SEISMICITY PREDICTION: A BAYESIAN APPROACH

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Synopsis. Paper aims at establishing probability distributions of intensities and other earthquake parameters which are related to the probability distributions of structural responses at given locations and times intervals. The normal scarcity of records precludes a straight-forward statistical treatment. Rather, prior distributions of magnitudes in relevant volumes of the earth are established on the basis of major geologic features; through use of Bayes' theorem, statistical information is incorporated to obtain posterior distributions of magnitudes; semiempirical formulas lead then to probability distributions of intensities and of other parameters at stations of interest. A new applications of Bayes' theorem permits incorporating available information about past earthquakes at those stations.

Introduction

Evaluation of seismic risk at given sites implies statistical prediction of the ground motion parameters that are relevant to structural behavior. Since only in exceptional cases do the available instrumental data suffice to quantitatively describe ground motions at the site, use has to be made of data other than local acceleration records.

Use of indirect data is mandatory. This work makes use of frequency-magnitude expressions as well as of correlations between spectral ordinates and maximum absolute values of ground acceleration, velocity, and displacement, and also between these and earthquake magnitude and focal distance.[1] The data used in those studies are affected by a great number of variables, including local ground conditions, nature of geologic formations crossed by the seismic waves, shock mechanism and many others. Consequently the ensuing correlations display great dispersion. In particular, the variability due to the nature of local ground properties has been partly eliminated by restricting the scope of such studies to sites with intermediate soil, comparable to a stiff clay or a compact conglomerate. Dispersion due to shock mechanism and geologic formations along the path of seismic waves remains, but its consequences are taken into account in this work.

Seismic risk prediction, as treated in this paper, starts from the formulation of a stochastic model of the process of earthquake occurrence (local seismicity) near a given station. Then these correlations are incorporated and a model is derived which includes probability distributions of maximum seismic excitations at the station for given time intervals[3,4].

The foregoing method has still an important drawback: seismic risk at a site is practically defined from the frequency and magnitude of earthquakes that *may originate in regions of relatively small dimensions, of the order* of 600 to 800 km in diameter, but statistical data often do not suffice to evaluate local seismicity throughout such regions. Earthquake resistant design cannot be exclusively based on direct use of statistical data, since

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their lack might lead designers to either of two positions: assuming zero seismicity or avoiding the decision. The former approach would frequently be strongly unconservative, whereas the latter would be unrealistic: engineering decisions cannot be postponed; they should be taken after making the most efficient use of every available piece of information, regardless of how incomplete it may be. Engineering judgement has played, and will continue to play even in the frame of modern analytical formulations of design, an important role in interpreting data other than statistical, or in extending or adapting to the region of interest the conclusions derived from other, better studied, regions of similar geotectonic properties. Formal manipulation of information of different nature, including statistical data, is accomplished through the use of bayesian statistics, as described below.

Correlations between earthquake properties

The scope of this paper will be limited to earthquakes of moderate duration (several dozen seconds) recorded on soils of intermediate properties, when focal distances are shorter than 600 km. The accelerograms of such motions are rather chaotic and justify the development of theoretical studies treating earthquakes as stochastic processes. As a result of such studies, it has been possible to establish correlations between the amplitude of ground motion (maximum absolute values of ground acceleration, velocity or displacement) and the expected ordinates of the response spectra, as well as the probability distribution of maximum spectral ordinates divided by their expectation[2].

Prediction of spectral ordinates according to the method adopted in this paper requires computation of maximum absolute values of ground acceleration, velocity and displacement in terms of magnitude and focal distance[II]. The empirical expressions have the form[1]

$$y = Ce^{kM} R^{-q} \qquad (1)$$

where C, k and q are empirical constants, M is the earthquake magnitude, y is the maximum absolute value of ground acceleration, velocity or displacement, and R is a modified focal distance, equal to $(x^2 + h^2 + r^2)^{\frac{1}{2}}$. Here, x is the epicentral distance and h the focal depth, both in km, and r is an empirical constant equal to 20 km.

Observed and computed values of a and v (maximum ground acceleration and velocity) were analyzed, and their ratios represented on normal probability paper following Gumbel's method[9] (fig 5). A distribution composed of two segments of the lognormal family was adjusted to each set of data. Standard deviations of $\ln(a/a_c)$ and $\ln(v/v_c)$ were found to be 1.5 and 1.0, respectively.[4] Here, subscript c stands for computed and ln means natural logarithm.

[II] These are not the only significant quantities. A more refined study might decrease the dispersion in the correlations, at the expense of considerably wider analysis of statistical data.

Local seismicity statistics

Instrumental data on earthquake magnitudes are available only for events occurred since the beginning of this century. For regions of relatively large dimensions (fig 1) or of high seismicity, these data suffice to obtain expresions for ν_M, the expected number of earthquakes, having magnitude greater than M, and originating per unit time in a given volume of the earth crust. The best known of them assumes a linear relationship between log ν_M and M.

$$\nu_M = \alpha e^{-\beta M} \qquad (2)$$

This equation may be objected since, according to Gutenberg and Richter's relationship between magnitude and energy, its right portion predicts and infinite amount of energy liberated in earthquake activity per unit time if β is not greater than 1.5 ln 10 = 3.46, while empirical data lead to β values usually smaller than 3. Fig 2 shows this, as well as the poor fit of eq 2 for magnitudes greater than 8. Even if the expression were valid for a macrozone such as the Circumpacific belt, it would then not be valid for portions of it, since the additions of terms similar to the second member of eq 2 does not give place to a function of the same form, unless β is the same for all regions. Although more adequate expressions may be adopted, eq 2 will be used in this paper, for the sake of simplicity, to illustrate the proposed formulation of seismic risk appraisal.

Bayesian estimation of local seismicity

Let H_i, i = 1, ..., n be a set of mutually exclusive hypotheses, $P(H_i)$ their associated initial or a priori probabilities of being true, and A an event that may occur in combination with anyone of the n hypotheses. Let $P(A|H_i)$ be the conditional probability of occurrence of A in case H_i were true. From the fundamental law of conditional probability,

$$P(H_i|A) = \frac{P(H_i)P(A|H_i)}{P(A)} = \frac{P(H_i)P(A|H_i)}{\Sigma P(H_j)P(A|H_j)} \qquad (3)$$

This equation is known as Bayes' theorem. In it, $P(H_i|A)$ is a modified, or posterior, probability of hypothesis H_i being true, once event A (statistical observation) is known to have occurred.[III] For its extension to local

[III] Severe criticism has been made of the use of Bayes' theorem, on the grounds of the apparent arbitrariness involved in choosing the prior distribution. However, alternate methods include concealed assumptions equivalent to arbitrary prior distributions.[5] The fact is that every decision we take, either in engineering or in any other rational discipline, is arrived at after an implicit, perhaps approximate, bayesian formulation for assimilating subjective concepts and factual evidence. A priori subjective information is far less arbitrary than it may seem at first sight if proper judgement is applied in extrapolating to the case of interest the results of similar, better studied phenomena.

seismicity estimation, let N $(M_1, M_2; t)$ be the number of earthquakes with magnitudes in the interval (M_1, M_2), that have occurred during time period t within a specified region of the earth crust. This number will be assumed to be a Poisson process with mean $\lambda\ (M_1, M_2)$ per unit time and volume, and independent of the corresponding processes in adjacent regions and in any other magnitude intervals: hence, $N(M, t) = N(M, \infty; t)$, the number of earthquakes exceeding a given magnitude during the specified time interval, will also be a Poisson process with expected value $V \lambda_M t$, where V is the volume of the region considered.[IV]

Assume that $V \lambda_M = \nu_M$ may be expressed as a function of known form and unknown parameters, $\nu_M = \nu_M (Z_1, \ldots, Z_m)$. Let $f(z_1, \ldots, z_m)$ be the a priori m-dimensional joint probability density function of the parameters, and A the event that during time interval t, $n_1, \ldots, n_k$ earthquakes have occurred with magnitudes in the intervals $(M_0, M_1) \ldots, (M_{k-1}, M_k)$, respectively. Then, if $\Delta_i \nu = \nu_{Mi} - \nu_{Mi-1}$, the probability of A, given the hypothesis $(Z_j = z_j,\ j = 1, \ldots, m)$ is obtained as follows.

$$P(A \quad z \quad \ldots., z_m) = \prod_{i=1}^{k} e^{-\Delta_i \nu t} \frac{(\Delta_i \nu t)^{n_i}}{n_i!} \tag{4}$$

Hence, the posterior distribution of the parameters would be

$$f(z_1, \ldots., z_m \mid A) = Kf(z_1, \ldots., z_m) \prod_{i=1}^{k} e^{-\Delta_i \nu t} \frac{(\Delta_i \nu t)^{n_i}}{n_i!} \tag{5}$$

where K is a normalizing constant. Practical use of eq 5 may be objected, since it does not produce a simple posterior distribution that may be expressed in terms of a few parameters. For applications, moments have to be computed through numerical integration. On the other hand, this approach permits simultaneous use of statistical data concerning all magnitude ranges to modify prior information, also for all magnitude ranges.

A second procedure, much simpler, although much more restricted, will be described, and some approximations introduced to widen its scope. It consists in determining in separate steps the (unidimensional) distribution of ν_M for each M of interest. Each successive application of Bayes' theorem deals with a given M and uses only the statistical information coming from events in which such M is exceeded. Great computational advantages are achieved if a gamma distribution is chosen for each ν_M. Our assumption that we deal with Poisson processes makes this choice "natural," as the gamma distribution is the conjugate of Poisson's and the choice brings about simplifi

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IV Data contradict the assumption of stochastic stationarity and time and space independence from past events.[3] There is an evolution in local seismicity, and earthquakes tend to cluster in time and space; this is true even if one disregards foreshocks and aftershocks in the analysis. However we shall retain the simplifying assumptions quoted, on the ground that they are probably adequate for the time intervals in which we are mostly interested-of the order of expected life spans of civil engineering works.

cations in numerical work and in interpretation[7]. Analytically,

$$f(\nu_M | r,t) = K_1 f(\nu_M) \frac{e^{-\nu_M t} (\nu_M t)^r}{r!} \tag{6}$$

where r is the number of earthquakes with magnitude greater than M which occurred during time interval t. If $f(\nu_M)$, the initial density function, is the gamma distribution with parameters r', t', the posterior distribution will be gamma with parameters r" = r' + r and t" = t' + t. [V] The initial and posterior expectations of ν_M will be r'/t' and r"/t", whereas the corresponding coefficients of variation will be $r'^{-1/2}$ and $r''^{-1/2}$, respectively.

The latter formulation fails to incorporate all significant statistical information. This becomes clear if one considers the discontinuities that would result, for instance, in the posterior expectation of ν_M at the magnitude value above which no events had been observed. Thus, any information about mean number of earthquakes of magnitude greater than 8 should convey additional, although somewhat vaguer, information concerning magnitude 8.5. This suggests the adoption of a conservative envelope of the results of the double criterion that follows. The first alternative is derived from the magnitude-by-magnitude procedure of eq 10. For the second, let M_0 be a reference magnitude for which ν_M may be estimated with relatively little uncertainty, and $M = M_0 + \Delta M$ a magnitude, the estimation of whose ν_M might be improved by using information relative to ν_{M_0}. From eq 2,

$$\nu_M / \nu_{M_0} = e^{-\beta (M - M_0)} \tag{7}$$

Therefore, according to our hypothesis, once ν_{M_0} is known, uncertainty in ν_M will come from that in β. If the latter variable is assumed to possess a normal distribution, the coefficient of variation of ν_M is given by eq 8e. In the bayesian treatment, both ν_{M_0} and β are assigned initial distributions. If no statistical information is obtained for $M > M_0$, a conservative estimation of ν_M in that range may be obtained if it is assumed that the distribution of β remains unaltered while that of ν_{M_0} may be modified. Prior and posterior expectations and coefficients of variation of ν_M would be

[V] These are appealing properties of the conjugate distribution in this case: prior and posterior belong to the same family; their parameters may be interpreted as fictitious times and numbers of events; and the order in which statistical data are incorporated is irrelevant.

$$E'(\nu_M) = E'(\nu_{Mo})\, e^{-\beta_o (M - M_0)} \qquad (8a)$$

$$c'^2(\nu_M) = c'^2(\nu_{Mo})\, c_1'^2 + c_1'^2 + c'^2(\nu_{Mo}) \qquad (8b)$$

$$E''(\nu_M) = E''(\nu_{Mo})\, e^{-\beta_o (M - M_0)} \qquad (8c)$$

$$c''^2(\nu_M) = c''^2(\nu_{Mo})\, c_1'^2 + c_1'^2 + c''^2(\nu_{Mo}) \qquad (8d)$$

where

$$c_1'^2 = \exp\left[(M - M_o)^2\, \sigma_\beta'^2\right] - 1 \qquad (8e)$$

and β_0 is a parameter of the initial distribution of seismicity. In these equations, prime and double prime mean initial and posterior, respectively. More refined treatments may be adopted to take into account possible modifications in the distribution of β.

In what follows, the three criteria described for application of Bayes' theorem will be referred to as the exact (eqs 4, 5), the step by step (eq 6) and the modified (eqs 8a-e) methods.

Bases for establishing initial distributions of local seismicity parameters

Some of the sources of significant information for local seismicity studies are of geophysical nature, such as geotectonic features, studies of regional strain and of energy available for sudden release; others are of statistical nature, such as magnitudes, focal coordinates, and energy released by earthquakes in different regions of the world. These are to be complemented by conclusions derived from similarity with other physical phenomena and by qualitative descriptions of earthquake history over long time periods. Efforts to interpret data from all these sources and to express them quantitatively in earthquake risk terms are in an embryonic stage. Consequently, let us for the moment limit our attention to the way in which conclusions might be obtained from the similarity of tectonic features. According to the present state of knowledge, only statements about seismicities to be of roughly the same order of magnitude may be made on the basis of this similarity. Accordingly, geotectonic information will only be used here for the purpose of dividing the crust into regions, without introducing prior distributions of local seismicity parameters on the basis of this information alone. The essence of our procedure for that effect will stand on the estimation of the seismicity of narrow zones from that of similar, but wider zones, for which statistical data permit a direct evaluation. Due account is taken of the uncertainties about the relationship between seismicities of both regions.

Take now the earth crust divided into three macrozones defined by the volumes that correspond to the Circumpacific belt, the Alpide belt, and the rest of the world, as shown in fig 1. For magnitudes in the range 6-8.5, statistical data are enough for a relatively precise estimate of ν_M for each macrozone. If a gamma distribution is assumed for ν_M, ignorance of prior information is tantamount to taking $r' = t' = 0$, wherefrom $r'' = r$, $t'' = t$, and, hence, the expected ν_M and its coefficient of variation depend only on statistical information.[VI] After independent treatment of ν_M for each M, a smooth curve is fitted to the computed values of $E''(\nu_M) = r''/t''$. Likewise, lower envelopes are fitted to the computed coefficients of variation $c'' = r''^{-\frac{1}{2}}$ for the various magnitudes (fig 2). Abrupt discontinuities in the computed values of c" are due to the different lengths of the observation periods that correspond to each magnitude range.[VII] The use of the lower envelope for representing the adopted coefficient of variation aims to account for more complete use of information. Now we may proceed to smaller volumes of the earth crust, by applying Bayes' theorem to each of them. Their prior distributions will be assumed to be gamma, the proper parameters being estimated from those of the macrozone including the zone of interest, as follows. Let V be the volume of the macrozone and V' that of the smaller one, and λ_M, λ'_M, the corresponding seismicities per unit volume. For any given magnitude M the following identity holds,

$$\lambda' \equiv (\lambda'/\lambda)\,\lambda$$

The spatial variation of seismicity is incorporated in the ratio λ'/λ while our ignorance of the macrozone's seismicity dictates the distribution of λ. Thus, λ'/λ and λ are independent variables. Hence,

$$E(\lambda') \equiv E(\lambda) \qquad (9)$$

$$c^2(\lambda') = c^2(\lambda'/\lambda)\,c^2(\lambda) + c^2(\lambda'/\lambda) + c^2(\lambda) \qquad (10)$$

The assumption of spatial independence of seismicity, coupled with the restriction that λ' should be equal to λ when $V' = V$, gives

$$c^2(\lambda'/\lambda) = (V/V')^{\gamma} - 1 \qquad (11)$$

where $\gamma(M)$ is a parameter characterizing the macrozone in question. Empirical estimates of $c^2(\lambda'/\lambda)$ in terms of V'/V for various M were obtained for the different macrozones (fig 3). The increasing deviation of the empirical values from the theoretical ones, when $V'/V \to 0$, are partly due to the increased error in estimation of ν' as r/t for very small volumes. Application of eqs 10 and 11 for magnitudes outside the interval 6 to 7 gives place to very high coefficients of variation, as compared with those within it. This again is a result of neglecting the information that earthquakes in a given magnitude range provide about other intervals. Proper account of that information may be taken in an approximate manner, if eqs 8 are applied, and M_0 is taken as the magnitude which minimizes $c^2(\nu')$ in eq 10.

VI Historic evidence might be included, leading to other values of r' and t'.

VII These periods are 3, 35 and 49 years for the intervals $6 \leq M < 7$, $7 \leq M < 7.8$ and $M \geq 7.8$, respectively.[10]

Example. Consider region 5 in the map of fig 1. According to geology, there seems to be no reason why seismicity should vary within it. Hence, uniform seismicity will be assumed throughout it. Its area is 129×10^3 km^2, as compared to 94×10^6 km^2 of the Circumpacific belt, to which this region belongs. Gutenberg and Richter[10] do not report any earthquake during the three-year period about which complete information is available for the range $6 \leq M < 7$. They report only one earthquake of magnitude 7.5 during the 35 period valid for the corresponding range. The prior information is shown in fig 4 by means of curves for $E'(\nu'_M)$ and $c'^2(\nu'_M)$. The first was obtained simply by multiplying $E(\nu_M)$, for the Circumpacific belt, by $V'/V = 129 / (94 \times 10^3) = 0.00137$. As regards the coefficient of variation, consider magnitude 7.5, por instance. From fig 2, $c^2(\nu_{7.5}) = 0.0034$, while from fig 3, $c^2(\nu'/\nu) = 9.6$.

Hence,

$$c^2(\nu_{7.5}) = 0.0034 \times 9.6 + 0.0034 + 9.6 = 9.63$$

Again, a lower envelope is fitted to the values of $c^2(\nu'_M)$ computed as shown, but again this is not enough to reflect all the information available. For application of the second alternative procedure, M_n is taken as 7.0, β_0 as 2.16 (the value for the Circumpacific belt as a whole) and $c'^2(\beta) = 0.10$. From fig 4, the prior expectation and coefficient of variation of $c\nu'_7$ are

$$E'(\nu'_7) = 0.0208$$

$$c'^2(\nu'_7) = 5.11$$

For M = 8, for instance, eqs 8 a-c lead to the following:

$$E'(\nu'_8) = 0.0208\ e^{-2.16\ (8-7)} = 0.0024$$

$$c'^2(\nu'_8 / \nu'_7) = \exp\left(0.10 \times 2.16^2\ (8-7)^2\right) - 1 = 0.6$$

$$c'^2(\nu'_8) = 0.6 \times 5.11 + 0.6 + 5.11 = 8.79$$

Coefficients of variation of ν'_M for other magnitudes, computed according to the two proposed methods, are shown in fig 4.

Posterior distribution of local seismicity

The different ways in which the prior distribution of local seismicity may be defined lead to alternate statements of Bayes' theorem. Each of them makes use in different degrees of the available prior and statistical information and leads to non-equivalent results expressed in various manners. Whatever the form in which these results are initially presented, they can always be converted to the distributions of ν'_M for all M of interest. This is not always the most useful way of presenting them for their applications to earthquake risk evaluation, but it permits an intuitevely more meaningful interpretation. Fig 4 shows the posterior expectations and variation coefficients of ν'_M for the example of last section, computed under several assumptions. Only for two magnitudes were non-zero statistical data available: 7 and 7.5. They correspond to one earthquake of magnitude 7.5. Since it did not occur during the three-year observation period considered for $M<7$, it was not taken into account in the application of the step-by-step criterion to that magnitude range.

The assumption that the distribution of β remains unaltered is conservative in this case for $M>7.5$. It permits to draw the line that represents the posterior expectation of ν'_M. This is a straight line parallel to that representing the prior expectation. Its vertical position was chosen by interpolating between the computed ν''_7 and $\nu''_{7.5}$. Again, lack of statistical information for magnitudes other than 7 and 7.5 caused the discontinuities of curve 6, the posterior coefficients of variation computed according to the step-by-step method. Curve 7 was obtained by the modified procedure, taking $M_0 = 7.3$ and applying eqs 8.

Distribution of maximum intensity at a given site

Let $N(y, t)$ be the number of earthquakes occurring during time interval t, and having intensity (in the generalized sense) greater than y at the site of interest; it can be obtained as the sum of a number of earthquakes of different magnitudes and focal coordinates, occurred during the same period.

If a deterministic correlation between M, R, and y is accepted, and if earthquake generation processes in different regions are independent from each other, and are assumed to be of the Poisson type, then $N(y, t)$ constitutes a Poisson process with mean rate of occurrence per unit time given by the following equation:

$$\nu_{Y_C}(y) = \int_V \lambda(M(y,R))\, dV \qquad (12)$$

Here, $M(y, R)$ is the magnitude that gives place to intensity $\underline{y}$ at the effective distance R, and $\lambda(M)$ is referred to unit volume. The integration is carried to the volume of the earth crust whose seismicity may appreciably contribute to the occurrence of intense earthquakes at the station. Subscript Y_C identifies computed intensities, predicted under the assumption of a deterministic correlation between M, R and y. Both mean and variance of $\nu_{Y_C}(y)$ as given by eq 12 may be computed[4]. For many decision problems concerning earthquake risk, only the expectation is significant; it may be obtained by using the expectation of $\lambda(M(y, R))$, instead of the variable itself, in the integral of eq 12. This was performed for maximum ground velocities and accelerations at point A in the map of fig 1. It was concluded that $E(\nu_{Y_C}(y))$ may be represented by an expression of the form Ky^{-r}. For the site selected, the corresponding values of these parameters are $K = 7500$, 5.18 and $r = 2.56$, 2.11 for maximum ground accelerations (cm/sec^2) and velocities (cm/sec), respectively.

The dispersion in the data of fig 5 may be taken into account by recalling that, under the assumption that each event of a Poisson process may give place to a new event with probability p, the process of the new events is also of the Poisson type, its mean rate being that of the original process multiplied by p. Hence, if $\nu_Y(y)$ is the mean number of earthquakes per unit time whose actual intensity exceeds y, the following equation applies.[4]

$$E[\nu_Y(y)] = \int_0^{\infty} \left(-\frac{d}{d\eta} E[\nu_{Y_c}(\eta)]\right) P[Y > y \mid Y_c = \eta]\, d\eta \qquad (13)$$

Expressions are also available for computing the variance of $\nu_Y(y)$.[4] Their evaluation is more complicated than that of eq 13 because of the appearance of the covariances between the values of $\nu_Y(y)$ for all pairs of η's. Hence, use will be made of bounds obtained for the two extreme cases of perfect and of null correlation.

The probability term in the integral of eq 13 is computed from the distributions in fig 5. If $E(\nu_{Y_c}(y)) = Ky^{-r}$, eq 13 leads to an expression of the ratio y/y_c of actual to computed intensities having equal recurrence periods:

$$y/y_c = \left(r \int_0^{\infty} z^{r-1}\, P\{Y/Y_c > 1/z\}\, dz\right)^{1/r} \qquad (14)$$

This ratio depends strongly on r, but not on y, if the distribution of Y/Y_c is independent from Y_c. In the case studied, this ratio took the values 2.4 and 1.9 for ground accelerations and velocities, respectively. Further applications of eq 13 are possible, for computing $E(\nu_Y)$ for other responses, such as spectral ordinates, whose distribution may be expressed in terms of the amplitude of the ground motion[2].

Use of regional data

Direct regional information at a site may be incorporated when it exists. Suppose that complete data of intensities Y are available during a given time period. A prior distribution of $\nu_Y(y)$ may be obtained, valid at the origin of the given time period, and the intensity data incorporated according to the bayesian formulation. The prior distribution of $\nu_Y(y)$ would be the result of obtaining the posterior distributions of ν_M at the neighboring regions, and then using the correlations between M, y and R and applying eqs 12 and 13. The posterior distribution of ν_M should not include statistical information of magnitudes occurred during the time interval for which intensity data will be used, in order to avoid double use of information.

At the beginning, mention was made of the importance of local soil conditions, and of their possible influence on the dispersion in the correlation between M, y and R. While the treatment described in this paper was restricted to intermediate soil, the same ideas may be extended to cases for which one may count on expressions similar to 1 and on the corresponding distributions of actual vs. computed values.

Final remarks

A method has been proposed to assimilate all available information for èstimating local seismicity. It provides means for assigning proper weights to data of different nature and for taking into account the corresponding uncertainties in the knowledge they supply. It gives place to use of intuition and subjective judgement, which has been the rule in problems of earthquake risk estimation. Unlike the traditional treatment, the proposed method concerns the criteria for expressing prior knowledge and its weight in a quantitative manner as well as the ways in which it should be modified as new information is gained. At the two ends of its scope the two traditional approaches are found: the purely intuitive and the purely statistical ones. Since the acquisition of direct statistical data has to be a slow process, earthquake risk prediction must rely on a mixed approach: this is the field of bayesian statistics.

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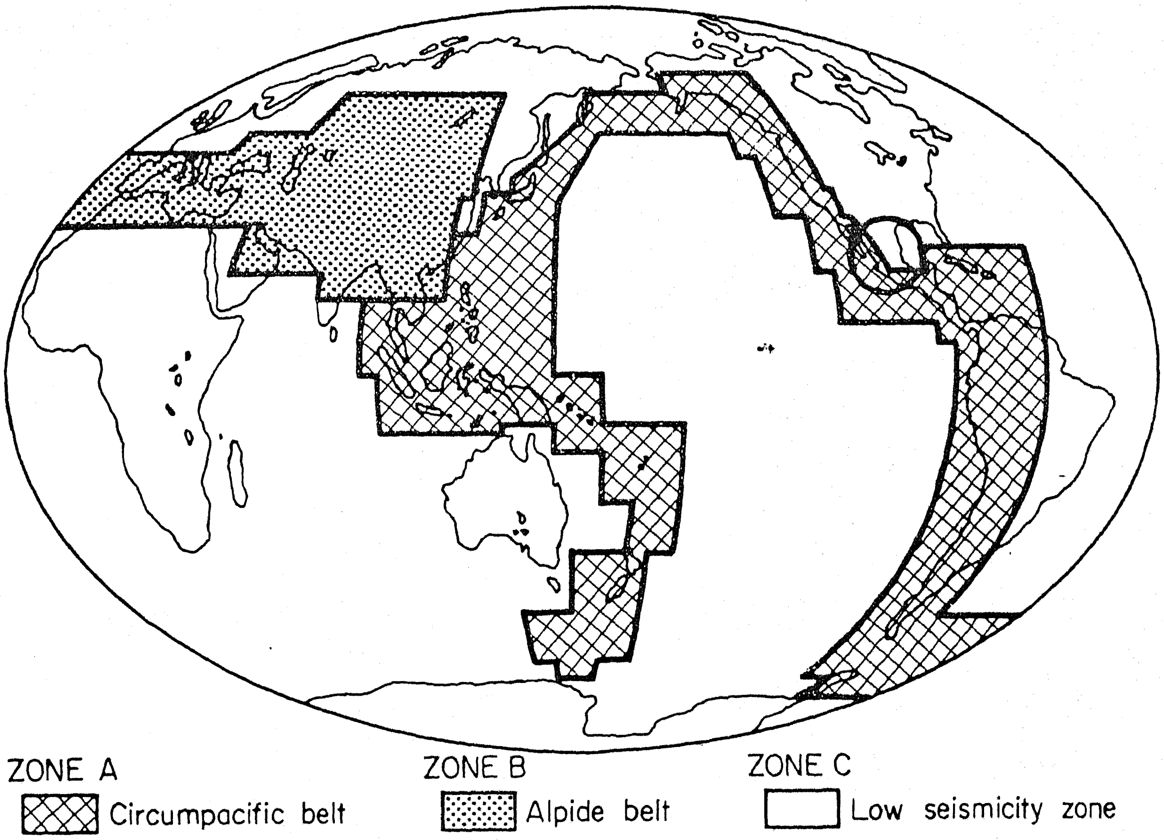
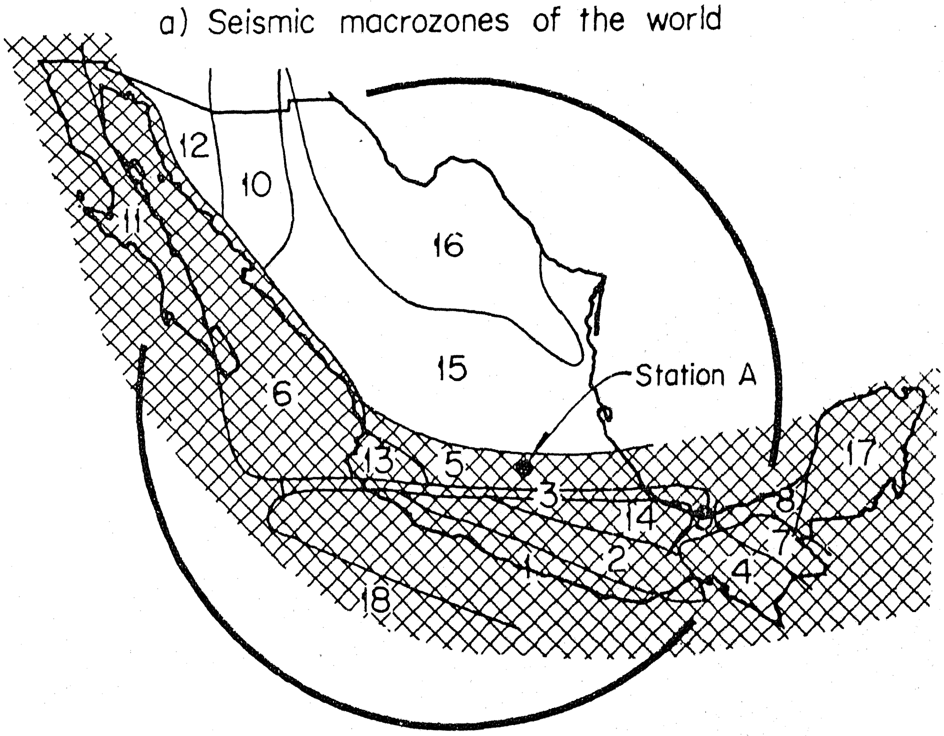
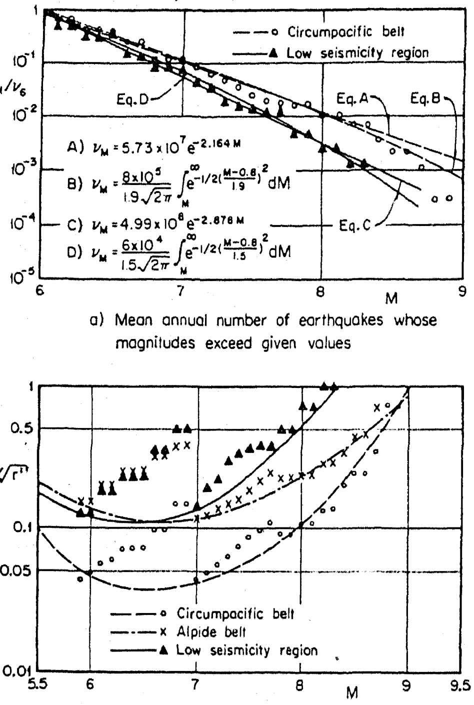


a) Seismic macrozones of the world



b) Geotectonic regions of Mexico

Fig 1 Seismic zones



a) Mean annual number of earthquakes whose magnitudes exceed given values

b) Coefficients of variation for ν_M in the three macrozones

Fig 2 Seismicity of macrozones

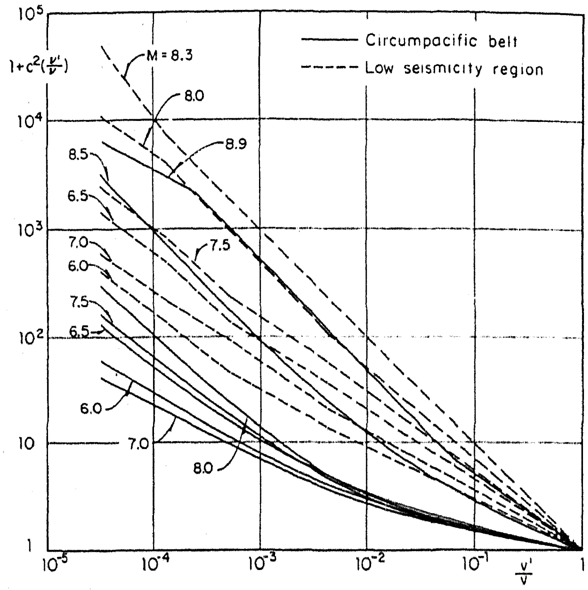
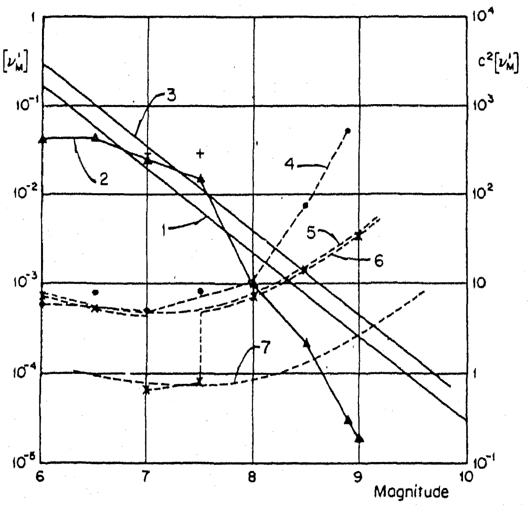


Fig 3 Variability of local seismicity within a given macrozone



EXPECTATIONS OF ν'_M

1 —— Initial

\+ Based on statistics only

2 —— ▲ Posterior, step by step method

3 —— Posterior, modified method

VARIATION COEFFICIENTS OF ν'_M

4 - - • Initial, neglecting prior information from other magnitude intervals

5 - - - Initial, including prior information from other magnitude intervals

6 - - x Posterior, neglecting statistical information from other magnitude intervals

7 - - - Posterior, including statistical information from other magnitude intervals

Fig 4 Bayesian estimation of local seismicity. Region 5

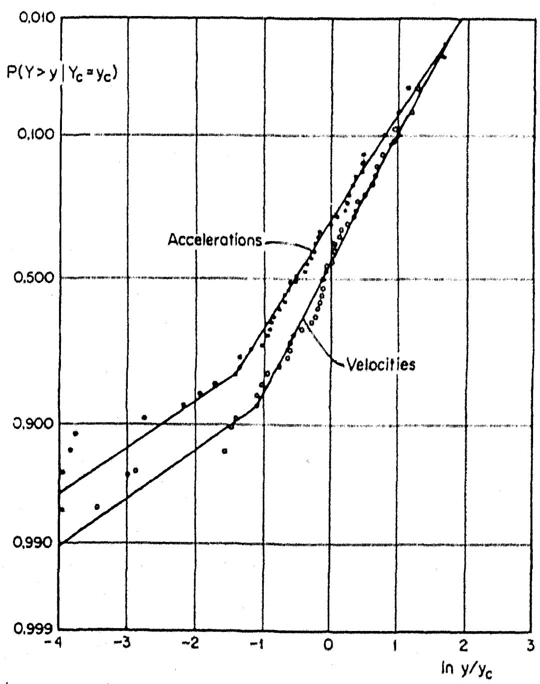


Fig 5 Uncertainty in correlations between magnitude, distance and intensity