Analysis of Strong-Motion Accelerograph Records

by

D. E. Hudson, N. C. Nigam, and M. D. Trifunac [(I)]

Abstract. Optimum data processing techniques for strong-motion earthquake accelerograph records from existing field instruments are investigated. It is shown that digitization errors of typical photographic records can account for variations in Fourier spectrum values of 4-8%, and that baseline corrections and initial condition uncertainties ordinarily introduce average variations not exceeding 5% into response calculations. The significance to be attached to integrated velocity and displacement curves is discussed. A new method of response spectrum computation is compared with other methods from the standpoint of time required for various levels of accuracy.

Introduction. Most of the existing knowledge of the nature of the motion of the ground during destructive earthquakes is provided by strong-motion accelerographs (1, 2, 3, 4)[(II)]. Because of the limited number of such instruments presently installed and likely to be available for some time in the future, it is of the utmost importance to derive the maximum amount of information from the relatively small number of strong-motion accelerograms that have been obtained in the past and that will be slowly accumulating in the years to come.

The first strong-motion accelerogram was obtained during the 1933 Long Beach earthquake in California, and since that time about 100 records have been obtained by the U. S. Coast and Geodetic Survey of earthquakes of a destructive or potentially destructive size. Several dozen of the most important of these records have been studied in considerable detail and have been used as input functions for structural response studies, response spectrum computations, ground velocity and displacement investigations, etc. The full potential of this basic data, however, has not yet been exploited, for several reasons. First, it has not yet been feasible to make detailed analyses of all of the records, and there are a number of good accelerograms of relatively large ground motions which have not yet been closely examined. Secondly, the methods of analysis have evolved slowly over the years, and there has been a considerable variation in the techniques of data handling which has made it difficult to make direct comparisons between the results of different investigations.

With the above situation in mind, the California Institute of Technology, in cooperation with the Seismological Field Survey of the U. S. Coast and Geodetic Survey, has embarked upon a program aimed at the preparation of past strong-motion earthquake data in a form which will make uniform processing techniques possible to all investigators for

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(I) Div. of Engineering and Applied Science, California Inst. of Technology.

(II) Numbers in parenthese refer to references at end of paper.

comparative studies. It is hoped that the data processing methods being developed for this purpose will also make it possible to keep the analysis of strong-motion accelerograms up-to-date as new records come in from the rapidly expanding network of strong-motion accelerographs.

The USCGS Strong-Motion Accelerograph Network. As of March, 1968, the Pacific Coast network of the U. S. Coast and Geodetic Survey consisted of about 90 USCGS accelerographs ("Montana" type), 115 type AR 240 Teledyne accelerographs, 7 MO 2 New Zealand type, and 12 AR 250 Teledyne accelerographs (6). In addition to these instruments, there are a dozen or so modified USCGS accelerographs located in other countries, and about 50 AR 240 Teledyne accelerographs, mainly in Canada, Central America, and in the Mexican network of some 30 accelerographs.

The USCGS accelerograph and the AR 240 produce paper photographic records of a size suitable for direct digitization. The paper speed of the USCGS instrument is 1 cm/sec, and of the AR 240 2 cm/sec. The instruments have sensitivities in the range 5.0 to 7.5 mm per 0.1 g. Since the existing U. S. network consists almost entirely of these two types which produce very similar records, the data processing discussion of the present paper will be confined entirely to this particular type of record. In 1968, the AR 240 was superseded by the RFT 250 which records on 70 mm perforated film with a corresponding reduction of sensitivity. With the additional step of a photographic enlargement of the 70 mm film record before digitizing, the data processing problems will be similar, so that the general conclusions as to digitization accuracy, etc., will also apply to the new RFT 250.

Within the next year an increasing use can be made of the new Teledyne RMT 280 accelerograph which records on a standard four-channel 1/4 in. magnetic tape cartridge. This will present new data processing problems, which will differ from past techniques because of the possibility of direct automatic equal-time interval digitization by an electronic analog-to-digital converter, which should significantly modify certain digital computational methods. Studies of such tape handling problems are now under way, and suitable data processing equipment will be available by the time the RMT 280 tape accelerograph is installed in the field in significant numbers.

Even though the tape-recording accelerographs of the near future are sure to permit greatly improved data handling and computational procedures, it is clear that the existing photographic recording devices will continue to operate in the field for many years to come. There is thus not only a major data processing problem involved in the treatment of past accelerograph records, but many of the accelerograms of the future are certain to be of the photographic type. It is for these reasons that the emphasis of the present paper on such photographic records is justified.

Criteria of Effectiveness and Accuracy. The number of strong-motion accelerographs will for a long time be small compared to the number needed to adequately monitor the seismic regions of the earth. This means that the chances of having one of a limited number of instruments sufficiently near the epicenter of a destructive earthquake so that a significant ground motion record will be obtained are not very high. It is thus imperative that the maximum field reliability be attained. To miss an important record because of an instrument malfunction is so distressing that every effort must be made to ensure the correct operation of an instrument which may have remained essentially inactive for many years. Field relaibility has thus always properly been placed ahead of ease of data processing and even of data accuracy.

In the thirty-five years since the first accelerograph was installed by the U. S. Coast and Geodetic Survey, the record of field reliability has been outstanding. There has been no important earthquake in California since that time for which there has not been at least one good ground acceleration record. The culmination of this experience was attained during the Borrego Mountain, California, earthquake of April 9, 1968, which triggered some 120 strong-motion accelerographs in southern California with only one instrument malfunction (7).

The accuracy of the ground acceleration measurement should be judged on the basis of the use to which the measurement is to be put. It is important to keep in mind that the main objective of measuring ground accelerations during destructive earthquakes in the past has been to provide input data for the calculation of structural response. The accuracy of the ground acceleration measurement should thus be evaluated in terms of the accuracy of the response calculations which can be made from it. It is for this reason that most past studies of instrument accuracy have been based on the degree of reproducibility of response spectrum curves (8). These response spectrum curves involve the same basic type of response calculations that enter into the calculation of dynamic response of all kinds, and can thus logically form a systematic basis for an evaluation of the overall "effective" accuracy of the input ground motion for the purpose at hand.

Although the original motivation and the most important application of the strong-motion accelerograph is to determine the time variations of ground acceleration from which structural response can be determined, there are secondary reasons for wishing to integrate ground accelerations to obtain ground velocities and displacements. It is well-known that the ground displacement curves as obtained from the ground acceleration by double integration may be subject to considerable error at long periods because of the limits of accuracy of baseline location (9, 10, 11). Since accurate calculations of structural response from ground displacements cannot be made in any event, this particular limitation of the basic data is not a critical one. It will be shown that valid ground velocity and displacement information can be derived over a wide frequency range, and the nature of the frequency limits on such data will be established.

Digitizing Procedures. The original accelerograms in the form of developed photographic traces on the recording paper are photographically copied on a full size contact film negatives which form the basic archival records. Special photographic copying materials of a high dimensional ability are used to reduce film distortion to a negligible level. Direct lm positive contact prints are then made on a frosted-base film (cronoex) which can then be placed directly on the illuminated table of the gitizing machine. This translucent lithographic film provides excellent ontrast and even illumination which facilitates the setting of the digitizg machine cross-hair on the accelerogram trace. By experimentation ith the time of exposure and development of the film positive, an optium balance between thickness of record line and trace blackness can be chieved. Records from the USCGS accelerograph and from the AR 240 an usually be directly digitized at the recorded scale without photoraphic enlargement. Smaller records from AR 250 or MO 2 acceleroraphs would usually be photographically enlarged to a suitable scale in he process of making the film positive.

The digitizing process itself is carried out on a Benson-Lehner 99D Datareducer unit. This device consists of a mechanical cross-head nd cross-hair system which can be manually set to successive x-y coorlinates on the record trace. By means of a magnetic readout head these -y coordinates are converted to digital position signals by an electronic ounter, and are stored in a 6-digit accumulator system from which they an be automatically read out to an electric typewriter or to a card punch.

The resolution of this system is such that one digital count corresponds to a least time interval of 0.0033 sec. Since one does not ordinarily try to read earthquake records to less than 0.010 sec, the resolution capabilities of the digitizing machine leaves a comfortable margin.

The back-illuminated glass light table on the digitizer unit will accommodate records up to 16 in. in width and 24 in. long. Since most of the records obtained on the USCGS accelerographs were recorded at 1 cm/sec, this means that about 60 sec of record length can be accommodated without changing the position of the record on the digitizer table, and about 30 sec for the AR 240 accelerograph. It is of course important to avoid if possible any change in position of the record since this will introduce further inaccuracies into the baseline corrections.

Digitizing Reproducibility. To give an idea of the overall reproducibility of the digitizing process, a number of repeated readings of the same earthquake record were made by the same person, and by various persons. For this purpose a typical earthquake accelerogram was selected which was of good record trace quality. Since the ultimate aim of the process is the calculation of structural responses, the reproducibility of the digitizing process is logically studied by a direct comparison of Fourier spectrum curves calculated from the independently digitized data.

In Fig. 1 are shown earthquake Fourier spectrum curves obtained from three independent digitizations by the same operator, and in Fig. 2 are shown similar curves obtained by three different operators. As would be expected, it is evident that there are larger variations between the different operators than between the repeat runs for the same operator. In Fig. 3 are compared the curves obtained by averaging the Fourier spectra for three independent runs for each of the three operators. These average values from different operators show smaller differences than the repeated runs of one operator. It should be mentioned that the three operators collaborated to the extent of mutual discussions of such digitizing techniques as setting the cross-hair on the record trace, etc. Fig. 2 probably represents about the best that can be expected in practice in response reproducibility as limited by the basic record digitizing process. In investigations involving large amounts of data, it usually cannot be expected that all of the record digitization would be done by one person. It is apparent that for certain comparative studies, however, it would be well to have at least a uniform program of instruction and training for the digitizer operators, and that some periodic check of the reproducibility of the process should be a routine part of the data processing operations.

The accelerograms for the above studies were all digitized on an unequal time basis. The coordinates of all points where changes of slope were judged to occur were picked, and as many intermediate points were spotted as were felt necessary for an accurate definition of the function. The various operators differed slightly in their judgement as to the number of points required for the digitization of the same record, the average number of points per second over the whole length of the record for the three operators being 30, 39, and 34, with a minimum spacing of 0.01 sec.

Baseline Corrections. The accelerogram obtained from the standard accelerographs contains an undeflected line for each component which is produced by a mirror attached rigidly to the frame of the instrument. In addition, the position of the light beam at zero acceleration is usually evident at the beginning and end of the record. In principle, therefore, the baseline of zero acceleration is unambiguously indicated, even in the presence of film warping or of erratic motions through the transport mechanism. To measure from the indicated baseline on the record, however, involves an additional digitization process with its attendant errors, and it has been found preferable in practice to measure the acceleration trace from an arbitrarily established straight line baseline inherent to the plotting machine, which is then adjusted by a computational technique.

It can be shown that the particular baseline correction technique used is not a critical matter so far as the calculation of response spectra or structural response are concerned (12). When double integration of the accelerogram to obtain ground displacements is to be carried out, however, more care must be given to the methods used, since the displacement results may be significantly altered for some parts of the frequency spectrum (11, 13, 14). Although the main objective of the data processing techniques herein described is the calculation of structural responses, it is thought desirable to introduce the type of baseline correction which will have the most meaningful interpretation for displacement calculations. In this way the maximum amount of information as to ground displacements will be preserved without any adverse effects on the accuracy of the response spectrum calculations.

A considerable effort has been made in the past to understand the details of the baseline correction problem, and a full discussion would give an undue importance to the subject in the present context (5, 11, 12, 13, 14, 15, 16, 17, 19). It is of importance, however, to indicate the procedures that are recommended for routine data processing.

In placing the earthquake accelerogram on the reading table of the digitizer machine, an attempt is made to align the straight line zero axis of the machine with the zero axis of the accelerogram as inferred from the fixed mirror traces and the general appearance of the record. Keeping in mind that an acceleration step of the order of magnitude of the width of the record trace will result in a significant shift in the calculated displacement, it will be evident that a visual alignment alone will not likely produce an optimum result. In fact, the straight line of the digitizer machine axis will inevitably make a small angle with the true baseline and will be translated slightly from it.

Accordingly, the simplest correction technique is to place the machine axis arbitrarily, and then to determine computationally the position of the best straight line adjustment to satisfy some optimization condition. Among various possible optimization conditions, the one which has seemed most suitable is to make the mean square value of the ground velocity a minimum. This is justified mainly by the feeling that most earthquake ground motions of significant time duration correspond to ground velocities which oscillate approximately symmetrically about a zero axis and approach zero velocity at the end of the earthquake. A similar condition could not of course be applied to the ground displacement since permanent ground displacements at the end of the earthquake could well occur. It should be kept in mind, however, that the imposition of the mean square ground velocity minimization does not correspond to any physical requirement, but simply provides a reasonable technique by means of which all investigators can at least produce similar results.

In the above remarks it has been assumed that the shape of the acceleration correction baseline is to be a straight line, since this corresponds to the physical process of placing the record on the digitizing machine. However, it could well happen that the displacement of the record on the digitizing machine is not the only type of error involved in the whole process. For example, a distortion of the paper record because of some action in the accelerograph transport mechanism, or because of the photographic developing process which involves wetting and drying the paper might introduce additional difficulties. With this in mind, it has commonly been supposed that the acceleration correction baseline has the shape of a second degree parabola (10, 11, 13). The practical effect of this parabolic acceleration baseline combined with a mean square ground velocity minimization is to cause the ground displacement curve to be more closely confined to the vicinity of the zero axis; i.e., longer periods of motion of the order of twice the record length tend to be suppressed from the displacement picture.

It would appear that the major practical decision involved in baseline corrections is whether to pick a straight line or a parabola for the acceleration correction axis. The straight line has the advantage of a more direct physical justification and a minimum introduction of extraneous information into the result. The disadvantage is that the long period components which are left in the ground displacement are of doubtful accuracy, and hence might be misleading. The disadvantage of the parabolic correction is that the long period motions which are suppressed may in fact be present.

To illustrate some effects of the above baseline correction techniques, Figs. 4 through 8 have been prepared. In all cases the mean square velocity minimization has been used, and the same original digitized accelerograms have been employed. Fig. 4 shows a typical response spectrum curve as calculated using a straight line compared with a parabolic acceleration baseline. A similar calculation for a 30 sec record length results in identical response spectrum curves that cannot be distinguished at the scale of the diagram. This and similar computations confirm previous studies that although a baseline correction of some sort is necessary the exact details are not of importance for response calculations.

Fig. 5 compares ground displacement curves computed with straight line and parabolic corrections for a typical short duration earthquake. The differences would not be considered significant. Figs. 6, 7 and 8 compare the ground displacement curves for a longer duration earthquake calculated for three different record lengths using straight line and parabolic corrections. It is evident that for the short record of Fig. 6 and the relatively long record of Fig. 8 there are no significant differences between the two correction curves. Fig. 7, however, indicates clearly that the longer period components of the motion are suppressed from the displacement curve for records of intermediate length. Although these conclusions are directly related to a particular earthquake record, the situation is believed to be a representative one.

As can be judged from Fig. 7, the presence of long period components in the ground displacement can markedly alter the appearance of the curve. These long period components are of course very much longer than the periods ordinarily of significance for structural response studies. The decision in past data-processing operations, therefore, has been to suppress such long period components by using the parabolic baseline correction. It is believed that this is also a sound procedure for future data processing, except for research studies for which the longer period content of the ground motion may be of some special significance. For such special studies, no simple type of baseline correction could be expected to be satisfactory, and a more elaborate type of frequency filtering analysis would be essential.

The results of the above studies of the baseline correction problem can be summarized in the following conclusions: (1) The "standard" baseline correction to be applied in a routine way to accelerograms at the Earthquake Engineering Research Laboratory of the California Institute of Technology is a parabolic acceleration baseline fixed on the basis of a mean square ground velocity minimization. (2) It is essential to indicate on all ground velocity and displacement curves the type of baseline correction used, and the accelerogram record length involved. (3) Uncorrected digitized accelerogram data measured from an arbitrary straight line should always be available for special investigations. No matter what aseline correction is ultimately used, there will be an obvious advantage or many investigations if all groups can start with the same uncorrected igitized accelerogram data. (4) Accurate photographic copies of original ccelerograms should always be used for special studies for which the ixed mirror traces may offer a chance for additional information on baseine location. (5) Although the ground displacement curves plotted from ntegrated accelerograms may look different superficially depending on he details of the data processing used, it is clear that the errors in such lisplacement curves are not necessarily large except at relatively long periods. For periods of the order of the record length or longer, it cannot be expected that accurate ground displacements can be calculated by routine procedures. For shorter periods, the ground displacements are less sensitive to the data processing details, and the magnitudes of the calculated ground displacements should have significance. It is thus evident that the question of the accuracy of ground displacement calculations becomes one of determining the period range over which an acceptable level of accuracy can be attained. Significant accuracy will depend upon the use to which the ground displacement data are to be put. It is recommended that in discussions of the accuracy of ground displacements as calculated from accelerograph records an indication be given of the purpose for which the ground displacements are being determined, and that an estimate be made of the accuracy which would be expected to have an engineering significance.

Length of Record. A decision involving considerable individual judgement which must be made before record digitization is the selection of the portion of the record to be used for analysis. The selection of both the initial point and the final point of the record may influence response and ground motion calculations in a significant way, and thus these points represent additional variable factors which should be accounted for.

Since the accelerograph is triggered by the earthquake itself, and requires about 0.1 sec to come to full recording capability after the initiation of the starting cycle, the very first onset of motion is inevitably lost. An inspection of past records indicates that little significant data has been lost in past earthquakes because of this starting delay, since the large motions are usually preceded by a number of cycles of gradually increasing motion which trigger the accelerograph. There has been, however, considerable thought applied to the problem of supplying the accelerograph with a short memory, so that the complete record will be obtained (1, 3). This could in principle be accomplished by a tape loop, with the starting device serving the function of stopping recording after one loop length to avoid erasing the tape. There seems to be a definite limit on the recording time which is feasible with such tape loops because of the build-up of deposits from the tape on the tape head, and this factor might well complicate the field servicing and maintenance problem. Although several such tape memory systems have been developed in the laboratory, none has so far seen service use (18). An alternative approach is to interconnect accelerographs located several miles apart so that whichever instrument triggers first will simultaneously trigger neighboring accelerographs. In this way the time delays inherent in wave propagation times would serve to give the initial motions on some instruments. Commercial equipment is available to make such interconnections either by radio link or by a leased telephone line.

To investigate the effects of record length, several computational experiments were carried out. In Fig. 9 is shown an accelerogram from the Parkfield Earthquake of June 27, 1966 (20). Three different initial points are identified as points O, A, and B, and three final points as C, D, and 20 sec. The effects of various initial points can be judged from the Fourier spectrum curves of Fig. 10. The three curves start at different points and end at the same point. Although this results in slightly different record lengths, this is evidently a secondary effect for the present investigation. In Fig. 10 it will be seen that the scatter in the three curves is in general less than that caused by digitizing errors as shown in Fig. 2, and hence it appears that in this case the selection of the exact starting point has a negligible effect on the Fourier spectrum curves.

In Fig. 11 are shown Fourier spectrum curves for various end points. All records started at point A. Here the effect of including a longer portion of the record which contains longer period components is clearly shown.

Additional indications of the effect of record length may be seen in Fig. 7, which shows the effect of record length on the double-integrated displacement curves. Here it is again clear that long period components may be altered in a significant way by changes in the record length. For complete clarity, recommended practice would be to reproduce the original accelerogram to scale with marks indicating the beginning and end of the record as used for analysis purposes.

Digital Computation of Response Spectra. One common method of numerical integration which has been used for earthquake response spectra is the Runge-Kutta scheme of integration in one of several versions. This method is well adapted to digital computer operations and has the advantage for earthquake inputs that unequal time intervals can easily be introduced (21, 22).

As an alternative solution method, the response spectrum calculations can be made using an exact solution of the system equation of motion for each time interval step. Since this direct method has a computing time advantage in some cases, it is the method which is currently being used at the California Institute of Technology. The particular numerical methods selected will, of course, depend upon the experience of the programmer and the special features of the available computing facilities.

A Direct Integration Scheme. For the direct method, the digitized points of the accelerograph record are assumed to be connected by straight line segments, so that for each time interval step the exciting function is linear. The solution to the equation of the single degree of freedom damped oscillator which defines the response spectrum can then be written in a convenient matrix form (22). This exact solution will involve two matrices of rank two whose elements are known functions of system frequency and damping, and of the digitizing time interval Δt_i. If Δt_i is constant, the matrices need to be calculated only once for each spectrum point calculation.

If unequal time intervals are used, new values of the matrices would in principle need to be calculated at each step of the integration. However, by rounding off the time coordinates of the record into a small number of discrete segments of standard length, the number of matrices which need to be calculated can be reduced to only a few which can be computed at the beginning of the operation and called out when needed.

The final step in the calculation of the response spectrum is a monitoring of response parameters to locate maximum values. Since the response is found at discrete points only, and the true maximum may occur between two points, there is a small discretization error such as is inherent in all numerical procedures. This response discretization error will depend upon the ratio of the time interval of integration to the period of the oscillator ($\Delta\tau/T$). Under the most pessimistic assumption that the actual maximum falls midway between two points, it follows that the discretization error is less than 4.9% for $\Delta\tau/T \leq 1/10$, 1.2% for $\Delta\tau \leq 1/20$, and 0.3% for $\Delta\tau/T \leq 1/40$. Unlike the Runge-Kutta methods, the direct method does not involve a truncation error, and therefore the choice of $\Delta\tau (\leq \Delta t_i)$ for a given T is governed only by an acceptable bound on the discretization error.

Equal and Unequal Time Digitizations. Record digitization at equal time intervals offers an advantage in computer programming which may be more than offset for typical earthquake records by the small time intervals which may be required for adequate definition of portions of the record which may result in a large number of total points. With accelerograms of average to good quality it is feasible to use a digitizing interval as small as 0.01 sec, although this is usually a smaller interval than is needed over the whole record. For efficient processing of magnetic tape records by electronic analog to digital converters it will of course be necessary to use equal time intervals.

When digital conversion is made automatically at equal time intervals as by an electronic analog to digital converter, it is necessary to be aware of the possibility of aliasing errors (25). Such errors are a consequence of the fact that two points on a sinusoidal wave one-half period apart do not define a unique sinusoid, but might equally well be sample points on a higher frequency sinusoid having a multiple period. If the sample time interval is τ, then information at frequencies higher than the Nyquist frequency $1/2\tau$ cannot be recovered from the data, and if it exists may introduce inaccuracies into the lower frequency data. For accelerogram digitization at 0.01 sec, the Nyquist frequency would be 50 cycles per second, considerably higher than is usually of structural interest in earthquake response. In fact, most modern strong-motion accelerograph transducers would have so much response attenuation above 50 cycles per second that low frequency distortion because of aliasing would be negligible.

It should be noted that a digitized visual analog trace contains in general more information than the digital points themselves. On the basis of the appearance of the analog trace between the digitized points it is ascertained that the assumption of a straight line between the points is a reasonable one. With this assumption, the possibility of data distortion below the Nyquist frequency is in effect ruled out.

Comparison of Computing Methods. To indicate typical errors involved in current computing techniques, a series of comparisons were made of the exact solution of the differential equation as outlined above with the Runge-Kutta method in the form used for the past few years at the California Institute of Technology. This is a third order Runge-Kutta scheme in the form developed by Heun (21, 28). As a test case, an "artificial" earthquake was used which had a frequency content similar to that of strong-motion earthquakes (23). Four intervals of integration were used for the Runge-Kutta calculation: $\Delta\tau \leq T/10$, $T/20$, $T/40$ and $T/80$ and for the direct method two intervals of $\Delta\tau \leq T/10$ and $T/20$. For equal time digitization at 0.025 sec, the direct method at $\Delta\tau \leq T/20$ requires an average execution time of about 1 sec per spectrum point for a 30 sec record length on an IBM 7090 computer.

These trial calculations suggest the following conclusions for equal time digitization: (1) The Runge-Kutta integration requires $\Delta\tau \leq T/80$ to achieve an accuracy of three significant figures; (2) For accuracy of three significant figures the direct method with $\Delta\tau \leq T/20$ is three to four times faster than the previously used Runge-Kutta method.

For unequal time digitization the direct method can easily be modified to preserve essentially the same computing advantage. If the inherent limitations of the digitized record are recognized an approximate method based on rounding off the time coordinates of the original record to a predetermined accuracy reduces the number of matrices to be computed to a point where the computing time is not significantly different from equal time interval computing. In fact an overall advantage is retained for unequal time digitization because a smaller number of points can be used. With these modifications, the only difference between equal time and unequal time computations is that the calculated interval matrices must be available for 4 or 5 different values of $\Delta\tau$ rather than for just one value. The number of matrices to be calculated will depend upon the acceptable level of the error introduced by rounding the time coordinates, which should be kept compatible with other errors in the method. For example, this error should certainly be reduced below the errors involved in the original digitization of the record.

The digitizing errors have been illustrated above in Figs. 1, 2 and 3 on the basis of a comparison of Fourier spectrum curves. For the present purpose an estimate of the digitization error is needed in terms of the acceleration-time coordinates themselves. Repeated digitization tests similar to those described above indicated that a round-off of the time coordinate to 0.01 sec would be compatible with digitizing errors (22). To provide a clear margin of safety against an undesirable accumulation of errors, it has been decided to use a round-off to 0.005 sec for routine calculations. Using an analysis similar to that of Berg (14), it can be shown that a round-off to 0.005 sec can be expected under typical conditions to correspond to a percentage error in response spectra of less than 2%.

Trial calculations comparing the above direct method with round-off for unequal time intervals with a third order Runge-Kutta method were made. Round-off was carried to 0.005 sec, using a typical subdivision scheme $\Delta\tau = 0.04$, 0.03, 0.02, 0.01, 0.005 sec, which requires the calculation of five sets of matrices for each spectrum point for a maximum interval of integration of 0.04 sec. The comparison indicates that the direct method retains its advantage in computing speed for the unequal time calculations. It is therefore concluded that the direct method is suitable for the routine calculation of earthquake response spectra, and that it has the advantage of reduced computing time as compared with past procedures. This is not to imply, however, that comparable developments of other computing techniques are not to be expected, and continual improvements in respect to reduced computing times may well be hoped for.

Calculation of Fourier Spectra. The basic digitization for the Fourier spectrum curves of Figs. 1, 2, and 3 was carried out on an unequal time interval basis and the points were assumed to be connected by straight lines. Equal time intervals were then constructed by an interpolation process. The 0.01 sec time interval was sufficiently small so that the discretization error was negligible, as discussed above. After a direct numerical integration of the defining integrals, the real and the imaginary parts of the solution were combined to give the Fourier amplitude spectra as plotted in the figures.

The Fourier amplitude spectra as plotted in Figs. 1, 2, and 3 have been smoothed by the relationship:

$$f_i^*(\omega) = \frac{1}{2} f_i(\omega) + \frac{1}{4}\left[f_{i-1}(\omega) + f_{i+1}(\omega)\right]$$

where the $f_i(\omega)$'s are the calculated points and the $f_i^*(\omega)$'s are the plotted smoothed points. This smoothing technique is the so-called "Hanning" type (25), and reduces the high frequency fluctuations in the spectra which might tend to obscure more significant frequency content.

Fourier spectrum curves can also be computed by another procedure which offers a considerable reduction of computing time in many situations. The Cooley-Tukey method requires that the total time of the record be divided up into (2^M) equal parts, where M is an integer, and the resulting Fourier spectra points are evaluated at (2^{M-1}) discrete frequencies (26). These features may be disadvantageous if the equi-spaced frequency points do not describe the spectrum in the desired way. However, even though a larger number of data points and of calculated frequency points might be required, the remarkable speed with which the computing procedures can be carried out using the Cooley-Tukey algorithm may give the method a very significant advantage. It is this Cooley-Tukey technique that is now being mainly used for general Fourier spectrum calculations at the California Institute of Technology.

Corrections for Accelerograph Dynamic Response. Most of the transducer elements in current strong-motion earthquake accelerographs have natural periods of about 15 cycles per sec and damping of approximately 60% critical. Ground accelerations in the frequency range of 0 to 12 cycles per sec thus have little amplitude and phase distortion. If there should be a significant signal content at frequencies higher than the 12 to 15 cps range, or if the damping in the transducer is significantly less than 60%, it may be desirable to introduce a correction to account for the dynamic response of the accelerograph element. If the instrument parameters are known, the dynamic response corrections can be made using the equations of motion of the transducer element. One method, used in connection with the above studies, involves an expansion of the recorded signal in a Fourier series, with a suitable amplitude and phase correction applied to each term. Such computations can easily be incorporated as an initial step in the response spectrum calculations. Another method which accomplishes essentially the same result by a rather different approach has been developed, and examples of the magnitude of the correction terms for typical response spectrum curves are available (27).

Conclusion. The accuracy of current recorded strong-motion accelerograms is such that structural response calculations and ground motion studies can be carried out at a very useful level of engineering significance. Present day computing techniques are adequate for this purpose, and with suitable indications of method details, a reasonable agreement should be attained between various groups engaged in such studies.

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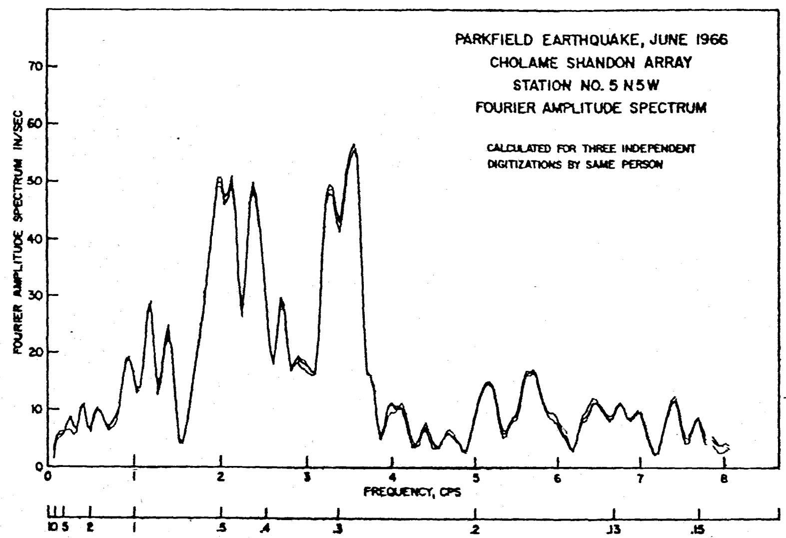

Fig. 1 COMPARISON OF RECORD DIGITIZATION - ONE OPERATOR

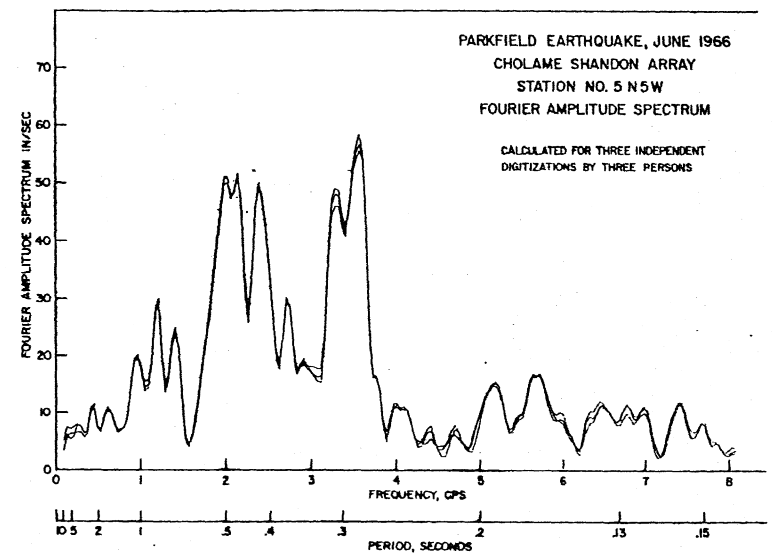

Fig. 2 COMPARISON OF RECORD DIGITIZATION - THREE OPERATORS

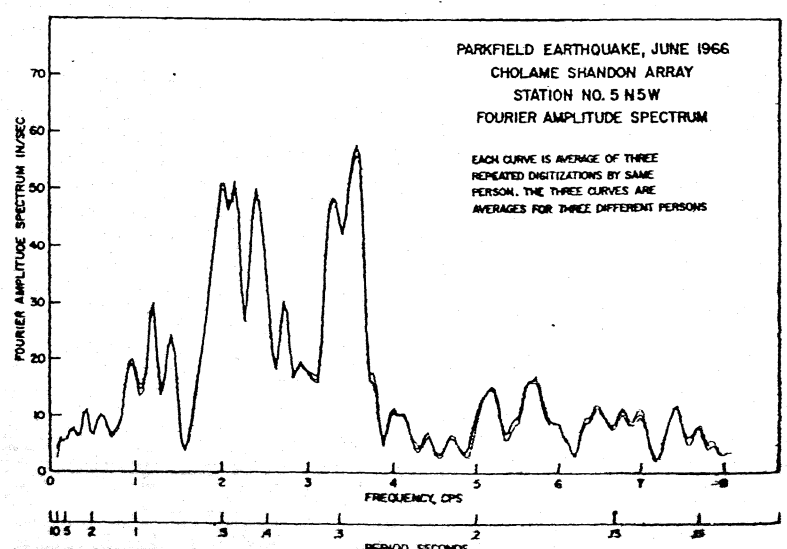

Fig. 3 COMPARISON OF AVERAGE RECORD DIGITIZATIONS

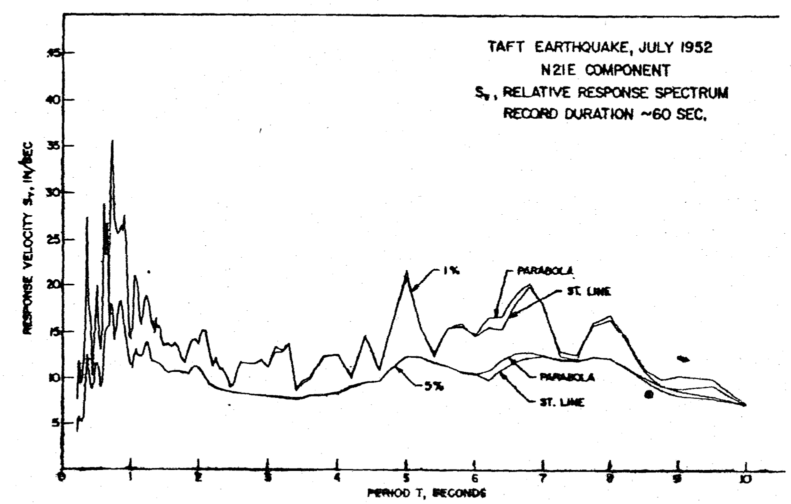

Fig. 4 COMPARISON OF PARABOLIC AND STRAIGHT LINE BASELINE CORRECTIONS

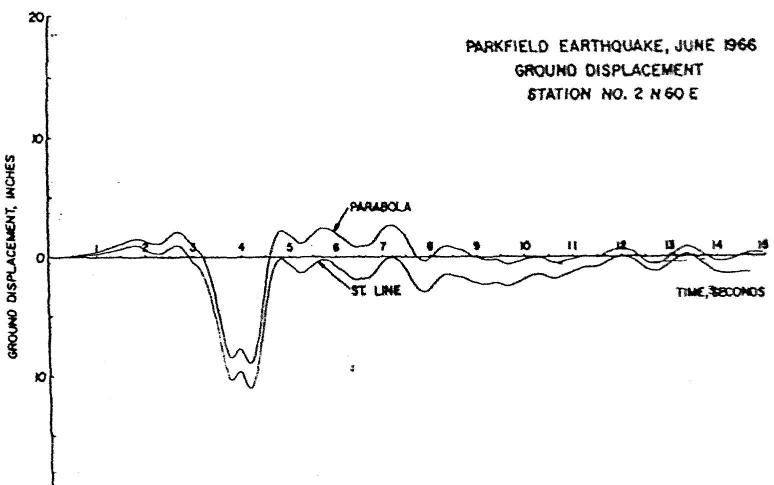

Fig 5 BASELINE CORRECTION COMPARISON

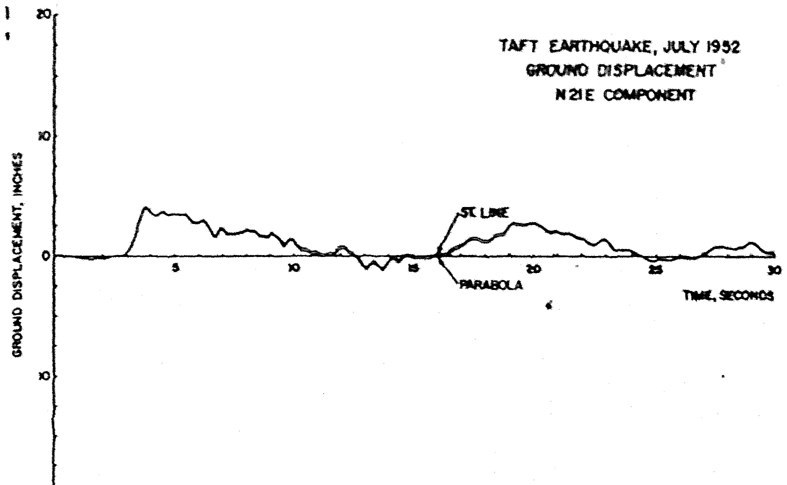

Fig. 6 BASELINE CORRECTION COMPARISONS-30 SEC. RECORD

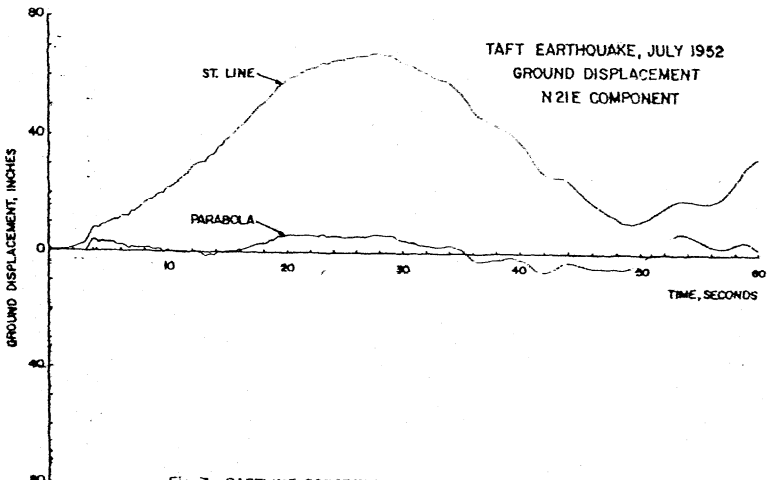

Fig. 7 BASELINE CORRECTION COMPARISONS-60 SEC. RECORD

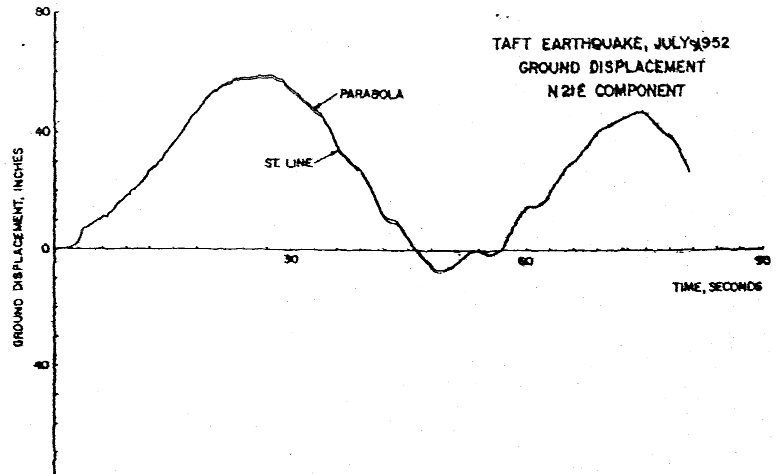

Fig. 8 BASELINE CORRECTION COMPARISONS-80 SEC. RECORD

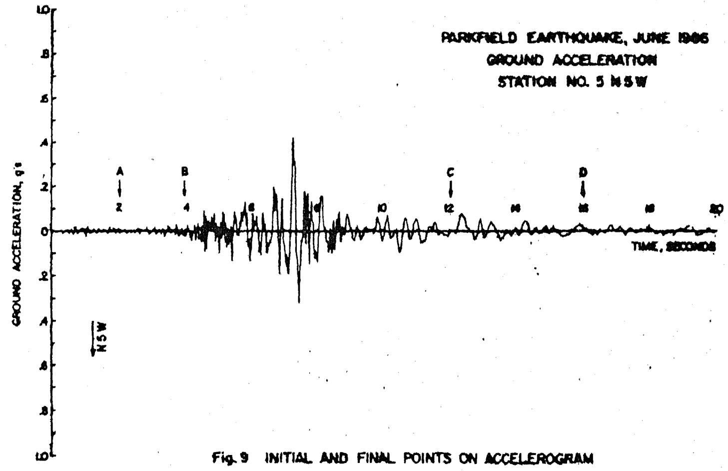

Fig. 9 INITIAL AND FINAL POINTS ON ACCELEROGRAM

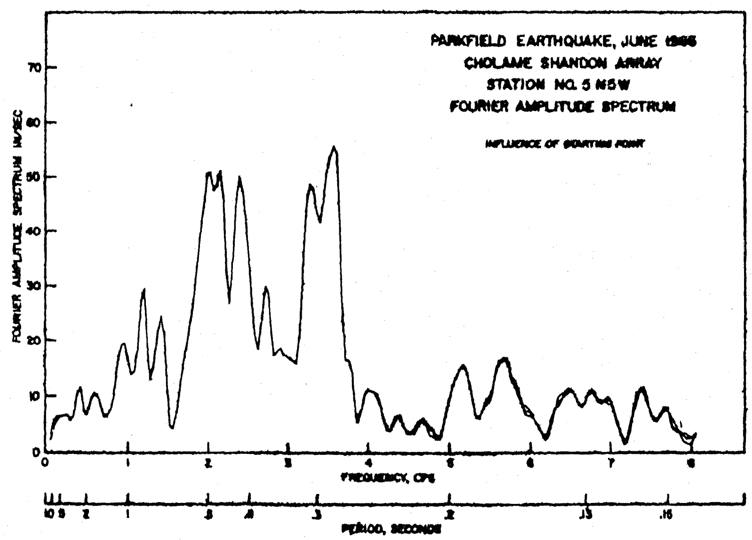

Fig. 10 EFFECT OF STARTING POINT ON RESPONSE SPECTRA

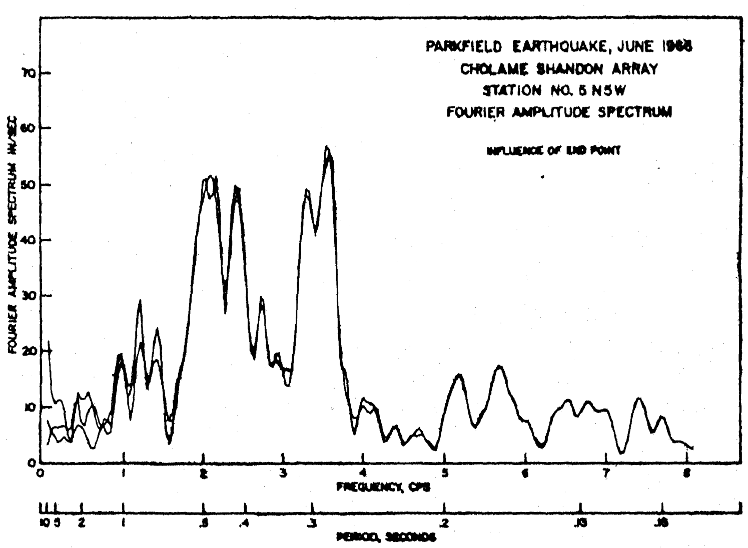

Fig. 11 EFFECT OF END POINT ON RESPONSE SPECTRA