RESPONSE SPECTRA FOR DIFFERENTIAL MOTION OF COLUMNS

Maria I TODOROVSKA[1] And Mihailo D TRIFUNAC[2]

SUMMARY

The common modeling assumptions in analyses of structural response considering the effects of soil-structure interaction are discussed, as well as their consequences. Issues such as spatial variability of strong ground motion and flexibility of building foundations are specifically addressed. Results are shown for the case when spatial variability is considered, but inertia interaction and kinematic interaction between the different footings is neglected.

INTRODUCTION

The term soil-structure interaction describes a collection of phenomena that are caused by the flexibility of the foundation soils. Their proper consideration requires introduction of additional degrees-of-freedom in the formulation of the equilibrium equations of a system and, depending on the model, it may call for methods of solutions based on wave propagation. In general terms, the soil-structure interaction will lengthen the apparent period of the system, will increase the relative contribution of rocking excitation of ground motion to the total response, and will usually reduce the maximum base shear [Todorovska and Trifunac, 1992]. The benefits of including soil-structure interaction in the design of structural systems result from the scattering of incident wave energy from the foundation, and from additional radiation of structural vibration energy into the soil. When the soil surrounding the foundation experiences small to modest levels of nonlinear response, the soil-structure interaction will lead to further significant loss of the available input energy. Since this energy loss occurs outside the structure, if will be one of the important challenges for future design of earthquake resistant structures to quantify this loss and to exploit it in design.

The simplest way to consider soil-structure interaction effects is to assume that the building is supported by a rigid foundation. This results in minimum number of additional degrees-of-freedom (three translations and three rotations), but may lead to restrictive and too simple representation. Studies which model flexible foundations are rare [Iguchi and Luco, 1982 Liou and Huang, 1994] and difficult to evaluate in absence of strong motion records. As far as we know, there exists no strong motion program to document distortions and warping of foundations of structures during the passage of strong seismic waves [Trifunac et. al, 1999]. Experimental studies of soil-structure interaction are best conducted in full-scale, in actual buildings during microtremors [Trifunac, 1970a,b; 1972], forced vibrations [Blume, 1936; Hudson, 1970], and earthquake excitation [Luco et al., 1987]. It is difficult to conduct soil-structure interaction tests in laboratories, not only because of the constraints imposed by the need to satisfy the similarity laws, but mainly because it is almost impossible to model the half space boundary conditions for the soils.

The degree to which the soil-structure interaction modifies the foundation and structural response depends mainly on the flexibility of the soil relative to the foundation and to the structure. In predictions by analytical and numerical models, it also depends significantly on the modeling assumptions. The rigid foundation assumption, for example, exaggerates the effects of scattering and radiation, and may overestimate the damping effects in the soil. It also overestimates the rocking and torsional response of the foundation. Large foundation rocking, coupled with the gravity forces and with the vertical accelerations, may be of concern for the stability of the structure, and may be a possible system failure mode, due to dynamic instability.

The perfect bond assumption is violated to a varying degree even during a single exposure to strong earthquake shaking. Variations of this bond with time lead to "spreading" of the peak in the structural response transfer-

[1] *Univ. of Southern California, Civil Eng. Dept., Los Angeles, CA 90089-2531, E-mail: mtodorov@usc.edu*
[2] *Univ. of Southern California, Civil Eng. Dept., Los Angeles, CA 90089-2531, E-mail: trifunac@usc.edu*

function corresponding to the fundamental system frequency over a range of frequencies. This effect can be classified as a "geometric nonlinearity" [Trifunac et al, 1999]. The material nonlinearity of soft soils also contributes to this "spreading" in the system transfer-function. Both the bonding and the characteristics of the material nonlinearity depend on the degree of consolidation of the soil, and it is reasonable to expect that these change with time, depending on the history of strong and weak ground shaking. This interpretation is supported by the results of a recent study of a 7-storey reinforced concrete building in Van Nuys, San Fernando Valley, California, of analysis of ground motion recorded at five stations of the Los Angeles and Vicinity Strong Motion Network, also in San Fernando valley, from the 1994 Northridge earthquake and all recorded aftershocks (both strong and weak), as well as during several other earthquakes preceding Northridge.

Of concern to this paper are the consequences of another effect ruled out by the rigid foundation assumption for buildings, and that is the phase delay between the excitation of different support points at the contact between the building and the foundation. There are three basic questions addressing the relevance of this issue: (1) are the consequences of this phase delay of any significance for the structural performance and integrity during strong shaking, (2) are building foundations indeed "so flexible" that the concerns raised in (1) would apply, and (3) is there a way to generalize and simplify these effects so that it is practical to introduce them in code procedures and in description of ground motion for design. In the following, we review results on the effects of phased excitation on response of analytical models and on experimental evidence for flexibility of building foundations, a proposed procedure how to handle these effects in a simplified way, via the response spectrum approach, and conclude with discussion of the research needed to fully address this issue and its consequences to design.

ANALYTICAL MODELING OF THE EFFECTS OF FLEXIBLE BASE ON BUILDINGS

One way to approach this problem is via wave propagation, and representation of the building as a continuum. Variations of one such model were studied by Todorovska and Trifunac [1989; 1990a,b] and Todorovska and Lee [1989]. These models are two-dimensional, are excited by horizontally propagating SH-waves, and neglect the scattering and inertia interaction. The base follows exactly the deformation of the ground. However, this representation of the building response can be used in a model that considers scattering and inertia interaction (if the resultant wave motion at ground level is expanded in a basis of harmonic horizontally propagating waves). Therefore, we can view their solution as response to a particular "mode" of the base excitation, just that the actual participation of these "modes" will be affected by the scattering and radiation. The results for this model show that nonharmonic "modes" of vibration also contribute to the response, and their participation is significant when the horizontal velocity of the wave at the base corresponds to incidence beyond critical angle. The deformation of the structure is such that the top moves very little, but the first storey sustains significant deformations to follow the motion at the soil. In this case, the base flexibility is beneficial for the upper storeys but may lead to critical conditions for the first storey columns. Figure 1a,b shows results for an anisotropic building model excited by plane SH-waves, with incident angle equal to (a) and grater than (b) the critical angle (for the interface between the building and the soil). Parts (a) and (b) show snap-shots of the model response at times equal to 0, ¼, ½, ¾ and $1\times T$, the period of the excitation [Todorovska and Trifunac, 1989; Todorovska and Lee, 1989].

EXPERIMENTAL EVIDENCE FOR FLEXIBILITY OF BUILDING FOUNDATIONS

An example of experimental evidence for flexibility of a building foundation is provided by detailed microtremor measurements of a 7-storey building in Van Nuys (mentioned also in the introduction). Figure 2 shows contours of relative amplitude and phase of the motion on the ground level in the building and in the area surrounding the building. The wave propagation is seen from the direction of the primary sources of the ambient noise-a busy freeway to the west and a busy street to the north of the building. The shape of these contours may change depending on the time of the day, but the evidence of wave propagation will be there. As mentioned in the introduction, there are no such data from strong shaking, but it is reasonable to expect that this would occur during strong motion. This building was severely damaged during the 1994 Northridge earthquake, at such locations and in such a way that the primary reasons are not clear at this time. The building is supported by friction piles, about 40 feet deep and spaced over a rectangular area 63 ft × 150 ft, symmetrically with respect to the two axes of symmetry of the building plan. In spite of the symmetry, recent microtremor measurements suggest eccentricity in the foundation response (for primarily NS excitation, the foundation rotated about a point outside and near the SE corner of the foundation; Fig. 2a). This behavior is possibly due to past strong motion excitation, uneven soil properties, or nonsymmetric damage of piles. Reexamination of recorded response to strong motion from several earthquakes also indicates asymmetry in the foundation response (Trifunac et al., 1999).

RESPONSE SPECTRA FOR DIFFERENTIAL MOTION OF COLUMNS

Trifunac and Todorovska [1997] searched for possibilities to describe the response of structures to differential ground motion via the familiar concept of response spectrum. The idea was that this concept is already familiar to designers and that the effects of differential motions could be considered with only minor changes in the current design procedures. To demonstrate first that this task is possible, they considered the simplest model, a structure on individual column supports excited by horizontal motion only (along the length of the structure, as shown in Fig. 3a). The scattering and inertial interaction were neglected. They showed how to express the new spectra in terms of the standard relative displacement spectra, the peak ground velocity, and a factor which depends on the distance of the column from a reference point and on the shear wave velocity in the top soil layer. Their model is shown in Fig. 3a. It can be a one storey or a multi-storey structure, the response of which is approximated by the first mode. The purpose is to define, via response spectrum, the shear force in the first storey columns. The following describes this approach.

In Fig. 3a, H_e and u_e are the height and relative response of the equivalent oscillator corresponding to the first mode of vibration, with period T. The relative response of the first storey is $d_1=\delta\, u_e$, where δ is a factor that depends on the height of the building and on the mode shape assumed. Due to strains in the soil, the supports of the columns move differently. For a 1-storey structure, the equation of motion is

$$\ddot{u}^r + 2\,\omega\,\varsigma\;\dot{u}^r + \omega^2 u^r = -\ddot{u}_0 \tag{1}$$

where $\omega=2\pi/T$ is the circular frequency of the structure, ζ is the damping ratio, u_0 is the absolute displacement of a conveniently chosen reference point on the ground, R, and u^r is the relative displacement of the structure with respect to reference point R. Eqn (1) is the classical equation of motion of a SDOF oscillator excited by synchronous acceleration of the base $\ddot{u}_0$. The peak of the relative response, $|u^r_{max}|$, is the spectral displacement $SD\,(T,\zeta)$ for the motion u_0 . The relative displacement of the columns is evaluated with the help of u^r and the relative displacement, u^r_i, of the bottom of the i-th column with respect to the reference point R (see Fig. 3a).

Trifunac and Todorovska [1997] showed that u_0 is a weighted average of the displacements at the base of the individual columns, u_i (the weighting factors are proportional to the stiffness of the columns). These motions are not completely random during an earthquake, and can be interrelated, for example, via the strain field in the ground [Trifunac and Lee, 1996; Todorovska and Trifunac, 1996; Trifunac et al., 1996]. For example, for a symmetric structure (symmetric distance between the columns and symmetric distribution of stiffness of the columns), R is at the mid-point of the structure. Let x_i be the distance from the i-th column to point R, and c_x be the "representative" phase velocity of the ground motion in the x-direction. When distance x_i for the end columns (x_1 and x_n in Fig. 1) is small compared to the wavelength of ground motion, c_xT, where T is the predominant period of ground motion, the displacement at the base of the i-th column, can be approximated by a second order Taylor series expansion of u_i about u_0

$$u_i(t) \approx u_0(t) + \frac{\partial u}{\partial x}x_i + \frac{1}{2}\frac{\partial^2 u}{\partial x^2}x_i^2 \tag{2}$$

Then, the relative motion of the base of the i-th column, u^r_i, (relative to point R) is

$$u_i^r(t) \approx \frac{\partial u}{\partial x}x_i + \frac{1}{2}\frac{\partial^2 u}{\partial x^2}x_i^2 \tag{3}$$

When, the wave nature of the motion in the soil is ignored or is not present (for example the phase velocity along the x-direction, c_x, is infinite), all $u^r_i = 0$ and the bases of all the columns move as u_0. Thus, u^r_i represent local relative motions caused by strains and by wave passage.

In eqns (2) and (3), $\partial u/\partial x$ is related to the axial strain, ε_{xx}. For a site on parallel layers, in the absence of complex three-dimensional interference and scattering, this strain component can be approximated by

$$\varepsilon_{xx}(t) = \frac{\partial u(t)}{\partial x} \approx -A\frac{v(t)}{\beta_{av}} \tag{4}$$

where β_{av} is the average shear wave velocity in the top 30 m of soil, $v(t)$ is the particle velocity, a function of time, and A is an empirical scaling function which can be estimated from numerical simulations of strong motion at a given site. Trifunac and Lee [1996] found A~0.4 for β_{av} = 300 m/s for a site in Westmoreland, Imperial Valley, California, and for synthetic strong motion consisting mainly of surface waves. For illustration purposes we will use this value of A in this outline. In general, the function A will depend on the specific earthquake and site conditions and should be evaluated for each site separately. Therefore, $(\partial u / \partial x)\, x_i$ in eqns (3) and (4) can be approximated by

$$\frac{\partial u}{\partial x} x_i \approx -A \frac{v(t)}{\beta_{av}} x_i = -v(t)\tau_i \tag{5}$$

where

$$\tau_i = A \frac{x_i}{\beta_{av}} \tag{6}$$

is the time (multiplied by factor A) it takes a wave propagating horizontally with velocity β_{av} to travel from the reference point R (see Fig. 3a) to the base of the i-th column. Using the same type of approximate analysis,

$$\frac{1}{2} \frac{\partial^2 u}{\partial x^2} (x_i)^2 \approx \frac{1}{2} a(t) \tau_i^2 \tag{7}$$

where $a(t) = \partial^2 u / \partial t^2$ is the acceleration of the ground. Then

$$u_i^r(t) \approx -v(t)\tau_i + \frac{1}{2} a(t) \tau_i^2 \tag{8}$$

Using this representation of the relative motion of the base of the columns, it can be shown that the shear forces, V_i, i=1,…, n, in the columns of a one-storey structure are equal to

$$V_i(t) = k_i \left[u^r(t) + v(t)\tau_i - \frac{1}{2} a(t) \tau_i^2 \right] \tag{9}$$

where $v(t) = \partial u_0(t)/\partial t$ and $a(t) = \partial^2 u_0(t)/\partial t^2$ are the ground velocity and the ground acceleration for reference point R moving as u_0.

To design the columns for maximum shear, the maximum relative displacement $u^r(t) = v(t)\tau_i + (1/2) a(t) \tau_i^2$ needs to be estimated. Trifunac and Todorovska [1997] defined a three parameter relative displacement spectrum, $SDC(T, \zeta, \tau)$, as

$$SDC(T, \varsigma, \tau_i) \equiv \max_{\forall t} \left[u^r(t) + v(t)\tau_i - \frac{1}{2} a(t) \tau^2 \right] \tag{10}$$

This spectrum can be computed on a routine basis during data processing and provided to the designer. Then, for a particular structure, the designer can evaluate τ for the end and other columns and read the maximum relative displacement from the corresponding response spectrum. For a multi-storey building,

$$SDC(T, \delta, \varsigma, \tau) \equiv \max_{\forall t} \left[\delta\, u_e(t) + v(t)\tau_i - \frac{1}{2} a(t) \tau^2 \right] \tag{11}$$

where δ is a the ratio of the relative displacement of the first storey to the relative displacement of the equivalent SDOF oscillator ($d_1 = \delta u_e$, see Fig. 3a). (For a one storey building, δ = 1). Assuming a shape for the first mode of vibration, and relating the natural period to the height of the building (number of storeys), δ can be expressed in terms of the period T of the fundamental mode. In eqn (11), u_e is the response of the equivalent SDOF oscillator for the multi-storey building to motion $u_0(t)$, and it can be calculated by the means of the Duhamel's integral.

An approximation for the *SDC*-spectrum defined by eqn (11) is [Trifunac and Todorovska, 1997]

$$SDC(T,\delta,\varsigma,\tau) \approx \left[\delta^2 \, \mathrm{SD}^2(T,\varsigma) + (v_{\max}\tau)^2 + \left(\frac{1}{2} a_{\max} \tau^2 \right)^2 \right]^{1/2} \tag{12}$$

where $SD(T,\zeta)$ is the classical relative spectral displacement and $v_{\max}$ and $a_{\max}$ are the peak velocity and acceleration of the motion of reference point *R*. Trifunac and Todorovska [1997] showed that the approximation in eqn (12) is very good and, moreover, that the contribution of the terms with peak acceleration is usually small and can be neglected. Fig. 3b shows $SDC\ (T,\delta,\zeta,\tau)$ for δ=1 (one-storey structure) and ζ=0.05, for different values of τ. The solid lines correspond to eqn (11) and the dashed lines to the approximation in eqn (12). These results were evaluated for horizontal motion of the 1994 Northridge earthquake (*M*=6.7) record at station USC No. 53 (epicentral distance 6 km). Fig. 3c shows the ratio $SDC\ (T,\delta,\zeta,\tau)/\delta\ SD\ (T,\zeta)$ for the same ground motion. It is seen that this ratio is significantly grater than one for short period (stiff) structures. Fig. 3d shows the ratio $SDC\ (T,\delta,\zeta,\tau)/\ \delta\ SD(T,\zeta)$ for a multi-storey building, also for the same record. The period of the fundamental mode of vibration, *T*, and factor δ are related by assuming a familiar relationship between number of storeys and fundamental period, and a shape for the first mode of vibration (for example, δ=1.5/(10*T*)). It is seen that the spectral ratio increases at both ends of the spectrum.

The model above, can be extended to out of plane excitation, and will eventually be extended to be applicable to consider more general excitation, e.g., Rayleigh waves. Also, it does not consider scattering and inertial interaction, but as more knowledge is obtained on this subject, it will be possible to define some scaling factors for these spectra to include in a simplified ways the associated effects.

CONCLUSIONS

A large body of literature exists on solving an elasto-dynamic problem (usually linear) to compute the foundation input motion and frequency dependent complex stiffness coefficients under the assumption of a rigid foundation. The results of models that consider flexible foundation-soil interface are case specific and cannot be easily generalized at this stage. Further research is needed to consider more realistic models and robust description of the associated effects, in a simple form applicable to design. Measurements of building response to shaking with different levels of severity are crucial to assess the degree to which these effects occur in the real world, and how they depend on the level of excitation. There is a misconception that in the laboratory we can simulate almost anything we want. Shaking table tests and soil boxes may be useful to understand some localized effects, but we should always bare in mind that the actual soil-structure interaction occurs in an infinite medium, and that the wave nature of the excitation and of the radiation damping are very difficult to capture in the laboratory [Trifunac and Todorovska, 1999].

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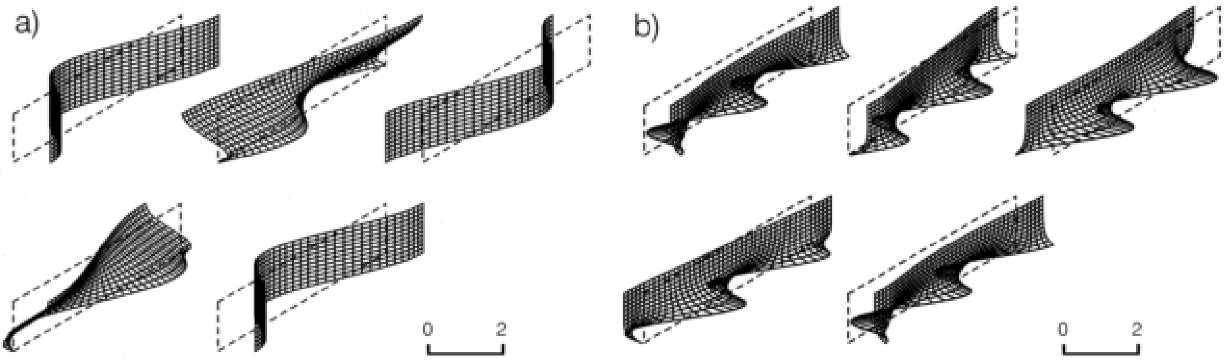

Fig. 1 Displacement response of a long building (represented by a homogeneous anisotropic plate; length–L, $\beta_x / \beta_z = 2$) to horizontally propagating monochromatic SH waves (phase velocity=c_x), at times t=0, T/4, T/2, 3T/4 and T (T–period of motion). (a) $c_x / \beta_z = 1$ (grazing incidence) and $\eta = L/(c_x T) = 1$; (b) $c_x / \beta_x = 0.05$ (incidence beyond critical angle) and $\eta = L/(c_x T) = 2$.

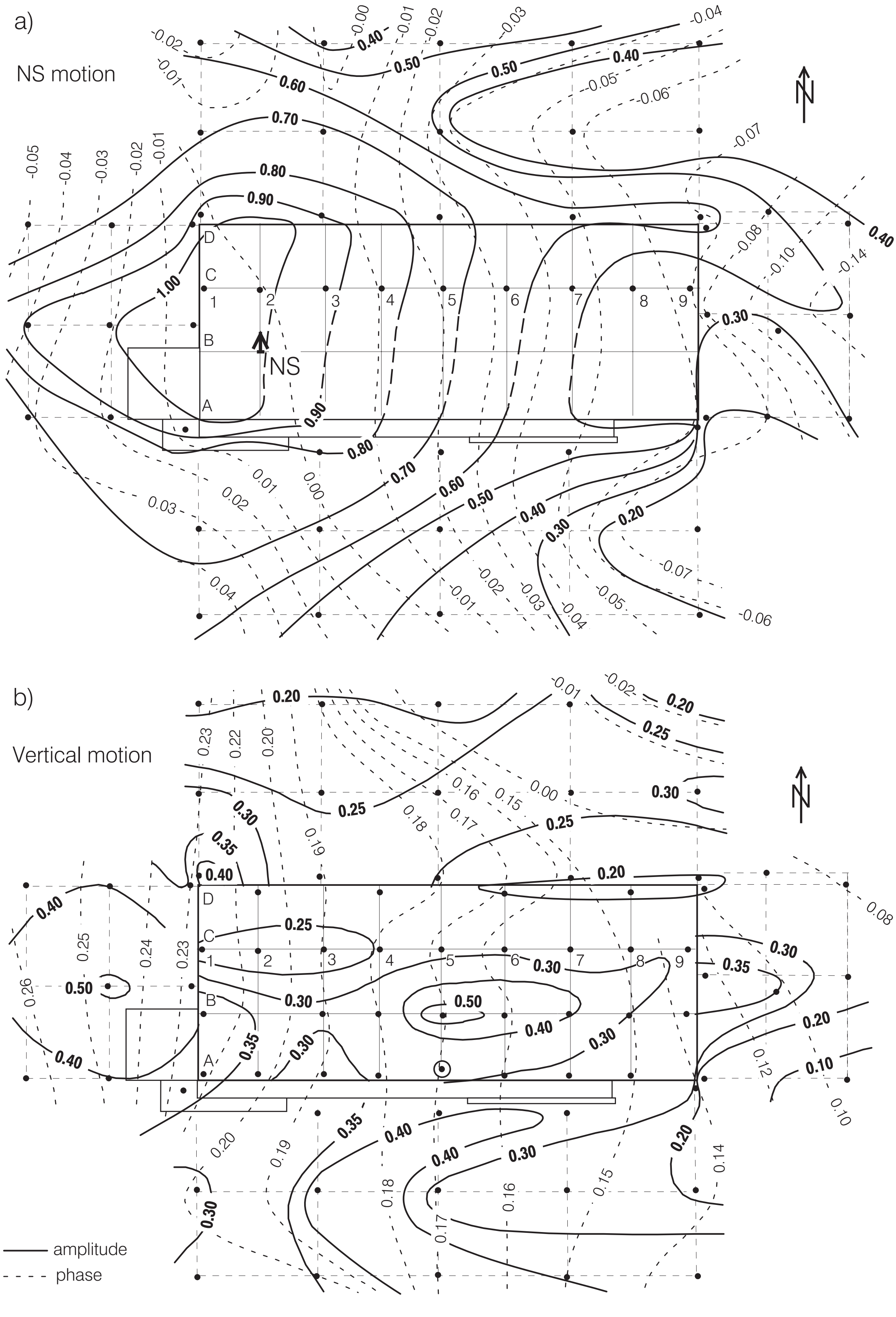

Fig. 2 Contours of $(R_{i,\ \mathrm{ref.}}(\tau))_{max}$ for NS motion (up) and vertical motion (down) (arbitrary normalized amplitudes, shown by heavy lines), and (τ) (in seconds) of $(R_{i,\ \mathrm{ref.}}(\tau))_{max}$ relative to the reference station (B2 for NS motion and A5 for vertical motion .

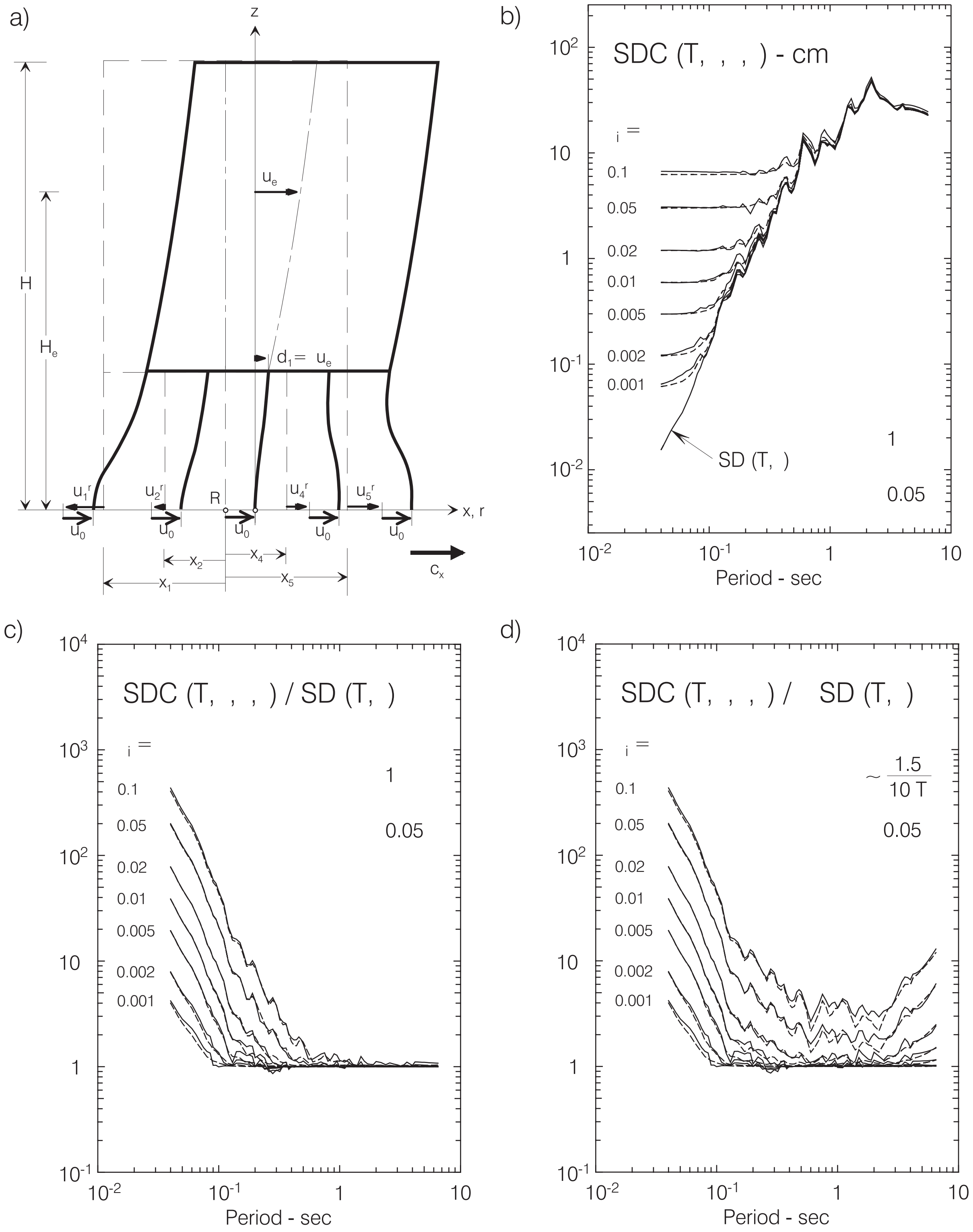

Figure 3. (a) The model. (b) The new displacement spectrum, SDC(T,δ,ζ,τ), for a one storey structure, and for horizontal motion from the 1994 Northridge at station USC #53. (c) Ratio of th new and conventional displacement spectra, for a one storey structure, and for horizontal motion from the 1994 Northridge at station USC #53.(d) Same as in (b) but for a multi-storey building.