An Experimental Study on the Horizontal Restoring Forces in Steel Frames under Large Vertical Loads

By

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Synopsis

An experimental study was made of the behavior of single bay, one- and two-storyed rectangular frames and cruciform frames, using mild steel models with wide flange sections. A varying horizontal force was applied to a frame model under a constant vertical load on the columns. From the horizontal force-displacement relation, a considerable reduction was observed in the restoring force due to the column loads, indicating the importance of the unstable character due to dead loads on the horizontal restoring force in tall buildings. Theoretical analysis had a reasonable agreement with the experimental results.

1. Introduction

Ordinary building structures are subjected to wind and earthquake forces as well as dead loads, snow loads, etc. Normally the latter act vertically and more or less constantly, while the former basically act in a horizontal direction and vary with time and space. Hence the strength of a structure hinges, among other things, on the behavior of the structural frame under constant vertical loads and varying horizontal forces. The existence of vertical loads somehow induces the unstable feature of a structure (1-3). The development of the plastic analysis of structures facilitates clarification of the ultimate state or the real strength of a structure. It usually neglects the instability effect of vertical loads. The destabilizing phenomenon, however, becomes more important in a higher structure, where the vertical loads are larger. This phenomenon is also significant when the horizontal displacement is large, even if vertical loads are small. In modern multi-story frames, more slender structural members are employed than before because of the adoption of high-strength steel, and this also makes destabilizing phenomena important in structural behavior.

A systematic study to investigate the behavior of a structure under such circumstances is imminent. It seems

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that no complete theoretical analysis is available, owing to computational difficulties (4). Plenty of tests have been carried out to check the validity of the plastic analysis of structures (5-8), but they have been mostly carried out under the conditions where vertical loads are either absent or so small that they have no significant effect on the overall response or the unstable character of frame behavior. The results of these tests, therefore, can not be directly applied to multi-story frames. A few tests have been conducted to investigate the frame instability caused by large vertical loads and horizontal loads. Low conducted model tests using miniature slender steel portal frames of three, five and seven stories under a constant horizontal load and a varying vertical load (9). A series of tests was conducted by Makino, et al., using miniature steel portal frames of two and three stories, under a constant vertical load and a repeated horizontal load (10). Igarashi, et al. conducted model tests using 1/4 scale mild steel one- and two-story portal frames with wide flange section under a constant vertical load and a repeated horizontal load (11,12). A full scale hybrid frame with wide flange sections was tested by Lu, et al. under a constant vertical load and a varying horizontal load (13). Prior to the experiment reported here, a series of preliminary tests was performed using miniature steel models for one-, three- and five-story portal frames (14-18). The destabilizing phenomena of the frame models were clearly observed in the horizontal force-displacement curves.

In this paper, the experimental results for one- and twe-story portal frames and cruciform frames with wide flange sections will be described. Vertical loads were applied constantly on the columns, and a varying horizontal force was applied, both at the top of a frame model. A theoretical analysis was made so as to study the elastic-plastic behavior of the frames for comparison with the experimental results. We attempted to include the strain-hardening effects in the inelastic range in an approximate way, in order to gain access to the real behavior of the frames tested. We conducted two kinds of analysis. The plastic hinge analysis takes into account the influence of axial forces and the strain hardening effect after mechanism state. The tri-linear analysis considers the development of a plastic and strain-hardening zone in member axis, using a trilinear moment-curvature relation. The former is simple and was used for the portal frames. The latter is more advanced but requires quite a computational effort, and was applied to the cruciform frames. The effect of shear deformation of the members is also taken into account in the latter analysis.

2. Description of the Tests

The behavior of a steel multi-story frame under a varying horizontal load was studied experimentally, using models of approximately 1/4-scale. In order to investigate the effects of the axial forces existing in the columns a variety of magnitudes were given for the vertical loads. The relative stiffness of the columns and beams was also varied.

The test program consisted of two series: Series I for one- and two-story portal frames and Series II for cruciform frames. The latter is typical of inner portion of multi-bay multi-story frames. Series I was composed of 4 one-story and 4 two-story portal frames and Series II was composed of 12 frames. The test frames and their loading condition are shown in Fig. 1 and Table 1. The magnitude of the vertical loads was decided so that it had approximately the same proportion to the yield load of the columns as in a lower story of an actual multi-story frame in Japan(iv). Test frames were fabricated by welding beams and columns of mild steel wide-flange sections and were designed to have the dimensions shown in Table 1. The material is called SS41, having proved yield point stress 2,500kg/cm^2 and ultimate stress somewhere between 4,100-5,200kg/cm^2. Most actual frames in this country have beam-to-column stiffness ratios in the range of those given here. The ratio of column height to the radius of gyration is also the representative value of multi-story frames. In order to prevent out-of-plane deformation, two identical plane frames were connected parallel to each other, to form a specimen, by welding a piece of wide flange section at several points across the two frames. The beam-to-column connections were stiffened by welding stiffners as shown in Fig. 1. Specimens I-2 and I-4 were annealed after they were constructed. The actual section dimensions and the material properties were observed for each frame member, as shown in Table 2. The effect of the rounded corners of a wide flange section at the flange-and-web connections is taken into account, since it turns out that the effect goes up to four per cent on the section properties. Tensile test specimens were taken from a flange of one of the stock wide-flanges of which the test frames were made.

The experimental arrangement is shown in Fig. 2. In the case of portal frames the test specimen was fixed on a L-shape supporting frame at the foot of the lower columns through 16 high-strength bolts. The supporting frame was set on an oil-pressure testing machine with 100 ton capacity. Rollers were placed between the supporting frame and the testing machine bed, in order to allow the supporting frame to move horizontally. The vertical load supplied by the testing machine is

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(iv) The ratio of the vertical load P to the elastic buckling load P_c of a frame is shown in Table 1. The ratio varies from 2 to 30%; the tangent modulus buckling load is not much different from the yield load P_Y. The frames are rather stubby, as compared with frames mostly used in foreign countries. The ratio P/P_Y in the lower columns in actual building is about 0.3-0.6 and the ratios in these tests was 0.17-0.34 for portal frames and 0-0.64 for crusiform frames as shown in Table 1.

distributed equally among the tops of the four upper columns. A horizontal force was applied by an oil jack with a capacity of 50 tons which was fixed on the supporting frame, and was also distributed equally between the two frames of the specimen at the upper beam-to-column connections. In the case of a cruciform frame, the vertical load was applied by an oil jack at the top of the columns. To eliminate constraints against rotation at both ends of the columns, universal ball bearings were used. The horizontal force was applied by an oil jack at the bottom of the columns. Rollers were used between the bottom of the columns and the supporting frame to eliminate constraints against sway. One end of the beam was adjusted vertically by an oil jack which was fixed beneath the supporting frame so that the both ends of the beams and the joint of beams and columns lie in a straight line during the test. The friction of the rollers was examined experimentally and the measured coefficient of friction was at maximum about 4/1000. As to the loading process, the vertical load was first set to the assigned value, and then the horizontal force was continuously varied in a quasi-static manner. The vertical load was maintained at the assigned value during the test by controlling the oil pressure. The load applied was measured by an oil pressure dial on the testing machine and by a load cell in the case of manual jacks as shown in Fig. 2. The horizontal relative displacements were measured by means of dial gauges with 1/100mm scale, and the strains were measured by wire strain gauges at the points also shown in Fig. 1.

The behavior of the frames observed is shown in Fig. 10 for portal frames and in Fig. 11 for cruciform frames. The dimensionless horizontal load was taken for the ordinate, and the dimensionless displacement for the abscissa. The hollow circles designate the experimental values. Throughout the tests, neither out-of-plane deformation nor local buckling phenomenon was observed. No appreciable deformation was observed at the beam-to-column connections. The strain measurements caught the process of yielding of critical sections one after another. The observed order of yielding shows agreement with the formation of plastic hinges in the theoretical prediction. The beams remain, after the removal of the loads, almost straight except for small regions at the ends where the permanent deformation is concentrated; in columns the plastic deformations extend over wider ranges near their ends. This is attributed to the axial forces in the columns. In Series I it is observed that the curvature is more pronounced in the columns on the side where the beam shear increases the column axial forces than in the columns on the other side. All the frames tested show configurations in which large plastic deformation has taken place at the member ends in accordance with the locations of plastic hinges. In Series II the upper and lower columns often showed different behavior after the attainment of the ultimate horizontal load in the case of specimens which were subjected to large axial forces as shown in Fig. 11. The rate of deformation of the column in which the ultimate capacity was attained earlier than in the other column was greater than that of the latter, although the test specimen was made so as to be symmetrial.

3. Theoretical Analysis

In order to determine the restoring-force characteristics, or the horizontal force-displacement relation under the constant vertical load, we first make the following general assumptions for two kinds of analysis to be introduced in this section.

1° The frame is composed of one dimensional members.
2° Although geometry change is considered, the deflections are so small that the cosine of the slope angle can be approximated as unity, and load directions are preserved.
3° The interaction of the moment and the axial force is considered for the fully plastic condition of the columns, the effect of shear being ignored.
4° Neither out-of-plane deformation nor any local buckling phenomena occur.
5° Effects of the residual stresses are negligible for the deformation characteristics of the frame.

3.1 Plastic hinge analysis

This approach was made for the frames of Series I, and was already described in detail in an earlier paper (18). In what follows, therefore, only the general principle is to be stated.

When the variation of the axial forces is neglected, we only have to consider the right or left half of the frame under consideration, taking account of the anti-symmetric property. As far as the loads are so small that the condition of linear elasticity is satisfied throughout the frame, the problem is solved by applying the slope-deflection method which takes care of the effect of the longitudinal force existing in a member. When the loads attain such magnitudes that a full plastic condition is satisfied at an end section of a member, forming a plastic hinge there, the appropriate end moment in the slope-deflection equation is set, in magnitude, to the full plastic moment of that member. We proceed to determine the elastic deformation of the rest of the frame, until another section reaches the state of full plasticity. A similar process is continued up to a mechanism state.

The elastic-perfectly plastic type of analysis does not describe the realistic behavior of a steel frame, because actual steel shows some strain-hardening behavior in a large deformation range; the elastic-perfectly plastic analysis does not agree very well with experimental results. Mild steel does not generally enter into the strain-hardening range until a certain amount of plastic deformation occurs. The full plastic state is reached in several sections at different stages of loading. When a sufficient number of cross sections reach the full plastic state so that the frame would be reduced to a kinematic mechanism according to the perfect plasticity theory, the strain-hardening may have been experienced in the sections where the full plastic state is reached at an earlier stage. However, those sections where the full plastic state is reached at a later stage may not have entered into the strain-hardening range. Therefore, we may take it that, on the average, the frame begins to suffer strain-hardening after the mechanism state. Thus, we assume that the frame gets strain-hardened upon arising of the mechanism state in the critical sections simultaneously. We approximate that the incremental moment and the incremental curvature have a linear relationship.

For this assumption, it may be reasonable to take it that the factor τEI relating the incremental bending moment to the incremental curvature (Fig. 3) is based on the average slope in the inelastic range of the stress-strain relationship, and we assume $\tau = 1/100$. In order to simplify the calculation in the strain-hardening stage, we assume that a plastic hinge spreads into a plastic zone in the member in which the full plastic state is reached, and it does not extend into the neighbouring members, Fig. 4. Noting that the incremental end angle $\Delta\theta$ is the same both for the beams and for the columns based on the assumption that the length of a plastic region is small in comparison with the length of a member, and that the additional elastic deformation after the arising of the mechanism state is negligible, we find the horizontal displacement of the joints from the geometry of the deflected frame. After expressing ΔM in terms of $\Delta\theta$ for each member, and then $\Delta\theta$ in terms of the incremental horizontal displacement, we can find the relation between the incremental load and the incremental horizontal displacement from the equilibrium conditions. The total horizontal displacement equals the sum of this incremental displacement and the displacement at the initiation of the mechanism state.

3.2 Tri-linear analysis

The cruciform frame is considered to be reduced to the assembly of a beam as in Fig. 5 and a column as in Fig. 6, if we assume the antisymmetric property. Therefore, we first investigate the behavior of a simply-supported beam and a cantilever-like column which have no intermediate loads. The relations among the bending moment M, the axial force P and the curvature κ are determined by the shape and dimensions of the cross section and the stress-strain relation of the material. In this analysis the relation is approximately replaced by a tri-linear relation as shown by dashed lines in Fig. 7. The tri-linear relation may be expressed as

$$\frac{dM}{d\kappa} = S_i\ , \quad (\kappa_{i-1} < \kappa < \kappa_i\ ;\ i = 1, 2, 3) \tag{1}$$

where κ_i designates the value of the curvature at which the slope of the $M-\kappa$ relation changes, and $\kappa_0 \equiv 0$.

First, we state the method of introducing the relationship between the end forces and the end deformation of the members, considering only bending deformation. If the end moment M_j is applied to end j, the letter j standing for the joint, of a simply-supported beam jc with length $\ell/2$ in Fig. 5, the curvature distribution corresponding to $M(x) = M_j 2x/\ell$ along the beam length is determined immediately in accordance with the $M-\kappa$ relation in Fig. 7. The end slope angle θ is, under the assumption of small deformation and of no unloading process, easily obtained from

$$\theta = \frac{2}{\ell}\int_0^{\ell/2} \kappa x dx \tag{2}$$

where x is the distance along the beam axis taken as shown in the figure.

We consider the column member Cj with length $h/2$ subjected to a constant compressive load P and a lateral load H at end C, which represent the point of contraflexure, Fig. 6. The loads P and H are in equilibrium with bending moment M_j at end j. Let θ be the angle between the tangent at end j of the deformed member axis and the acting direction of the compressive load P. Defining the x axis as shown in Fig. 6, the equilibrium equation for an element of the member is expressed as follows

$$\frac{d^2M}{dx^2} + P\kappa = 0 \tag{3}$$

Since

$$\frac{d^2M}{dx^2} = \frac{d}{dx}\left(\frac{dM}{d\kappa}\frac{d\kappa}{dx}\right)$$

Eq. (3) becomes

$$S_i\frac{d^2\kappa}{dx^2} + P\kappa = 0 \ , \ (\kappa_{i-1} < \kappa < \kappa_i) \tag{4}$$

It is convenient to write the general solution of Eq. (4) as

$$\kappa = A_i \sin\omega_i(x - x_i) + B_i\cos\omega_i(x - x_i), \tag{5}$$

$$(\kappa_{i-1} \leq \kappa \leq \kappa_i \ ; x_{i-1} \leq x \leq x_i)$$

where $\omega_i^2 \equiv P/S_i$; the interfaces x_i and the integral constants A_i and B_i are determined by the boundary conditions at both ends of the column and by the continuity conditions of the bending moments and shearing forces at the interfaces $(x_0 \equiv 0)$. The displacement Δ and slope angle α at end C are obtained from the geometric conditions

$$\Delta = \int_0^{\ell/2}\kappa x dx + \theta\ell/2 \ ; \ \alpha = \int_0^{\ell/2}\kappa dx + \theta \tag{6}$$

or, from the equilibrium conditions

$$P\Delta + Hh/2 = M_j \ ; \ H + P\alpha = \frac{dM}{dx}\bigg|_{x=0} = S_i\frac{d\kappa}{dx}\bigg|_{x=0} \tag{7}$$

The $M_j - \theta$ diagram of the beam and $M_j - \Delta$, $H - \Delta$ of the column are drawn in Figs. (a), (b) and (c), respectively, of Fig. 8.

In order to obtain the $H - \Delta$ relation of a cruciform frame subjected to a constant vertical load P and a variable horizontal force H , we assume the joint rotation θ , and let it be, say, θ_1 . The end moment M_j of the beam is determined from the $M_j - \theta$ diagram of Fig. 8(a). The end moment of the column is equal in magnitude to that of the beam according to the equilibrium of moments at the joint. Therefore, the displacement Δ is obtained from the $M_j - \Delta$ diagram for the case of $\theta = \theta_1$ as shown in Fig. 8(b), and the corresponding horizontal load H from the $H - \Delta$ diagram drawn in Fig. 8(c). Thus we get a point on $H-\Delta$ curve of the cruciform frame. The same procedure is repeated for the values of $\theta = \theta_2$, $\theta = \theta_3$, etc. Thus $H - \Delta$ curve of the cruciform frame is obtained for increasing deformation as shown by a dashed line in Fig. 8(c).

In the afore-mentioned analysis the shearing deformation has been neglected. Considering the small shear rigidity of a wide flange section and the rather small length-to-depth ratio of the members employed, the shear effect may decrease the frame rigidity to a considerable amount. This effect is, therefore, to be analized for a member subjected to axial force also as a general case. Total curvature κ is expressed as

$$\kappa = \kappa^b + \kappa^s \tag{8}$$

where κ^b is the curvature due to bending deformation alone and κ^s to shearing. We assume that the moment-curvature relation is tri-linear, i.e.,

$$\frac{dM}{d\kappa^b} = S_i^b, \ (\kappa_{i-1}^b < \kappa < \kappa_i^b \ ; \ i = 1, 2, 3) \tag{9}$$

we define β to be the angle between the undeformed axis and the normal to the cross section after deformation and γ to be the additional slope angle of member axis caused by the shearing deformation, Fig. 9. κ^b and κ^s are then found from

$$\kappa^b = -\frac{d\beta}{dx} \quad , \quad \kappa^s = -\frac{d\gamma}{dx} \tag{10}$$

Assuming linearity of the relation between γ and the shear Q , we have $\gamma = \lambda Q$, where λ is a constant depending on the cross section and the material of the member. Hence, $\kappa^s = -\lambda dQ/dx$. Reference to Figs. 6 and 9 gives us the relation $Q = H + P\beta$. Now,

$$\frac{d^2M}{dx^2} = \frac{d}{dx}\left(\frac{dM}{d\kappa^b}\frac{d\kappa^b}{dx}\right) = S_i^b \frac{d^2\kappa^b}{dx^2} = \frac{S_i^b}{1+\lambda P}\frac{d^2\kappa}{dx^2}, (\kappa_{i-1}^b \le \kappa^b \le \kappa_i^b) \tag{11}$$

and when the above equation is substituted into Eq. (3), which is also valid for the case containing shearing deformation, it follows that

$$\frac{S_i^b}{1+\lambda P}\frac{d^2\kappa}{dx^2} + P\kappa = 0 \tag{12}$$

If we write

$$S_i = \frac{S_i^b}{1+\lambda P} \tag{13}$$

then Eq. (12) becomes identical with Eq. (4). Therefore, we may use S_i modified as in Eq. (13), in place of S_i^b which is based on bending deformation alone, for the analysis accounting for shearing deformation as well. The restoring force characteristics of the frame can be obtained by using a similar procedure mentioned above, taking into account the fact that the rotation angle of the rigid joint is not θ but β, when the shearing deformation is considered as well as bending.

4. Results and Discussion

In Fig. 10 the relations between the horizontal force H and the displacement Δ of Series I are shown. H_p is the collapse load under the horizontal force alone, based on the assumption of perfect plasticity. Δ_1 and Δ_2, in Figs. 10(e)$\sim$(h), are the horizontal displacements of the first and second floors, respectively, and h is the column height. Solid lines refer to the experimental values and dashed lines to the theoretical predictions introduced in the preceding section, the variation of the axial forces being disregarded. The solid circles indicate the formation of plastic hinges. The dot-and-dash lines refer to the rigid-perfectly plastic theory (see the top of Fig. 10(a)), and coincide, after the arising of the mechanism state, with the elastic-perfectly plastic theory for the top floor displacement and also approximately for the first floor displacement of two-storeyed frames. For the theoretical curve in the process of decreasing displacement, under constant vertical loads, it is simply assumed that the relation between the decrement in the horizontal force and the decrement in the displacement is linear, and is identical with that between the increments for the initial elastic state.

Both the experimental and theoretical results show as a general characteristic that the restoring force of a frame is reduced by the vertical loads; the slope of the horizontal force-displacement curve decreases as the displacement increases. A rough agreement is seen between the experiment and the theory which takes care of the strain-hardening effect. In the small deformation range, the theory predicts an appreciably higher slope than the experiment, in particular for the frames with large beam-to-column stiffness ratios (see Fig. 10 and Table 1). The difference comes from the shearing deformation of members the effect of which will be discussed for the cruciform frames. It is to be noted that perfect plasticity underestimates the restoring force by a considerable amount after the arising of the mechanism state and it does not fully describe the frame behavior for a large deformation. For one-storyed frames, the theory overestimates the restoring force consistently in the inelastic range, by about 10%. Considering that this is not seen for two-storyed frames, the cause might be concerned with the indeterminancv of the frames but it is not conclusive at this stage of the investigation. The maximum hotizontal force is of main interest. This can be clearly observed in the experiment in the case where the stiffness ratio is small and the vertical load is large. This displacement is nearly at the displacement of $\Delta_1/h = 0.01 \sim 0.015$. This displacement is nearly two times the displacement at which the first theoretical plastic hinge arises in the frame under consideration (see Figs. 10(a), 10(b) and 10(f)). No difference is observed in frame behavior for specimens either with or without heat treatment.

In Fig. 11 the relations between the horizontal force H and the displacement Δ of each column for Series II are shown. Δu and $\Delta \ell$ are the horizontal displacements of the upper and the lower end of the columns, respectively. For the frame in which both column showed almost the same horizontal displacement, only the mean value of Δu and $\Delta \ell$ is shown. In the figure the theoretical curves are also drawn from the tri-linear theory. The solid lines are determined by considering bending deformation only. The dot-and-dash lines are evaluated by the theory which takes account of the shearing deformation also by assuming the shear is carried only by the web and that the shearing stress, which is correlated with the shearing strain by the law of elasticity, is uniformly distributed in the web. In both cases the panel at the beam-to-column joints are assumed to be rigid as compared with members. As described in section 2, the deformation of the two columns in some model frames were not identical, and in these cases the experimental curve for the column which showed larger deformation is compared with the theory described in the preceding section[v].

A reasonable agreement is seen between the experiment and the theory and it is seen that the discrepancy becomes small if the effect of shearing deformation is taken into account as well as the strain-hardening effect.

5. Summary and Conclusions

Models of single bay, one-and two-story rigid frames and cruciform frames were tested, using wide flange sections, from which the characteristic behavior of a multi-story frame was observed. Large vertical loads have induced a significant reduction in the restoring force or unstable character in frame behavior, indicating the importance of dead loads for the horizontal restoring force in tall buildings.

The elastic-plastic behavior of the portal frames was reasonably well predicted, until the arising of the mechanism state, by the slope-deflection method taking into consideration the axial forces, when the formation of plastic hinges was considered. The mild steel frames showed more favorable behavior than the theory of perfect plasticity predicts, after the mechanism state was reached. A rough, yet rational, in-

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(v) In the analysis it is assumed that the deformations of the two columns are identical. The assumption is not strictly correct for the frames tested. However, the error of the theoretical $H-\Delta$ relation for the column which undergoes larger deformation than the other column due to this assumption may not be too large, because in these particular frames the beams remain in the elastic range and the increment of the joint rotation is observed to be small after the attainment of the maximum horizontal force. The increment of Δ after this stage is, therefore, mainly caused by the curvature increment of the column (see the first equation of Eq. (6)).

clusion of the strain-hardening effect in the analysis showed a general agreement on the overall behavior of a frame with the experimental results, the closest agreement being found for two-storyed frames. It was observed in this experiment with portal frames that annealing does not affect overall frame behavior.

For cruciform frames the restoring force characteristics are reasonably well predicted by the theory which takes account of the strain-hardening effect and shearing deformation of members using the tri-linear moment-curvature relation. This theory would have predicted the behavior of portal frames also, better than the plastic hinge analysis introduced in this paper.

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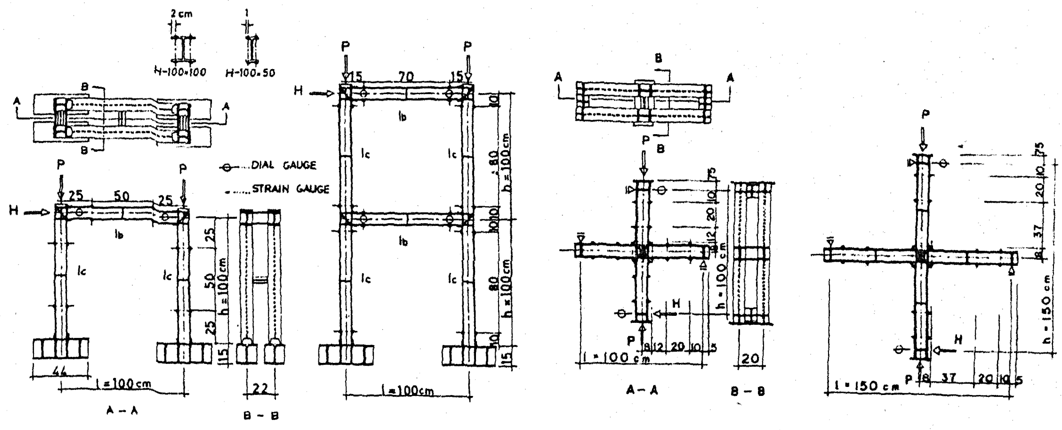
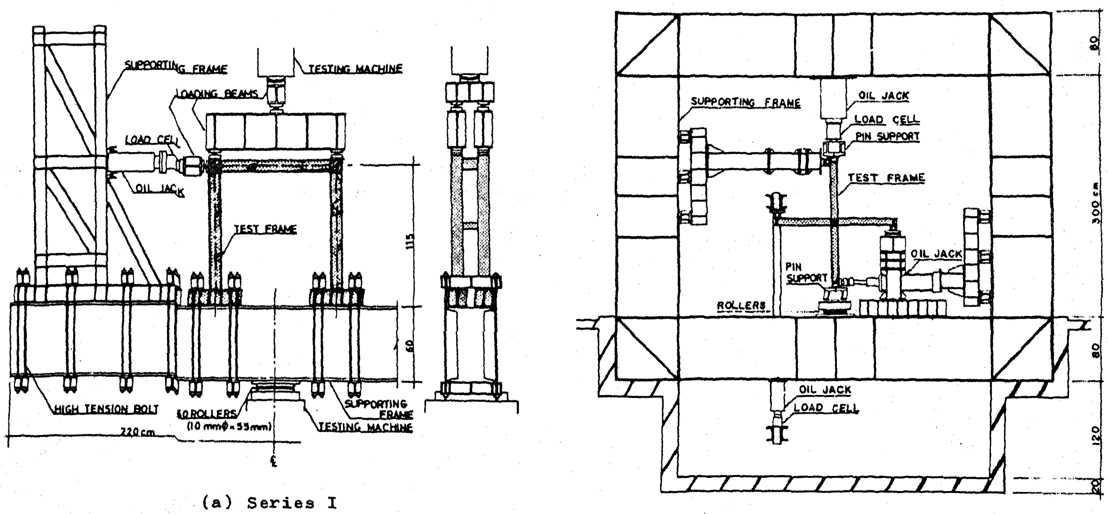


I-1∼I-4 I-5∼I-8 II-1∼II-6 II-7∼II-12

(a) Series I (b) Series II

Fig. 1 Test Models.



(a) Series I (b) Series II

Fig. 2 Loading Arrangement.

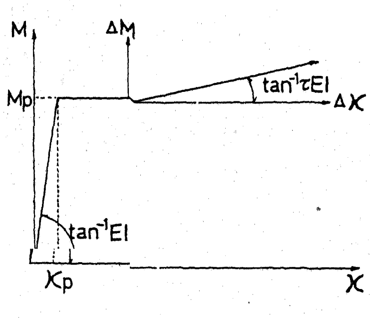


Fig. 3 Assumed Moment-Curvature Relation.

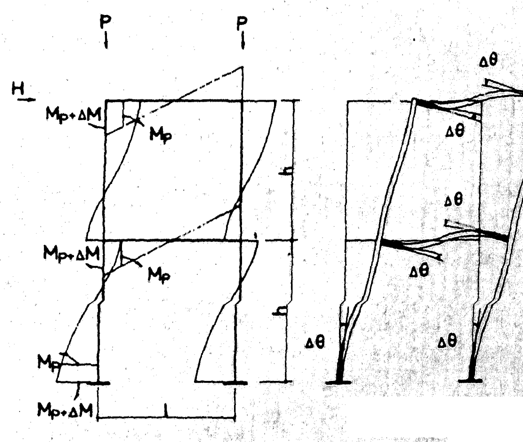


Fig. 4 Moment Distribution and Plastic Zones.

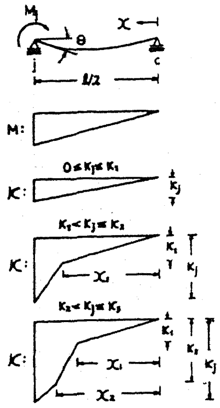


Fig. 5 Beam Member.

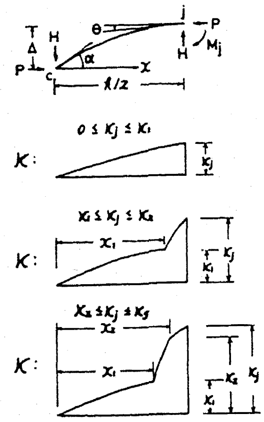


Fig. 6 Column Member.

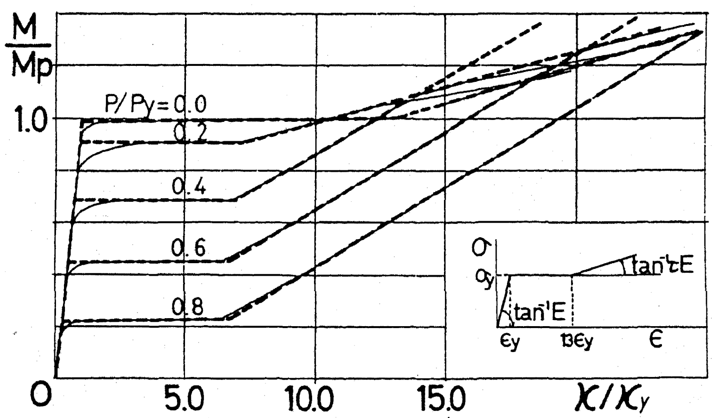


Fig. 7 Moment-Curvature Relation.

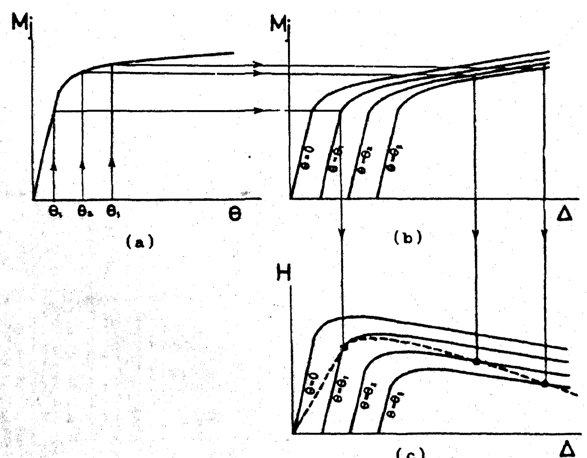


Fig. 8 Method of Analysis for Cruciform Frames.

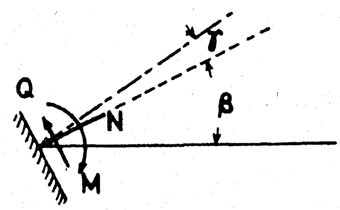


Fig. 9 Shear Deformation

Experimental
Elastic-Plastic
Rigid-Perfectly Plastic

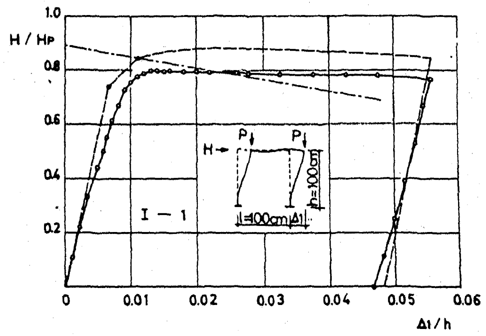


Fig. 10(a) I-1

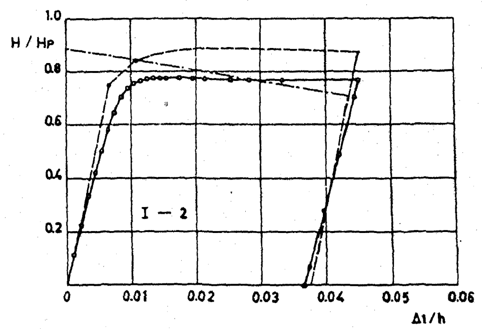


Fig. 10(b) I-2

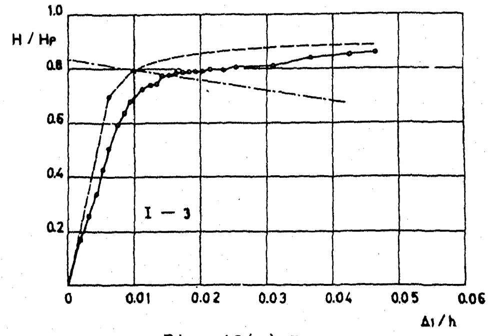


Fig. 10(c) I-3

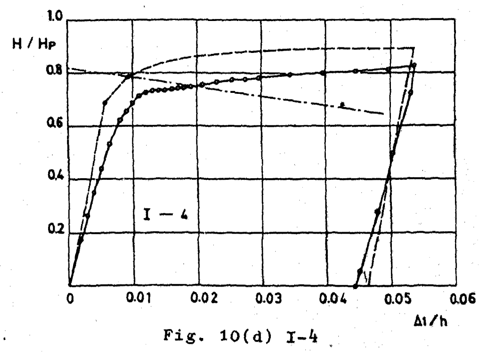


Fig. 10(d) I-4

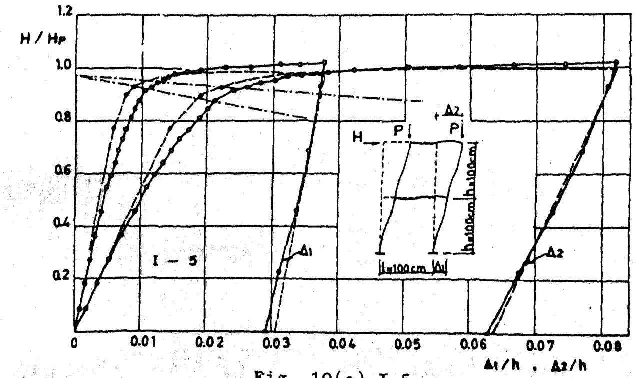


Fig. 10(e) I-5

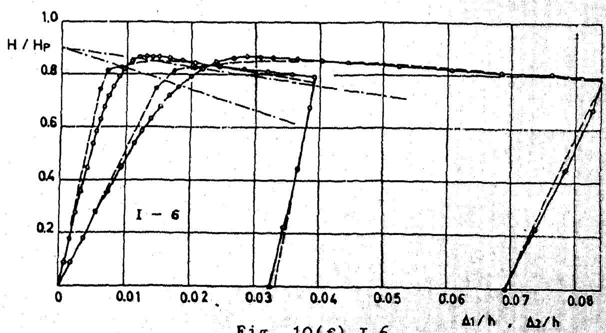


Fig. 10(f) I-6

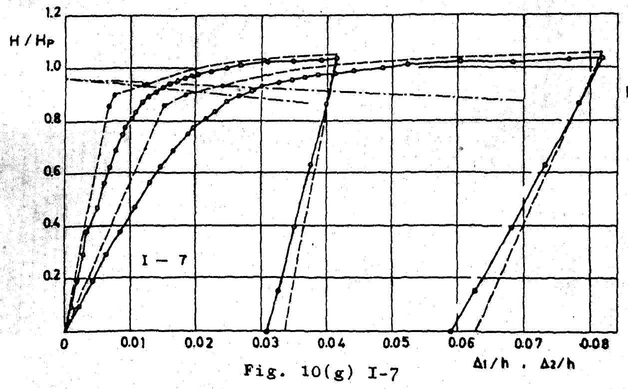


Fig. 10(g) I-7

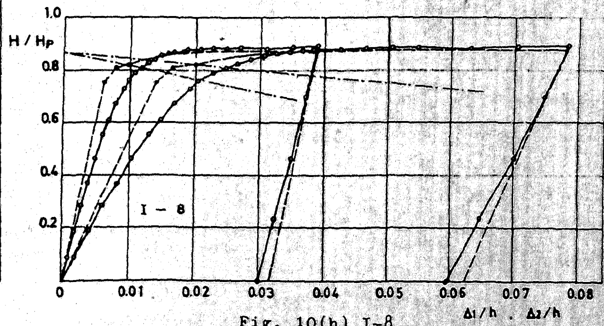


Fig. 10(h) I-8

Fig. 10 Experimental Results and Theoretical Predictions for Portal Frames.

—o—o— Experimental (Δ_l)
..o...o.. Experimental (Δ_u)
——— Theoretical(bending only)
— — — Theoretical(shear effect considered)

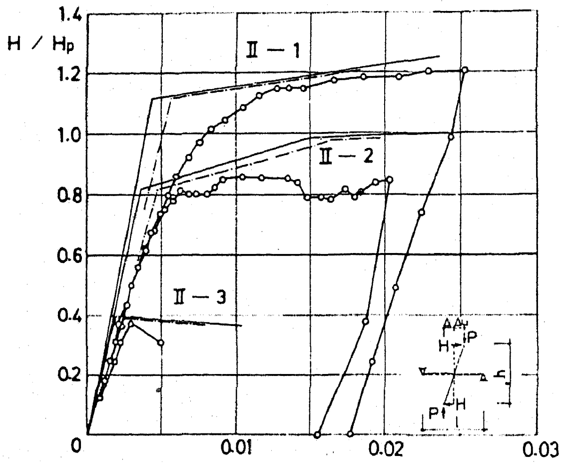


Fig. 11(a) II-1~II-3

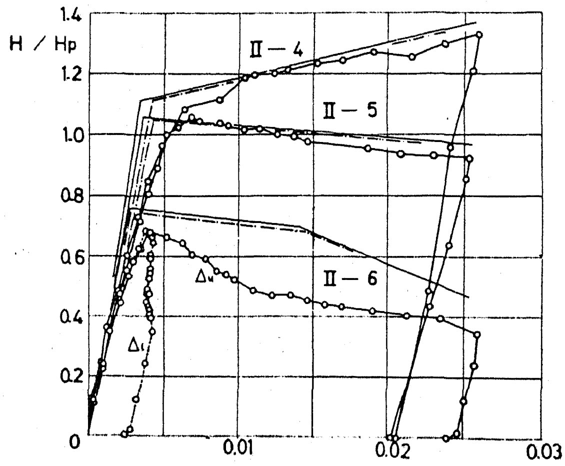


Fig. 11(b) II-4~II-6

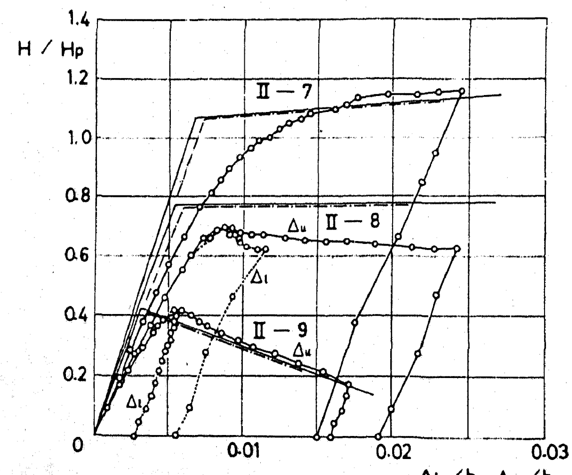


Fig. 11(c) II-7~II-9

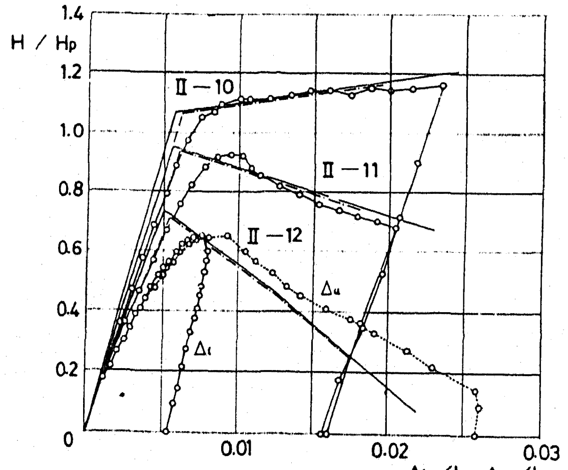


Fig. 11(d) II-10~II-12

Fig. 11 Experimental Results and Theoretical Predictions for Cruciform Frames.

Table 1. Nominal Dimensions of Test Frames

Specimen number	P (ton)	$\frac{P}{P_Y}$	$\frac{P}{P_c}$	$\frac{h}{i}$	Column (mm)	Beam (mm)	$\frac{I_b \times h}{I_c \times \ell}$	Heat treatment
I-1	20	0.31	0.04	24	H-100×100×6×8	H-100×50×4×6	0.43	none
I-2	20	0.31	0.04	24	〃	〃	0.43	annealed
I-3	20	0.29	0.03	24	〃	H-100×100×6×8	1.00	none
I-4	20	0.30	0.03	24	〃	〃	1.00	annealed
I-5	10	0.17	0.04	24	〃	H-100×50×4×6	0.43	none
I-6	20	0.34	0.07	24	〃	〃	0.43	none
I-7	10	0.17	0.02	24	〃	H-100×100×6×8	1.00	none
I-8	20	0.33	0.05	24	〃	〃	1.00	none
II-1	0	0	0	24	H-100×100×6×8	H-100×100×6×8	1.00	none
II-2	20	0.32	0.04	24	〃	〃	1.00	〃
II-3	40	0.64	0.09	24	〃	〃	1.00	〃
II-4	0	0	0	24	〃	H-100×50×4×6	0.45	〃
II-5	20	0.32	0.07	24	〃	〃	0.45	〃
II-6	40	0.64	0.13	24	〃	〃	0.45	〃
II-7	0	0	0	36	〃	H-100×100×6×8	1.00	〃
II-8	20	0.32	0.10	36	〃	〃	1.00	〃
II-9	40	0.64	0.19	36	〃	〃	1.00	〃
II-10	0	0	0	36	〃	H-100×50×4×6	0.45	〃
II-11	20	0.32	0.15	36	〃	〃	0.45	〃
II-12	40	0.64	0.30	35	〃	〃	0.45	〃

P : column load
h : column height
ℓ : beam length
P_Y : yield load of a column
P_c : elastic buckling load of a frame
i : radius of gyration of a column for in-plane flexure
I : sectional moment of inertia
$I_b \times h/(I_c \times h)$: beam-to-column stiffness ratio

Table 2. Actual Properties of Frame Members

Specimen number	h (cm)	ℓ (cm)	Column A (cm²)	Column I (cm⁴)	Column σ_y (t/cm²)	Column σ_u (t/cm²)	Beam A (cm²)	Beam I (cm⁴)	Beam σ_y (t/cm²)	Beam σ_u (t/cm²)
I-1	100.0	100.0	22.0	385	2.92	4.55	11.1	185	3.54	4.72
I-2	100.0	99.0	22.2	387	2.90	4.52	11.0	183	3.44	4.63
I-3	99.8	100.1	23.2	408	2.98	4.75	23.2	409	2.98	4.74
I-4	99.9	99.9	23.1	406	2.86	4.63	23.1	407	2.86	4.63
I-5	99.9	98.8	21.9	392	2.64	4.22	10.8	181	3.04	4.27
I-6	99.9	99.6	22.2	397	2.67	4.21	10.7	181	3.08	4.40
I-7	100.3	99.6	22.0	393	2.73	4.31	22.2	398	2.63	4.23
I-8	99.9	99.8	22.3	397	2.73	4.26	22.2	397	2.73	4.26
II-1	100.0	99.9	22.3	397	2.85	4.44	22.5	401	2.98	4.56
II-2	100.2	99.9	22.2	393	2.90	4.47	22.4	381	2.90	4.49
II-3	99.9	99.9	22.1	391	2.80	4.43	22.7	403	2.95	4.49
II-4	99.9	99.9	22.1	396	2.95	4.46	10.6	177	3.18	4.55
II-5	99.8	99.8	22.2	397	2.95	4.46	10.6	179	3.23	4.57
II-6	100.0	99.9	22.0	392	2.79	4.44	10.7	179	3.18	4.53
II-7	150.0	149.8	22.1	396	2.98	4.56	21.9	390	2.85	4.44
II-8	150.1	149.8	22.5	395	2.90	4.49	22.1	394	2.90	4.47
II-9	149.9	150.0	21.5	400	2.95	4.49	21.9	390	2.88	4.43
II-10	149.8	148.6	21.8	390	2.85	4.44	10.6	176	3.18	4.55
II-11	148.5	148.7	21.9	391	2.88	4.48	10.7	179	3.23	4.57
II-12	151.9	148.7	22.0	394	2.88	4.47	10.6	176	3.18	4.53

A : cross-sectional area
σ_y : yield point strees
σ_u : tensile strength