EVALUATION OF INELASTIC SEISMIC DEFLECTION OF REINFORCED CONCRETE FRAMES BASED ON THE TESTS OF MEMBERS

by: Hajime Umemura (I) and Hiroyuki Aoyama (II)

ABSTRACT

This paper presents a practical method to evaluate the effect of severe earthquakes, in terms of inelastic deflection, on the reinforced concrete frames.

Numbers of response spectrum analysis of earthquake records taken at various locations in recent years made it possible to evaluate the seismic effect on the structures, if the dynamic characteristics of the structures could be established. On the other hand, much experiments have been carried out on the reinforced concrete structural members as to their strength and deformation characteristics in the inelastic range.

The authors attempted to combine these two, with the aim of evaluating more realistic inelastic deflection than the estimation based on the idealized elastic or elasto-plastic analysis. As the first step to this problem, only framed structures were considered in this paper, treating a typical portion of uniform frames.

Inelastic behavior of framing members was formulated on the basis of tests of members (Figs. 1 & 2). Test data of simple beams with or without axial load were reviewed in terms of moment vs. rotation angle, leading to a set of empirical equations for the moment vs. rotation relationship. A general evaluation of the deformation of beam-column connections was also made using test results (Figs. 3 & 4).

Inelastic behavior of frames was then analyzed considering flexural and shear deformation of members as well as shear deformation of connections (Fig. 5). Thus the stiffness reduction due to cracking was appropriately taken into account. It was suggested that reduced stiffness be used in the dynamic analysis, rather than the elastic stiffness based on the uncracked sections.

Study on the velocity spectra for several recorded earthquakes (Fig. 6) resulted in empirical formulas for estimation of response values on the safe side. Using the dynamic property of frames as determined above, the maximum relative story displacement was obtained, and the associated behavior of members was compared with test results.

It was found that cracking of beams would occur in the very early stages, cracking of columns was also inevitable, and yielding of members would take place partially in case of severe earthquakes. However, large plastic deformation as would be predicted by elasto-plastic analysis was not likely to occur, if the inelastic behavior of reinforced concrete was duly accounted for.

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SYNOPSIS

This paper presents practical method to evaluate inelastic seismic deflection. Empirical equations were derived from test data for inelastic moment-rotation relation of frame members as well as shear stress-strain relation of beam-column connection panels. Simplified method of analysis was developed for load-deflection relation of frames. Then, based on empirical equations for spectral displacement, seismic deflection was estimated and the associated behavior was discussed. It was found that yielding of members would take place only partially in case of severe earthquakes, but large plastic deformation was not likely to occur.

INTRODUCTION

Numbers of response spectrum analysis of earthquake records taken at various locations in recent years made it possible to evaluate the seismic effect on the structures, if the dynamic characteristics of the structures could be established. On the other hand, much experimental works have been carried out on the reinforced concrete structural members as to their strength and deformation characteristics in the inelastic range. Now it is fairly well known that reinforced concrete structures soften considerably in advance of general yielding.

The authors attempted in this paper to evaluate the maximum deflection of reinforced concrete structures subjected to earthquakes. As the first step to this problem, only framed structures were considered. The maximum expected deflection will provide an important design criterion from practical point of view.

The evaluation should be based on the realistic restoring force characteristics. Hence the first part of the study was devoted to the survey of test data of framing members, to derive a set of empirical equations for the inelastic moment-rotation relation of members. The behavior of beam-column connections was also studied.

Load-deflection relation of frames was then analyzed. As to the typical portion of uniform frames, which assumes anti-symmetric deformation, very simple analysis gives the estimation of inelastic load-deflection relation. A more generalized method of analysis is also available.[12]

Inelastic response of frames may be obtained by the vibration analysis of a system having restoring force characteristics thus derived. As far as the maximum deflection is concerned, however, the use of available response spectra would suffice, and an empirical equation for this purpose was also developed.

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:lastic Stiffness

It was found that the rational expression for elastic stiffness agrees ıirly well with initial stiffness up to cracking of test specimens.

$$M/\theta = 6EK/(1+2\gamma') \tag{4}$$

rhere $\gamma' = 6EI_e\kappa/(GA_e\ell^2)$

nd E and G are the modulus of elasticity and rigidity of concrete, $K = I_e/\ell$, I_e, effective second moment of section, and κ, shape factor for shear deforma-ion. In case of rectangular members where length to depth ratio is more than ı, shear deformation may be disregarded at the risk of error less than 10%.

(ield Moment

Numerous studies on the ultimate flexural strength of reinforced concrete ɔoint out that the yield moment, which is very close to the ultimate moment for ;ections below the "balance" condition, can be evaluated very accurately from naterial property. The senior author had developed the "e-function theory,"(11) which employs the following expression for stress-strain relation of concrete,

$$\frac{\sigma}{{}_c\sigma_B} = 6.75\left\{\exp\left(-0.812\frac{\varepsilon}{{}_c\varepsilon_B}\right) - \exp\left(-1.218\frac{\varepsilon}{{}_c\varepsilon_B}\right)\right\} \tag{5}$$

where ${}_c\varepsilon_B$ is the concrete strain at maximum strength.

Although this theory is generally applicable to members with or without axial load, authors attempted to estimate the yield moment with much simpler approach. For members without axial load a simplified e-function equation by the senior author is available.(11)

$$M_y = 0.95\{1-0.43p_t(1-30p_c)\,{}_s\sigma_y/{}_c\sigma_B\}\,p_t\,{}_s\sigma_y\,bd^2 \tag{6}$$

where ${}_s\sigma_y$ is the yield stress of steel, b and d, width and effective depth of section, p_t and p_c, tension and compression steel ratio.

For members with axial load, an assumption was made that both tension and compression steel yield, or at least that the moment corresponding to above condition approximates the yield moment. Then we obtain

$$M_y = \{g_1 p_t\,{}_s\sigma_y/{}_c\sigma_B + 0.5\eta_0(1-\eta_0)\}\,{}_c\sigma_B\,bD^2 \tag{7}$$

where g_1 is the distance between tension and compression steel divided by depth of section, b and D, width and depth of section, and $\eta_0 = N/(bD\,{}_c\sigma_B)$. Eq. 7 is applicable to columns below the balance axial load, and $p_t = p_c$.

Fig. 8 indicates that the yield moment of both beams and columns show satisfactory agreement to the above-mentioned simplified estimation. For the evaluation of inelastic deformation of frames subjected to earthquakes, this approximation may be used instead of more complicated ultimate strength theories.

$$\tau = (1-u-v)Qh/V \qquad (10)$$

where $V = t_u l_v h$ (volume of connection panel).

From tests summerized in Table 2,[8~10] it was found that general trend of shear stress and strain in the panel would be idealized by tri-linear relation as Fig. 4. Cracking shear stress τ_c, initial stiffness G, yield shear stress τ_y, and stiffness reduction β_y completely define this relation.

Cracking Shear Stress

A rational expression for cracking shear stress τ_c is

$$\tau_c = \sqrt{\sigma_t^2 + \sigma_t \sigma_n} \qquad (11)$$

where σ_t is the tensile strength of concrete, and σ_n, normal stress due to column load (compression positive). This expression was directly derived from theory of elasticity assuming that crack occurs when the principal stress reaches tensile strength of concrete.

A reasonably good fit to test data was obtained by assuming

$$\sigma_t = 1.8\sqrt{{}_c\sigma_B}. \qquad (\mathrm{kg/cm^2}) \qquad (12)$$

The comparison of measured and computed τ_c is given in Fig. 10.

Elastic Stiffness

The test data confirmed that the initial slope of Fig. 4 could be represented by the modulus of rigidity of concrete G, or E/2.3.

Yield Shear Stress

It appeared in the test data that yield shear stress τ_y is little affected by amount of axial load N, or axial stress σ_n. The plotting of measured τ_y against concrete strength resulted in the following empirical equation.

$$\tau_y = 0.65\,{}_c\sigma_B - 0.0014\,{}_c\sigma_B^2 \leq 75 \quad (\mathrm{kg/cm^2}) \qquad (13)$$

This equation may be used for ${}_c\sigma_B = 150 \sim 300\ \mathrm{kg/cm^2}$. Measured and computed τ_y are also compared in Fig. 10.

Stiffness Reduction at Yielding

It was found that shear stress reaches above 90% of its maximum value when shear strain γ reaches 0.4%, regardless of concrete strength. As the *yielding of connection panel is the result of diagonal compression failure of* concrete, and the concrete stress reaches its maximum when strain is about 0.2%, above statement sounds reasonable for nearly square connection panels.

The stiffness reduction at yielding is then expressed as follows.

$$\beta_y = \tau_y/(\gamma_y G) \qquad (14)$$

where $\gamma_y = 0.004$.

INELASTIC DEFLECTION OF FRAMES

Components of Deflection

Consider a typical portion of frame subjected to horizontal loading and deformed into an anti-symmetric configuration. The deformation of the frame consists of flexural deflection in Fig. 5(a) and deflection due to shear deformation of connection panel in Fig. 5(b). Both of these deformations are in general inelastic.

Flexural deflection δ_F results from the deformation of beams and columns as in Fig. 5(a). Denoting the column rotation angle due to flexure by R_F, we obtain

$$R_F = \delta_F / h = \theta_B + \theta_C \qquad \text{(c)}$$

The rotation of beam end θ_B and column end θ_C must be estimated based on the moment-rotation relation as in Fig. 2. On this occasion, the portions of each member within the connection panel may be regarded as perfectly rigid against bending moment.

On the other hand, the deflection due to shear deformation of connection panel δ_P is obtained from the shear strain of panel γ as follows.

$$R_P = \delta_P / h = (1 - u - v)\,\gamma. \qquad \text{(d)}$$

The shear strain γ must be estimated based on the relation as in Fig. 4.

The total deflection of the frame δ, or column rotation angle R, may be expressed as follows.

$$R = \delta / h = \theta_B + \theta_C + (1 - u - v)\,\gamma \qquad (15)$$

End Rotation of Beam

Since a beam in the frame has rigid zones at both ends, the inelastic moment-rotation relation as outlined in Fig. 2 should be thought of that for the deformable portion of members (Fig. 11). What we need is the moment-rotation $(M_B - \theta_B)$ relation at the beam end including end rigid zones.

Let M_{BC} and M_{BY} denote apparent cracking and yield moments with respect to beam end whereas M_{BC}' and M_{BY}' denote true cracking and yield moments.

$$M_{BC} = M_{BC}' / (1 - u) \qquad \text{(16a)}$$

$$M_{BY} = M_{BY}' / (1 - u) \qquad \text{(16b)}$$

In the elastic range up to $M_B = M_{BC}$, eq. 4 may be rewritten as follows.

$$\theta_B = M_B / (6EK_o C_B k_B) = M_B / (6EK_o k_{BE}) \qquad \text{(e)}$$

where $k_{Be} = C_B k_B$

$k_B = I_B/(l K_0)$

$K_0 =$ standard rigidity of member (constant)

$$1/C_B = (1-u)^3(1+2\gamma_B') \tag{17}$$
$$\gamma_B' = 6EI_B K/\{GA_B(1-u)^2 l^2\}.$$

k_{Be} is so-called effective rigidity ratio of member, considering end rigid zones and shear deformation. C_B in eq. 17 is essentially same to the coefficient in Dr. Muto's literature.[13]

In the elasto-plastic range of $M_{BC} < M_B \leq M_{BY}$, it is more convenient to define stiffness reduction factor α_B as follows, rather than to use α_y in Fig. 2. It is the ratio of slopes of moment-rotation line and initial elastic line. It is obtained by

$$\alpha_B = \frac{1 - M_{BC}/M_{BY}}{1 - \alpha_{BY} M_{BC}/M_{BY}} \alpha_{BY} \tag{18}$$

Then

$$\theta_B = \frac{M_B}{\alpha_B \cdot 6EK_0 k_{Be}} - \theta_{B0} \tag{r}$$

where

$$\theta_{B0} = \frac{1-\alpha_B}{\alpha_B} \frac{M_{BC}}{6EK_0 k_{Be}} \tag{19}$$

This θ_{B0} indicates the location of the intersection of extended moment-rotation line in the elasto-plastic range and the abscissa.

If a convention is made that $[\alpha_B]$ and $[\theta_{B0}]$ have differently specified value according to the end moment, then eq. 20 may be used to represent the whole range of moment up to yielding. Note that $M_B = Qh/2$.

$$\theta_B = \frac{Qh}{[\alpha_B] \cdot 12EK_0 k_{Be}} - [\theta_{B0}] \tag{20}$$

where

$$[\alpha_B] = \begin{cases} 1, \\ \alpha_B, \end{cases} \quad [\theta_{B0}] = \begin{cases} 0, & \text{for } M_B \leq M_{BC} \\ \theta_{B0}, & M_B > M_{BC} \end{cases}$$

End Rotation of Column

Above discussion is directly applicable to columns, and the resulting equation may be expressed as follows.

$$R = \delta/h = \left\{ \frac{1}{[\alpha_B]K_{Bc}} + \frac{1}{[\alpha_c]K_{ce}} + \frac{H}{[\beta]} \right\} \frac{Qh}{12EK_o} - \left\{ [\theta_{Bo}] + [\theta_{co}] + (1-u-v)[\gamma_o] \right\} \quad (24)$$

where $$H = (1-u-v)^2\, 12EK_o/GV \quad (25)$$

' As stiffness reduction factors α_B , α_c and β are successively introduced with increasing horizontal load Q , eq. 24 effectively expresses inelastic deformation, or decaying stiffness, of typical portion of frames where antisymmetric deformation takes place. For other portion of frames such as the first story, some modification to above equations should be made, by assuming the location of the point of contraflexure in the columns. As an alternative to this method of analysis, a more elaborate method of inelastic frame analysis is available.[12]

Example of Frame Analysis

Analysis carried out on the typical portion of frame shown in Fig. 12 will be introduced here. Material property assumed in this analysis was also shown in Fig. 12.

Case 1 is assumed to represent the third floor from the top of the building, and Case 2 the sixth floor from the top. In both cases the yield level is set about 20% of the vertical load, or the seismic coefficient of 0.2. It is seen in Fig. 13 that the yield deflection is about 4 times the elastic (uncracked) deflection in both cases. Although the absolute value of R depends on the initial stiffness and steel ratio, the stiffness reduction at yielding of frame seems to be about 1/4 in most practical cases.

FRAME DEFLECTION DUE TO EARTHQUAKES

Response of One-Mass System to Earthquakes

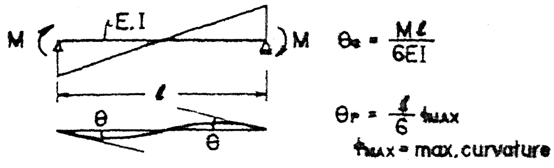
There have been many studies about the response spectra for earthquake ground motion. Examples of several response spectra are shown in Fig. 6.

The earthquakes considered here are recent medium earthquakes recorded in Japan, which have been reported in SERAC reports. Shown here is the velocity spectra for maximum ground acceleration increased to gravity acceleration. Damping coefficient of 5% critical damping was taken for all records.

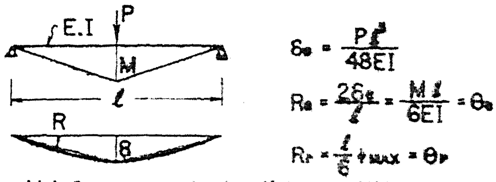
Different marks used in the plotting of Fig. 6 indicates the location of accelerogram. By the way the instruments(SMAC) were all installed on the foundation floor. It will be seen that buildings having deep basement stories respond less than buildings without basement.

The heavy lines in the Fig. 6 are drawn to indicate a safe side estimation excluding some cases of buildings without basement. In terms of displacement, these lines are expressed as follows.

$$\left.\begin{array}{llll} \delta_d = 45TR & (\mathrm{cm}) & \text{for} & T > 0.5 \text{ sec} \\ \delta_d = 90T^2R & (\mathrm{cm}) & \text{for} & T \leq 0.5 \text{ sec} \end{array}\right\} \quad (26)$$



(a) Anti-Symmetric Loading



(b) Concentrated Loading at Midspan

FIG. 1 FRAME MEMBER AND SIMPLE BEAM

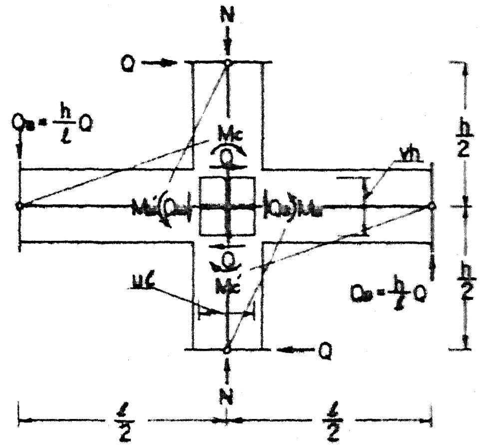


FIG. 3 FORCES AROUND BEAM-COLUMN CONNECTION

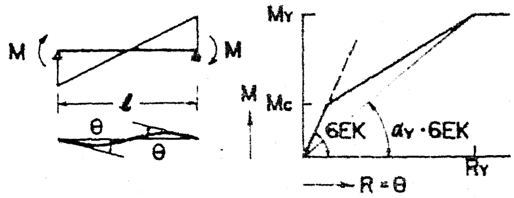


FIG 2 MOMENT ROTATION RELATIONSHIP

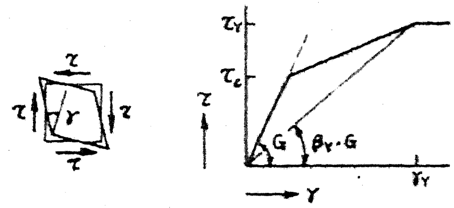
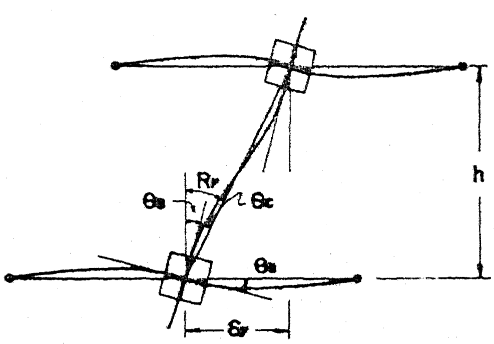
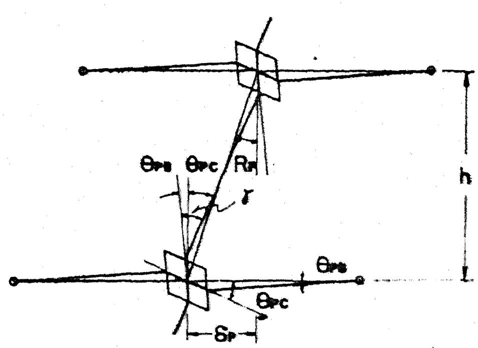


FIG. 4 CONNECTION PANEL DEFORMATION

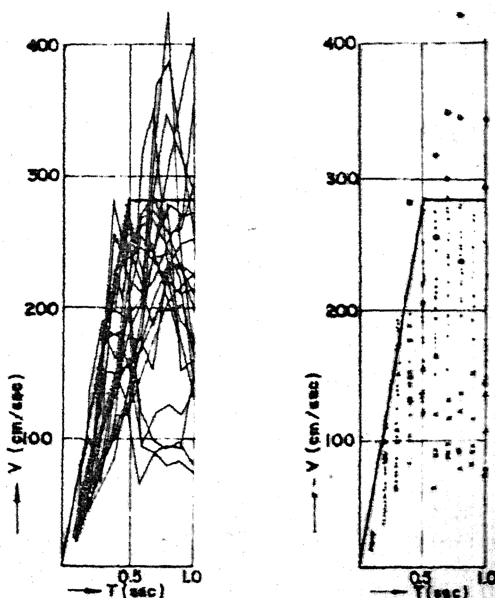


(a) FLEXURAL DEFLECTION



(b) CONNECTION PANEL DEFLECTION

FIG. 5 FRAME DEFLECTION



Max. Acceleration of Earthquake = 1.0 g
× Underground 3, 4 Floor
· Underground 1, 2 Floor
○ No Basement
Measurement at Base Floor

FIG. 6 VELOCITY SPECTRA