STATISTICAL INFERENCE OF THE FUTURE EARTHQUAKE GROUND MOTION

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SYNOPSIS

Presented are the results of a theoretical analysis of the statistical properties of the ground motion in future strong earthquakes. The statistical model of earthquakes is proposed so as to be consistent with the past records of occurrence of earthquakes and with strong motion accelerograms, on the basis of which the methods are discussed to find the probability distribution of the maximum ground motion in a single earthquake and that for a certain future period. Also are discussed the r.m.s. intensity of the ground motion corresponding to an earthquake of arbitrary intensity scale. Numerical results are given in the form of charts and seismic maps.

1. INTRODUCTION

From the randomness of the sequence of strong earthquakes and of the ground motion in earthquakes, it is essential to make a statistical evaluation of the intensity of the earthquake for which structures are to be designed. The structure may be designed for the earthquake with the maximum ground motion corresponding to a certain assigned probability of excess or for a set of earthquakes with a certain assigned r.m.s. intensity. Whatever the measure of the earthquake intensity may be, there must be provided sufficient statistical information based on the data of occurrence of past strong earthquakes and seismograph records. Due to the fact, however, that even a most active seismic region is struck by catastrophic earthquakes at intervals of tens of years, the record of earthquakes before modern science is required. However, one can readily understand that it is extremely difficult to obtain a complete one of such a record. Fig.1, for example, shows the number of earthquakes felt to be of intensity V or stronger in the JMA scale(III) in every 200 years in Tokyo and Kyoto computed by H. Kawasumi's method[1] on the basis of the list of destructive earthquakes in the Chronological Table of Science, 1966 (edited by the Tokyo Astronomical Observatory). It is noted that for the most recent 200 years far more earthquakes are recorded than for other ages. It would be more reasonable to interpret this as being that some strong earthquakes which occurred in older ages are missing in this record than to believe that it is a precise description of past seismic activities. The statistical model of occurrence of earthquakes used in this study takes account of this possible time-dependence of the record of past earthquakes which has been neglected in former studies[1].

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(III) Unless otherwise is stated, the phrases "intensity V, intensity VI, etc." shall be used to represent the earthquake intensity in the JMA (Japan Meteorological Agency) scale.

In analyzing the statistical properties of the ground motion in a single earthquake, the seismic wave has been treated as a Gaussian random process with the power spectrum determined from those for strong motion accelerograms.

With the aid of these statistical models of earthquakes, the probability distribution of the earthquake ground motion in an arbitrary future period has been derived. Numerical results have been obtained for the main islands of Japan.

2. STATISTICAL MODEL OF EARTHQUAKES

(1) Sequence of Earthquakes

Divide the past ages into r intervals $B_1, B_2, \cdots, B_r$ of arbitrary length $S_1, S_2, \cdots, S_r$, respectively, and suppose that at a certain locality N_k, (k=1,2, $\cdots, r$), earthquakes occurred in the interval B_k, as shown in Fig.2. Let N denote the total number of earthquakes for this locality and let $n_{I1}, n_{I2}, \cdots$, n_{Im} of these N earthquakes were felt to be of intensity $I_1, I_2, \cdots, I_m$, respectively. Then we have

$$N = \sum_{k=1}^{r} N_k = \sum_{j=1}^{m} n_{Ij}$$

Our statistical model for this record shall be described as follows. *It is assumed that this record is a realization of N Bernoulli trials of which $n_{I1}, n_{I2}, \cdots n_{Im}$ trials are made for earthquakes of intensity $I_1, I_2, \cdots I_m$, respectively. The probability P_k that the earthquake in one trial will occur in the interval B_k is given by*

$$P_k = N_k / N \quad \cdots\cdots\cdots\cdots\cdots\cdots \quad (1)$$

The probability P_k is considered to represent the moment of the interval B_k in the whole record. By making a weighted evaluation of earthquake danger in each interval with the aid of such P_k, the usefulness of the record of past earthquakes whose accuracy is likely to vary with ages would be improved to a considerable extent.

If we assume that the intensity of seismic activity in the most recent interval B_r will not vary through a future interval B_f of length S_f for which the earthquake danger is to be estimated, then it suffices to analyze a subinterval B_f' of B_r of length S_f. The probability P_f that the earthquake will occur in the interval B_f' in one Bernoulli trial is given from Eq.(1) by

$$P_f = P_r S_f / S_r = N_r S_f / N S_r \quad \cdots\cdots\cdots\cdots\cdots\cdots \quad (2)$$

From the above discussions the probability $P_c(k_{I1}, k_{I2}, \cdots, k_{Im})$ that k_{I1}, $k_{I2}, \cdots, k_{Im}$ earthquakes of intensity $I_1, I_2, \cdots, I_m$, respectively, will occur in the interval B_f', hence in B_f also, is given by

$$P_c(k_{I1}, k_{I2}, \cdots, k_{Im}) = \mathrm{P}[\bigcap_{j=1}^{m} (k_{Ij}\text{e.q.'s of intensity } I_j \subset B_f')]$$

$$= \prod_{j=1}^{m} \left\{ \binom{n_{Ij}}{k_{Ij}} P_f^{k_{Ij}} (1-P_f)^{n_{Ij}-k_{Ij}} \right\} = \prod_{j=1}^{m} \mathrm{b}(k_{Ij}; n_{Ij}, P_f) \quad \cdots\cdots\cdots\cdots\cdots\cdots \quad (3)$$

where $b(k;n,p)$ is the binomial distribution function. The return period T_{rIj} of earthquakes of intensity I_j or stronger is obtained as

$$T_{rIj} = NS_r/N_r \sum_{l=j}^{m} n_{Il} \quad \cdots\cdots\cdots\cdots\cdots\cdots (4)$$

(2) Description of the Ground Motion.

It is clear that the time history of a strong earthquake motion is statistically nonstationary. However, it has been known from strong motion seismograph records that its strongest part of duration of, say several or ten and several seconds, are fairly stationary. Since we are concerned with the intensity of the ground motion in earthquakes, it would be reasonable to deal only with this strong part of the seismogram. Thus the earthquake acceleration $x(t)$ shall be represented by

$$x(t) = \beta f(t;\tau) g(t) \quad \cdots\cdots\cdots\cdots\cdots\cdots (5)$$

$$f(t;\tau) = u_-(t) - u_+(t-\tau) = \begin{cases} 1;\ 0 \leqq t \leqq \tau \\ 0;\ t<0,\quad t>\tau \end{cases}$$

where $g(t)$ is a nondimensional stationary random process with zero mean value and the variance of unity, β is a constant with the dimension of acceleration, and $u_\pm(t)$ are the asymmetrical unit step functions. Hence $x(t)$ becomes a stationary process in the interval $0 \leqq t \leqq \tau$.

From the test of significance on typical strong motion accelerograph records, it has been concluded that their amplitude characteristics are nearly Gaussian[2]. Hence we assume the Gaussian distribution for $g(t)$, hence for $x(t)$ also. Then the probability density $\phi(x;t)$ of $x(t)$ and the joint probability density $\phi_j(x,\dot{x};t)$ of $x(t)$ and $\dot{x}(t)$ are represented by

$$\phi(x;t) = \frac{1}{\sqrt{2\pi}\sigma_1} \exp\left\{-\frac{1}{2}\left(\frac{x}{\sigma_1}\right)^2\right\} \quad \cdots\cdots\cdots\cdots\cdots\cdots (6)$$

$$\phi_j(x,\dot{x};t) = \frac{1}{2\pi\sigma_1\sigma_2\sqrt{1-\rho_{12}^2}} \exp\left[-\frac{1}{2(1-\rho_{12}^2)}\left\{\left(\frac{x}{\sigma_1}\right)^2 - 2\rho_{12}\left(\frac{x}{\sigma_1}\right)\left(\frac{\dot{x}}{\sigma_2}\right) + \left(\frac{\dot{x}}{\sigma_2}\right)^2\right\}\right] \cdots\cdots (7)$$

where

$$\left.\begin{aligned} \sigma_1 &= \sigma_1(t) = \left\{E[x^2(t)]\right\}^{1/2} = \beta\sigma_{g_1} f(t;\tau) \\ \sigma_2 &= \sigma_2(t) = \left\{[E\dot{x}^2(t)]\right\}^{1/2} = \beta\left\{\sigma_{g_1}^2 \dot{f}^2(t;\tau) + \sigma_{g_2}^2 f^2(t;\tau)\right\}^{1/2} \\ \rho_{12} &= \rho_{12}(t) = \frac{E[x(t)\dot{x}(t)]}{\sigma_1\sigma_2} = \beta^2 \frac{\sigma_{g_1}^2}{\sigma_1\sigma_2} f(t;\tau)\dot{f}(t;\tau) \end{aligned}\right\} \quad \cdots\cdots\cdots\cdots\cdots\cdots (8)$$

The standard deviations σ_{g_1} and σ_{g_2} of $g(t)$ and $\dot{g}(t)$, respectively, are given by

$$\left.\sigma_{g_1}^2 = E[g^2(t)] = \int_0^\infty S_g(\omega)\, d\omega \quad\right\} \quad \cdots\cdots\cdots\cdots\cdots\cdots (9)$$

$$\sigma_{g_2}{}^2 = \mathrm{E}[\dot{g}^2(t)] = \int_0^\infty \omega^2 S_g(\omega)\, d\omega$$

where $S_g(\omega)$ is the power spectrum of $g(t)$. From the results of the spectral analysis of strong motion accelerograph records, $S_g(\omega)$ has been assumed to take the form[1),2)]

$$S_g(\omega) = \frac{128}{3\omega_0}\left(\frac{\omega}{\omega_0}\right)^4 e^{-4\frac{\omega}{\omega_0}} \qquad \cdots\cdots\cdots\cdots\cdots\cdots (10)$$

where ω_0 is the predominant frequency. The auto-correlation function $R(\tau)$ associated with Eq.(10) is obtained as its Fourier inverse transform:

$$R(\tau) = \frac{1}{\sigma_{g_1}{}^2}\int_0^\infty S_g(\omega)\cos\omega\tau\, d\omega = \left\{1-10\left(\frac{\omega_0\tau}{4}\right)^2+5\left(\frac{\omega_0\tau}{4}\right)^4\right\}\Big/\left\{1+\left(\frac{\omega_0\tau}{4}\right)^2\right\}^5 \cdots\cdots (11)$$

Substitution of Eq.(10) into Eqs.(9) yields

$$\sigma_{g_1} = 1\,, \quad \sigma_{g_2} = \frac{\sqrt{30}}{4}\omega_0 \qquad \cdots\cdots\cdots\cdots\cdots\cdots (12)$$

The same discussions can be made for the earthquake velocity $v(t)$. By analogy to Eq.(5), we set

$$v(t) = \beta f(t;\tau)\int_{-\infty}^t g(t)\, dt \qquad \cdots\cdots\cdots\cdots\cdots\cdots (13)$$

The probability densities related to Eq.(13) are obtained as

$$\phi(v;t) = \frac{1}{\sqrt{2\pi}\,\sigma_0}\exp\left\{-\frac{1}{2}\left(\frac{v}{\sigma_0}\right)^2\right\} \qquad \cdots\cdots\cdots\cdots\cdots\cdots (14)$$

$$\phi_j(v,\dot{v};t) = \frac{1}{2\pi\sigma_0\sigma_1\sqrt{1-\rho_{01}{}^2}}\exp\left[-\frac{1}{2(1-\rho_{01}{}^2)}\left\{\left(\frac{v}{\sigma_0}\right)^2 - 2\rho_{01}\left(\frac{v}{\sigma_0}\right)\left(\frac{\dot{v}}{\sigma_1}\right)+\left(\frac{\dot{v}}{\sigma_1}\right)^2\right\}\right] \cdots (15)$$

where

$$\left.\begin{aligned}
\sigma_0 &= \sigma_0(t) = \left\{\mathrm{E}[v^2(t)]\right\}^{1/2} = \beta\sigma_{g_0} f(t;\tau)\\
\sigma_1 &= \sigma_1(t) = \left\{\mathrm{E}[\dot{v}^2(t)]\right\}^{1/2} = \beta\left\{\sigma_{g_0}{}^2\dot{f}^2(t;\tau)+\sigma_{g_1}{}^2 f^2(t;\tau)\right\}^{1/2}\\
\rho_{01} &= \rho_{01}(t) = \frac{\mathrm{E}[v(t)\dot{v}(t)]}{\sigma_0\sigma_1} = \beta^2\frac{\sigma_{g_0}{}^2}{\sigma_0\sigma_1} f(t;\tau)\dot{f}(t;\tau)
\end{aligned}\right\} \qquad \cdots\cdots\cdots\cdots\cdots\cdots (16)$$

and

$$\sigma_{g_0}{}^2 = \mathrm{E}\left[\left\{\int_{-\infty}^t g(t)\, dt\right\}^2\right] = \int_0^\infty \frac{S_g(\omega)}{\omega^2}\, d\omega \qquad \cdots\cdots\cdots\cdots\cdots\cdots (17)$$

On substitution from Eq.(10) to Eq.(17), we obtain

$$\sigma_{g_0} = \frac{2}{\sqrt{3}\,\omega_0} \qquad \cdots\cdots\cdots\cdots\cdots\cdots (18)$$

3. PROBABILITY DISTRIBUTION OF THE MAXIMUM GROUND MOTION IN A SINGLE EARTHQUAKE

(1) Basic Analysis

In this section, we shall derive the probability distribution of the maximum ground motion in a single earthquake for the statistical model proposed in the previous chapter. First, let α denote the maximum ground acceleration in a single earthquake described by Eq.(5). Then the probability distribution $\Phi_S(\alpha)$ of α is represented by

$$\Phi_s(\alpha) = P[\max|x(t)| \leqq \alpha;\ 0 \leqq t \leqq \tau] = P[|x(0)| \leqq \alpha]\ P_0(\alpha;\tau) \quad \cdots\cdots\cdots\cdots (19)$$

where

$$P_0(\alpha;t) = P\left[\left\{\max|x(t)| \leqq \alpha;\ 0 \leqq t \leqq t\right\}\Big|\,|x(0)| \leqq \alpha\right]$$

Since $x(t)$ is a continuous process, we have, from the fundamental differential equation for a birth process[4],

$$\frac{d}{dt} P_0(\alpha;t) = -C_0(\alpha;t)\ P_0(\alpha;t) \quad \cdots\cdots\cdots\cdots (20)$$

in which

$$c_0(\alpha,t)dt = P[\{|x(t+dt)|>\alpha\,|\max|x(t'')|\leqq\alpha\};\ 0\leqq t''\leqq t]$$

$$= \frac{P[\{|x(t+dt)|>\alpha \cap \max|x(t'')|\leqq\alpha\};\ 0\leqq t''\leqq t]}{P[\max|x(t'')|\leqq\alpha;\ 0\leqq t''\leqq t]} \quad \cdots\cdots\cdots\cdots (21)$$

Solving Eq.(20) under the initial condition $P_0(\alpha;0)=1$, and substituting into Eq.(19), we obtain

$$\Phi_s(\alpha) = P[|x(0)| \leqq \alpha]\ \exp\left\{-\int_0^\tau C_0(\alpha;t)\,dt\right\} \quad \cdots\cdots\cdots\cdots (22)$$

Inasmuch as it is an extremely difficult task to find the exact solution for $c_0(\alpha;t)$, we make an assumption that $c_0(\alpha;t)$ is independent of wheather $\max|x(t')|\leqq\alpha$ or not in the interval $0\leqq t'<t$. Thus the process of crossing of $x(t)=\pm\alpha$ becomes the Poisson process, and we obtain an approximation $\Psi_S(\alpha)$ to $\Phi_S(\alpha)$ as

$$\Psi_s(\alpha) = P[|x(0)| \leqq \alpha]\ \exp\left\{-\int_0^\tau C_0^*(\alpha;t)\,dt\right\} \quad \cdots\cdots\cdots\cdots (23)$$

where

$$C_0^*(\alpha;t)dt = P[|x(t)| \leqq \alpha \cap |x(t+dt)| > \alpha] \big/ P[|x(t)| \leqq \alpha]$$

$$= N_{c\alpha}(t)\,dt \big/ P[|x(t)| \leqq \alpha] \quad \cdots\cdots\cdots\cdots (24)$$

Here $N_{c\alpha}(t)$ denotes the probability that $x(t)$ will cross a positive threshold valuε with positive slope or a negative value $-\alpha$ with negative slope in the unit time, hich is represented by[5]

$$N_{c\alpha}(t) = \int_{-\infty}^{0} |\dot{x}| \phi_j(-\alpha, \dot{x}; t) d\dot{x} + \int_{0}^{\infty} \dot{x}\phi_j(\alpha, \dot{x}; t) d\dot{x} \qquad \cdots\cdots (25)$$

It is noted in passing that in the interval $0 \leq t \leq \tau$ where $x(t)$ and $v(t)$ are stationary, Eqs.(8) and (16) are simplified as

$$\left.\begin{aligned} &\sigma_0 = \frac{2\beta}{\sqrt{3}\,\omega_0} = \frac{\beta T_0}{\sqrt{3}\,\pi}, \quad \sigma_1 = \sigma_1 = \beta, \quad \sigma_2 = \frac{\sqrt{30}}{4}\beta\omega_0 = 2.7386\frac{\pi\beta}{T_0}, \\ &\rho_{01} = \rho_{12} = 0 \end{aligned}\right\} \cdots (26)$$

On applying the statistical model of the ground motion proposed in 2.(2) to these results, Eq.(23) yields

$$\Psi_s(\alpha) = \mathrm{erf}\left(\frac{\alpha}{\sqrt{2}\,\beta}\right) \exp\left[-2.7386\frac{\tau}{T_0} \exp\left\{-\frac{1}{2}\left(\frac{\alpha}{\beta}\right)^2\right\} \Big/ \mathrm{erf}\left(\frac{\alpha}{\sqrt{2}\,\beta}\right)\right] \cdots\cdots (27)$$

Likewise, an approximation $\tilde{\Psi}_s(v_m)$ to the probability distribution $\tilde{\Phi}_s(v_m)$ of the maximum velocity v_m is obtained as

$$\tilde{\Psi}_s(v_m) = \mathrm{erf}\left(\frac{\sqrt{3}\,\pi}{\sqrt{2}\,\beta T_0}v_m\right) \exp\left[-\sqrt{3}\frac{\tau}{T_0} \exp\left\{-\frac{1}{2}\left(\frac{\sqrt{3}\,\pi}{\beta T_0}v_m\right)^2\right\} \Big/ \mathrm{erf}\left(\frac{\sqrt{3}\,\pi}{\sqrt{2}\,\beta T_0}v_m\right)\right] \cdots\cdots (28)$$

If we set

$$\zeta = \frac{\alpha}{\sigma_1} = \frac{\alpha}{\beta}, \quad \eta = \frac{v_m}{\sigma_0} = \frac{\sqrt{3}\,\pi}{\beta T_0}v_m \qquad \cdots\cdots (29)$$

then $\Psi_s(\alpha)$ and $\tilde{\Psi}_s(v_m)$ are normalized as

$$\Psi_{sn}(\zeta) = \mathrm{erf}\left(\frac{\zeta}{\sqrt{2}}\right) \exp\left\{-2.7386\frac{\tau}{T_0} \exp\left(-\frac{\zeta^2}{2}\right) \Big/ \mathrm{erf}\left(\frac{\zeta}{\sqrt{2}}\right)\right\} \cdots\cdots (30)$$

and

$$\tilde{\Psi}_{sn}(\eta) = \mathrm{erf}\left(\frac{\eta}{\sqrt{2}}\right) \exp\left\{-\sqrt{3}\frac{\tau}{T_0} \exp\left(-\frac{\eta^2}{2}\right) \Big/ \mathrm{erf}\left(\frac{\eta}{\sqrt{2}}\right)\right\} \cdots\cdots (31)$$

Fig.3 are the plots of $\Psi_{sn}(\zeta)$ and $\tilde{\Psi}_{sn}(\eta)$ for various values of τ/T_0.

The Poisson process approximation made in the derivation of Eqs.(27)∿(31) are considered to become better 1) as the power spectra of $x(t)$ and $v(t)$ flatten, 2) for such a large α and v_m that the crossing of $x(t)=\pm\alpha$ or $v(t)=\pm v_m$ occurs almost independently of the previous crossing of the same level, and 3) for properly large τ/T_0 for which $\Psi_{sn}(\zeta)$ and $\tilde{\Psi}_{sn}(\eta)$ at relatively lower level of ζ and η, respectively, almost vanish. Here the accuracy of these formulae is examined in two ways; i.e., they are compared with the theoretical upper and lower bounds and with the results of a numerical simulation. It suffices to make these tests for the lower limit of τ/T_0, since the larger τ/T_0 is, the more accurate the approximation becomes. In most major earthquakes τ/T_0 would fall into the range of some 10∿100. Hence in the present test of accuracy, τ/T_0 is taken as 10 and 30; the latter value is used for the application in the next chapter. A method for determining the lower and upper bounds of the probability of excess in a random process has been presented by M. Shinozuka[6]. In this study, his method has been improved on so as to give closer bounds to the real value.

Choose the instants τ_k such that

$$0 = \tau_1 < \tau_2 < \cdots < \tau_{n+1} = t$$

as illustrated in Fig.4. In this figure, τ_0 is the value of τ at which $R(\tau)$ first crosses the τ-axis, and τ_c is the correlation time for which $R(\tau)$ becomes so small that $g(t)$ and $g(t+\tau_a)$ can be treated as independent for any $\tau_a \geqq \tau_c$. It is noted that $n(t)$ must be integral times three. Then the lower bound $\Phi_{sl}(\alpha)$ and the upper bound $\Phi_{su}(\alpha)$ of $\Phi_s(\alpha)$ are obtained as[2),3)]

$$\Phi_{sl}(\alpha) = 1-\int_{-\infty}^{\infty} \mathrm{P}[\bigcap_{k=1}^{n(t)} |x(\tau_k)| \leqq \alpha]\cdot N_{ca}(t)\, dt$$

$$= \mathrm{erf}\left(\frac{\alpha}{\sqrt{2}\ \beta}\right) - 2.7386\frac{\tau}{KT_0}\frac{1-\{p_s(\alpha)\}^K}{1-p_s(\alpha)}\exp\left\{-\frac{1}{2}\left(\frac{\alpha}{\beta}\right)^2\right\} < \Phi_s(\alpha) \quad \cdots\cdots\cdots (32)$$

$$\Phi_{su}(\alpha) = \mathrm{P}[\bigcap_{k=1}^{n(\bar{\tau})} |x(\tau_k)| \leqq \alpha] = \{p_s(\alpha)\}^{K+1} > \Phi_s(\alpha) \quad \cdots\cdots\cdots (33)$$

where

$$p_s(\alpha) = \mathrm{erf}\left(\frac{\alpha}{\sqrt{2}\ \beta}\right)\cdot\frac{1}{\sqrt{2\pi}}\int_0^{\alpha/\beta}\exp\left(-\frac{\xi^2}{2}\right)\left[\mathrm{erf}\left\{\frac{1}{\nu}\left(\frac{\alpha}{\beta}+R(2\tau_0)\cdot\xi\right)\right\} + \mathrm{erf}\left\{\frac{1}{\nu}\left(\frac{\alpha}{\beta}-R(2\tau_0)\cdot\xi\right)\right\}\right]d\xi$$

$$\nu = \sqrt{2(1-\{R(2\tau_0)\}^2)}\ ,\quad K = n(\tau)/3\ ,\quad \bar{\tau} = \tau+\tau_c+2\tau_0$$

In the integration of Eq.(32), Eqs.(26) are no longer valid and Eqs.(8) are involved. The normalized form of Eqs.(32) and (33) analogous to Eq.(30) is obtained, respectively, as follows:

$$\Phi_{sln}(\zeta) = \mathrm{erf}\left(\frac{\zeta}{\sqrt{2}}\right) - 2.7386\frac{\tau}{KT_0}\frac{1-\{p_{sn}(\zeta)\}^K}{1-p_{sn}(\zeta)}\exp\left(-\frac{\zeta^2}{2}\right) \quad \cdots\cdots\cdots (34)$$

$$\Phi_{sun}(\zeta) = \{p_{sn}(\zeta)\}^{K+1} \quad \cdots\cdots\cdots (35)$$

where

$$\zeta = \frac{\alpha}{\beta},\quad p_{sn}(\zeta) = p_{sn}\left(\frac{\alpha}{\beta}\right) = p_s(\alpha)$$

From Eq.(11), we obtain

$$\tau_0/T_0 = 0.2068\ ,\quad R(2\tau_0) = -0.4005$$

and it would suffice to take the correlation time τ_c as

$$\tau_c/T_0 = 3.5$$

Using these data, the lower and upper bounds of $\Phi_s(\alpha)$ can be computed for any τ/T_0 with the aid of Eqs.(32) and (33) or of Eqs.(34) and (35), examples of which are shown in Fig.5 for τ/T_0=10 and 30 in comparison with $\Psi_{sn}(\zeta)$ in Eq.(30). It is observed that $\Phi_{sln}(\zeta)$ and $\Psi_{sn}(\zeta)$ almost coincide in the range of the non-excess probability of about 0.8. Thus it could be stated at least for this range that the approximation by Eq.(30) is good enough, since from the statistical meaning of $\Phi_{sl}(\alpha)$ and $\Psi_s(\alpha)$ we can assert that they both will furnish good approximations to $\Phi_s(\alpha)$ for a high probability level. For the lower range of ζ corresponding to a lower probability level, the question how well $\Psi_{sn}(\zeta)$ approximates the real distribution does not pose a serious problem, since it almost vanishes for such ζ, and indeed so does the upper bound $\Phi_{sun}(\zeta)$.

Next, the numerical simulation was made on a digital computer, in which 219 sample accelerograms were generated for τ/T_0=10 and 73 of them for τ/T_0=30. Fig.6 shows the experimental distribution thus obtained compared with $\Psi_{sn}(\zeta)$, which demonstrates that the experimental and theoretical values are in fairly good agreement.

The same discussions have been made on the maximum velocity, for which similar results have been obtained. From these discussions, it would be appropriate to conclude that Eqs.(27)∿(31) due to the Poisson process approximation is available with sufficient accuracy to represent the probability distribution of the maximum ground motion in a single earthquake.

(2) The Intensity Parameter β

The parameter β in Eq.(5) is to be related to the intensity of earthquakes. In this study, β_I which denotes the value of β for the earthquakes of intensity I is determined so that the mean value of the maximum ground acceleration in a single earthquake of intensity I may equal the earthquake acceleration α_I obtained for the same intensity from the empirical formula proposed by seismologists; i.e., we set

$$E[\alpha] = \int_0^\infty \{1-\Phi_s(\alpha)\}\,d\alpha = \alpha_I \quad \cdots\cdots\cdots\cdots\cdots\cdots (36)$$

If we use the approximate distribution functions $\Psi_s(\alpha)$ and $\Psi_{sn}(\zeta)$, we have

$$E[\alpha] \simeq \int_0^\infty \{1-\Psi_s(\alpha)\}\,d\alpha = \beta_I\int_0^\infty \{1-\Psi_{sn}(\zeta)\}\,d\zeta = \alpha_I$$

Hence β_I is obtained as

$$\beta_I = \alpha_I \Big/ \int_0^\infty \{1-\Psi_{sn}(\zeta)\}\,d\zeta \quad \cdots\cdots\cdots\cdots\cdots\cdots (37)$$

As we see in Eq.(26), β_I^2 represents the variance of the stationary part of the accelerogram. Hence β_I would serve as a direct measure of the earthquake intensity in the structural response analysis in which the structure is subjected to an ensemble of random earthquakes with a certain assigned r.m.s. intensity. Fig.7 is a plot of β_I/α_I versus τ/T_0. It is noted that β_I does not change greatly in the range τ/T_0=10∿100 which is of interest to us. This fact simplifies the problem since we can assert that the duration of the earthquake has little effect upon the mean value of the maximum ground motion.

(3) Determination of α_I

The intensity parameter β_I (or β) is affected directly by what formula we use for the determination of α_I. In the present study, the empirical formula proposed by K. Kanai[7] shall be adopted. Kanai has proposed a formula with an account for the predominant period T_0 on the basis of the data of earthquakes with I_{MM}=6~8, (I_{MM}: modified Mercalli intensity scale):

$$\alpha_{I_{MM}} = 0.62\ T_0^{-1.316}\ 10^{0.238 I_{MM}}\ (\mathrm{cm/sec^2}),\quad T_0\text{: in sec} \qquad \cdots\cdots (38)$$

For I_{MM}=8; i.e., for I=V in the JMA scale, we have

$$\alpha_{V} = 50\ T_0^{-1.316}\ (\mathrm{cm/sec^2}) \qquad \cdots\cdots (39)$$

For higher intensities, the values of acceleration proposed by JMA[8] shall be refered to. The middle value of the upper and lower bounds of the JMA accleration for I=V is given as 170cm/sec^2, which corresponds to T_0=0.395≃0.4sec in Eq.(39). We assume that the JMA values for higher I correspond to T_0=0.4sec as well and that, in this range of I, the earthquake acceleration is also proportional to $T_0^{-1.316}$. The middle value of the JMA acceleration for I=VI is 320cm/sec^2. Hence we have

$$\alpha_{VI} = 320\left(\frac{T_0}{0.4}\right)^{-1.316} = 96\ T_0^{-1.316}\ (\mathrm{cm/sec^2}) \qquad \cdots\cdots (39')$$

Extrapolating the same rule for the case of I=VII,we obtain the similar formula for I=VII as

$$\alpha_{VII} = 470\left(\frac{T_0}{0.4}\right)^{-1.316} = 140\ T_0^{-1.316}\ (\mathrm{cm/sec^2}) \qquad \cdots\cdots (39'')$$

4. PROBABILITY DISTRIBUTION OF THE MAXIMUM GROUND MOTION AT A CERTAIN LOCALITY IN A FUTURE PERIOD

(1) Formal Representation

If the probability distribution of the maximum ground motion in a single earthquake discussed in the previous chapter is known, the probability distribution of the maximum earthquake ground motion to occur at a certain locality in a future period can be determined with the aid of the statistical model of occurrence of earthquakes proposed in 2.(1).

Let α_f denote the maximum earthquake acceleration to occur in a future interval B_f of length S_f, and A_f denote its realized value. If $R_f(k_{I_1},k_{I_2},\cdots,k_{I_m})$ represents the event that $k_{I_1},k_{I_2},\cdots,k_{I_m}$ earthquakes of intensity $I_1,I_2,\cdots,I_m$, respectively, occur in B_f, then the conditional probability distribution $\Phi_c(\alpha_f|k_{I_1},k_{I_2},\cdots,k_{I_m})$ of α_f on the hypothesis of $R_f(k_{I_1},k_{I_2},\cdots,k_{I_m})$ is given by

$$\Phi_c(\alpha_f|k_{I_1},k_{I_2},\cdots,k_{I_m}) = \mathrm{P}[A_f \leqq \alpha_f | R_f(k_{I_1},k_{I_2},\cdots,k_{I_m})] = \prod_{j=1}^{m}\{\Phi_s(\alpha_f;I_j)\}^{k_{I_j}} \qquad \cdots\cdots (40)$$

The events $R_f(k_{I_1},k_{I_2},\cdots,k_{I_m})$ and $R_f(l_{I_1},l_{I_2},\cdots,l_{I_m})$ are exclusive if $\bigcup_{j=1}^{m}(k_{I_j}\neq l_{I_j})$ holds. With this and Eq.(3), we can derive the probability distribution $\Phi_f(\alpha_f)$ of the maximum earthquake acceleration in the future interval B_f in the following form:

$$\Phi_f(\alpha_f) = \mathrm{P}[\bigcup_{k_{I_1}=0}^{n_{I_1}}\bigcup_{k_{I_2}=0}^{n_{I_2}}\cdots\bigcup_{k_{I_m}=0}^{n_{I_m}}\{A_f \leqq \alpha_f \cap R_f(k_{I_1},k_{I_2},\cdots,k_{I_m})\}]$$

$$= \sum_{k_{I1}=0}^{n_{I1}} \sum_{k_{I2}=0}^{n_{I2}} \cdots \sum_{k_{Im}=0}^{n_{Im}} P[A_f \leqq \alpha_f \cap R_f(k_{I1}, k_{I2}, \cdots, k_{Im})]$$

$$= \sum_{k_{I1}=0}^{n_{I1}} \sum_{k_{I2}=0}^{n_{I2}} \cdots \sum_{k_{Im}=0}^{n_{Im}} \{ \Phi_c(\alpha_f | k_{I1}, k_{I2}, \cdots, k_{Im}) P_c(k_{I1}, k_{I2}, \cdots, k_{Im}) \}$$

$$= \sum_{k_{I1}=0}^{n_{I1}} \sum_{k_{I2}=0}^{n_{I2}} \cdots \sum_{k_{Im}=0}^{n_{Im}} \{ \prod_{j=1}^{m} [\{\Phi_s(\alpha_f; I_j)\}^{k_{Ij}} \mathrm{b}(k_{Ij}; n_{Ij}, P_f)] \} \quad \cdots\cdots\cdots\cdots\cdots \quad (41)$$

Likewise, the approximate distribution function $\Psi_f(\alpha_f)$ of the future maximum acceleration is obtained as

$$\Psi_f(\alpha_f) = \sum_{k_{I1}=0}^{n_{I1}} \sum_{k_{I2}=0}^{n_{I2}} \cdots \sum_{k_{Im}=0}^{n_{Im}} \{ \prod_{j=1}^{m} [\{\Psi_s(\alpha_f; I_j)\}^{k_{Ij}} \mathrm{b}(k_{Ij}; n_{Ij}, P_f)] \} \quad \cdots\cdots\cdots\cdots\cdots \quad (42)$$

In the same manner, the approximate distribution function $\tilde{\Psi}_f(v_{mf})$ of the future maximum earthquake velocity v_{mf} is represented in the following form:

$$\tilde{\Psi}_f(v_{mf}) = \sum_{k_{I1}=0}^{n_{I1}} \sum_{k_{I2}=0}^{n_{I2}} \cdots \sum_{k_{Im}=0}^{n_{Im}} \{ \prod_{j=1}^{m} [\{\tilde{\Psi}_s(v_{mf}; I_j)\}^{k_{Ij}} \mathrm{b}(k_{Ij}; n_{Ij}, P_f)] \} \quad \cdots\cdots\cdots\cdots\cdots \quad (43)$$

In Eqs.(41)∿(43), $\Phi_s(\alpha_f;I_j)$, $\Psi_s(\alpha_f;I_j)$, and $\tilde{\Psi}_s(v_{mf};I_j)$ are the values , respectively, of $\Phi_s(\alpha_f)$, $\Psi_s(\alpha_f)$, and $\tilde{\Psi}_s(v_{mf})$ in which β_{I_j} is used for β.

The mean values of α_f and v_{mf} are represented in terms, respectively, of $\Psi_f(\alpha_f)$ and $\tilde{\Psi}_f(v_{mf})$ by

$$\left. \begin{aligned} \mathrm{E}[\alpha_f] &\simeq \int_0^\infty \{1-\Psi_f(\alpha_f)\} d\alpha_f \\ \mathrm{E}[v_{mf}] &\simeq \int_0^\infty \{1-\tilde{\Psi}_f(v_{mf})\} dv_{mf} \end{aligned} \right\} \quad \cdots\cdots\cdots\cdots\cdots \quad (44)$$

(2) Data of Past Earthquakes Prepared for Numerical Application

The results of analysis in the present study shall be applied to the data of earthquakes available for the main islands of Japan. Here we deal with earthquakes of intensity V, VI, and VII. Earthquakes of lower intensities are neglected because of lack of data of such earthquakes in former times. This, however, will not greatly affect the result since the acceleration for lower intensities rapidly decreases and $\Phi_s(\alpha_f;I_j)$ for such an intensity tends to unity even for small α_f.

Every locality on the main islands of Japan are represented by grid points taken at every 30′ in latitude and longitude. The shock intensities felt at these grid points were computed by means of Kawasumi's method[1] for all earthquakes indicated in the Chronological Table of Science (1966) whose dates, locations and magnitudes are known.

From the statistical point of view, the length S_r of the interval B_r should be sufficiently large compared with S_f. However, S_r must not be so large that B_r stretches over the ages in which the accuracy of the data of past earthquakes greatly varies. In this study, we take S_r=150years for Hokkaido and S_r=200years for other districts. Thus the numbers of earthquakes N, N_r, n_V, n_{VI}, n_{VII} and the return periods T_{rV}, T_{rVI}, T_{rVII} were obtained, some of which are shown in Table 1.

(3) Effect of τ/T_0

In computing the probability distribution of the future maximum ground motion, we must assign some proper value for τ/T_0. We have seen in 3.(2) that β_I, hence the expected value E[ζ] also, does not change greatly with τ/T_0 in the range of our interest. This is also true when the shape of $\Psi_s(\alpha;I)$ is concerned, which gives us a prospect for the validity of fixing the value of τ/T_0.

The approximate distribution $\Psi_f(\alpha_f)$ of the future maximum acceleration for Tokyo and Kyoto computed from Eq.(42) for the seismic data discussed in the previous section is shown in Fig.8 for T_0=0.5sec, S_f=75years and τ/T_0=10~100. Variation of $\Psi_f(\alpha_f)$ for different τ/T_0 would be small enough for engineering purposes. The expected value E[α_f] computed from these distribution functions with the aid of Eq.(44) also varies in a small range; 328~336cm/sec^2 for Tokyo and 255~262cm/sec^2 for Kyoto. In what follows, we use τ/T_0=30 which gives nearly the middle value of β_I between those for τ/T_0=30 and τ/T_0=100.

(4) Local Seismicity and Seismic Maps

The probability distributions $\Psi_f(\alpha_f)$ and $\tilde{\Psi}_f(v_{mf})$ of the future maximum ground motion and their expected values E[α_f] and E[v_{mf}] were computed for all grid points covering Japan. The length of the future interval B_f was taken as S_f=75years. Some examples of $\Psi_f(\alpha_f)$ are shown in Fig.9, which shall be discussed along with Fig.8.

The finite value of $\Psi_f(0)$ results from casting earthquakes of lower intensities away, and it represents the probability that no earthquake of intensity V or above it will occur in the interval B_f. It is also noted that the shape of $\Psi_f(\alpha_f)$ varies greatly with localities. For example, in spite of the fact that E[α_f] for Kyoto is larger than that for Miyazaki, the value of α_f corresponding to the non-excess probability of 90% for Miyazaki is larger than that for Kyoto. Thus it would be hard to find out some standard shape of $\Psi_f(\alpha_f)$, and therefore, when a precise probabilistic judgment must be made for α_f, reference should be made not only to the expected value E[α_f] but also to the distribution function $\Psi_f(\alpha_f)$ particular to each locality.

With this fact in mind, there is no doubt that the expected value E[α_f] serves as a direct measure of the future earthquake danger. Fig.10 is a seismic map showing the distribution of E[α_f] over the main islands of Japan. In Fig.10, also are shown the seismic map for the velocity E[v_{mf}].

All discussions in this section have been made for the predominant period of 0.5sec. Hence, when earthquakes with other predominant period T_0 is in question, all values of α_f and E[α_f] in this section must be multiplied by $(T_0/0.5)^{-1.316}$, and v_{mf} and E[v_{mf}], by $(T_0/0.5)^{-0.316}$; in both cases T_0 is given in sec.

5. CONCLUSIONS

From the results of analyses in this study, following conclusions may be derived.

1) With the aid of the statistical model of earthquakes proposed in 2., one can make a probabilistic analysis of the occurrence of strong earthquakes and of the ground motion in earthquakes with due consideration of the possible time-dependence of the accuracy of the record of past earthquakes and with that for a certain future period.

2) On the basis of this statistical model of earthquakes, methods have been discussed for deriving the probability distribution of the maximum ground motion in a single earthquake and that for a certain future period.

3) It has been proved by a theoretical analysis and the result of numerical simulation that the distribution functions defined by Eqs.(26)∿(30) due to the Poisson process approximation are good approximations to the probability distribution of the maximum ground motion in a single earthquake.

4) The parameter β (or β_I) discussed in 3.(2) would serve as a direct measure of the earthquake intensity in the structural response analysis in which the structure is to be subjected to an ensemble of earthquakes with a certain assigned r.m.s. intensity.

5) The shape of the probability distribution function of the maximum ground motion in a single earthquake and its expected value are not greatly affected by τ/T_0 in the range τ/T_0=10∿100 which is of our interest.

6) It can be stated from the numerical results that when a precise probabilistic judgment of the future maximum ground motion in earthquakes is required, reference should be made not only to its expected value but also to its distribution function which has a shape particular to the locality under discussion.

7) A rough prediction of the maximum earthquake ground motion in future can be made by means of the seismic map showing its expected value. For this purpose, the authors would like to recommend to use Fig.10 which are based or the method of analysis in this study.

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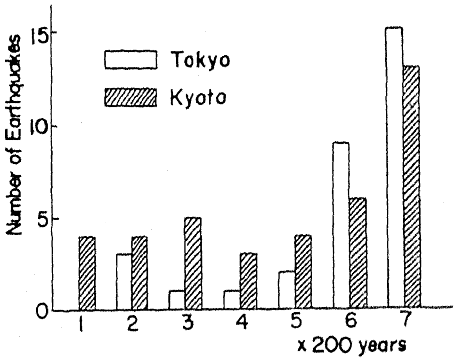

Fig.1 Number of Earthquakes in Every 200 Years
(I≥V ; A.D.565 ~ 1964)

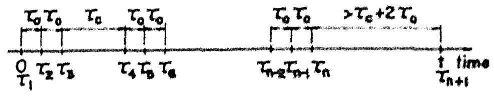

Fig.4 Illustration of the Instants τ_k

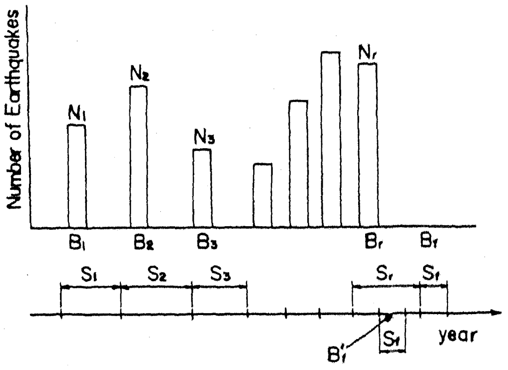

Fig.2 Division of Time Axis and Number of Earthquakes

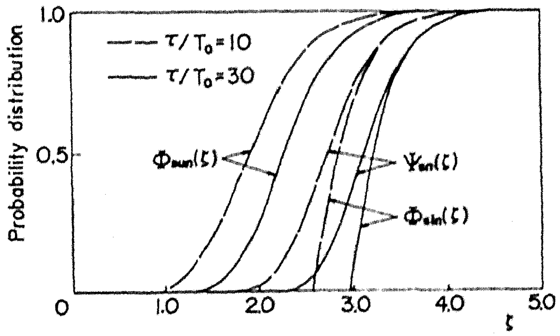

Fig.5 Upper and Lower Bounds of the Probability Distribution of the Maximum Acceleration in a Single Earthquake

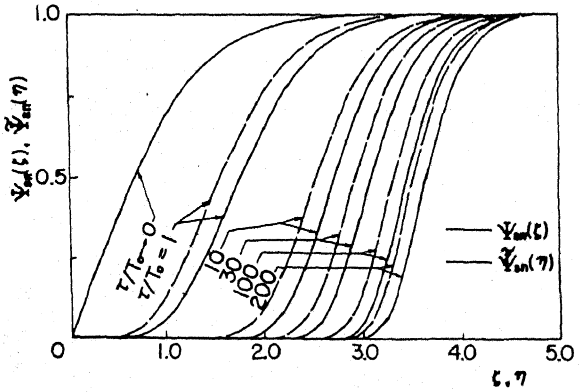

Fig.3 Approximate Probability Distribution of the Maximum Ground Motion in a Single Earthquake

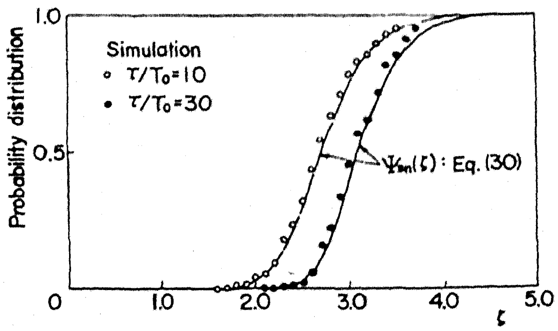

Fig.6 Experimental Distribution of the Maximum Acceleration in a Single Earthquake

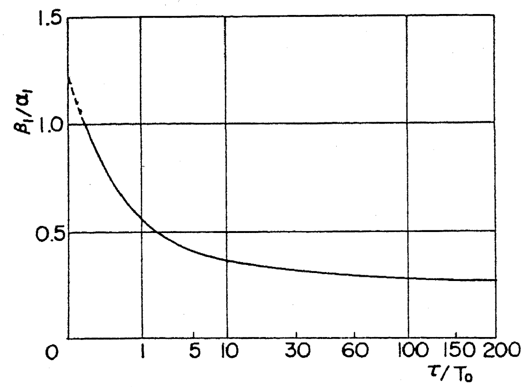

Fig.7 Intensity Parameter β_1
(cubic-root scale for abscissa)

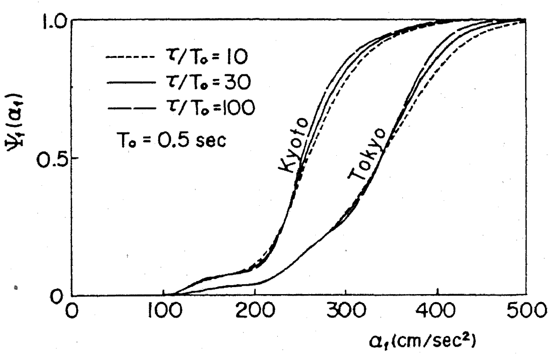

Fig.8 Probability Distribution of the Maximum Earthquake Acceleration to Occur in 75 Years

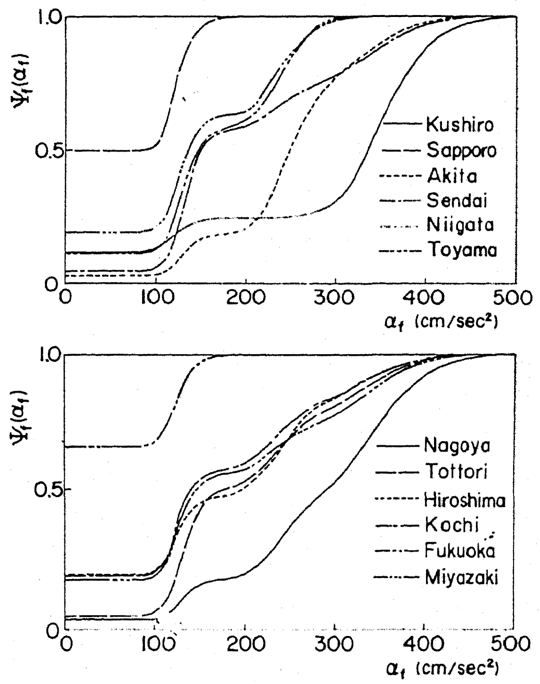

Fig.9 Probability Distribution of the Maximum Earthquake Acceleration to Occur in 75 Years ($T_0 = 0.5$ sec, $\tau/T_0 = 30$)

Table 1 Data of Past Earthquakes

Location		Number of Earthquakes					Return Period (year)			S_r (year)
No. in Fig.10	Name	N	N_r	n_V	n_{VI}	n_{VII}	T_{rV}	T_{rVI}	T_{rVII}	
1	Kushiro	3	3	1	0	2	50	75	75	150
2	Sapporo	1	1	1	0	0	150	∞	∞	150
3	Akita	14	8	7	6	1	25	50	350	200
4	Sendai	11	7	9	1	1	29	157	314	200
5	Tokyo	31	15	14	10	7	13	24	59	200
6	Niigata	8	5	6	2	0	40	160	∞	200
7	Toyama	14	4	10	4	0	50	175	∞	200
8	Nagoya	19	8	9	6	4	25	48	119	200
9	Kyoto	39	13	20	18	1	15	32	600	200
10	Tottori	13	7	10	2	1	29	124	371	200
11	Hiroshima	9	4	5	3	1	50	113	450	200
12	Kochi	9	4	6	2	1	50	150	450	200
13	Fukuoka	2	1	2	0	0	200	∞	∞	200
14	Miyazaki	6	4	4	1	1	50	150	300	200

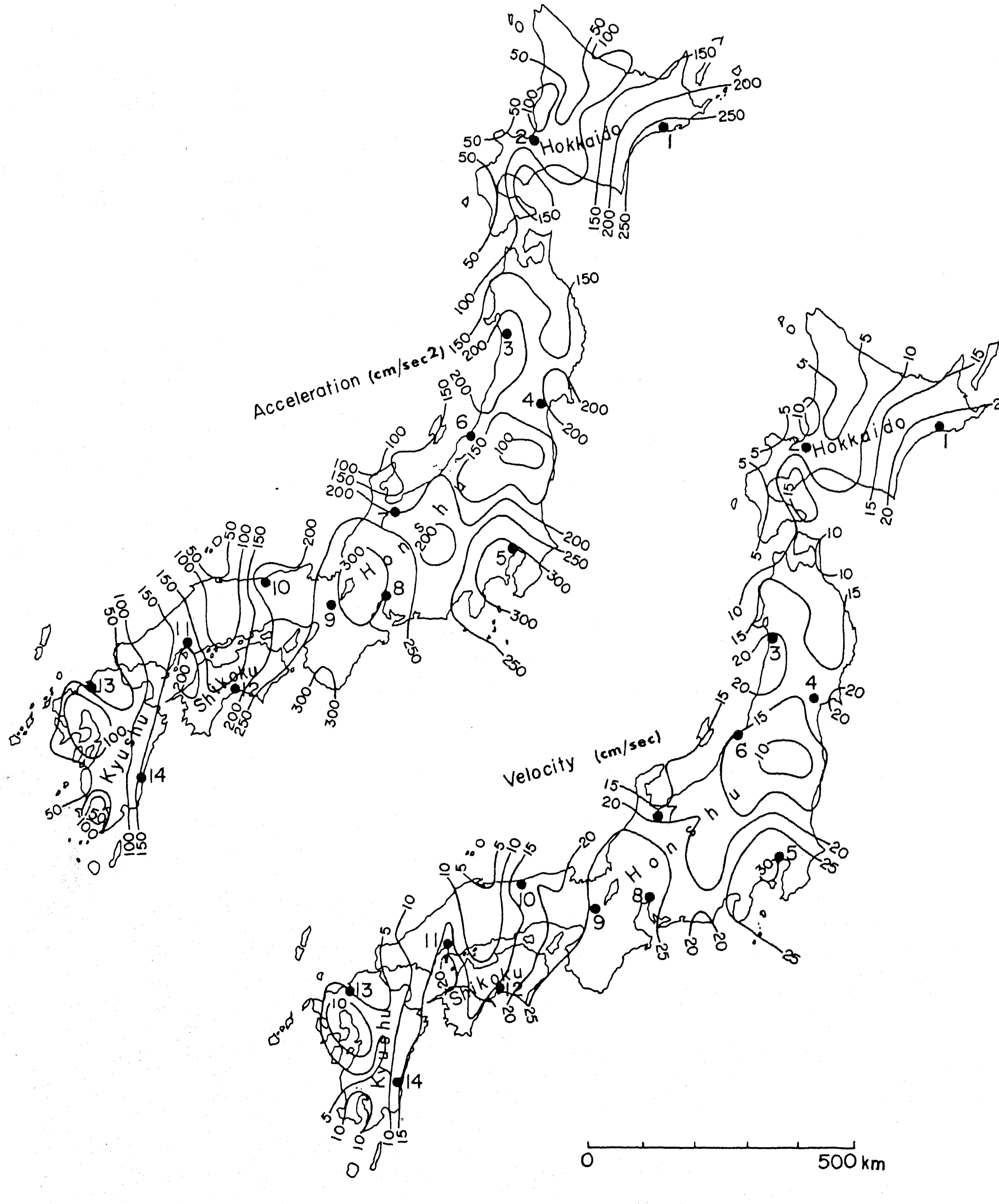

Fig.10 Seismic Map for the Expected Value of the Maximum Ground Motion to Occur in 75 Years (T_0 = 0.5sec, τ/T_0 = 30)