THE MAJOR INFLUENCES ON SEISMIC RISK

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Synopsis: A new, analytical method of seismic risk analysis is used to study the sensitivity of the peak ground acceleration values associated with prescribed design return periods to the assumptions which must be made when analyzing a site. The predominent influences on risk are thereby demonstrated quantitatively. It is shown that typically the more frequent, but smaller earthquakes contribute more to the risk than larger, more broadly destructive earthquakes. The importance of collecting instrumental data to provide more accurate attenuation laws for near-focus conditions (even for smaller earthquakes) is emphasized when the major influences on seismic risk are appreciated.

Introduction: A newly developed method[1,2,3] makes it possible to put design for earthquakes on a basis comparable to that presently used by engineers for large winds and floods, namely design for a seismic input of a specified (mean) return period. It is the purpose of this paper to extend that method, to demonstrate its usefulness in interpreting the relative influence of various factors of the seismic risk of a particular site, and to evaluate the sensitivity of the conclusions to inevitable errors in estimating the significant parameters. It is desired to leave the engineer with an appreciation for these factors so that he can better apply this or other statistically based methods[4,5] and, more generally, so that he understands better the nature of seismic risk.

Site Parameters: To determine the seismic risk at a particular site, the method requires that the engineer assign to each of the various potential earthquake sources (be they known fault lines, relatively concentrated "points", or arbitrarily shaped areas, Fig. 1) an activity level in the form of the average annual rate of occurrence of earthquakes of magnitude m_0 or greater. The rate estimates are presumably based on the tectonics and the seismic history of this and geophysically similar regions. (These rates and other necessary parameters might eventually be stipulated in regional codes.) The method provides the theory to combine this information with the geometrical relationships between these sources and the particular site of interest. The integration of the influences of all the various potential sources yields the seismic risk of the site, or more specifically, the probability that in a period of t years the maximum of all earthquake motions (as measured, for example, by their peak ground accelerations) at the site will exceed a particular level.

The method requires stipulation of a law relating the chosen earthquake motion variable, Y, to earthquake magnitude, M, and focal distance, R. A common[6,7] and convenient functional form for peak ground accelera-

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tion, velocity, or displacement is

$$Y = b_1 e^{b_2 M} R^{-b_3} \quad (1)$$

in which the parameters b_1, b_2, and b_3 are subject to estimation over rather broad geographical regions. For example, Esteva and Rosenblueth(6) have estimated that in the western U.S. $b_1 = 2000$, $b_2 = 0.8$, $b_3 = 2$ for Y denoting peak ground acceleration in cm/sec^2, with R in kilometers, and M in the Richter scale.

In addition the method requires an assumption on the relative frequency distribution of earthquake magnitudes (larger than the minimum value of interest, m_o). The most popular choice is that due to Gutenberg and Richter(8) which can be stated

$$F_M(m) = 1 - e^{-\beta(m-m_o)} \qquad m_o \le m \le \infty \quad (2)$$

in which $F_M(m)$ is the fraction of earthquakes smaller than m. (The parameter β is the product of the natural logarithm of 10 and b, the slope in Gutenberg and Richter's familiar equation(8) $\log_{10} N - a = bM$). The stability from region to region of the parameter β (or b) has been widely studied (e.g., by Isacks and Oliver(9)); it therefore can be estimated with some confidence. In this paper a generalization of this distribution will be considered also. It limits the magnitude, M, to finite values less than m_1;

$$F_M(m) = k_{m_1} [1 - e^{-\beta(m-m_o)}] \qquad m_o \le m \le m_1 \quad (3)$$

in which a factor

$$k_{m_1} = [1 - e^{\beta(m_1-m_o)}]^{-1} \quad (4)$$

is needed to normalize the probability function to unity at m_1, the maximum possible value of M.

It is assumed, at least for the larger magnitude earthquakes of interest, that the engineer is justified in considering the occurrences in time at the i^{th} potential source as "random" (specifically as Poisson arrivals) with constant average arrival rate ν_i per year. The implication is that the number of occurrences of earthquakes from any source during an interval of time, t, is random with a Poisson distribution (mean equal to $\nu_i t$.) In addition, it is assumed for the spatially distributed potential sources (line or area) that these sources have been defined so that, given that an earthquake has occurred somewhere within that source, it is equally likely to have occurred anywhere within that source. A number of these assumptions are easily relaxed(2).

<u>Basic Results</u>: The results(2) for the magnitude distribution in Eq. 2 are that, $F_{\tilde{Y}}(y)$, the probability that $\tilde{Y}$, the <u>maximum</u> value of the random number of site motions, Y, is less than any value y is

$$F_{\tilde{Y}}(y) = \exp\left[- e^{\beta m_o} \, t \, \nu G \, (y/b_1)^{-\beta/b_2}\right] \quad (5)$$

in which the product νG is a sum over the n potential sources

$$\nu G = \sum_{i=1}^{n} \nu_i G_i \quad (6)$$

In this equation the G_i are geometry dependent factors for the various sources. For example, for a point source at focal distance r

$$G_i = r^{-\rho} \quad (7)$$

while for an infinitely long line (fault) source a perpendicular (slant) distance d from the site

$$G_i = \frac{2\pi}{(2d)^{\rho-1}} \; \frac{\Gamma(\rho-1)}{[\Gamma(\rho/2)]^2} \quad (8)$$

in which $\Gamma(\cdot)$ is the complete gamma function and ρ is a commonly recurring ratio

$$\rho = \beta \, b_3/b_2 \quad (9)$$

Any G_i is, in fact, just the expected value, given an occurrence in the i^{th} source, of $R^{-\rho}$. Letting

$$\nu = \sum_{i=1}^{n} \nu_i \quad (10)$$

note that the sum in Equation 6 can be written

$$\sum_{i=1}^{n} \nu_i G_i = \nu \sum_{i=1}^{n} (\nu_i/\nu) \, G_i$$

But ν_i/ν is just the probability, given an earthquake occurrence somewhere, that the event took place in the i^{th} source. Therefore

$$G = \sum_{i=1}^{n} (\nu_i/\nu) \, G_i \quad (11)$$

is simply the unconditional expected value of $R^{-\rho}$, the focal distance of an arbitrary event raised to the power, $-\rho$. It is clear that an entire probability distribution of this pertinent geometry factor, $R^{-\rho}$, could be derived from the assumptions, but it is significant that only its mean or expected value affects the final result, Eq. 5. A seismological engineer might be prepared for design purposes to estimate this factor directly, at least for one site relative to another. (It is perhaps prudent to state here that the expected value of $R^{-\rho}$ is not the $-\rho$ power of the expected value of R, unless the coefficient of variation of R is small.)

Strictly, the simple (Type II Extreme Value) form of $F_{\tilde{Y}}(y)$ holds only for y larger than $\tilde{y}'$, the largest y' of the various line sources. For any such source[2]

$$y' = b_1 e^{b_2 m_o} d^{-b_3} \tag{12}$$

The form of the distribution of $\tilde{Y}$ and the easily computed (or graphed) G_i factors make the risk results quickly obtainable and readily accessible to interpretation. For example, the probability that the annual maximum of Y exceeds y (a probability which is the reciprocal of the mean annual return period of level y) is approximately

$$1 - F_{\tilde{Y}}(y) \cong e^{\beta m_o} (y/b_1)^{-\beta/b_2} \sum_{i=1}^{n} \nu_i G_i \tag{13}$$

for the small values of this probability of interest in design. In this sense, the sources contribute in an approximately additive way to the seismic risk at a site. For this reason it is possible to study the influences of various potential sources independently.

For the ultimate purpose of this paper, namely for the study of the influence of the various parameters, it is desirable to write and plot the geometry factors for various cases as follows. The geometrical parameters for each are defined in Fig. 2;

Point Source:

$$G_i^P = r^{-\rho} = h^{-\rho} (1 + a^2)^{-\rho/2} = h^{-\rho} g_\rho^P (a) \tag{14}$$

Line Source:

$$G_i^L = \frac{d^{-\rho}}{a' - a''} c_\rho^L [g_\rho^L(a') - g_\rho^L(a'')] \tag{15}$$

Area Sources:

$$G_i^A = \frac{h^{-\rho}}{(a')^2 - (a'')^2} c_\rho^A [g_\rho^A(a') - g_\rho^A(a'')] \tag{16}$$

in which the functions $g_\rho^P(\cdot)$, $g_\rho^L(\cdot)$, and $g_\rho^A(\cdot)$ are plotted in Fig. 3, 4, and 5 respectively for various values of ρ, and in which the factors c_ρ^L and c_ρ^A (to be associated with sources of infinite extent) are

$$c_\rho^L = \frac{1}{2} \sqrt{\pi}\ \Gamma\left(\frac{\rho - 1}{2}\right) / \Gamma\left(\frac{\rho}{2}\right) \tag{17}$$

$$c_\rho^A = \frac{2}{\rho - 2} \tag{18}$$

These functions are plotted in Fig. 4 and 5 respectively.

This completes the review of the basic results, given here in a new form and with a new interpretation more suitable for present purposes. An example will follow in which the terms in these equations will be interpreted.

Limited Magnitude Distribution: It is important to be able to consider situations in which the size of the largest earthquake which can occur is limited to a finite value, m_1. Although its value is not known, there is undoubtedly some physical bound on the size of all earthquakes. In any local region or for any particular potential source there may be historical or geophysical reasons (such as a short fault length) to bound the magnitudes which can potentially occur. In any case, owing to scarcity of data, the frequency distribution is not well established in this upper tail and it is important to understand how this uncertainty influences site risk estimates. This can be accomplished by altering the distribution in the tail, here, by eliminating the tail entirely.

It is easily demonstrated[10] that the limited distribution, Eq. 3, can be treated by a minor modification of the method outlined above for the unlimited distribution, Eq. 2. Notice that the assumptions imply that the limiting magnitude, m_1, defines a focal distance, r_y, beyond which it is impossible for an earthquake to cause a ground motion in excess of level y. This distance is (Eq. 1)

$$r_y = (y/b_1)^{-1/b_3} e^{b_2 m_1/b_3} \tag{19}$$

To determine $F_{\tilde{Y}}(y)$ when magnitudes are limited, it is only necessary 1) to restrict attention to sources within the radius r_y (that is to find ν and G or $E[R^{-\rho}]$ conditional on an event occurring within r_y) and 2) to modify Eq. 5 for the normalizing factor k_{m_1} in Eq. 4. The result is

$$F_{\tilde{Y}}(y) = \exp[-\nu_y\, t(1-k_{m_1})]\, \exp[-k_{m_1}\, e^{-\beta m_o}\, t\, \nu_y G_y\, (y/b_1)^{-\beta/b_2}] \tag{20}$$

in which $\nu_y G_y$ (Eq. 6) is now found from a sum over all sources within radius r_y. It is important to recognize that r_y is a function of the ground motion level of interest, y, and that, therefore, $\nu_y G_y$ may also depend on y. It will also be found that k_{m_1} is very nearly unity in most cases of interest. The implications of these facts will be discussed subsequently.

Illustration: As an example of the method, the seismic risk at the site in Fig. 1 will be investigated. Using the unlimited distribution (with $m_o = 4$ and $\beta = 2.3$) for magnitudes (Eq. 2), Table 1 shows the calculation of νG of Eq. 5 for peak ground acceleration and peak ground velocity. It is assumed that $\rho = 5.0$ and 3.4 for acceleration and velocity respectively. For acceleration (assuming $b_1 = 2000$, $b_2 = 0.8$, and $b_3 = 1.75$) Eq. 13 becomes

$$1 - F_{\tilde{Y}}(y) \cong e^{-\beta m_o}\, \nu G\, (y/b_1)^{-\beta/b_2}$$

$$= 2.5 \times 10^4\, y^{-2.8} \tag{21}$$

implying that for this site the peak acceleration with a 50-year return period ($1-F_Y(y) = 0.02$) is about 130 cm/sec^2 or 0.13g. Using Newmark's recommendations(11) the corresponding design value for psuedo-acceleration response of a short period structure with 5% damping would be (2.6)(0.13g)= 0.34g. For ground velocity (with $b_1 = 16$, $b_2 = 1.0$, and $b_3 = 1.5$) the corresponding equation is

$$1 - F_{\tilde{Y}}(y) = 4.0\ y^{-2.3} \qquad (22)$$

and the velocity with a 50-year return period is about 10 cm/sec. Under Newmark's recommendations this value would dictate design response values for structures in the intermediate period range.

In addition to noting the simple form of Eq. 21 and 22 and the relative size of the exponents for peak acceleration and velocity, it is instructive to study Table 1 in some detail, for the terms in these calculations point to a number of general conclusions. Compare, for example, the relative values of the G_i terms. Study of these values will show the influence of source distance and of "smearing" the activity over line or area sources. The form of the equations is designed to help in this study. Consider, for example, the line source, No. 4. The number $d^{-\rho}$(= 17 x 10^{-9}) is the value that G_i would have if the activity of the line were concentrated at the point on the line closest to the source. (Compare G_1 for point source 1.) The factor 1/a' (= 1/0.7) in effect converts the total activity, into the activity per unit length. Multiplying by the factor $c_\rho^L = 0.65$ gives the value that G_i would have if the fault stretched for an infinite length with the same activity per unit length. The final factor, $g_\rho^L(a')$(= 0.86) gives the fraction of the hypothetical infinite line source that the true, finite length of source contributes. Inspection of Fig. 4 reveals how rapidly this approaches unity especially for the larger values of ρ associated with peak acceleration. The implication is that the contribution to the risk at the site comes only from the very closest part of a fault, that within d to 2d of the closest point for $\rho = 5$ (acceleration), that within 2d to 3d of the closest point for $\rho = 3.4$ (velocity). Thus the risk of larger accelerations at the example site would not be significantly less if Source 5 were only a third as long (if its activity per unit length were unchanged). Fig. 4 can be used also to determine the fraction of the risk contributed by various portions of the fault. For example, for Source 4, of the total value of $g_\rho^L(.7)$ (= 0.86) an amount $g_\rho^L(0.5)$ (= 0.66) comes from the interval 0 to 0.5, or about (0.86 - 0.66)/0.86 = 23% comes from the portion of the fault between 0.5d and 0.7d (18 and 25 km.). The curves for areal sources can be interpreted in an analogous way. (Note, incidentally, that the angle ω does not enter the calculations).

Recall that G can be interpreted as the expected value of $R^{-\rho}$. In this case, since for acceleration (Table 1) $\nu = 0.66$, $G = 2.2 \times 10^{-9}$. Relating this value to a point source, *the combination of sources is* "equivalent" to a single point source at an epicentral distance of about 50 kilometers. A similar argument for velocity finds the equivalent point source at a distance of 60 kilometers.

Risk analysis when a limited magnitude distribution governs can be illustrated by considering the same case but with $m_1 = 6$. This is a severe assumption, but one that might be considered in only mildly seismic regions. We seek the return period at the site of a ground acceleration of 0.13g, keeping all other parameters the same, including the seismic activities, ν_i. An earthquake with focal distance more than (Eq. 19)

$$r_{130} = \left(\frac{130}{2000}\right)^{-1/1.75} e^{(0.8)(6)/1.75} = 74 \text{ km}$$

or epicentral distance $\sqrt{74^2 - 25^2} = 70$ km cannot cause an acceleration greater than 130 cm/sec^2 under the governing assumptions. Restricting the original problem to sources less than this distance, the original problem becomes a new one problem without Source 2, with Source 3 restricted to a range between 25 and 70 km, and with Source 5 shortened from 125 km to $\sqrt{70^2 - 25^2} = 65$ km. The total activity along the remaining portion of Source 5 is $\nu_5' = (65/125)(0.5) = 0.025$ per year, while the total activity in the restricted portion of Source 3 is $\nu_3' = 0.014$.

Table 2 shows the calculations for $\nu_{130}\, G_{130}$. Since (Eq. 4) $k_{m_1} = (1 - \exp[-2.3(6-4)])^{-1} = 1.01$, the probability the maximum peak acceleration will exceed $y = 130$ cm/sec^2 in any year becomes (Eq. 20, approximated for small probabilities).

$$1 - F_{\tilde{Y}}(y) \cong \nu_y(1 - k_{m_1}) + k_{m_1} e^{-\beta m_o} \nu_y G_y (y/b_1)^{-\beta/b_2} \tag{23}$$

$$= .099(1 - 1.01) + 1.01 \left(\frac{\nu_{130} G_{130}}{\nu G} 0.02\right)$$

$$= -0.001 + 1.01 \left(\frac{1.3 \times 10^{-9}}{1.5 \times 10^{-9}} 0.02\right)$$

$$= -0.001 + 0.017 = 0.016$$

This is a change in only 20% from the value of 0.2 obtained with an unlimited distribution.

Inspection of these calculations reveals that, since k_{m_1} is very nearly unity, the major factor in the change in the probability is the reduction in $\nu_y G_y$ from νG. Study, next, of Table 2 in comparison with Table 1 shows that, while certain ν_i's were reduced, the corresponding G_i's increased, leaving the products $\nu_i G_i$ virtually unchanged. The exception, and the major cause for the reduction in $\nu_y G_y$, is Source 2, which was eliminated completely. It is concluded, then, that the major influence on the risk of decreasing (increasing) the upper bound m_1 is to delete (add) potential sources or portions of the sources within the radius, r_y. Since those sources are a significant distance from the site, their influence may not be important.

These observations are further evidence that only the closer portions of sources contribute significantly to the risk at a site. Consider, for example, line sources 4 and 5. Earlier inspection of the function $g_o^L(a)$, coupled now with the observation that elimination of larger quakes ($M > 6$) does not substantially change the risk contribution, give strong support to the general hypothesis that <u>only the closer, smaller, more frequent</u>

earthquakes are significant in contributing to the risk of large ground accelerations at a site. For ground velocities, with their typically smaller attenuation constants, b_3, distant sources can be more important (see Table 2) and the hypothesis, while still generally true, is not as markedly so. In subsequent pages these observations will be reinforced.

Sensitivity of Estimates to Parameters: Both for better understanding of seismic risk and for confidence in engineering design conclusions, it is important to consider the sensitivity of final estimates to the various parameters which must be assigned. Since sources contribute almost additively to (small) risks, one can study individual sources independently. For most parameters it suffices to consider the simple point source.

Assuming that the design ground motion parameter, y*, will be that for a specified return period, t*, the former value is found by re-arranging Eq. 13 and 14 (for unlimited magnitudes):

$$y^* = b_1 e^{b_2 m_o} (t^* \nu G)^{b_2/\beta} \qquad (24)$$

The ratio of the two magnitude related parameters, b_2/β, is an important factor. A good estimate is important. Fortunately both parameters are widely studied, and both may change relatively little from region to region. But, since typical values of the ratio are of the order of 1/3 to 1/2 for acceleration and velocity, it is clear that the design value y* is not sensitive to ν or G. A 25% error in either factor would cause an error of 10% or less in y*. The errors are smaller for acceleration than velocity. The value adopted for m_o is not important, since an increase in m_o would decrease ν by a factor which would leave y* unchanged. On the other hand, y* is directly proportional to b_1, and, more importantly, if G is replaced (Eq. 14) by $r^{-\rho}$, y* is found to be proportional to r^{-b_3}. It is clear that accurate estimates of the focal distance, r, and of b_3 may be important. The former factor requires careful definition of potential sources by seismologists. Since, as was demonstrated above, closer sources are so influencial, the more important values of focal distance, r, will be sensitive to focal depth, h, which may be difficult to estimate but is relatively stable within regions. The latter factor, b_3, is more difficult to estimate closely as it is regionally dependent. Only more strong motion instrumental data will alleviate this situation. Indeed such data will undoubtedly modify the attenuation law (Eq. 1) in the important near-source zone. Esteva[3], for example, has suggested replacing the true h by an "effective" focal depth of $\sqrt{h^2 + 20^2}$ to enhance the accuracy of the law in this zone. Other new suggestions have also been made recently for short focal distances[12,13]. The basic method of risk analysis can accommodate such changes easily[2].

The sensitivity of the risk or the design value to the assumed value of m_1, the bound on the magnitude, is best studied using a single line or areal source. Consider an infinitely long line source with a fixed activity per unit length μ. Then if the distribution of M is limited at m_1, r_y is given by Eq. 19 and the "restricted" length, $a_y d$, of the fault, i.e., that portion which contributed to the risk has length (Fig. 2b)

$$a_y d = \sqrt{r_y^2 - d^2}$$

or

$$a_y = \sqrt{r_y^2/d^2 - 1} \qquad r_y \geq d \qquad (25)$$

which is a function of m_1. Since the total activity is $\nu_y = \mu\, a_y d$, Eq. 23 can be written

$$1 - F_{\tilde{Y}}(y) \cong \mu\, a_y d\, (1 - k_{m_1}) + k_{m_1}\, e^{\beta m_o} (y/b_1)^{-\beta/b_2}\, \mu\, a_y d\, \frac{d^{-\rho}}{a_y}\, c_\rho^L\, g_\rho^L(a_y) \qquad (26)$$

For most values of m_o and m_1 of interest k_{m_1} will be sufficiently close to unity that this equation can be written

$$1 - F_{\tilde{Y}}(y) = e^{-\beta m_o} (y/b_1)^{-\beta/b_2}\, \mu d^{-\rho+1}\, c_\rho^L\, g_\rho^L\, (ay) \qquad (27)$$

in which a_y is the only factor dependent on m_1. Therefore inspection of the function g_ρ^L in Fig. 4 will indicate directly the sensitivity of the risk to the assumed value of m_1. If two estimates for m_1 are under consideration, say 8 and 8.5, with their corresponding values for a_y, and a_y', then the relative value of the risks associated with these two assumptions can be found by simply calculating the ratio $g_\rho^L(a_y)/g_\rho^L(a_y')$. As long as r_y is large enough (i.e., as long as the m_1's are large enough and y is small enough, Eq. 19) to keep a_y equal to at least 1.5 to 2.5 (depending on ρ), the influence of changing m_1 is small. Since k_{m_1} is virtually unity, what change in risk there is only that of changing a_y, the effective length of the fault, a change which can be associated with increasing or decreasing the extent of potential sources threatening the site. The implication is that the contribution of the closer more influential portion of fault is not (significantly) altered when m_1 is changed (or incorrectly estimated). Areal sources can be studied in a parallel manner using $g_\rho^A(a_y)$ with similar general conclusions. These conclusions of insentivity to m_1 for the risk are even stronger for the design value y* since it is, in turn, relatively insensitive to the risk (Eq. 24).

Sensitivity of Estimates to Source Modeling: A more difficult question than sensitivity of risk and design values to parameter values is that of the influence of alternative assumptions as to geometrical configuration of the potential sources. In practice one must be guided by tectonic maps and seismic history maps of the region. These may produce good estimates of the total activity ν, but it may not be clear how to model the geometry of the sources. Should a certain cluster of historical activity be concentrated at a point source or smeared along a fault passing near the cluster? A more critical question in regions of low or moderate seismic activity, where it is difficult to deliniate major, active faults, or where thick overlying deposits obscure tectonic evidences, is whether the possibility of occurrence of an earthquake directly below the site should be considered. Any areal source which includes the site admits this event as an (improable) possibility.

To approach such questions, consider (Fig. 2c) a segmental (a" = 0) areal source centered at the site. The source has a fixed total activity ν and some radius ah. The question of sensitivity to modeling can be idealized to that of sensitivity to a. (Note the distinction between this and the previous study where the activity per unit length or unit area was held fixed.) For unlimited magnitudes, the risk becomes (Eq. 13 and 16)

$$1 - F_{\tilde{Y}}(y) \cong e^{\beta m_o} (y/b_1)^{-\beta/b_2} \nu h^{-\rho} \left[\frac{c_\rho^A g_\rho^A (a)}{a^2}\right] \tag{28}$$

By comparison with point source results it is clear that if the term in square brackets is dropped the risk is that which would be obtained if the most conservative assumption were adopted, namely that all the activity is concentrated at the closest point directly below the site. The term in brackets shows the influence in the risk of assuming this activity is distributed uniformly over an area of radius ah about the site. This term is plotted in Fig. 6b (and its counterpart for line sources in Fig. 6a). Inspection of Fig. 6b reveals that, depending on ρ, the risk estimated will be very sensitive to the assumed radius if it is within the range of 3 to 4 times the focal depth, otherwise not. (Conversely, for fixed radius, in this case of a source close to the site, the sensitivity of risk to the focal depth h is again important.) Again the relative risk associated with two assumptions, a and a', can be found by calculating the ratio of the factor in the brackets evaluated from Fig. 6b at the two values, a and a'. For example if $\rho = 4$, for a = 2 versus a' = 3, the risk will be 0.2/0.1 or twice as large. A design value, y*, will, of cource, be considerably less dependent than the risk on an error made in modeling the source (Eq. 24). For example, even the factor of two in the risk just calculated is only a factor about 25% in y* if $\beta/b_2 = 3$.

The marked sensitivity of risk to nearby sources was again demonstrated in the modeling study in the previous paragraph. Therefore the assumption of a source extending under or close to the site will give extremely conservative results if, in fact, it is not a correct assumption. To understand better how close "close" is in this modeling procedure, consider next an areal source which is a sector of an annulus (Fig. 2c) with a fixed ν and a fixed outer radius a', and some inner radius a" to be chosen. How sensitive are the results to the engineer's assumption about the location of the closest point of the source? The risk (Eq. 13 and 16) can be written

$$1 - F_{\tilde{Y}}(y) \cong e^{\beta m_o} (y/b_1)^{-\beta/b_2} \nu h^{-\rho} \frac{c_\rho^A g_\rho^A(a')}{(a')^2\{1-(a''/a')^2\}} [1-g_\rho^A(a'')/g_\rho^A(a')] \tag{29}$$

If larger values of a' are considered, $g_\rho^A(a')$ is virtually unity; if the values of a" consided are relatively small compared to a', $(a''/a')^2$ is negligible. Under these circumstances,

$$1 - F_{\tilde{Y}}(y) \cong e^{\beta m_o} (y/b_1)^{-\beta/b_2} \nu h^{-\rho} \frac{c_\rho^A}{(a')^2} [1 - g_\rho^A(a'')] \tag{30}$$

Thus the sensitivity of risk to a" can be observed directly in Fig. 4. If smaller values of a" (i.e., values of the inner radius about equal to h) are under consideration, the risk will fall off approximately linearly in a". If these smaller values need not be considered, however, the risk and the design value y* are not particularly sensitive to the assumption of the closest extent of the source.

Conclusions: From the numerical illustration and from the analytical studies, the following general conclusions follow:

1. The seismic risk (i.e., the probability that a particular peak ground acceleration, say, will be exceeded in any period of time) can be easily estimated for a site under any of a very general set of assumptions as to the geometry and activity of regional sources of activity, and under a general class of assumptions as to the frequency distribution of magnitudes and as to the relationship of peak acceleration to magnitude and focal distance.

2. For common assumptions and for the smaller values of the risk of engineering interest, the probability is calculated from a Type II extreme value distribution.

3. Each of the various potential sources contributes in an approximately additive way to the risk through a factor which is the product of ν, the source's average activity, and G, the expectation (over the source) of the focal distance to the minus ρ power.

4. The major contribution to the risk comes from the more frequent, smaller earthquakes, at the closer sources. (See Fig. 4 or 5.) This conclusion is not sensitive to reasonable modifications to the attenuation law.

5. The sensitivity of the estimate of the risk or of the design value to any factor depends on the degree to which it influences these smaller earthquakes and closer sources. Thus the risk may be sensitive to the focal depth, h, to the attenuation coefficient, b_3, and to the "magnitude" coefficient β/b_2, but it is seldom sensitive to the form of the upper tail of the magnitude distribution (e.g., to m_1, the limit on this distribution), or to the details of modeling sources which are more than two times the focal depth from the site.

6. Whereas the risk is proportional to such factors as the total activity, ν, and the geometry factor, G (or $E[R^{-\rho}]$), the design value, y*, associated with a particular risk or return period, t*, is proportional to only the b_2/β power of these factors. This power may vary from about 1/3 to 1/2, being smaller for ground accelerations than ground velocities. The design value is proportional to the design return period raised to this same power.

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Source	Type	Key Distances (Fig.2)	ν_i	Acceleration G_i	Acceleration $\nu_i G_i$	Velocity G_i	Velocity $\nu_i G_i$
1	Point	ah=1h	0.05	$(10^{-7})(0.17)=17\text{x}10^{-9}$	10^{-9}	$54\text{x}10^{-7}$	$2.7\text{x}10^{-7}$
2	Point	ah=3h	0.5	$(10^{-7})(0.004)=0.4\text{x}10^{-9}$	$0.2\text{x}10^{-9}$	$5.6\text{x}10^{-7}$	$2.8\text{x}10^{-7}$
3	Area	a"h=1h a'h=5h	0.05	$\frac{10^{-7}}{(5^2-1)}(.65)(1.0-.65)=1\text{x}10^{-9}$	$0.05\text{x}10^{-9}$	$6\text{x}10^{-7}$	$0.3\text{x}10^{-7}$
4	Line	d=1.4h a'd=0.7d	0.01	$\frac{17\text{x}10^{-9}}{0.7}(.65)(.86)=14\text{x}10^{-9}$	$0.14\text{x}10^{-9}$	$50\text{x}10^{-7}$	$0.5\text{x}10^{-7}$
5	Line	d=1.4h a'd=3.5d	0.05	$\frac{17\text{x}10^{-9}}{3.5}(.65)(1.0)=3\text{x}10^{-9}$	$0.15\text{x}10^{-9}$	$12\text{x}10^{-7}$	$0.6\text{x}10^{-7}$
			$\bar{\nu} = \sum_i \nu_i = 0.66$		$\overline{\nu G} = \sum_i \nu_i G_i = 1.5\text{x}10^{-9}$		$\overline{\nu G} = 6.8\text{x}10^{-7}$

Unlimited Magnitudes

Table 1

Source	Key Distances	ν'_i	Acceleration G_i	Acceleration $\nu'_i G_i$	Velocity G_i	Velocity $\nu'_i G_i$
1	ah=1h	0.05	$17\text{x}10^{-9}$	10^{-9}	$54\text{x}10^{-7}$	$2.7\text{x}10^{-7}$
2.	does not contribute.					
3	a"h=1h a'h=2.8h	0.014	$\frac{10^{-7}}{2.8^2-1}(.65)(.96-.65)=3\text{x}10^{-9}$	$0.04\text{x}10^{-9}$	$16\text{x}10^{-7}$	$0.22\text{x}10^{-7}$
4	d=1.4h a'd=0.7 d	0.01	$14\text{x}10^{-9}$	$0.14\text{x}10^{-9}$	$50\text{x}10^{-7}$	$0.5\text{x}10^{-7}$
5	d=1.4h a'd=1.8d	0.025	$\frac{17\text{x}10^{-9}}{1.8}(.65)(.96)=6\text{x}10^{-9}$	$0.15\text{x}10^{-9}$	$22\text{x}10^{-7}$	$0.55\text{x}10^{-7}$
		$\bar{\nu}_{130}=0.099$		$\overline{\nu_{130}G_{130}}= 1.3\text{x}10^{-9}$		$\overline{\nu_{130}G_{130}}= 4.0\text{x}10^{-7}$

Limited Magnitudes : $m_1 = 6$

Table 2

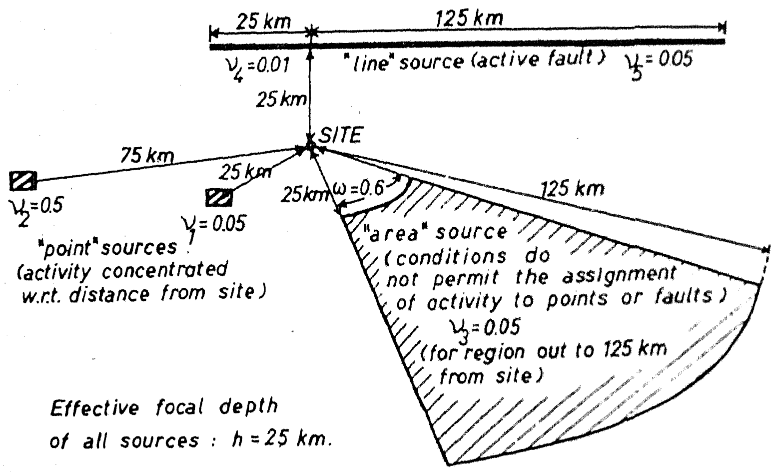


FIG.1 - A SET OF POTENTIAL SOURCES OF EARTHQUAKES

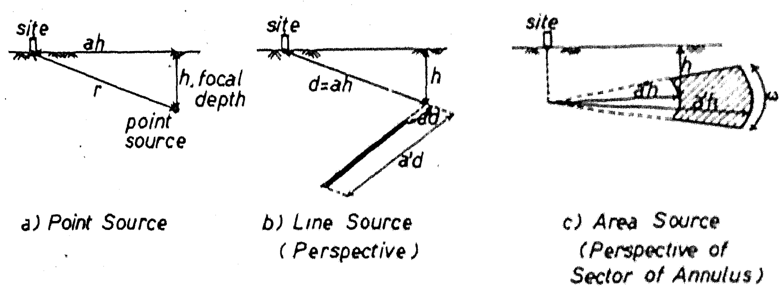


FIG. 2 - DEFINITION OF GEOMETRY PARAMETERS

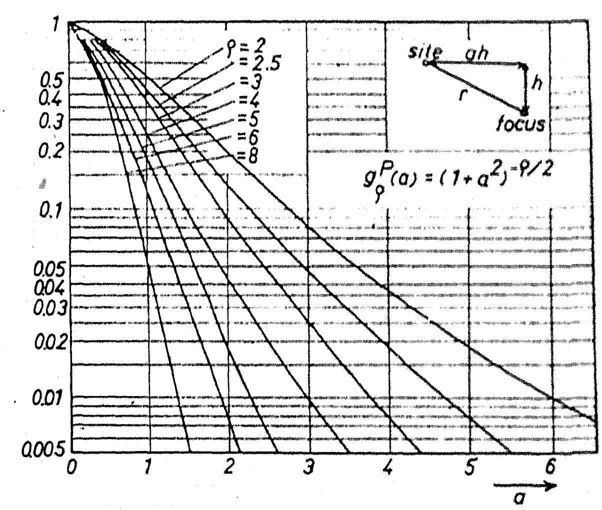


FIG.3 - FACTOR $g_\rho^P(\cdot)$ FOR POINT SOURCES

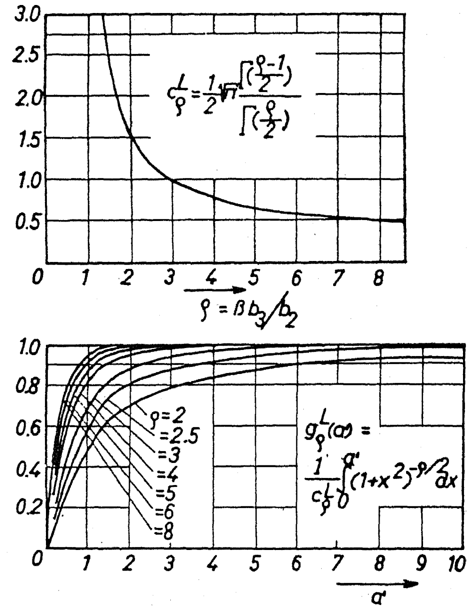


FIG. 4 - FACTORS c_ρ^L AND $g_\rho^L(\cdot)$ FOR LINE SOURCES

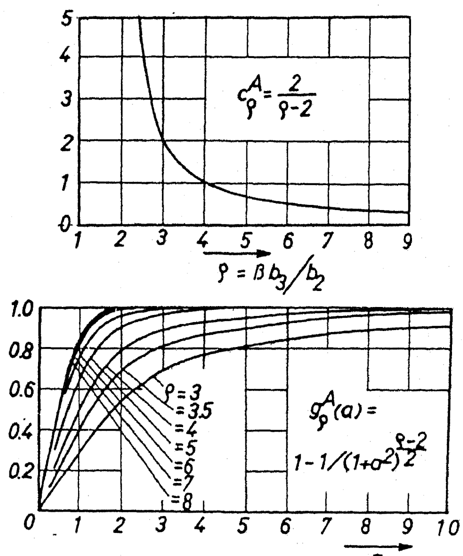
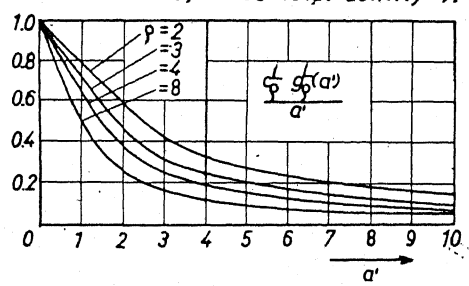


FIG. 5 - FACTORS c_ρ^A AND $g_\rho^A(\cdot)$ FOR AREA SOURCES

a) Point vs Line; Fixed total activity ν.



b) Point vs Circle; Fixed total activity ν

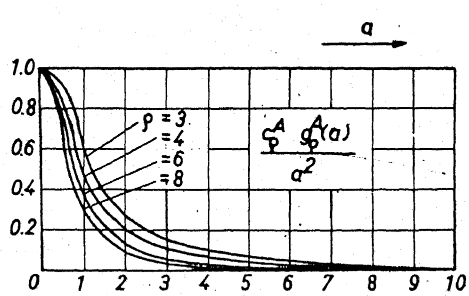


FIG. 6 SENSITIVITY OF RISK TO SOURCE MODELING