EARTHQUAKE PROBABILITY

by

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ABSTRACT

Earthquakes which have occurred in the earthquake zones along the west coast and the northeastern regions of North America are subjected to a statistical analysis. Shock amplitudes which may be expected are computed for several locations within each region by two methods. The first uses the distribution of average acceleration amplitudes to determine the average annual number of times the ground acceleration has exceeded some standard value. The second method uses the annual extremes of acceleration amplitude to compute expected accelerations with specific return periods. The results are displayed in the form of contour maps by which the relative seismicity of the regions may be compared.

INTRODUCTION

The current approaches to defining earthquake risk have been directed to zonal predictions. The determination of the average annual rate of strain energy release per unit area is one example of a method for estimating the earthquake risk over a large area within an earthquake belt (1, 2). This paper describes a different approach in that the individual location is the center of interest. The method is particularly useful in defining relative earthquake risks for seismic zoning purposes.

Statistical methods are employed to describe amplitudes of earth shocks that may occur at specific locations in areas where a reasonably complete catalogue of earthquakes is available. The amplitudes which are used in this study are the peak accelerations at sites where the foundation material is typical of the region. This is usually defined as firm ground. Accelerations at sites on bedrock, or on extremely loose material can be inferred from those calculated by these methods by references to papers such as that by Gzovsky (3).

Discussion in this paper is limited to the active earthquake regions along the St. Lawrence River in eastern Canada, and along the west coast of North America from the Yukon Territory to the southern boundary of

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California. Smith (4, 5) has published a list of the earthquakes in eastern Canada. The list for the western United States has been compiled from the standard earthquake catalogues, from a list of earthquakes in southern California which has been kindly supplied by the Seismological Laboratory at the California Institute of Technology, and by reference to such specific papers as the California Resources Agency Bulletin 116-2 (1964). None of the catalogues, though as complete as possible, list all the earthquakes of all magnitudes for any area for the whole period 1899 to 1960. It has been necessary to assign an arbitrary low amplitude cut-off in the mathematical solutions to compensate for this omission of earthquakes of small magnitude.

STATISTICAL METHODS

In calculating the peak acceleration at any location, it is necessary to assume that the acceleration is a function of magnitude and epicentral distance. This relationship may be expressed as

Acceleration Amplitude = F(magnitude, epicentral distance)

$$A = F(M, \Delta)$$

This assumption neglects some important factors which affect amplitudes such as local tectonic features, mechanism of the fault break, and focal depth of the earthquake. The soil condition is not considered at this time other than the remark that the soil at the site is typical of the area. This soil factor is an extremely important item upon which much research needs to be conducted. However, in this paper it is assumed that the above relationship does exist, but the same relationship does not necessarily hold for different regions. A value for A for one location can be calculated for each earthquake for which the magnitude and epicentral coordinates are known. A set of values of A can be obtained for the one location. Statistical analyses may be performed upon this set of values of A, or acceleration in this case. The two methods discussed in this paper are first the average amplitude distribution, and second the distribution of extreme amplitudes.

Let N(A) be the average number of shocks per annum which have an acceleration amplitude greater than A at one location. For several natural phenomena such as wind, floods, microearthquakes, etc., the value of N(A) can be represented by an equation of the form

$$N(A) = \left(\frac{A}{C}\right)^{\alpha}$$

or

$$\log_e A = \log_e C + \frac{1}{\alpha} \log_e N(A)$$

where α, and C are constants, particular to one location.

The values of α, and $\log_e C$ are obtained by a least squares solution. Prior to solving for the constants, the ordered set of values of A have been cut off at the low end at an arbitrary value so that the slope of the curve is not unduly influenced by numbers which are not complete. The criterion used is that each earthquake must contribute an acceleration great enough to be felt at the location. That is, the acceleration must be greater than .15% gravity or the earthquake is deleted from the set for the least squares solution.

Values of α, and $\log_e C$ can be computed by a digital computer for many locations in a region. Contour maps can be drawn to show the regional variation of these constants. By combining the two contour maps of the parameters, predictions may be made concerning earthquake accelerations at any site. These maps are shown in a paper by Milne and Davenport (7). The results of combining these parameters mathematically, that is, the values of N(A) for a specific acceleration can also be presented in the form of contours on a map. In this paper, the contour map which is presented shows the distribution of the reciprocal of the values of N(A = 10%g), that is, the return periods of an acceleration of 10% gravity.

The second method concerning the distribution of annual extremes is developed from the same initial equation as used in the first method. Milne and Davenport (7) show that for this second method

$$\log_e A = U - \frac{1}{a}\left(\log_e(-\log_e P)\right)$$

where U and 1/a are again constants or parameters which are assigned to one location. If the occurrences of large earthquakes are assumed to have a Poisson distribution, the values of U and $\log_e C$, and similarly -1/a and $1/\alpha$, should correspond for one location. P is the probability that the acceleration amplitude A will not be exceeded in any given year. As before, the values of the parameters are obtained by the least squares method. A digital computer is used to determine the parameters at a number of sites so that a grid is obtained over a large map area. Contour maps of U and 1/a may be drawn. In this paper, the contour map which is presented for this method, shows the values of acceleration as a percentage of gravity for a specific value of P. The value of P used in the calculations is such that the return period is 100 years.

<u>Relationship for Determining A</u>

It was recognized that the relationship between magnitude, distance and acceleration amplitude was not the same for each area. It must be assumed for this paper that one relationship applies along the west coast of North America, and that a different relationship may exist for the St. Lawrence valley. Several authors have deduced equations relating these constants; the one which Richter employed in developing the magnitude scale for local earthquakes in California is typical of these. Milne and Davenport (7) have used an equation of the form

$$A = \frac{.69e^{1.64M}}{1.1e^{1.10M} + \Delta^2}$$

Δ is in kilometers and A is expressed as a percentage of gravity. The relationship has been developed from observations of strong earthquakes in California, but the resulting accelerations are within the range of observed accelerations of the few earthquakes in British Columbia for which data are available. This relationship is seen to fit the Cloud (8) data over most magnitude and distance ranges in figure 1. The equation appears to produce too high an acceleration for moderate earthquakes at short distances, but recent observations of the Parkfield (1966) earthquake in California suggest that the equation need not be altered at the present time. The equation makes no allowance for the duration of maximum shaking which is a very important factor from the intensity viewpoint.

It is obvious when reading the reports of observations of earthquakes in eastern Canada, that the above equation cannot be used in this area. The formula shows that the radius of perception of a magnitude 7 earthquake is near 500 kilometers, but in 1925 the St. Lawrence earthquake that was of this magnitude, was felt to a distance of 2000 kilometers. There are no accelerograms available for eastern Canada for relating acceleration and distance in a direct manner. However, there are many observations of intensity for several well documented earthquakes covering several areas of this large region. The authors (7) have developed a relationship between intensity, distance, and magnitude from these observations. Intensities are converted to accelerations by the formula

$$\log_{10} A = \frac{I}{3} - 1.5$$

The modified Mercalli intensity scale of 1931 is used throughout this work.

The above equation may not be valid in eastern Canada, but there is as yet no reason to suspect that it cannot be used. Errors may be introduced into the results because this method makes the isoseismal lines become circles, whereas in practice, isoseismals are often elliptical in shape. It has not been possible to derive an equation to fit the eastern observations so a table containing the observations is read into the computer. The resulting intensity or acceleration versus distance curves for integral magnitude values for eastern Canada are shown in figure 2. The equivalent curve for the west coast for an earthquake of magnitude 7 is superimposed upon the eastern curves to indicate that there is a marked difference in the acceleration relationship between the two areas.

RESULTS

a) Eastern Canada

The statistical analysis is carried out on the list of earthquakes from 1899 to 1963 inclusive for eastern Canada and to 1960 inclusive for western North America. It is possible to compare the predictions from calculations on the above list with a historical record in eastern Canada which dates from 1638. This comparison is not possible in other areas. The eastern data are being discussed first because the comparison between predictions based upon current activity and past observations is encouraging.

The data for Quebec City are presented in more detail than for any other location because it is for this area that the historical record is most complete. The values of the constants, $\log_e C$ and $1/\alpha$, which are obtained by using the average acceleration amplitude distribution, are calculated by the least squares method from the data which are plotted as dots on figure 3. The curve is drawn through these points, and then the observations of the historical data are plotted as x's upon the same diagram. The past experiences fit the current predictions very well for large earthquakes, and fall below the curve for small events because the historical catalogue probably does not contain all the small earthquakes. The value of $\alpha = -1.63$ compares closely with the value of -1.9 found by Hashisume, Oike, and Kishimoto (9) from their study of microearthquakes in Japan. The values of these constants are found for many sites in the map area, so that a grid is obtained over the whole eastern region. The value of the return periods of accelerations of 10% gravity are computed for each location. These return periods are plotted as contours on a map of the area which is shown as figure 4. The contours are plotted with a broken line when the number of entries for the least squares solution falls below 10.

The second method of computing earthquake probabilities uses the distribution of the annual extremes of acceleration amplitudes. Again, the data for Quebec City have been presented in more detail. Values of U, and 1/a, the parameters, are found from the least squares solution of the data which are plotted as dots on figure 5. The resulting curve is drawn, and again, the historical observations are plotted as x's on the same figure. The large events during the historical period follow the slope of the curve of recent activity, although the values are slightly lower. The historical points fall below the present curve in the small magnitude range where the historical catalogue is obviously incomplete. Values of U and 1/a are computed for a number of points forming a grid over eastern Canada. Accelerations which have a return period of 100 years are calculated for these points. The resulting contour map is shown as figure 6.

The two methods show similar results. There is a small area east of Quebec City where the probability of an earthquake near magnitude 7 is high. The return period for an event of this magnitude is less than 100 years. There is a large area around this where the expected accelerations for a 100-year return period are near 3% gravity. Midway between Montreal and Quebec, where recent activity is a minimum, the predicted event is slightly lower than this. Two earthquakes in eastern Canada in

1935 and 1944, which have received much publicity, have occurred within the area where the prediction is near 3% gravity. The results of the statistical study do not rule out the possibility of acceleration amplitudes of this magnitude at any site in this area, but they do indicate that the probability of earthquakes which can cause as much damage as these two is very small. The area near the Grand Banks earthquake of 1929 appears as an area where the risk is great, although the number of entries for the least squares solution is small and the results are not as reliable as in the central area of the region.

b) Western Canada

The area included in this section extends from within the Yukon Territory to the 47th parallel of latitude. The area in Canada which frequently is disturbed by the earthquakes in Montana falls within this section. The earthquake list includes the great earthquakes in Alaska in 1899, but it does not include the recent Anchorage event in 1964. It is expected that because there are several very large earthquakes in this north-east Pacific area, one event, even such a strong event as that in 1964, will alter the slope of the curves only very little.

The contour map of the return periods of acceleration of 10% gravity which are computed by the average amplitude distribution method is presented in figure 7. No historical data are available for comparison purposes for the whole western region. A second contour map is presented for this region in figure 8. It shows the accelerations with a return period of 100 years which are computed from the distribution of the annual extremes. Both maps show that the area near the Alaska, British Columbia, Yukon border is a very high risk zone. Accelerations greater than 10% gravity occur here more often than once in 50 years, or according to the second method, accelerations slightly greater than that of gravity are experienced once in one hundred years. The probability of an earthquake near magnitude 8 is high throughout the zone. The earthquake risk decreases to the south in the western Canada region, although the north shore of the Queen Charlotte Islands must still be considered to be in a very dangerous area. Most of Vancouver Island falls in the zone where accelerations are predicted to reach 20% gravity once per one hundred years. Victoria, and Vancouver both fall slightly outside this zone, between 10 and 15%. The earthquake zone follows the coastline in British Columbia very closely. There is little activity east of the coast range of mountains.

c) Western United States

The catalogue of earthquakes in the United States west of 110° west longitude has been compiled from international catalogues, from data supplied by the California Institute of Technology, and from the Bulletin of the Berkeley Seismographic Stations. This list was then checked by reference to many papers which contain lists of earthquakes for specific areas. Foremost among the individual papers is the California Resources Agency Bulletin 116-2 (1964). The list of earthquakes for the California area is very long because there have been excellent seismograph networks operated by Pasadena and Berkeley for many years. It was necessary to delete from the list of earthquakes those with magnitudes less than 4, in order that the memory of the computer would not be exceeded and that the program could be run in an economical time. This limitation in the list does not make the California study different from that in other areas because it can be demonstrated that only the earthquakes with magnitudes greater than 4 would contribute to the calculation for one site.

Figure 9 shows the contour map of return periods for an acceleration of 10% gravity for western United States. These data were computed by the average amplitude distribution method. The contour map of figure 10 shows the results obtained by the second method, or by reference to the distribution of the annual extremes of acceleration amplitudes. The contours are for values of acceleration with a 100-year return period.

The two methods show similar results. The zone where the results are strongly influenced by the earthquakes associated with the San Andreas fault zone shows up very clearly. The low zone in the central valley of California is not as evident as on some zoning maps, but the values of acceleration fall nearly to 10% gravity here and reach to greater than 20% along the mountains on either side of the valley. The earthquake areas of Nevada, and Montana are evident. The probability of there being a large earthquake in the State of Oregon at this time appears to be very small. The Puget Sound basin in the State of Washington is one where the earthquake risk is again very high.

Comparison of Seismicity

The earthquakes in the California region have been well documented, and the tectonic features are all mapped, thus this region can be used as a base to which other areas can be compared. The location where the acceleration amplitude reaches a value of 10% gravity for a 100-year return period provides a measure for comparing earthquake risk.

This level of acceleration amplitude is evident in all areas of California except the north-east corner, and a very small zone along the border of Arizona. The extremes of acceleration amplitude reach to nearly the value of gravity in isolated active areas such as the Imperial Valley. The other way of expressing the risk is that nearly all of California can expect to experience accelerations greater than 10% gravity on the average once per 100 years. This acceleration is experienced more often than once in 50 years at many sites along the San Andreas fault zone. This level of earthquake activity is not reached elsewhere in the western United States.

Much of Nevada, and some of Montana fall within the 10% acceleration contour on figure 9. In both areas, the maximum acceleration can exceed 50% gravity, but this figure is reached in only a very small sector of Montana. There are no areas in Oregon where the predicted accelerations approach those of California. The Puget Sound Basin of Washington does fall in an area where accelerations are greater than 10% gravity per 100 years. The maximum value reached here is in excess of 50% gravity for the areas where the strong earthquakes of 1949, 1965 and others have occurred. The contour map of return periods shows that accelerations of 10% gravity can occur more often than once in 50 years in the central part of the Puget Sound Basin.

The contour maps for the west coast of Canada have indicated that the major earthquake activity is limited to a narrow zone along the coast and beneath the coastal waters. Within this narrow earthquake zone, the predicted acceleration amplitudes are of the same order as those found for California. Almost the whole coastal area falls within the 10% gravity zone. The maximum acceleration reached in the Vancouver Island zone, where we find most of the population, is approximately 20% gravity. This value increases more to slightly in excess of the value of gravity near the Alaska, Yukon, British Columbia boundary where the great earthquakes have occurred. Figure 9, which is the contour map of return periods for an acceleration of 10% gravity, shows that the earthquake zone can in general expect this acceleration each 100 years. Some areas of Vancouver Island which are influenced by the earthquakes of 1918 and 1946 have somewhat shorter return periods. The return period for an acceleration of 10% gravity decreases for the Queen Charlotte Islands area, and decreases again to less than 25 years in the area where the very severe earthquakes of 1899 have occurred.

The region in eastern Canada where earthquakes are prevalent covers a large area. Within the area, only a very small section east of Quebec City is predicted to experience acceleration amplitudes in excess of 10% gravity per 100 years. The maximum value reached within this section approaches 20% gravity near the epicenter of the 1925 earthquake. Accelerations predicted for a large area surrounding this high risk area do not reach 5% gravity very often, although there are a few isolated sites where the value is between 5% and 10%. The return periods for accelerations of 10% gravity in the maximum zone are slightly less than 100 years, possibly as low as 75 years. Throughout the remainder of the area the return periods are large. The possibility of an earthquake producing acceleration amplitudes of 10% gravity does exist in these areas in eastern Canada, and in fact did happen at one site in 1935 and at a different site in 1944, but the probability that this will happen at a specific site in any year is very small.

Applications to Structural Engineering

While the estimates of peak ground acceleration for various return periods, given by the regionalization maps can only be regarded as fairly approximate, it is felt that they provide a reasonable index for structural design needs. While the peak ground acceleration amplitude may be regarded as a measure of the shock amplitude, other factors need to be considered in order to find the effective inertia loads acting on the structure. Both the dynamic response characteristics of the structure and the soil may amplify or attenuate the level of input acceleration.

The dynamic response characteristics of the structure may adequately be represented by Housner's acceleration response spectra, such as that given in figure 11. If the ordinate of this response spectrum, which is normalized by the peak ground acceleration, is multiplied by the peak ground acceleration estimated from the regionalization maps in this paper, the resultant peak acceleration of the structure may be found. For a single degree of freedom structure, this peak acceleration (in percentage of gravity) multiplied by the mass constitutes the effective inertial load to be applied in design. For multi degree of freedom structures, allowance must be made for the modal response characteristics. Similarly, it may be possible to account for some of the characteristics of ground response.

A further, more general, statistical approach might be developed using statistical representations of the shock processes themselves. An example of such a discussion is included in a paper by Housner and Jennings (10).

Summary

The probability that an earthquake will occur which can create acceleration amplitudes of sufficiently high values to cause damage, has been computed by two related but somewhat different methods. Almost all of California can expect to experience earthquakes which will cause damage according to these predictions. The central area of Nevada, and a small portion of Montana may also experience strong accelerations. A narrow zone of activity extends along the north-west coast of North America. The activity in the southern area which begins in Puget Sound is somewhat lower than in California, but at the very north section of this coast, the predicted acceleration amplitudes are more severe, and the strong amplitudes more frequent than in California. Maximum acceleration amplitudes in eastern Canada are lower than the maximum values predicted for California, and the frequency of accelerations which can cause damage is also lower in eastern Canada.

ACKNOWLEDGEMENTS

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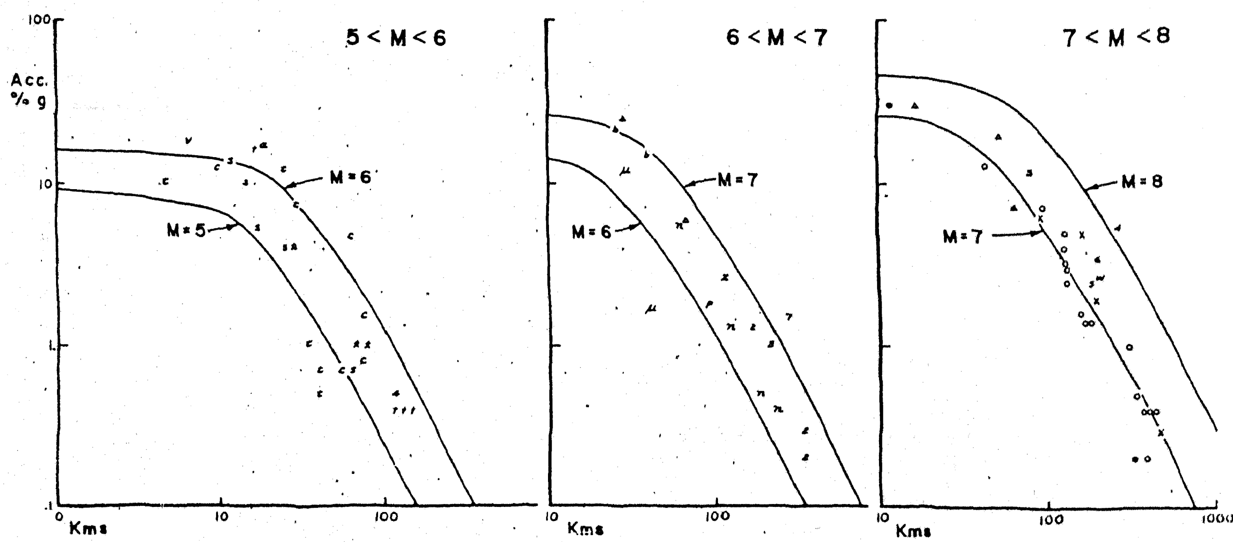

Figure 1. Relationship between acceleration and distance in western North America

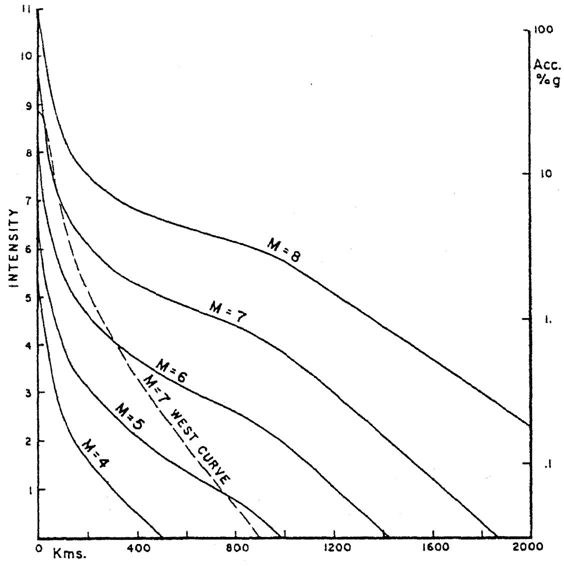

Figure 2. Relationship between acceleration and distance in eastern Canada

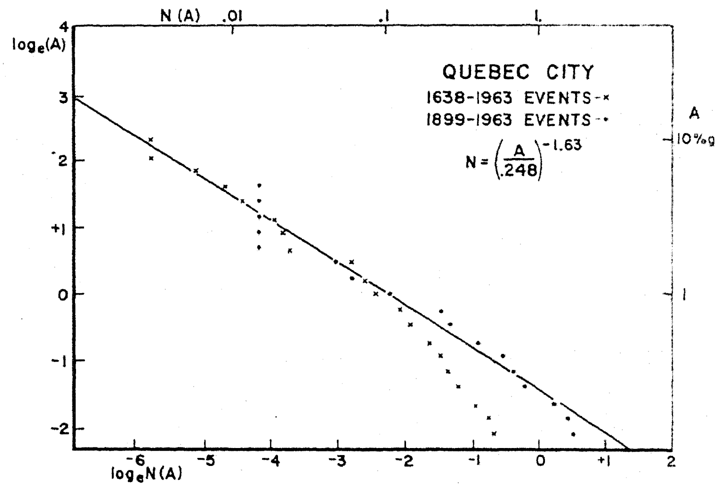

Figure 3. Average distribution plots for Quebec City

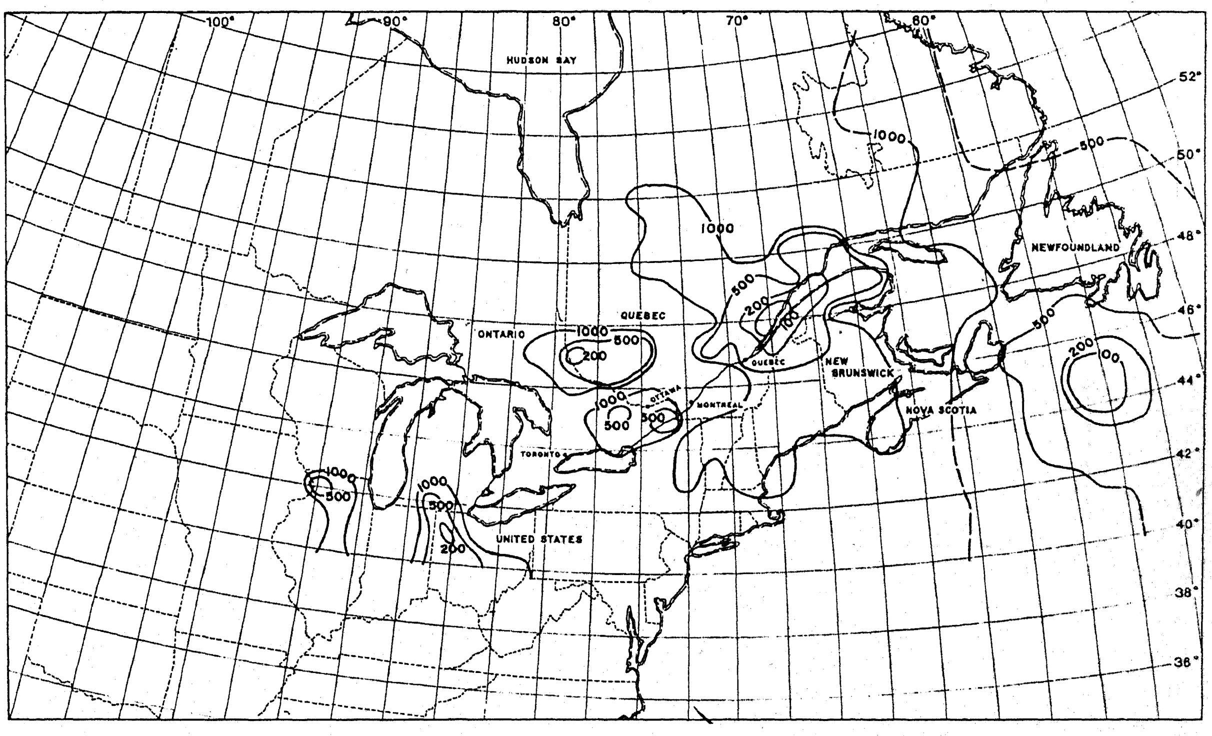

Figure 4. Return periods, in years, for acceleration of 10% gravity in eastern Canada.

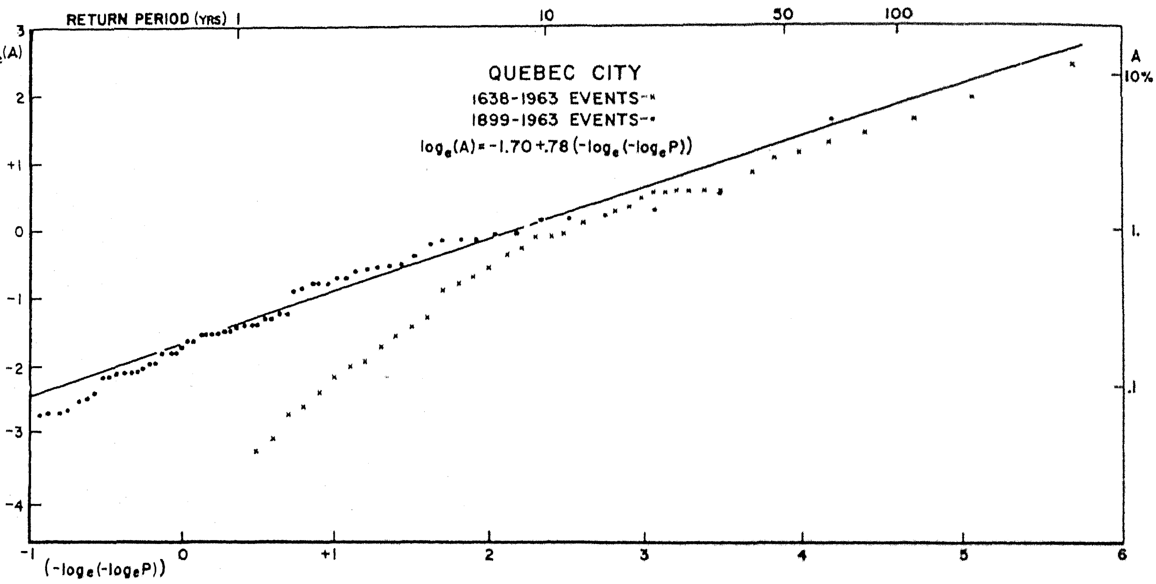

Figure 5. Extreme value distribution plot for Quebec City

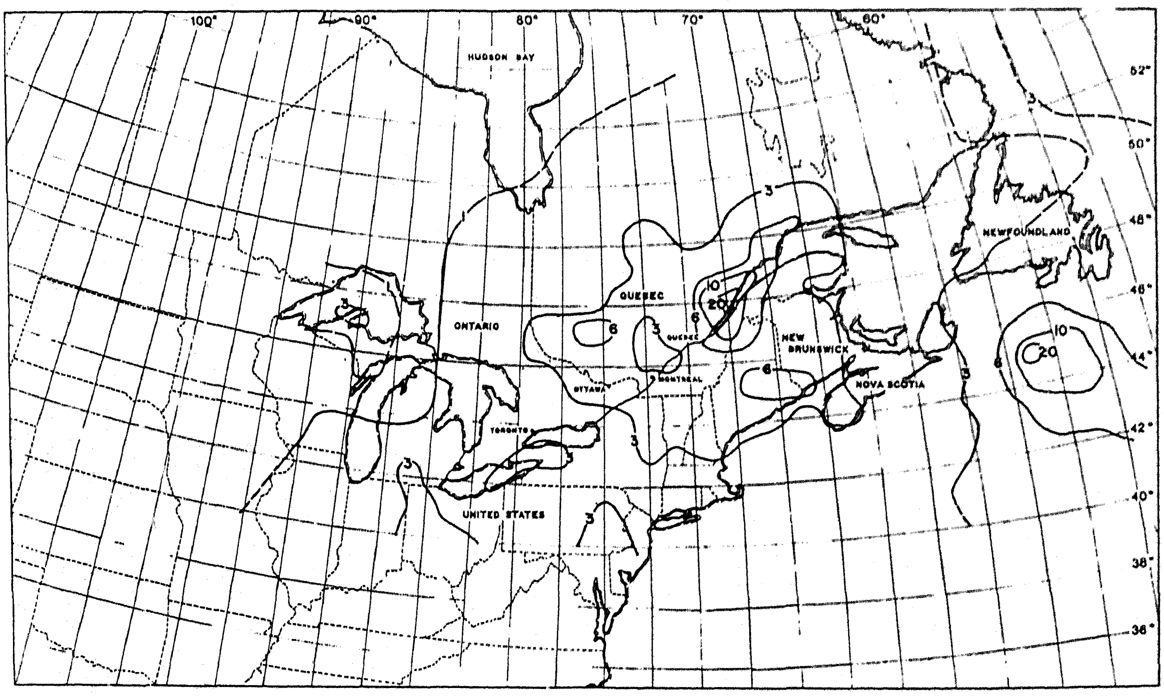

Figure 6. Accelerations, in percent of g, with a 100 year return period for eastern Canada

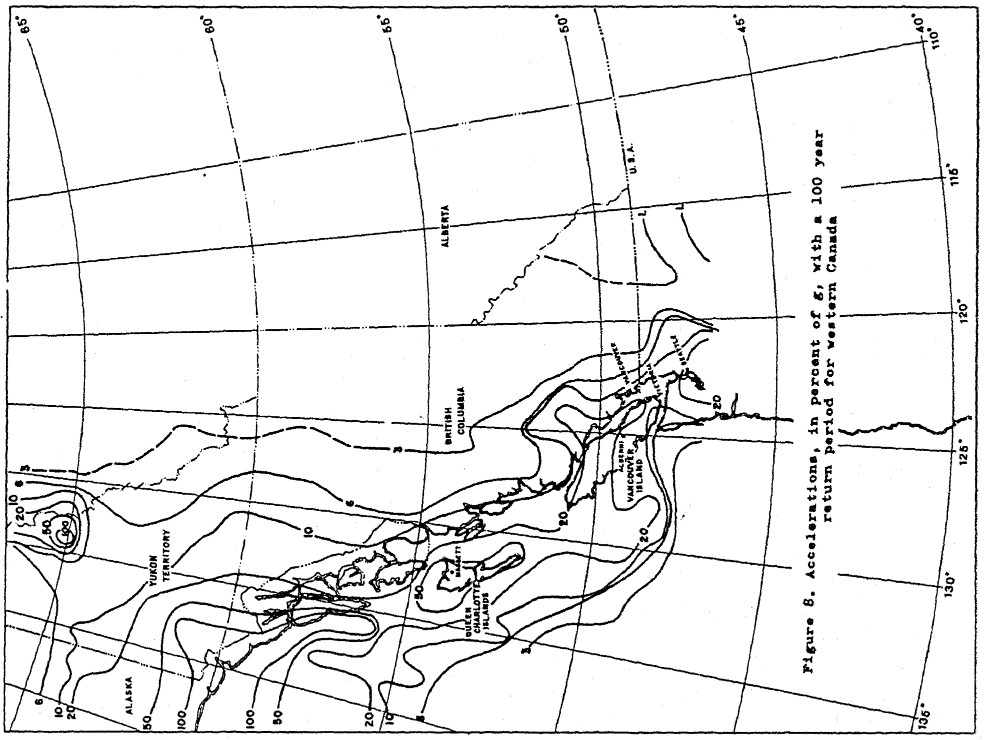

Figure 8. Accelerations, in percent of g, with a 100 year return period for western Canada

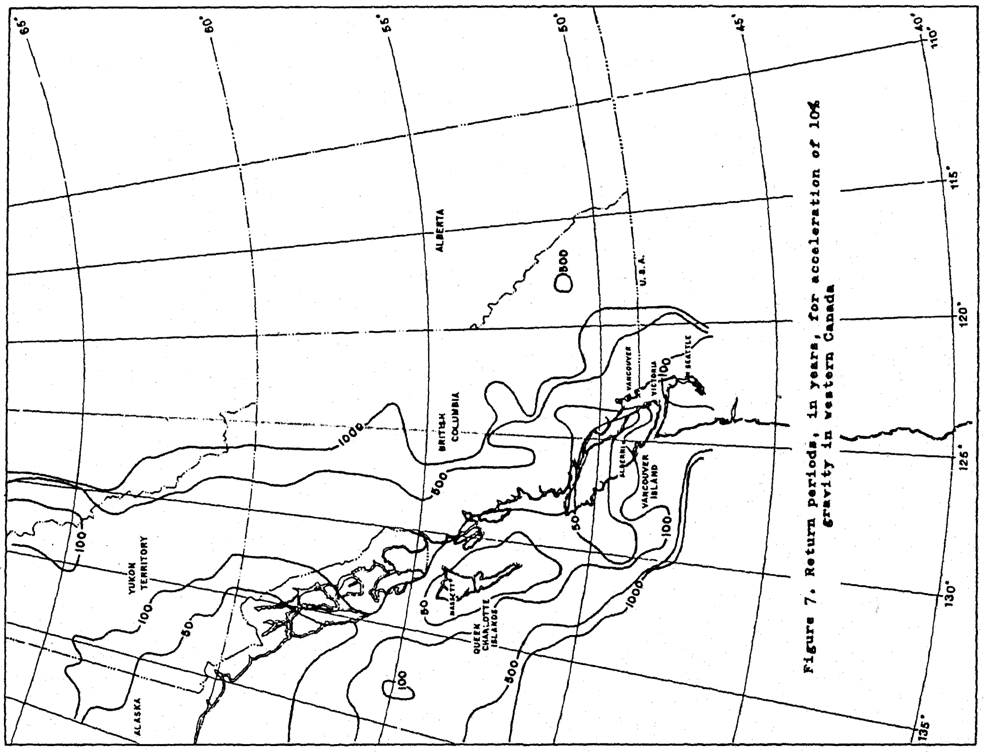

Figure 7. Return periods, in years, for acceleration of 10% gravity in western Canada

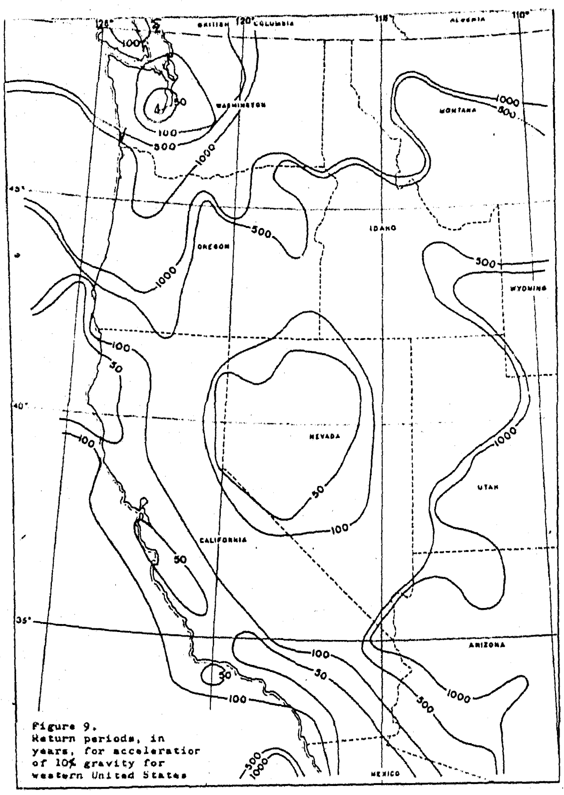

Figure 9. Return periods, in years, for acceleration of 10% gravity for western United States

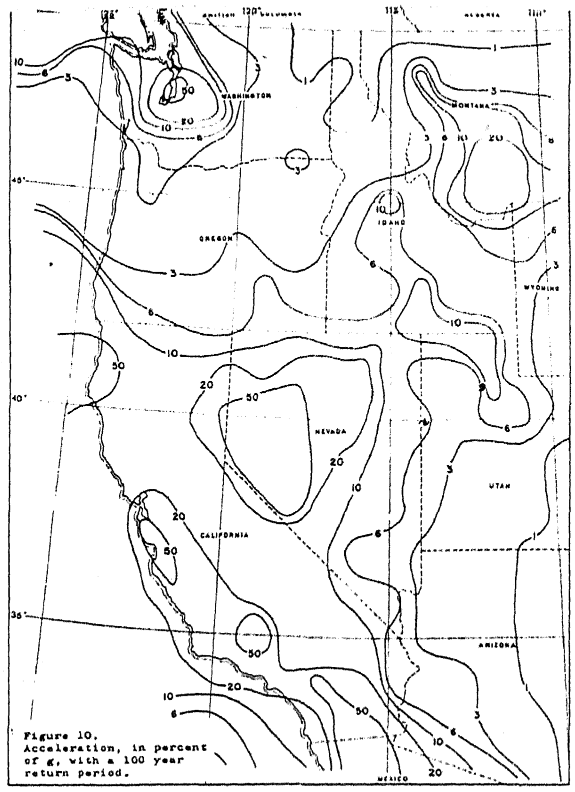

Figure 10. Acceleration, in percent of g, with a 100 year return period.

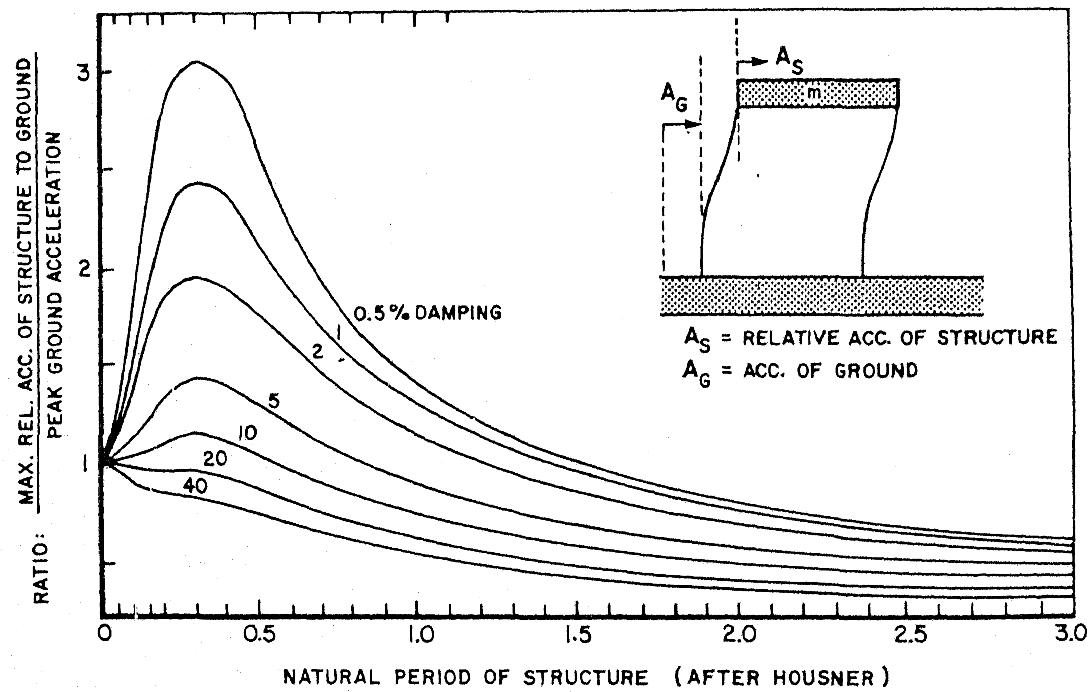

Figure 11. Earthquake acceleration response spectrum (after Housner)