wk_h + P_{AE}\cos(\alpha+\delta_A) - P_{PE}\cos(\beta-\delta_p) - P_v\tan(\phi_b) \tag{5}$$

2. Settlement (vertical equilibrium)

$$P_v = W - k_v W + P_{AE}\sin(\delta_A + \alpha) + P_{PE}\sin(\beta - \delta_p) - \frac{W}{g}R\dot{\theta}^2\sin\eta \tag{6}$$

3. Rotation (moment equilibrium)

$$\begin{aligned}\frac{W}{g}R\sin(\eta)\ddot{x} + \left(I_{cg} + \frac{WR^2}{g}\right)\ddot{\theta} &= k_h WR\sin(\eta R - \frac{W}{g}R\cos(\eta\cos - k_v g] + P_{AE}(mH)\cos(\alpha+\delta_A) - P_{AE}\sin(\alpha+\delta_A)[R\cos(\eta) \\ &+ a - mH\tan(\alpha)] - M_{yo} - P_{PE}(nD)\cos(\beta - \delta_P)[B - R\cos(\eta) - a - nD\tan(\beta)]\end{aligned} \tag{7}$$

Here CG is the center of gravity of the wall; R = distance from CR to CG; I_{cg} = mass moment of inertia about the CG, g is the gravitational acceleration constant; $\ddot{X}_g(t)$ and $\ddot{Y}_g(t)$ are horizontal and vertical of ground motion; δ_A = the active wall-backfill friction angle and δ_P = the passive wall-backfill friction angle. The foundation soil friction angle is ϕ_b,, and M_{yo} is the soil moment resistance.

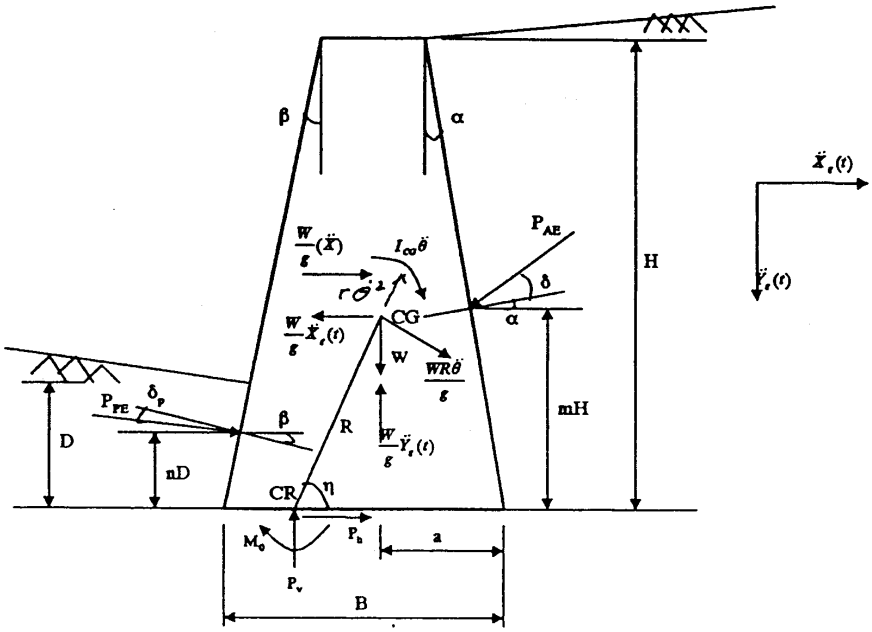

Figure 1. Free Body Diagram of Retaining Wall

Tilting With P-Δ Effects

If the vertical resultant reaction at the base of the wall, P_v, lies outside the middle third of the wall footing, then the heel may lift off as the wall rotates with a local bearing failure at the toe. Each increment of rocking makes the overturning moment more critical as the center of gravity of the wall moves over the toe, and the seismic thrust, P_{AE}, becomes more horizontal. Thus, there is the potential for a large P-Δ effect. In fact if rotation of the wall is large enough to cause the line of action of the wall weight to contribute to the overturning moments, the wall will become unstable and overturn.

Neither Siddharthan et al (1992) nor Steedman and Zeng (1996) consider the P-Δ effect. Referring to Figure 1, the P-Δ effect can be included by using the following simplified procedure. After each time step, if the wall rotates, calculate the increment of the rotation angle, Δθ. Then modify the wall geometry as follows:

$$\eta_{new} = \eta_{old} + \Delta\theta \tag{8a}$$
$$\alpha_{new} = \alpha_{old} + \Delta\theta \tag{8b}$$
$$\beta_{new} = \beta_{old} - \Delta\theta \tag{8c}$$

Threshold Acceleration

The threshold acceleration is that level of acceleration beyond which the resistance of the wall in terms of sliding, overturning or bearing capacity is overcome and permanent seismic induced deformations accumulate. The initial threshold acceleration and corresponding mode of deformation may be determined by evaluating Equations (5), (6) and (7) with the terms for relative and angular motion of the wall set equal to zero. The threshold acceleration for sliding k_h^{slide} , and for tilting k_h^{tilt} are determined by finding the ground acceleration that satisfies Equations (5) and (6), receptively. The threshold acceleration for overturning, $k_h^{O.T.}$, is the ground acceleration that satisfies Equation (7) with the center of rotation at the toe of the wall and M_{yo} equal to zero. The lowest of the three computed threshold accelerations k_h^{slide} ,k_h^{tilt}, $k_h^{O.T.}$ defines the initial threshold acceleration and the mode of deformation.

Subsequent modes of deformation may occur at ground acceleration in excess of the initial threshold. Subsequent threshold accelerations are determined by satisfying Equations (5) , (6), or (7) with the appropriate wall motion included. It should be noted that during the sliding mode of deformation neither the wall nor the backfill within the failure wedge behind the wall may sustain accelerations beyond the threshold value. Therefore, both the forces k_h^{slide} (W) and P_{AE} remain constant during sliding. Based on this, if sliding occurs first, the overturning mode of failure cannot subsequently occur. However, a bearing capacity (tilting) failure may occur due to the seismic inertial response of the foundation soil.

At each excursion of the ground acceleration beyond the threshold value the wall will undergo increments of seismic induced permanent deformation which may be determined by numerical integration of Equations (5), (6), and (7).

SIMPLE EXAMPLE PROBLEM

To illustrate the concepts, the analysis is applied to the simple retaining wall shown in Figure 2. For this example there is no passive restraint at the toe. First, assuming that there is no rotation or vertical movement of the wall, $k_h^{slide} = 0.2$, is determined from Equation (5) with $P_v = W$ and $\ddot{x}, \ddot{\theta}, \text{and } \dot{\theta}$ equal to zero. Using Equation 7, and assuming m = 0.5 and the point of rotation at the toe of the footing such that $R\sin(\eta) = H/2$, and $R\cos(\eta) = B/2$, $k_h^{O.T.}$ is computed as 0.16.

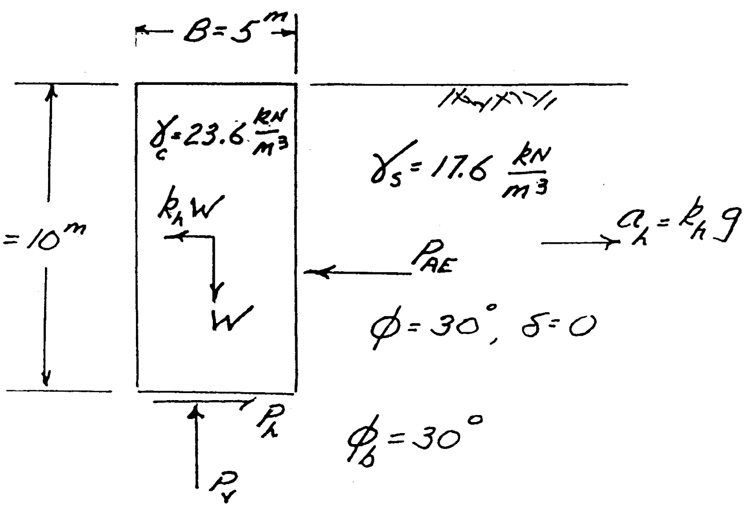

Figure 2. Retaining Wall for Simple Example Problem

To determine k_h^{tilt}, the bearing capacity factor must be determined, which depends on k_h and the shear transfer coefficient. For our simple case

$$f = \frac{P_h}{k_h P_v} = \frac{k_h W + P_{AE}}{k_h W} = 1 + \frac{P_{AE}}{k_h W} \tag{9}$$

Assuming $k_h = 0.1$, then from Equations (1) and (2) $P_{AE} = 350$ kN; and with W = 1180 kN, f is computed as 4.15. Figure 3 presents the ratios of $N_{\gamma S}/ N_{\gamma E}$ versus k_h for a variety of friction factors from which, with f = 4.15 and $k_h = 0.1$, a ratio of approximately 4 is obtained. According to Vesic (1972), for $\phi_b = 30°$, $N_{\gamma S} = 22$ and therefore $N_{\gamma E} \approx 5.5$. The seismic bearing capacity is computed as $p_{IE}B = 0.5\gamma B^2 N_{\gamma E} = 0.5(17.6)(5)^2(5.5) = 1210$ kN which is approximately equal to $P_v = W = 1180$ kN. For the example problem $k_h^{tilt} = 0.1 < k_h^{O.T.} < k_h^{slide}$. Thus, the initial threshold acceleration is 0.1g, and the initial mode of failure is by loss of bearing capacity and corresponding tilting of the retaining wall.

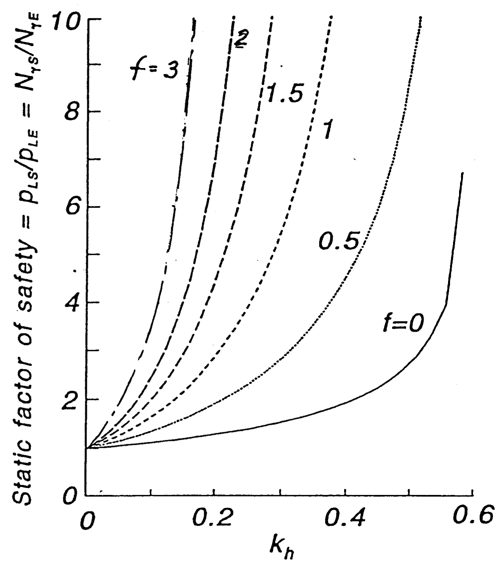

Figure 3. Bearing Capacity Factors for $\phi_b = 30°$

To calculate the displacement of the wall once k_h exceeds 0.1, assumptions must be made concerning the position of the vertical reaction, P_v, and its magnitude; the induced vertical acceleration, $\ddot{Y}$, and the position of the center of rotation, CR. As shown by Richards et al (1990), when bearing capacity is lost, the wall will move at constant velocity like a body in a viscous fluid obeying Stokes Law. The vertical motion in a strong earthquake is almost entirely due to *viscous flow*. This is because for high friction factors the difference between initial yield, or initial fluidization, and general fluidization is small.

Experiments on heavy cylinders settling in seismically fluidized, dry sand (Richards et al., 1990) gives a constant viscosity of $\mu \approx 0.14$ (N)sec/mm^2 . To estimate the constant settlement velocity for a long wall one can assume that the flat contact surface captures a half-cylinder of immobile sand as it settles and rotates. From Stokes Law $v \approx F/6\mu = 1180/6(.14) = 0.7$ m/sec decelerating as buoyancy and viscous drag on the vertical walls reduce the driving force. Thus, $\ddot{Y}$ is small and P_v is equal to W even though the bearing capacity has been exhausted.

For the simple block wall at $k_h = 0.1$, $P_h = P_{AE} + k_hW = 350 + 118 = 468$ kN, $P_v = 1180$ kN; so the eccentricity of P_v with respect the center of the wall footing, e, equals 1.98 for moment equilibrium. Assuming the center of rotation is at e =1.98, apply Equation (7) with $\ddot{x} = 0$, $I_{cg} = 10.4(W/g)$, and the step function for ground acceleration shown in 4(a) as input, then $\ddot{\theta} = 0.634$ for the first half cycle of ground motion.

At $t = t_1 = 0.5$ sec the acceleration reverses direction and the horizontal inertia force combines with the weight to slow the rotation to zero. Using Equation (7) with the term $k_hWR\sin\eta$ of the opposite sign renders $\ddot{\theta} = 0.86$ and the rotation continues until the relative velocity becomes zero. Figure 4(b), (c) and (d) show the results of computing the angular acceleration over time and the integrations that render the angular velocity and rotation. Results are presented with and without including the P-Δ effect. For the simple problem, Figure 4 demonstrates that the P-Δ effect is significant for wall rotation beyond approximately 10°. For the relatively severe ground motion considered ($K_h^{max} = 0.6g$ and period 1 Hz.) computations that do not include the P-Δ effect demonstrate that the wall will reach instability after approximately 2.5 cycles of ground motion; and with the P-Δ effect instability is achieved after only approximately 2 cycles of strong ground motion.

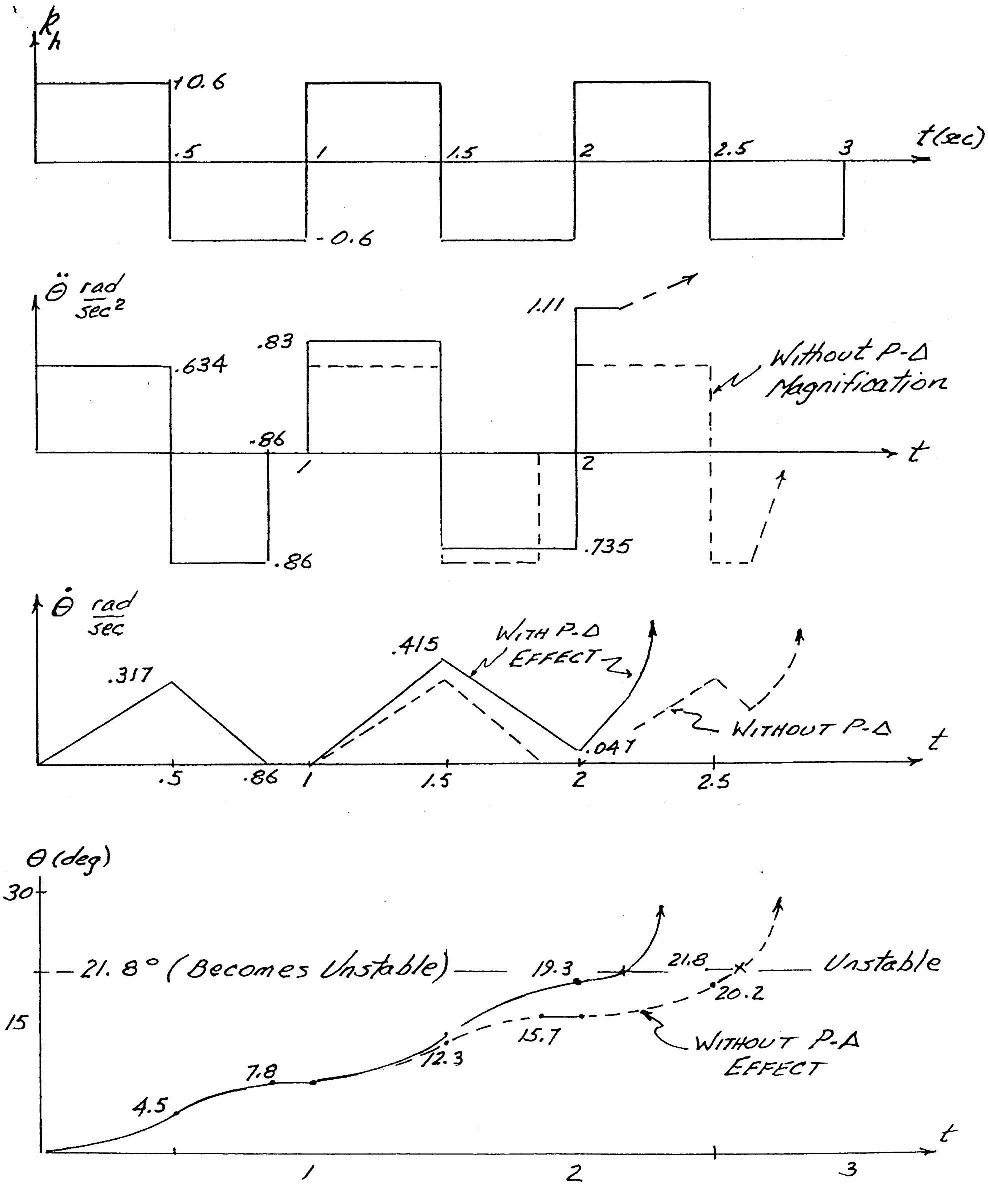

Figure 4. Integration of Angular Deformation of Retaining Wall with Bearing Capacity Failure.

APPLICATION TO KOBE WALL

During the 1995 Hyogoken-Nambu (Kobe) Earthquake, many embedded retaining walls suffered seismic induced deformation (Tateyama et al., 1995). Some retaining walls failed by tilting to the point that they overturned. Comparison of the computed behavior of these walls to that actually observed allows a validity check of the analysis models currently proposed.

In this study a cantilever-type reinforced concrete retaining wall located along the main line of the Hansin Railway Company, adjacent to the Ishiyaga Station will be investigated. A sketch of the wall and the condition of the wall observed after the earthquake is shown in Figure 5. During the earthquake these walls tilted outwards considerably inducing large settlement at the crest of the embankment and heaving of the sidewalk in front of the wall. The retaining wall orientation is approximately in the east-west direction, thus the north-south component of the acceleration time history is used as input in the analysis. The earthquake record was obtained from a recording station located at the site of longitude 135.18° and latitude 34.69°. As shown in Figure 6, the time history has peak ground acceleration of +0.59g, and -0.834g. It is important to point out that the Kobe earthquake is characterized not only by strong ground shaking, but also by a characteristically long period. Long period earthquakes are more damaging to retaining structures, particularly if they yield, since the amount of seismic induced deformation is proportional to the period of the ground motion squared.

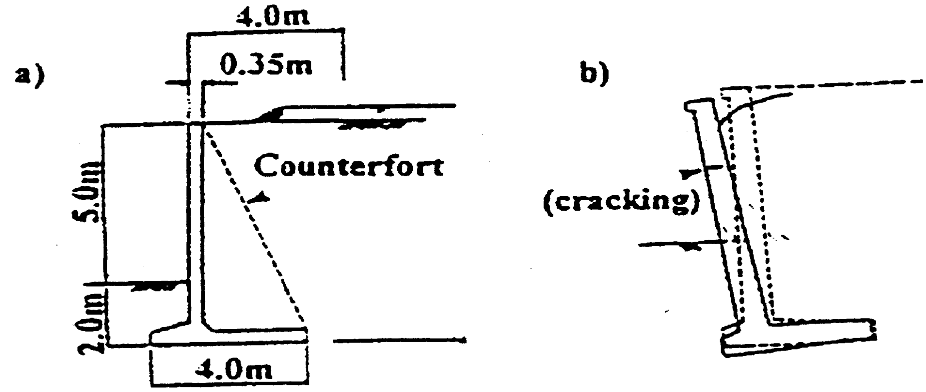

Figure 5. Sketch of Reinforced Concrete Cantilever Retaining Wall near Ishiyaga Station

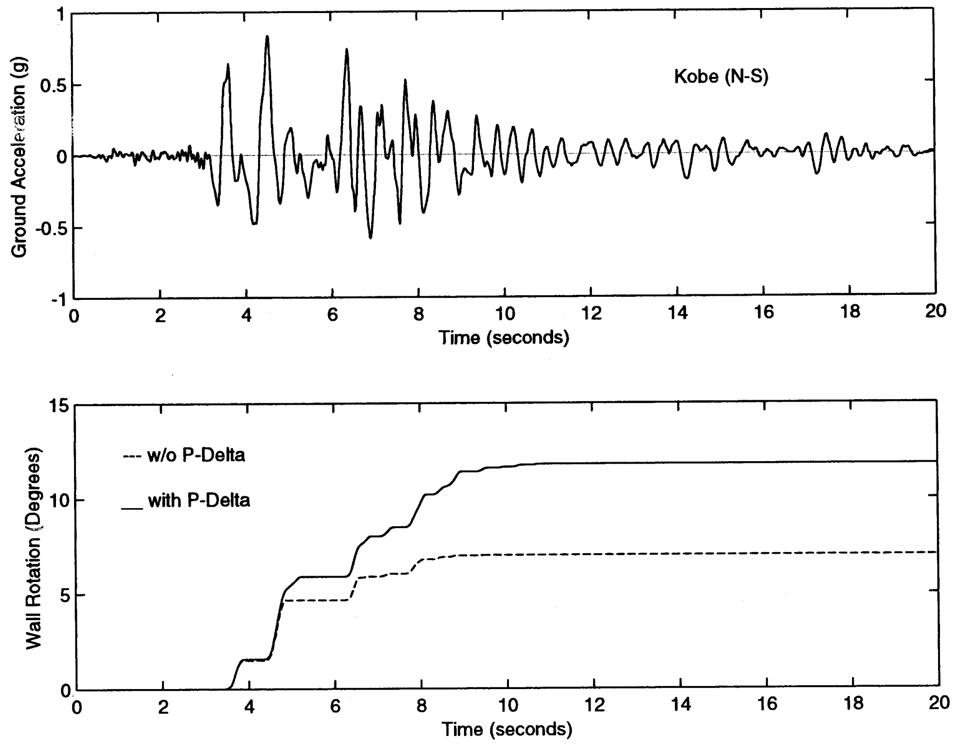

Figure 6. Kobe 1995 Earthquake, N-S Component Acceleration Time History and the Rotational Response of the Example Retaining Wall

Table 1 is a summary of the soil properties describing the backfill and foundation soil and the results of the analysis of the wall. The computed threshold acceleration for sliding and overturning is 0.46g and 0.28g, respectively. The threshold acceleration for bearing capacity was evaluated for two different assumptions : (a) considering the contact pressure to be uniform at yield and b) considering the resultant of the contact pressure at the base of the footing to be eccentric. The effect of eccentricity is evaluated using the procedure proposed by Meyerhof (1953) for static loading by which an effective footing width is used in the bearing capacity equation such that B' = B-2e. If the effective footing width is included in the calculations the critical threshold acceleration for bearing capacity is computed as 0.21g.

With $k_h^{tilt} = 0.21$, Equation (7) is used with the Kobe earthquake record shown in Figure 6 as input. The results of integrating Equation (7) twice with respect to time to determine rotation are also shown in Figure 6. The total rotation of the wall is calculated as approximately 7° without considering the P-Δ effect and 11.8° with the P-Δ effect included in the analysis. This compares very well with the observed deformation of the wall (Tateyama et al ,1995).

CONCLUSIONS

A general model for computing seismic induced deformation of retaining walls is presented. The model can describe failure by sliding, overturning or loss of bearing capacity and includes the detrimental P-Δ effect during rotation. The analysis is applied to walls that failed by tilting in the Kobe earthquake and the results are in good agreement with observations.

Table 1. Results from Analysis of Kobe Wall

	P_{PE} (kN)	P_{PS} (kN)	n	P_{AE} (kN)	P_{AS} (kN)	m	P_h (kN)	P_v (kN)	$F.S.$ slide	f	k_h^{tilt}	Rotation	
												Without P-Δ	with P-Δ
B'=B	141	182	0.28	378	191	0.43	325	453	1.03	1.6	0.46	3.8°	4.4°
B'=B-2e	165	182	0.31	253	191	0.38	118	395	2.42	1.4	0.21	**7.0°**	**11.8°**

Materials: Soil Density = 18.1 N/m^3 , ϕ = 42 °, $\delta_a = \delta_p$ = 0°, ϕ_b = 36°. Concrete Density = 23.5 kN/m^3
k_h^{slide} = 0.6, $k_h^{O.T.}$ = 0.28, x_0 = 1.56m, y_0 = 2.35m, CR = 0.3m

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